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Boundary terms in the decomposition of nucleon spin

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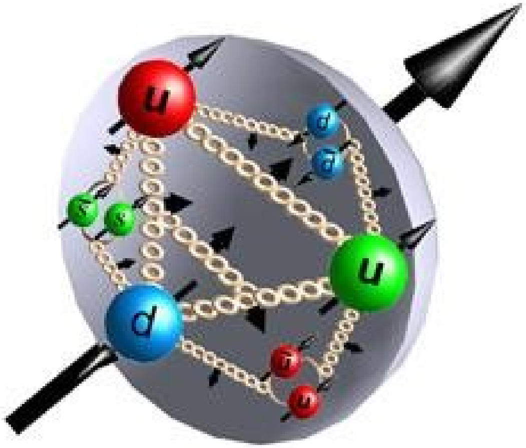
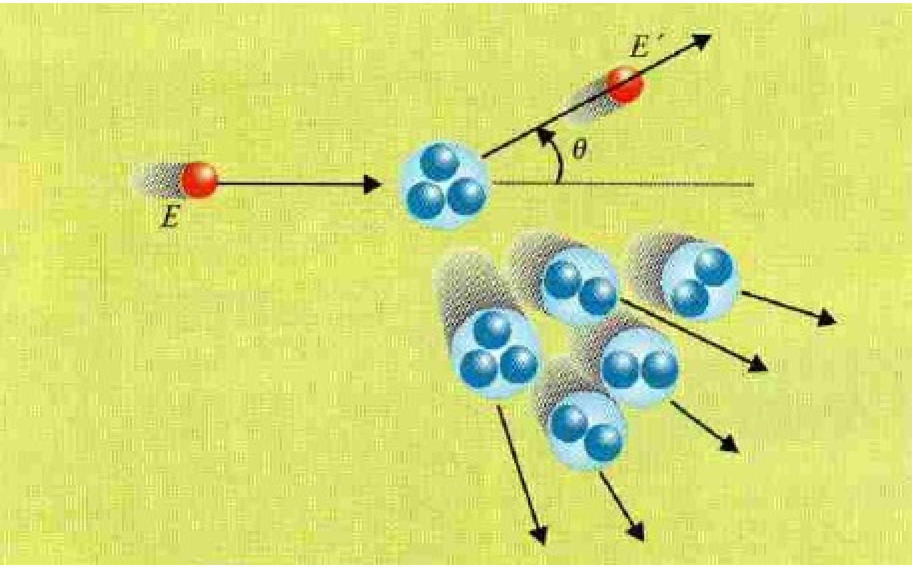
Outline

1. The proton spin crisis
2. Decomposition of J_{QCD}
3. Spatial boundary terms in QFT
4. Conclusions



1. The proton spin crisis

- In 1989 the European Muon Collaboration (EMC) published results [Ashman et al. (1989)] which suggested that quark spin S_q accounted for only a small amount of the spin of the proton



- This sparked many attempts to explain the 'crisis', including the proposed existence of unaccounted low energy corrections, and the violation of certain symmetries [Jaffe (1995)]

[M. J. Ashman et al., *Nucl. Phys.* **B328**, 1 (1989).]
[R. L. Jaffe, *Physics Today*, No. 9, 24 (1995).]

1. The proton spin crisis

- Most current approaches involve decomposing the QCD angular momentum operator J_{QCD}, and forming a sum rule [Jaffe, Manohar (1990); Jaffe (1996); Leader, Lorcé (2014)]

e.g. Jaffe-Manohar: $S_{q/g} = \frac{\langle p,s|S^3_{q/g}|p,s\rangle}{\langle p,s|p,s\rangle}$

$$\frac{1}{2} = S_q + L_q + S_g + L_g$$

Is the rest here?

