

The muon g-2: status from a theorist's point of view

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Physics of fundamental Symmetries and Interactions
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Theory of the g-2: the beginning

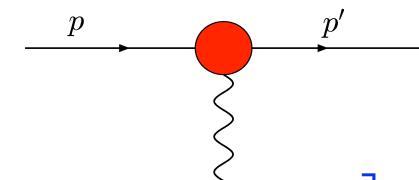
- Kusch and Foley 1948:

$$\mu_e^{\text{exp}} = \frac{e\hbar}{2mc} (1.00119 \pm 0.00005)$$

- Schwinger 1948 (triumph of QED!):

$$\mu_e^{\text{th}} = \frac{e\hbar}{2mc} \left(1 + \frac{\alpha}{2\pi}\right) = \frac{e\hbar}{2mc} \times 1.00116$$

- Keep studying the lepton- γ vertex:



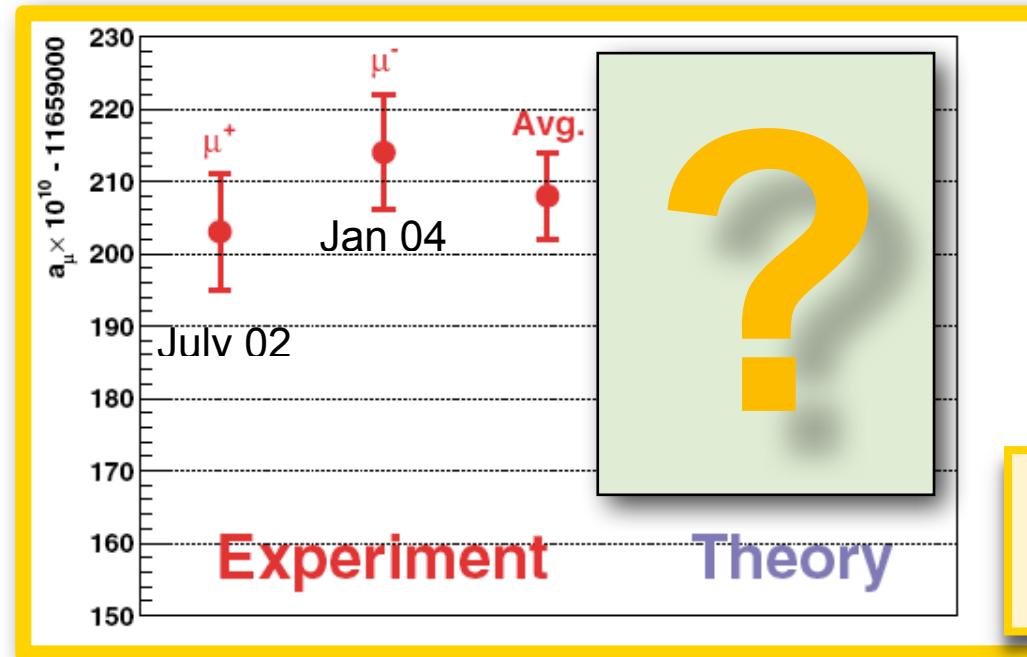
$$\bar{u}(p')\Gamma_\mu u(p) = \bar{u}(p') \left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m} F_2(q^2) + \dots \right] u(p)$$

$$F_1(0) = 1 \quad F_2(0) = a_l$$

A pure “quantum correction” effect!

The muon g-2: experimental status

μ



See Hertzog's,
Marshall's and
Saito's talks

- Today: $a_\mu^{\text{EXP}} = (116592089 \pm 54_{\text{stat}} \pm 33_{\text{sys}}) \times 10^{-11}$ [0.5ppm].
- Future: new muon g-2 experiments at:
 - Fermilab E989: aiming at $\pm 16 \times 10^{-11}$, ie 0.14ppm.
Beam expected next year. First result expected in 2018 with a precision comparable to that of BNL E821.
 - J-PARC proposal: aiming at 2019 Phase 1 start with 0.4ppm.
- Are theorists ready for this (amazing) precision? Not yet

The muon g-2: the QED contribution

μ

$$a_\mu^{\text{QED}} = (1/2)(\alpha/\pi) \quad \text{Schwinger 1948}$$

$$+ 0.765857426 (16) (\alpha/\pi)^2$$

Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; MP '04

$$+ 24.05050988 (28) (\alpha/\pi)^3$$

Remiddi, Laporta, Barbieri ... ; Czarnecki, Skrzypek; MP '04;
Friot, Greynat & de Rafael '05, Mohr, Taylor & Newell 2012

$$+ 130.8773 (61) (\alpha/\pi)^4$$

Kinoshita & Lindquist '81, ... , Kinoshita & Nio '04, '05;
Aoyama, Hayakawa, Kinoshita & Nio, 2007, Kinoshita et al. 2012 & 2015;
Lee, Marquard, Smirnov², Steinhauser 2013 (electron loops, analytic),
Kurz, Liu, Marquard, Steinhauser 2013 (τ loops, analytic);
Steinhauser et al. 2015 & 2016 (all electron & τ loops, analytic).

$$+ 752.85 (93) (\alpha/\pi)^5 \text{ COMPLETED!}$$

Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta,
Karshenboim, ..., Kataev, Kinoshita & Nio '06; Kinoshita et al. 2012 & 2015

Adding up, we get:

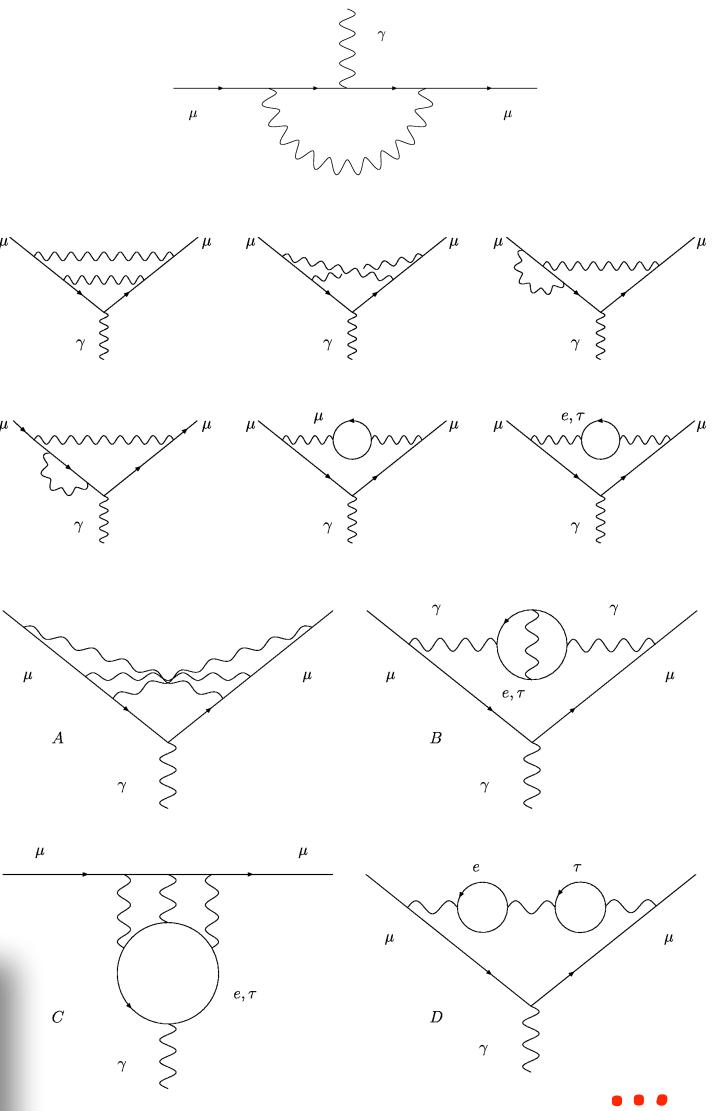
$$a_\mu^{\text{QED}} = 116584718.941 (21)(77) \times 10^{-11}$$

from coeffs, mainly from 4-loop unc



from δα(Rb)

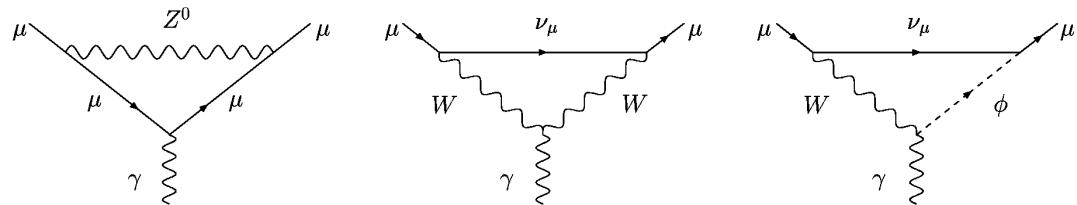
$$\text{with } \alpha = 1/137.035999049(90) [0.66 \text{ ppb}]$$



The muon g-2: the electroweak contribution

μ

- One-loop term:



$$a_\mu^{\text{EW}}(\text{1-loop}) = \frac{5G_\mu m_\mu^2}{24\sqrt{2}\pi^2} \left[1 + \frac{1}{5} (1 - 4 \sin^2 \theta_W)^2 + O\left(\frac{m_\mu^2}{M_{Z,W,H}^2}\right) \right] \approx 195 \times 10^{-11}$$

1972: Jackiw, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda;
Studenikin et al. '80s

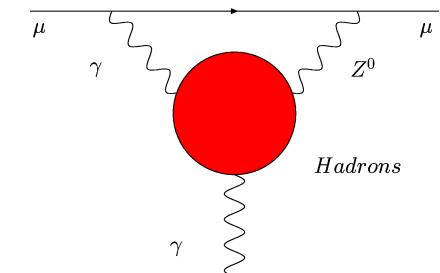
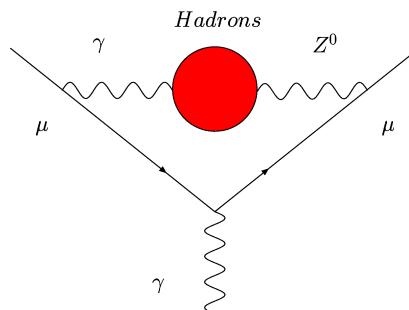
- One-loop plus higher-order terms:

$a_\mu^{\text{EW}} = 153.6 (1) \times 10^{-11}$

with $M_{\text{Higgs}} = 125.6 (1.5) \text{ GeV}$

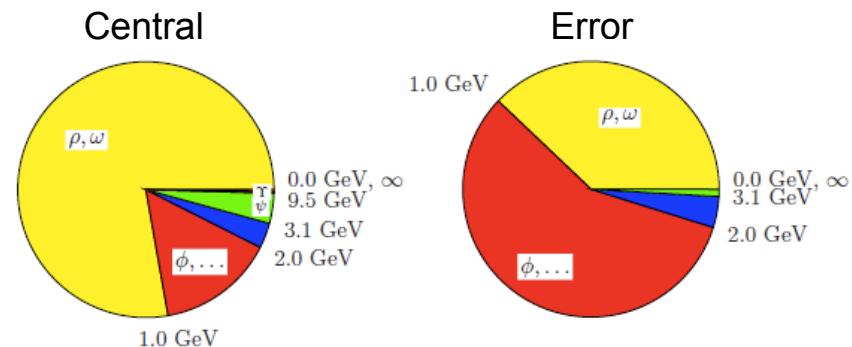
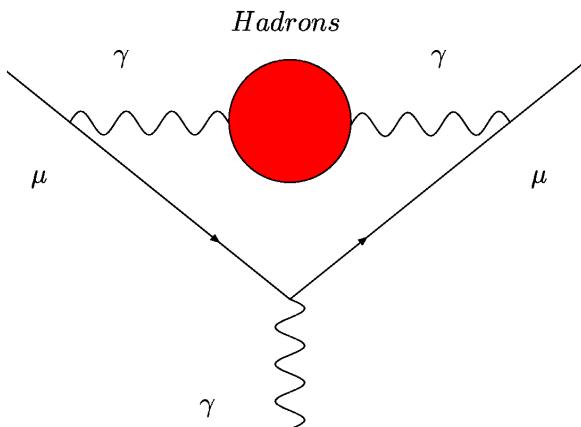
Hadronic loop uncertainties
and 3-loop nonleading logs.

Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano and Vainshtein '02; Degrassi and Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk and Czarnecki '05; Vainshtein '03; Gnendiger, Stockinger, Stockinger-Kim 2013.



