NLO prediction for $\mu \to e\gamma\nu\bar{\nu}$ and $\mu \to e(e^+e^-)\nu\bar{\nu}$ decays in the SM

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PSI 2016, October 19th, 2016

1506.03416, 1602.00457

work in collaboration with:
C. Greub, L. Mercolli, M. Passera.

See also the poster by Y. Ulrich, A. Signer, M. Pruna

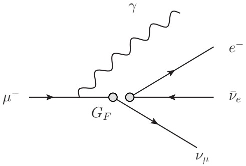
Exclusive decays of the muon

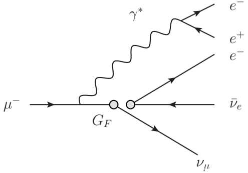
Radiative decay

$$\mu \to e\nu\bar{\nu}\gamma$$

Rare decay

$$\mu \to e\nu\bar{\nu}(e^+e^-)$$





- Very clean, can be predicted with very high precision.
- TH formulation in terms of Michel parameters allow to test couplings beyond the SM $V-A$; additional Michel param. accessible in RMD.
- Precise data on τ radiative decays may allow to determine its g–2.
  Eidelman, Epifanov, MF, Mercolli, Passera, JHEP 1603 (2016) 140

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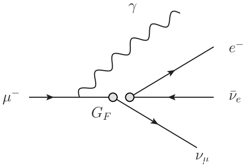

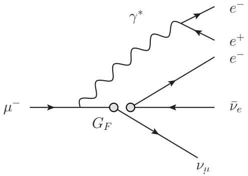

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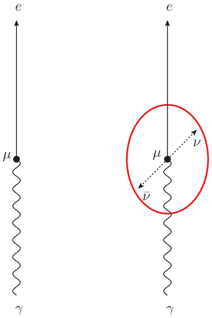
  Eidelman, Epifanov, MF, Mercolli, Passera, JHEP 1603 (2016) 140
- SM background for μ and τ flavour violating decays: $\mu \to e\gamma, \mu \to eee$.

Time-correlated background: MEG

see talks by A. Papa

- $\mu^+ \to e^+\gamma$ background:
  - Accidental coincidence of a positron and a photon (RMD or AIF).
  - RMD where $\not{E} \to 0$,
- Time- and vertex- correlated.
- Indistinguishable except for energy carried out by neutrinos: $\not{E} = m_\mu - E_{\text{vis}}$.



- Energy and $t_{e\gamma}$ calibration.
- Normalization:

$$N_\mu = \frac{N^{e\nu\bar{\nu}\gamma}}{\mathcal{B}^{e\nu\bar{\nu}\gamma}} \times \varepsilon_{\text{exp}}$$

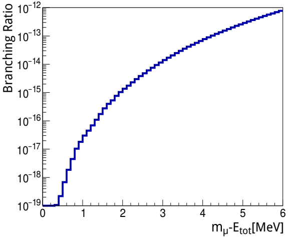
$\mathcal{B}^{\text{exp}}(\mu^+ \to e^+\nu\bar{\nu}\gamma, \omega_0 \geq 40\text{ MeV}, E_e \geq 45\text{ MeV}) = 6.03\,(14)_{\text{st}}\,(53)_{\text{sys}} \times 10^{-8}$
MEG collaboration, EPJ C 76 (2016) 108

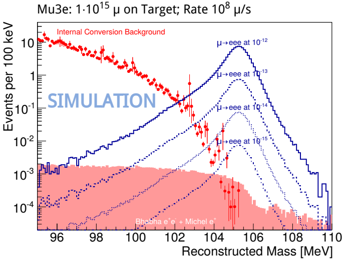
Time-correlated background: Mu3e

see talks by N. Berger

Background:

- Accidental combination two positron and an electron,
- Rare decay: $\mu^+ \to e^+ e^- e^+ \nu \bar{\nu}$.
- Background suppression with $m_\mu - E_{\text{vis}} \leq \not{E}_{\max}$





Mu3e collaboration,
EPJ Web Conf. 118 (2016) 01028.