→ Problem: *...there are many ways to do this!*

$$J^i_{QCD} = S^i_{q,\text{JM}} + L^i_{q,\text{JM}} + S^i_{g,\text{JM}} + L^i_{g,\text{JM}}$$ [Jaffe, Manohar (1990)]

$$J^i_{QCD} = S^i_{q,\text{Ji}} + L^i_{q,\text{Ji}} + J^i_{g,\text{Ji}}$$ [Ji (1997)]

$$J^i_{QCD} = S^i_{q,\text{Wak}} + L^i_{q,\text{Wak}} + S^i_{g,\text{Wak}} + L^i_{g,\text{Wak}}$$ [Wakamatsu (1997)]

→ *which of these J_{QCD} decompositions (if any) are correct?*

[R. L. Jaffe and A. Manohar, *Nucl. Phys. B* **337**, 509 (1990).]
[R. L. Jaffe, *Phys. Lett. B* **365**, 359 (1996).]
[E. Leader and C. Lorcé, *Phys. Rept.* **541**, 163 (2014).]
[X. Ji, *Phys. Rev. Lett* **78**, 610 (1997).]
[M. Wakamatsu, *Int. J. Mod. Phys. A* **29**, 1430012 (2014).]

2. Decomposition of J_{QCD}

- Let's take the Jaffe-Manohar (JM) decomposition as an example:

$$J^i_{QCD} = \underbrace{\epsilon^{ijk}\int d^3x \left[\frac{i}{2}\overline{\psi}\gamma^0\left(x^j\partial^k\right)\psi + \text{ h.c.}\right]}_{:=L^i_q} + \underbrace{\epsilon^{ijk}\int d^3x \left[\frac{1}{4}\epsilon^{0jkl}\overline{\psi}\gamma_l\gamma^5\psi\right]}_{:=S^i_q}$$
$$\underbrace{-\epsilon^{ijk}\int d^3x\left[F^{0la}\left(x^j\partial^k\right)A^a_l\right]}_{:=L^i_g} + \underbrace{\epsilon^{ijk}\int d^3x\left[F^{0ka}A^{ja}\right]}_{:=S^i_g}$$
$$\underbrace{-\frac{i}{16}\epsilon^{ijk}\int d^3x\ \partial_l\left[x^j\overline{\psi}\{\gamma^k,[\gamma^0,\gamma^l]\}\psi\right]}_{:=\mathcal{S}^i_1} + \underbrace{\epsilon^{ijk}\int d^3x\ \partial_l(x^jF^{0la}A^{ka})}_{:=\mathcal{S}^i_2}$$

…it only holds if these spatial boundary terms vanish!

- Stokes' theorem is invoked to justify the vanishing of these terms – "*fields vanish at spatial infinity*"

Problem: this is a purely classical argument

→ *applying classical reasoning to the full quantum theory is problematic!*

3. Spatial boundary terms in QFT

- Quantum fields $\varphi(x)$ are *operator-valued distributions*, and so in general are not point-wise defined

  → *the question of when operators of the form:* $\int d^3x\, \partial_i B^i$ *vanish is therefore more subtle in QFT*

- It *is* possible to rigorously make sense of these operators though, by smearing with suitable test functions [Kastler et al. (1966); Ferrari et al. (1977); Kugo, Ojima (1979)]

- Using this more rigorous QFT approach the following condition can be derived [Lowdon (2014)]:

$$\int d^3x\, \partial_i B^i \quad vanishes \quad \Longleftrightarrow \quad \int d^3x\, \partial_i B^i|0\rangle = 0$$

→ which can then be applied to the boundary terms $\mathcal{S}_1^i$ and $\mathcal{S}_2^i$

[D. Kastler, D. W. Robinson and J. A. Swieca, *Commun. Math. Phys.* **2**, 108 (1966).]
[R. Ferrari, L. E. Picasso and F. Strocchi, *Il Nuovo Cim.* **39A**, 1 (1977).]
[T. Kugo and I. Ojima, *Prog. Theor. Phys. Suppl.* **66** (1979).]
[P. Lowdon, *Nucl. Phys. B* **889**, 801 (2014).]

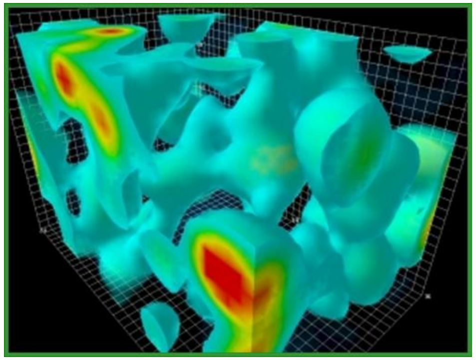
3. Spatial boundary terms in QFT

- It turns out that $\mathcal{S}_1^i$ is non-zero, and that this is implied by the non-vanishing of the condensate: $\langle 0|\mathcal{S}_1^i|0\rangle \sim \epsilon^{ijk}\epsilon^{0jkl}\underbrace{\langle 0|\overline{\psi}\gamma^l\gamma^5\psi|0\rangle}$