The muon g-2: the hadronic LO contribution (HLO)

μ



F. Jegerlehner and A. Nyffeler, Phys. Rept. 477 (2009) 1

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)(s/m^2)}$$

$$a_\mu^{\text{HLO}} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^\infty ds K(s) \sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^\infty \frac{ds}{s} K(s) R(s)$$

$$a_\mu^{\text{HLO}} = 6870 (42)_{\text{tot}} \times 10^{-11}$$

F. Jegerlehner, arXiv:1511.04473 (includes BESIII 2 π)

$$= 6928 (33)_{\text{tot}} \times 10^{-11}$$

Davier et al, Tau2016, Beijing, Sep 2016, Preliminary

$$= 6949 (37)_{\text{exp}} (21)_{\text{rad}} \times 10^{-11}$$

Hagiwara et al, JPG 38 (2011) 085003



Radiative Corrections are crucial. S. Actis et al, Eur. Phys. J. C66 (2010) 585

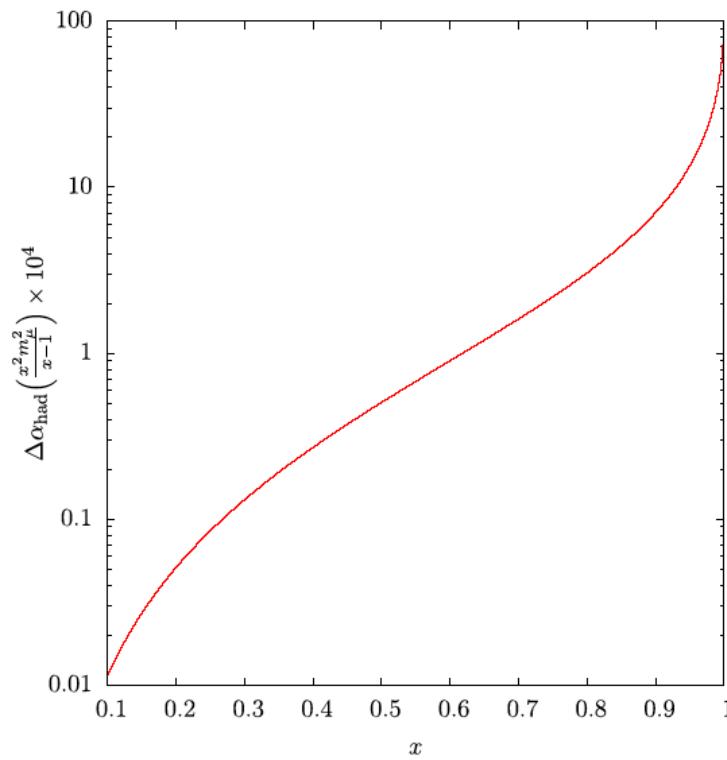
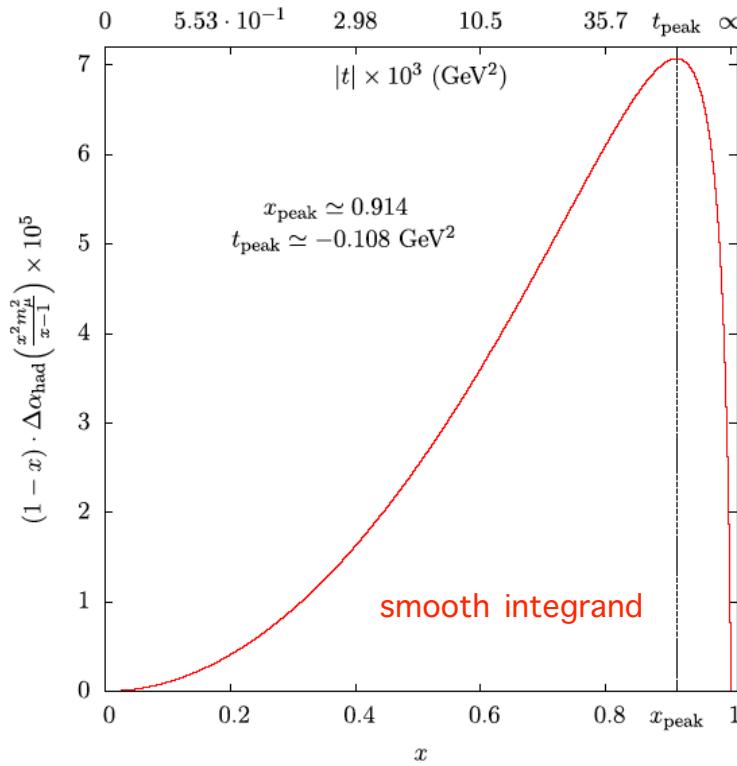


Lots of progress in lattice calculations. T. Blum et al, PRL116 (2016) 232002

- Alternatively, exchanging the x and s integrations in a_μ^{HLO} :

$$a_\mu^{\text{HLO}} = \frac{\alpha}{\pi} \int_0^1 dx (1-x) \Delta\alpha_{\text{had}}[t(x)] \quad t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

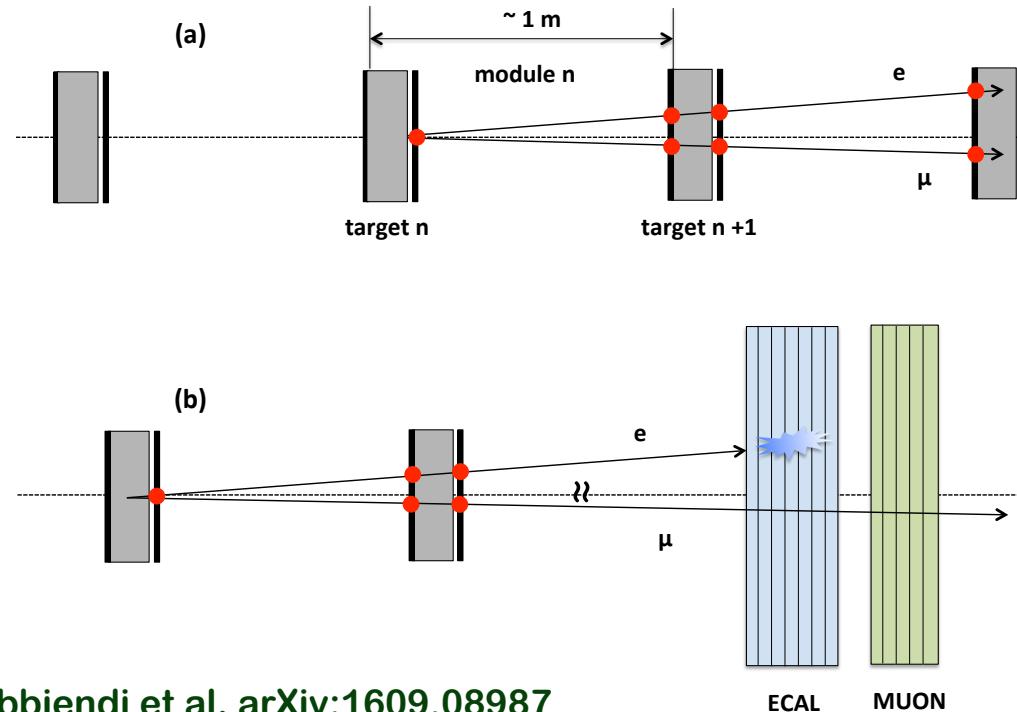
which involves $\Delta\alpha_{\text{had}}(t)$, the hadr. contrib. to the running of α in the space-like region. It can be extracted from Bhabha scattering data!