Babar's measurements of $\tau \to \ell\gamma\nu\bar{\nu}$ decays

B.R. of radiative τ leptonic decays ($E_\gamma^{\rm min} = 10$ MeV)		
	$\tau \to e\bar{\nu}\nu\gamma$	$\tau \to \mu\bar{\nu}\nu\gamma$
$\mathcal{B}_{\rm EXP}$	$1.847\,(15)_{\rm st}(52)_{\rm sy} \times 10^{-2}$	$3.69\,(3)_{\rm st}(10)_{\rm sy} \times 10^{-3}$

BABAR coll., PRD 91 (2015) 051103

- Babar experimental precision around 3%.
- More precise than CLEO results: T. Bergfeld et al., PRL 84 (2000) 830
  $1.75\,(6)_{\rm st}(17)_{\rm sy} \times 10^{-2}\,(\tau \to e\gamma\nu\bar{\nu})$,
  $3.61\,(16)_{\rm st}(35)_{\rm sy} \times 10^{-3}\,(\tau \to \mu\gamma\nu\bar{\nu})$.

PHYSICAL REVIEW VOLUME 113, NUMBER 6 MARCH 15, 1959

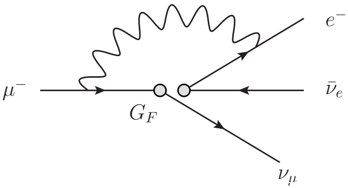
Radiative Corrections to Fermi Interactions*

TOICHIRO KINOSHITA, *Laboratory of Nuclear Studies, Cornell University, Ithaca, New York*

AND

ALBERTO SIRLIN, *Physics Department, Columbia University, New York, New York*

(Received October 23, 1958)



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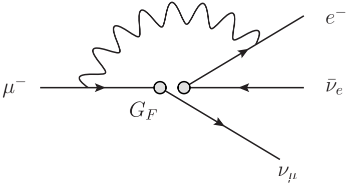


T. van Ritbergen, R. Stuart, PRL 82 (1999) 488

2-loop QED contributions to the muon lifetime in the Fermi model:

Why NLO?

- Decay rates at LO:
  - $\mu \to e\gamma\nu\bar{\nu}$ Kinoshita, Sirlin, PRL 2 (1959) 177; Fronsdal, Uberall, PR 133 (1959) 654; Eckstein, Pratt, Ann. Phys. 8 (1959) 297; Kuno, Okada, RMP 73 (2001) 151; (one-loop) Fischer et al., PRD 49 (1994) 3426; Arbuzov, Scherbakova, PLB 597 (2004) 285.
  - $\mu \to e(e^+e^-)\nu\bar{\nu}$ Bardin, Istatkov, Mitselmakher, Yad. Fiz. 15 (1972) 284; Fishbane & Gaemers, PRD. 33 (1986) 159; van Ritbergen & Stuart, NPB 564 (2000) 343; Djilkibaev & Konoplich, PRD 79 (1009) 073004.
- $\alpha/\pi \sim 0.002$
- NLO enhancement (up to a relative $\mathcal{O}(10\%)$ correction) due to
  - collinear photons: $\alpha \ln m_e/Q$.
  - soft photons: $\alpha \ln \omega_0/Q$.
- Babar's BRs must be compared with SM branching ratio at NLO $(\alpha/\pi)\ln(m_l/m_\tau)\ln(\omega_0/m_\tau)$, $\sim 10\%$ for $l = e$, $\sim 3\%$ for $l = \mu$.
- For per-cent accuracy, leading-log resummation or even $O(\alpha^2)$ correction are relevant.
- Reduce error on the TH prediction:
  - Unknown higher order corrections,
  - μ_R dependence in $\overline{\text{MS}}$,
  - α or $\alpha(q^2)$?