Non-vanishing suggested by [Pasupathy, Singh (2006)]

→ which suggests: $$J^i_{QCD} \neq S^i_q + L^i_q + S^i_g + L^i_g$$

→ *what's interesting about the apparent failure of this decomposition is that it follows from the non-trivial structure of the QCD vacuum*



[The University of Adelaide (2015)]

- The spin decomposition also appears to not hold on a matrix element level
  - → both these results cast doubt on the validity of the JM spin sum rule

$$\frac{1}{2} \overset{\textbf{?}}{=} S_q + L_q + S_g + L_g$$

[J. Pasupathy and R. K. Singh, *Int. J. Mod. Phys. A* **21**, 5099 (2006).]

4. Conclusions

- Spatial boundary terms play an important role in the determination of decompositions of J_{QCD}, and the subsequent spin sum rules
- The issue of whether or not these terms vanish is subtle, but can be addressed using a more rigorous QFT approach
- A necessary and sufficient condition for operators of the form: $\int d^3x \, \partial_i B^i$ to vanish is that they must annihilate the vacuum
- Applying this condition to the spatial boundary terms that arise in the Jaffe-Manohar J_{QCD} decomposition suggests they are non-vanishing, and hence: $J^i_{QCD} \neq S^i_q + L^i_q + S^i_g + L^i_g$
- Moreover, it appears that the obstruction to such a decomposition arises because of the non-trivial vacuum structure of QCD

Backup

- The derivative of distributions is defined by:

$$\int d^4x \; \varphi'(x) f(x) = -\int d^4x \; \varphi(x) f'(x)$$

→ *there are no boundary terms because distributions are in general not point-wise defined!*

- Distributions only make sense when smeared with suitable test functions:

$$\int d^3x \; \partial_i B^i \quad \longleftrightarrow \quad \lim_{R\to\infty} \int d^4x \; \alpha(x_0) f_R(\mathbf{x}) \partial_i B^i(x)$$

where: $\int dx_0 \; \alpha(x_0) = 1, \qquad f_R(\mathbf{x}) = \begin{cases} 1, & |\mathbf{x}| < R \\ 0, & |\mathbf{x}| > R(1+\varepsilon) \end{cases}$

with $\alpha \in \mathcal{D}(\mathbb{R})$ $(\mathrm{supp}\,(\alpha) \subset [-\delta, \delta],\ \delta > 0)$ and $f_R \in \mathcal{D}(\mathbb{R}^3)$

Backup

- Derivation of J_{QCD} from the QCD energy-momentum tensor:

$$T^{\mu\nu}_{QCD} = T^{\mu\nu}_{\text{phys}} - \left\{Q_B,\ (\partial^\mu \overline{C}^a)A^{\nu a} + (\partial^\nu \overline{C}^a)A^{\mu a} + g^{\mu\nu}\left(\frac{1}{2}\xi \overline{C}^a B^a - (\partial^\rho \overline{C}^a)A^a_\rho\right)\right\}$$

$$T^{\mu\nu}_{\text{phys}} = \frac{1}{2}\overline{\psi}\left(\frac{i}{2}\gamma^\mu(\overrightarrow{\partial}^\nu - \overleftarrow{\partial}^\nu) + gT^a A^{\nu a}\gamma^\mu\right)\psi + (\mu \leftrightarrow \nu) + F^{\mu a}_{\ \ \rho}F^{\rho\nu a} + \frac{1}{4}g^{\mu\nu}F^a_{\alpha\beta}F^{\alpha\beta a}$$