New space-like proposal for HLO (2)

μ

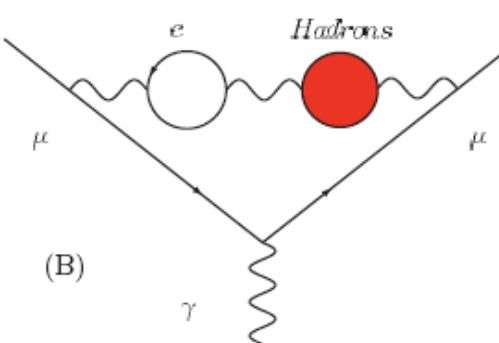
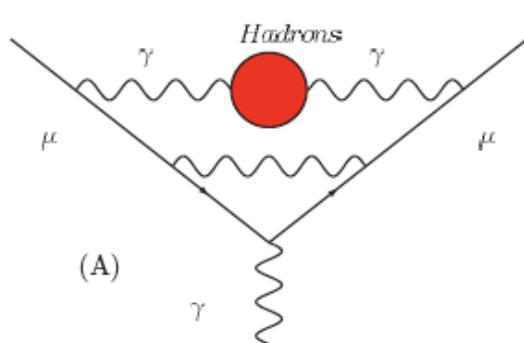
- $\Delta\alpha_{\text{had}}(t)$ can also be measured via the **elastic scattering** $\mu e \rightarrow \mu e$.
- Scattering a beam of muons of 150 GeV, available at CERN's North Area, on a fixed electron target, $0 < x < 0.93$ (peak at 0.91).



G. Abbiendi et al, arXiv:1609.08987

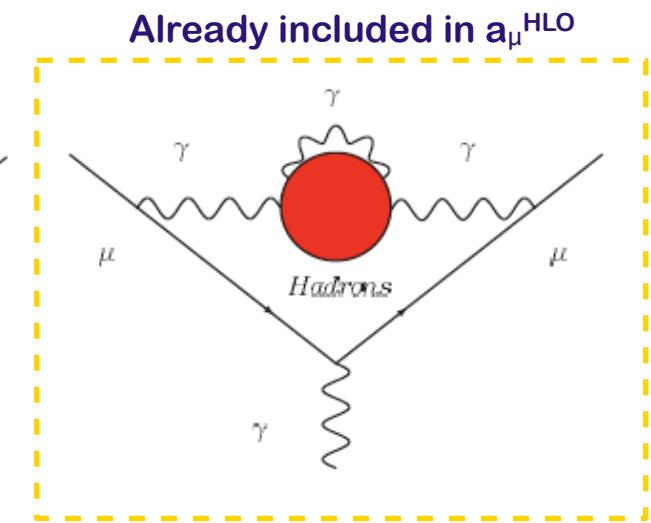
- With CERN's 150 GeV muon beam ($1.3 \times 10^7 \mu/\text{s}$ average) a statistical uncertainty of $\sim 0.3\%$ ($\sim 20 \times 10^{-11}$) can be reached on a_μ^{HLO} with 2 years of data taking. 10ppm systematic accuracy needed at peak.

- HNLO: Vacuum Polarization



(A)

(B)



$\mathcal{O}(\alpha^3)$ contributions of diagrams containing hadronic vacuum polarization insertions:

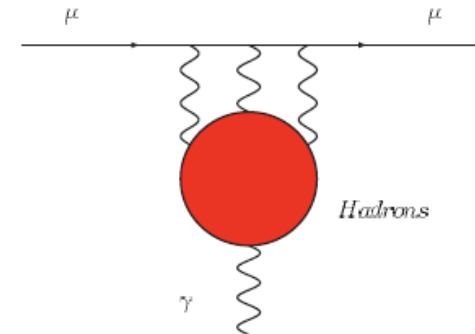
$$a_\mu^{\text{HNLO(vp)}} = -98 (1) \times 10^{-11}$$

Krause '96, Alemany et al. '98, Hagiwara et al. 2011

- HNLO: Light-by-light contribution

- Unlike the HLO term, the hadronic l-b-l term relies at present on theoretical approaches.

- This term had a troubled life! Latest values:



$$a_\mu^{\text{HNLO}(\text{lbl})} = +80(40) \times 10^{-11} \quad \text{Knecht \& Nyffeler '02}$$

$$a_\mu^{\text{HNLO}(\text{lbl})} = +136(25) \times 10^{-11} \quad \text{Melnikov \& Vainshtein '03}$$

$$a_\mu^{\text{HNLO}(\text{lbl})} = +105(26) \times 10^{-11} \quad \text{Prades, de Rafael, Vainshtein '09}$$

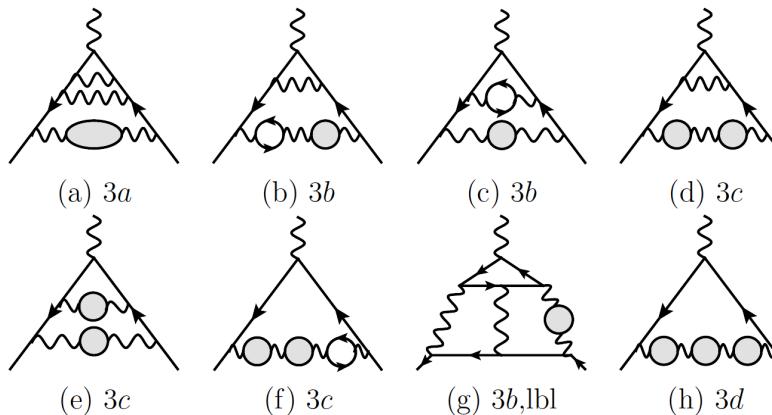
$$a_\mu^{\text{HNLO}(\text{lbl})} = +102(39) \times 10^{-11} \quad \text{Jegerlehner, arXiv:1511.04473}$$

Results based also on Hayakawa, Kinoshita '98 & '02; Bijnens, Pallante, Prades '96 & '02

- Improvements expected in the π^0 transition form factor A. Nyffeler 1602.03398
- Dispersive approach proposed Colangelo et al, 2014 & 2015, Pauk & Vanderhaeghen 2014.
- Progress on the lattice: $+53.5(13.5)\times 10^{-11}$. Statistical error only, finite-volume and finite lattice-spacing errors being studied. Omitted subleading disconnected graphs still need to be computed.

Blum, Christ, Hayakawa, Izubuchi, Jin, Jung, Lehner, arXiv:1610.04603

- HNNLO: Vacuum Polarization



$\mathcal{O}(\alpha^4)$ contributions of diagrams containing hadronic vacuum polarization insertions:

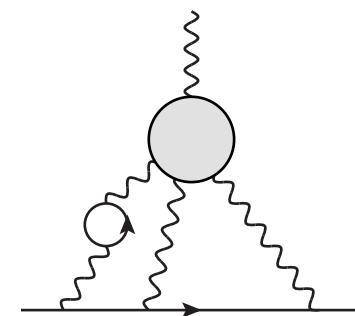
$$a_\mu^{\text{HNNLO(vp)}} = 12.4(1) \times 10^{-11}$$

Kurz, Liu, Marquard, Steinhauser 2014

- HNNLO: Light-by-light

$$a_\mu^{\text{HNNLO(lbl)}} = 3(2) \times 10^{-11}$$

Colangelo, Hoferichter, Nyffeler, MP, Stoffer 2014



The muon g-2: SM vs. Experiment

μ

Comparisons of the SM predictions with the measured g-2 value:

$$a_\mu^{\text{EXP}} = 116592091 (63) \times 10^{-11}$$

E821 – Final Report: PRD73
(2006) 072 with latest value
of $\lambda = \mu_\mu / \mu_p$ from CODATA'10

$a_\mu^{\text{SM}} \times 10^{11}$	$\Delta a_\mu = a_\mu^{\text{EXP}} - a_\mu^{\text{SM}}$	σ
116 591 761 (57)	$330 (85) \times 10^{-11}$	3.9 [1]
116 591 820 (51)	$271 (81) \times 10^{-11}$	3.3 [2]
116 591 841 (58)	$250 (86) \times 10^{-11}$	2.9 [3]

with the recent “conservative” hadronic light-by-light $a_\mu^{\text{HNLO(lbl)}} = 102 (39) \times 10^{-11}$ of F. Jegerlehner arXiv:1511.04473, and the hadronic leading-order of:

- [1] Jegerlehner, arXiv:1511.04473.
- [2] Davier et al, Tau2016, Beijing, Sep 2016, Preliminary.
- [3] Hagiwara et al, JPG38 (2011) 085003.

Brief digression: the electron g-2

The electron g-2: SM vs Experiment

e

- The 2008 measurement of the electron g-2 is:

$$a_e^{\text{EXP}} = 11596521807.3 (2.8) \times 10^{-13}$$

Hanneke, Fogwell, Gabrielse
PRL100 (2008) 120801

- Using $\alpha = 1/137.035\ 999\ 049\ (90)$ from h/M measurement of ^{87}Rb (2011), the SM prediction for the electron g-2 is

$$a_e^{\text{SM}} = 115\ 965\ 218\ 16.5 (0.2) (0.2) (0.2) (7.6) \times 10^{-13}$$

δC_4^{qed} δC_5^{qed} δa_e^{had} from $\delta \alpha$

- The EXP-SM difference is (note the negative sign):

$$\Delta a_e = a_e^{\text{EXP}} - a_e^{\text{SM}} = -9.2 (8.1) \times 10^{-13}$$

The SM is in very good agreement with experiment (1σ).

- The present sensitivity is $\delta\Delta a_e = 8.1 \times 10^{-13}$, ie (10^{-13} units):

$$(0.2)_{\text{QED4}}, \quad (0.2)_{\text{QED5}}, \quad (0.2)_{\text{HAD}}, \quad (7.6)_{\delta\alpha}, \quad (2.8)_{\delta a_e^{\text{EXP}}}$$

$\overbrace{\qquad\qquad\qquad}^{(0.4)_{\text{TH}}} \quad \leftarrow \text{may drop to } 0.2$

- The $(g-2)_e$ exp. error may soon drop below 10^{-13} and work is in progress for a significant reduction of that induced by $\delta\alpha$.
→ sensitivity of 10^{-13} may be reached with ongoing exp. work
- In a broad class of BSM theories, contributions to a_l scale as

$$\frac{\Delta a_{\ell_i}}{\Delta a_{\ell_j}} = \left(\frac{m_{\ell_i}}{m_{\ell_j}} \right)^2 \quad \text{This Naive Scaling leads to:}$$

$$\Delta a_e = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 0.7 \times 10^{-13}$$

- The experimental sensitivity in Δa_e is not too far from what is needed to **test if the discrepancy in $(g-2)_\mu$ also manifests itself in $(g-2)_e$** under the naive scaling hypothesis.
- NP scenarios exist which **violate Naive Scaling**. They can lead to larger effects in Δa_e and contributions to EDMs, LFV or lepton universality breaking observables.
- Example: In the MSSM with non-degenerate but aligned sleptons (vanishing flavor mixing angles), Δa_e can reach 10^{-12} (at the limit of the present exp sensitivity). For these values one typically has breaking effects of lepton universality at the few per mil level (within future exp reach).

Giudice, Paradisi, MP JHEP 2012

Back to the muon g-2

- Can Δa_μ be due to hypothetical mistakes in the hadronic $\sigma(s)$?
- An upward shift of $\sigma(s)$ also induces an increase of $\Delta \alpha_{\text{had}}^{(5)}(M_Z)$.
- Consider:

$$\begin{aligned} a_\mu^{\text{HLO}} &\rightarrow a = \int_{4m_\pi^2}^{s_u} ds f(s) \sigma(s), \quad f(s) = \frac{K(s)}{4\pi^3}, \quad s_u < M_Z^2, \\ \Delta \alpha_{\text{had}}^{(5)} &\rightarrow b = \int_{4m_\pi^2}^{s_u} ds g(s) \sigma(s), \quad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)}, \end{aligned}$$

and the increase

$$\Delta \sigma(s) = \epsilon \sigma(s)$$

($\epsilon > 0$), in the range:

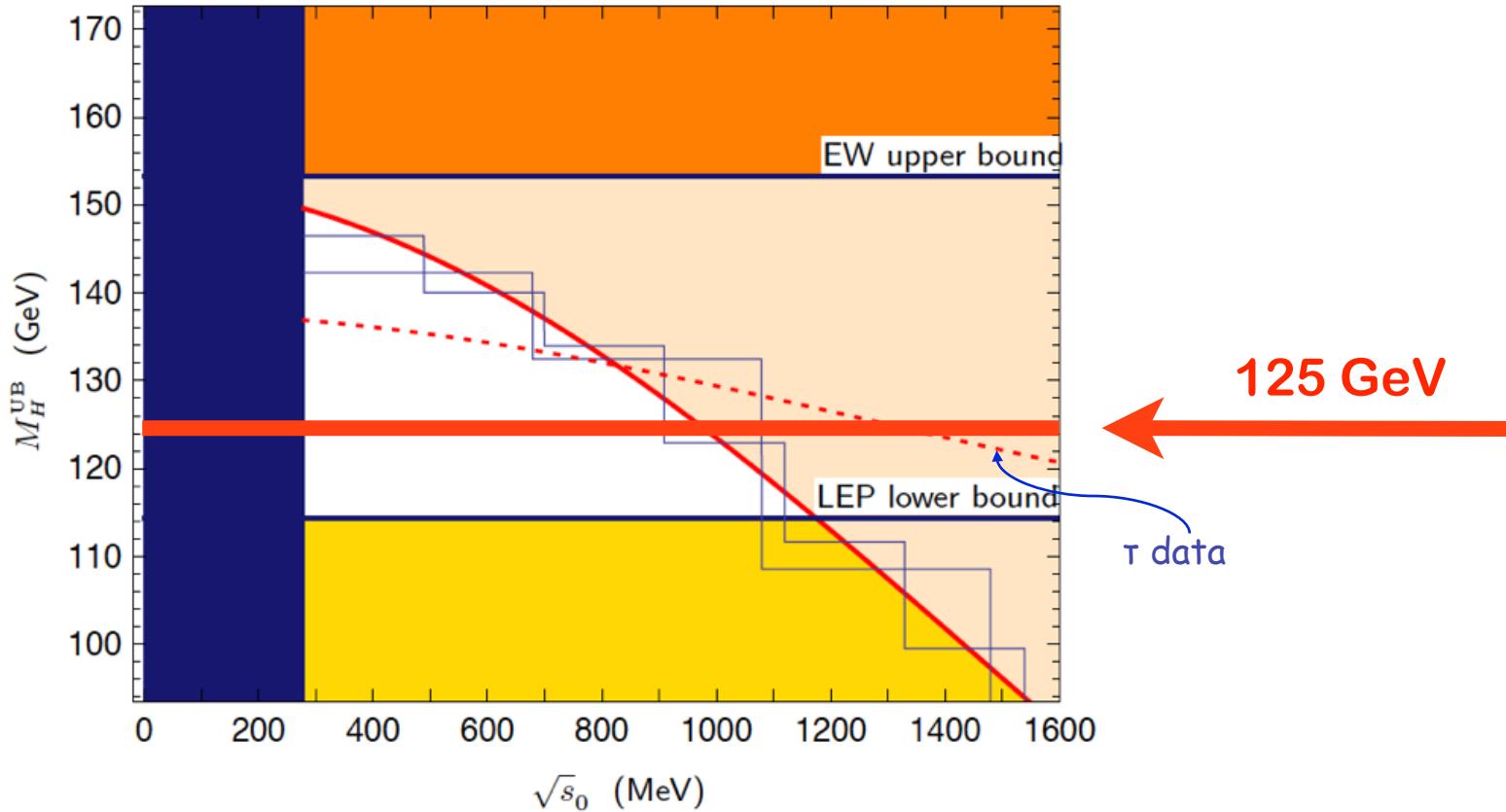
$$\sqrt{s} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2]$$



The muon g-2: connection with the SM Higgs mass

μ

- How much does the M_H upper bound from the EW fit change when we shift $\sigma(s)$ by $\Delta\sigma(s)$ [and thus $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$] to accommodate Δa_μ ?

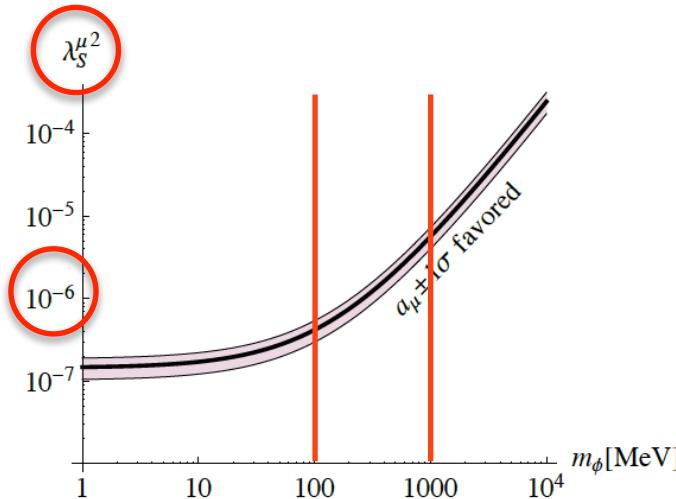
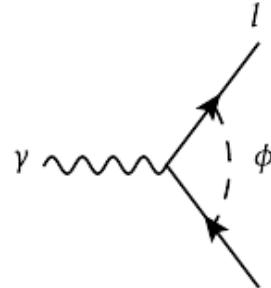


W.J. Marciano, A. Sirlin, MP, 2008 & 2010

- Given the quoted exp. uncertainty of $\sigma(s)$, the possibility to explain the muon g-2 with these very large shifts $\Delta\sigma(s)$ appears to be very unlikely.
- Also, given a 125 GeV SM Higgs, these hypothetical shifts $\Delta\sigma(s)$ could only occur at very low energy (below ~ 1 GeV) where $\sigma(s)$ is precisely measured.
- Vice versa, assuming we now have a SM Higgs with $M_H = 125$ GeV, if we bridge the M_H discrepancy in the EW fit decreasing the low-energy hadronic cross section, the muon g-2 discrepancy increases.