$$M^{\mu\nu\lambda} := x^\nu T^{\mu\lambda} - x^\lambda T^{\mu\nu}$$

$$\begin{aligned} M^{\mu\nu\lambda}_{QCD} &= x^\nu\left[\frac{1}{2}\overline{\psi}\left(\frac{i}{2}\gamma^\mu(\overrightarrow{\partial}^\lambda - \overleftarrow{\partial}^\lambda) + gT^a A^{\lambda a}\gamma^\mu\right)\psi + (\mu \leftrightarrow \lambda) + F^{\mu a}_{\ \ \rho}F^{\rho\lambda a} + \frac{1}{4}g^{\mu\lambda}F^a_{\alpha\beta}F^{\alpha\beta a}\right] \\ &\quad - x^\lambda\left[\frac{1}{2}\overline{\psi}\left(\frac{i}{2}\gamma^\mu(\overrightarrow{\partial}^\nu - \overleftarrow{\partial}^\nu) + gT^a A^{\nu a}\gamma^\mu\right)\psi + (\mu \leftrightarrow \nu) + F^{\mu a}_{\ \ \rho}F^{\rho\nu a} + \frac{1}{4}g^{\mu\nu}F^a_{\alpha\beta}F^{\alpha\beta a}\right] \\ &= x^\nu F^{\mu a}_{\ \ \rho}F^{\rho\lambda a} - x^\lambda F^{\mu a}_{\ \ \rho}F^{\rho\nu a} + \frac{1}{4}F^a_{\alpha\beta}F^{\alpha\beta a}(x^\nu g^{\mu\lambda} - x^\lambda g^{\mu\nu}) \\ &\quad + \frac{i}{4}\left[x^\nu\overline{\psi}(\gamma^\mu D^\lambda + \gamma^\lambda D^\mu)\psi - (\nu \leftrightarrow \lambda)\right] + \text{h.c.} \end{aligned}$$

$$J^i_{QCD} := \frac{1}{2}\,\epsilon^{ijk}\int d^3x\ M^{0jk}_{QCD}(x)$$

$$\begin{aligned} J^i_{QCD} = &\underbrace{\epsilon^{ijk}\int d^3x\left[\frac{i}{2}\overline{\psi}\gamma^0\left(x^j\partial^k\right)\psi + \text{h.c.}\right]}_{:=L^i_q} + \underbrace{\epsilon^{ijk}\int d^3x\left[\frac{1}{4}\epsilon^{0jkl}\overline{\psi}\gamma_l\gamma^5\psi\right]}_{:=S^i_q} \\ &\underbrace{-\epsilon^{ijk}\int d^3x\left[F^{0la}\left(x^j\partial^k\right)A^a_l\right]}_{:=L^i_g} + \underbrace{\epsilon^{ijk}\int d^3x\left[F^{0ka}A^{ja}\right]}_{:=S^i_g} \\ &\underbrace{-\frac{i}{16}\epsilon^{ijk}\int d^3x\ \partial_l\left[x^j\overline{\psi}\{\gamma^k,[\gamma^0,\gamma^l]\}\psi\right]}_{:=\mathcal{S}^i_1} + \underbrace{\epsilon^{ijk}\int d^3x\ \partial_l(x^j F^{0la}A^{ka})}_{:=\mathcal{S}^i_1} \end{aligned}$$

Backup

- The matrix elements for explicitly *x*-dependent spatial boundary terms (as in the J_{QCD} decompositions) are given by:

$$\langle p| \int d^3x \ \partial_i \left(x^j B^{k0i}(x) \right) |0\rangle = \begin{cases} \lim_{R\to\infty} \int d^3x \ f_R(\mathbf{x}) \langle 0|B^{k0j}(0)|0\rangle, & p = 0 \\ \lim_{R\to\infty} \int d^4x \ \alpha(x_0) f_R(\mathbf{x}) e^{ip_\mu x^\mu} \big[\langle p|B^{k0j}(0)|0\rangle & \\ \qquad\qquad + ip_i \langle p|x^j B^{k0i}(0)|0\rangle \big], & p \neq 0 \end{cases}$$

- The vacuum expectation values of boundary terms in the Jaffe-Manohar decomposition have the explicit form:

$$\langle 0|\mathcal{S}_1^i|0\rangle = \lim_{R\to\infty} \int d^3x \ \frac{1}{4} f_R(\mathbf{x}) \epsilon^{ijk} \epsilon^{0jkl} \langle 0|\overline{\psi}\gamma^l \gamma^5 \psi|0\rangle$$

$$\langle 0|\mathcal{S}_2^i|0\rangle = \lim_{R\to\infty} \int d^3x \ f_R(\mathbf{x}) \epsilon^{ijk} \langle 0|F^{0ja} A^{ka}|0\rangle$$