W.J. Marciano, A. Sirlin, MP, 2008 & 2010

- Light spin 0 scalars & pseudoscalars (axion-like-particles or ALPs), contribute to a_μ . We consider ALPs in the mass range $\sim[0.1\text{--}1] \text{ GeV}$, where experimental constraints are rather loose.
- A possible resolution of Δa_μ by 1-loop contributions from scalar particles with relatively large Yukawa couplings to muons, of $O(10^{-3})$, was analyzed by Chen, Davoudiasl, Marciano & Zhang, PRD 93, 035006 (2016):



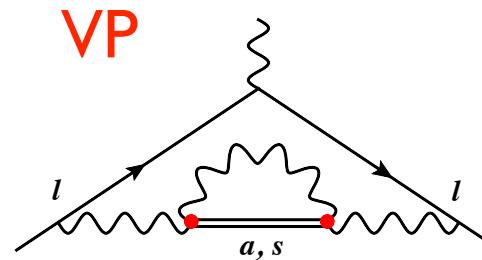
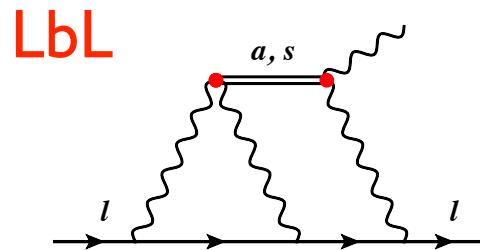
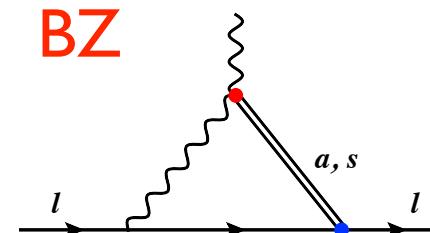
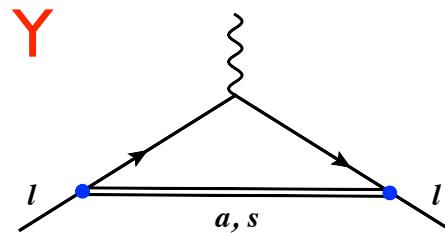
- For a **pseudoscalar**, the 1-loop contribution has the wrong sign (negative) to resolve the discrepancy on its own.

Consider ALP- $\gamma\gamma$ couplings as well as Yukawa couplings:

$$\mathcal{L}_a = \frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu} + i y_{a\psi} a \bar{\psi} \gamma_5 \psi,$$

$$\mathcal{L}_s = \frac{1}{4} g_{s\gamma\gamma} s F_{\mu\nu} F^{\mu\nu} + y_{s\psi} s \bar{\psi} \psi$$

New, potentially important, ALP contributions to a_μ :



Marciano, Masiero, Paradisi, MP, arXiv:1607.01022

Y
BZ
LbL
VP

$$a_{\ell,a}^Y < 0$$

$$a_{\ell,a}^{\text{BZ}} \simeq \left(\frac{m_\ell}{4\pi^2} \right) g_{a\gamma\gamma} y_{a\ell} \ln \frac{\Lambda}{m_a}$$

$$a_{\ell,a}^{\text{LbL}} \simeq 3 \frac{\alpha}{\pi} \left(\frac{m_\ell g_{a\gamma\gamma}}{4\pi} \right)^2 \ln^2 \frac{\Lambda}{m_a} > 0$$

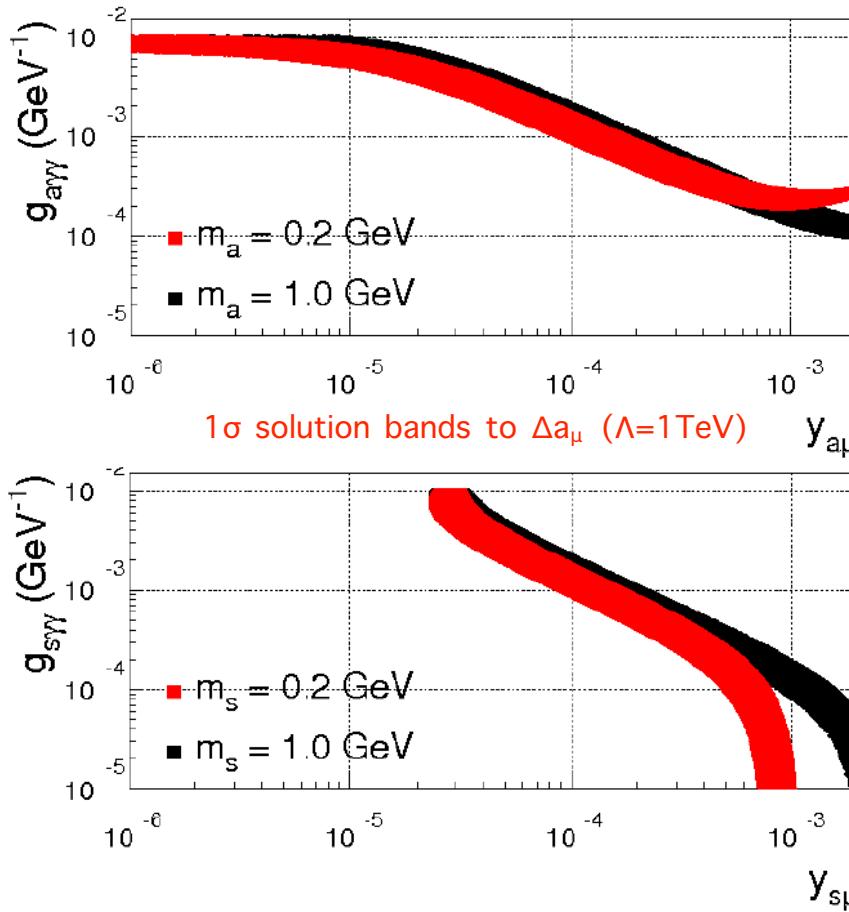
$$a_{\ell,a}^{\text{VP}} \simeq \frac{\alpha}{\pi} \left(\frac{m_\ell g_{a\gamma\gamma}}{12\pi} \right)^2 \ln \frac{\Lambda}{m_a} > 0$$

leading log-enhanced terms

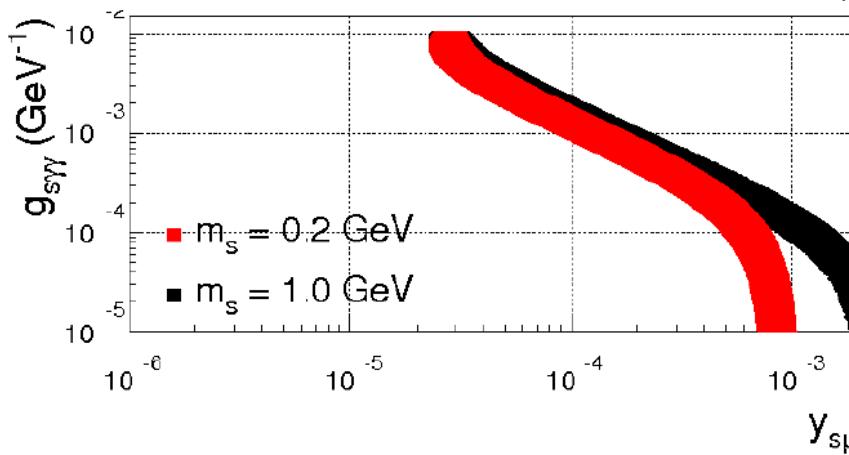
Pseudoscalar

- ➊ For a **scalar ALP**, change the signs of Y & LbL.
- ➋ The sign of BZ depends on the couplings. We assume it's > 0.
- ➌ VP is positive both for scalar & pseudoscalar, but negligible.

Pseudoscalar



Scalar



- ➊ Both pseudoscalar and scalar ALPs can solve Δa_μ for masses and couplings allowed by current exp. constraints.
- ➋ They can be tested at present low-energy e^+e^- colliders through dedicated $e^+e^- \rightarrow e^+e^- + \text{ALP}$ searches.

Conclusions

- The lepton g-2 provide beautiful examples of interplay between theory and experiment, and different areas of Physics.
- Muon g-2: $\Delta a_\mu \sim 3.5 \sigma$. New upcoming experiment: QED & EW ready. Lots of progress in the hadr sector, but not yet ready!
- New proposal to measure the leading hadronic contribution to the muon g-2 via $\mu\text{-}e$ elastic scattering at CERN.
- Electron g-2: Does the discrepancy in $(g-2)_\mu$ also manifests itself in $(g-2)_e$? NP sensitivity limited by exp. uncertainties, but a strong exp. program is under way to improve both α & a_e .
- Could Δa_μ be due to mistakes in the hadronic $\sigma(s)$? Very unlikely. Also, given a 125 GeV SM Higgs, these hypothetical shifts $\Delta\sigma(s)$ could only occur at very low energies ($\leq 1\text{GeV}$).
- Light spin 0 scalars & pseudoscalars can solve Δa_μ for masses and couplings allowed by current experimental bounds. Dedicated searches can test them at low-energy e^+e^- colliders.

The End

Backup

The QED prediction of the electron g-2

e

$$a_e^{\text{QED}}$$

$$= + (1/2)(\alpha/\pi) - 0.328\ 478\ 444\ 002\ 55(33) (\alpha/\pi)^2$$

Schwinger 1948

Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; CODATA Mar '12

$$A_1^{(4)} = -0.328\ 478\ 965\ 579\ 193\ 78\dots$$

$O(10^{-18})$ in a_e

$$A_2^{(4)} (m_e/m_\mu) = 5.197\ 386\ 68 (26) \times 10^{-7}$$

$$A_2^{(4)} (m_e/m_\tau) = 1.837\ 98 (33) \times 10^{-9}$$

$$+ 1.181\ 234\ 016\ 816(11) (\alpha/\pi)^3$$

$O(10^{-19})$ in a_e

Kinoshita; Barbieri; Laporta, Remiddi; ... , Li, Samuel; MP '06; Giudice, Paradisi, MP 2012

$$A_1^{(6)} = 1.181\ 241\ 456\ 587\dots$$

$$A_2^{(6)} (m_e/m_\mu) = -7.373\ 941\ 62 (27) \times 10^{-6}$$

$$A_2^{(6)} (m_e/m_\tau) = -6.5830 (11) \times 10^{-8}$$

$$A_3^{(6)} (m_e/m_\mu, m_e/m_\tau) = 1.909\ 82 (34) \times 10^{-13}$$

$$- 1.91206(84) (\alpha/\pi)^4$$

$0.2\ 10^{-13}$ in a_e

Kinoshita & Lindquist '81, ... , Kinoshita & Nio '05; Aoyama, Hayakawa, Kinoshita & Nio 2012 & 2015;
Kurz, Liu, Marquard & Steinhauser 2014: analytic heavy virtual lepton part.

$$+ 7.79 (34) (\alpha/\pi)^5$$

Complete Result! (12672 mass indep. diagrams!)

Aoyama, Hayakawa, Kinoshita, Nio, PRL 109 (2012) 111807; PRD 91 (2015) 3, 033006

$0.2\ 10^{-13}$ in a_e

NB: $(\alpha/\pi)^6 \sim O(10^{-16})$

The SM prediction of the electron g-2

e

The SM prediction is:

$$a_e^{\text{SM}}(\alpha) = a_e^{\text{QED}}(\alpha) + a_e^{\text{EW}} + a_e^{\text{HAD}}$$

The EW (1&2 loop) term is: Czarnecki, Krause, Marciano '96 [value from Codata10]

$$a_e^{\text{EW}} = 0.2973(52) \times 10^{-13}$$

The Hadronic contribution, at LO+NLO+NNLO, is:

Nomura & Teubner '12, Jegerlehner & Nyffeler '09; Krause'97; Kurz, Liu, Marquard & Steinhauser 2014

$$a_e^{\text{HAD}} = 17.10(17) \times 10^{-13}$$

$$a_e^{\text{HLO}} = +18.66(11) \times 10^{-13}$$

$$a_e^{\text{HNLO}} = [-2.234(14)_{\text{vac}} + 0.39(13)_{\text{lbf}}] \times 10^{-13}$$

$$a_e^{\text{HNNLO}} = +0.28(1) \times 10^{-13}$$

Which value of α should we use to compute a_e^{SM} ?

- The 2008 measurement of the electron g-2 is:

$$a_e^{\text{EXP}} = 11596521807.3 \text{ (2.8)} \times 10^{-13} \quad \text{Hanneke et al, PRL100 (2008) 120801}$$

vs. old (factor of 15 improvement, 1.8σ difference):

$$a_e^{\text{EXP}} = 11596521883 \text{ (42)} \times 10^{-13} \quad \text{Van Dyck et al, PRL59 (1987) 26}$$

- Equate $a_e^{\text{SM}}(\alpha) = a_e^{\text{EXP}}$ → best determination of alpha:

$$\alpha^{-1} = 137.035\ 999\ 158 \text{ (33)} \quad [0.24 \text{ ppb}]$$

- Compare it with other determinations (independent of a_e):

$$\alpha^{-1} = 137.036\ 000\ 0 \text{ (11)} \quad [7.7 \text{ ppb}] \quad \text{PRA73 (2006) 032504 (Cs)}$$

$$\alpha^{-1} = 137.035\ 999\ 049 \text{ (90)} \quad [0.66 \text{ ppb}] \quad \text{PRL106 (2011) 080801 (Rb)}$$

Excellent agreement → beautiful test of QED at 4-loop level!