Technical Ingredients

Width @ NLO

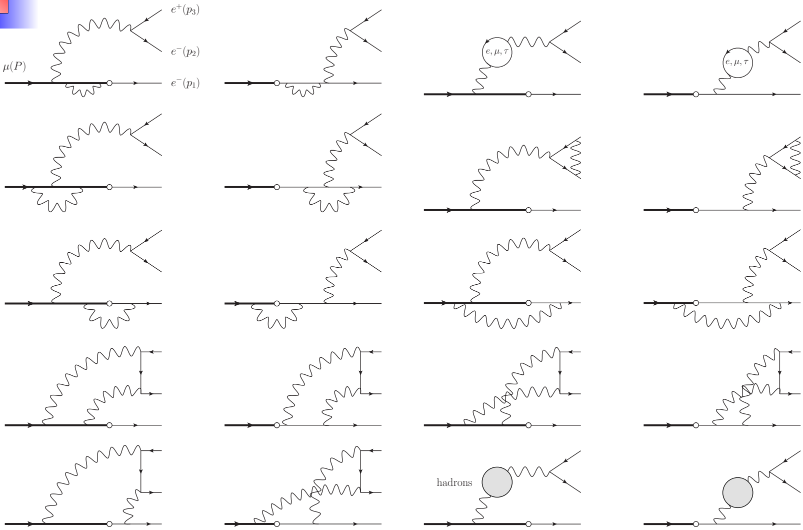
$$\Gamma_{\rm NLO} = \int d\Phi_n \left[|\mathcal{M}_{\rm LO}|^2 + 2\,{\rm Re}(\mathcal{M}_{\rm virt}\mathcal{M}^*_{\rm LO}) \right] + \int d\Phi_{n+1} |\mathcal{M}_{\rm real}|^2$$

- NLO correction computed with Fermi Lagrangian.
- Virtual corrections are finite after e and m renormalization.
- finite terms $\propto m_e$ cannot be neglected:

$$\frac{d\Gamma}{d\theta_{l\gamma}} \sim \frac{(m_l/E_l)^2}{((m_l/E_l)^2 + \theta^2_{l\gamma})^2}$$

T. D. Lee, M. Nauenberg, PR 133 (1964) B1549
L. M. Sehgal, PLB 569 (2003) 25
V. S. Schulz, L. M. Sehgal, PLB 594 (2004) 153

Virual Corrections

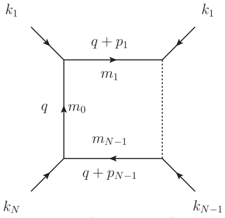


Virual Corrections

$$T^{N,\mu_1\ldots\mu_P} = \frac{(2\pi\mu)^{4-D}}{\mathrm{i}\pi^2}\int \mathrm{d}^D q\, \frac{q^{\mu_1}\cdots q^{\mu_P}}{N_0 N_1 \ldots N_{N-1}}$$

$$N_i = (q+k_i)^2 - m_i^2 + \mathrm{i}\varepsilon, \quad i = 0,\ldots,N$$

- Lorenz decomposition of $T^{N,\mu_1\ldots\mu_P}$



$$T^{N,\mu} = \sum_{i_1=1}^{N-1} p_{i_1}^{\mu}\, T_{i_1}^{N}, \qquad T^{N,\mu\nu} = \sum_{i_1,i_2=1}^{N-1} p_{i_1}^{\mu} p_{i_1}^{\nu}\, T_{i_1 i_2}^{N} + g^{\mu\nu}\, T_{00}^{N},$$

$$T^{N,\mu\nu\rho} = \sum_{i_1,i_2,i_3=1}^{N-1} p_{i_1}^{\mu} p_{i_1}^{\nu} p_{i_3}^{\rho}\, T_{i_1 i_2 i_3}^{N} + \{gp\}_{i_1}^{\mu\nu\rho}\, T_{00 i_1}^{N}, \quad \text{etc.}$$

- Tensor coefficients are evaluated numerically (e.g. via Passarino-Veltman reduction).
  - *LoopTools*, T. Hahn, hep-ph/9807565
  - *Collier*, Denner et al. hep-ph/1604.06792

Real Emission

- Processes with additional soft photon emission are experimentally undistinguishable.
- Logarithmic IR singularity when photon energy $k_0 \to 0$.

$$\Gamma_{\mathrm{real}} = \int d\Phi_{n+1} |\mathcal{M}_{\mathrm{real}}|^2$$

Real Emission

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- Logarithmic IR singularity when photon energy $k_0 \to 0$.

$$\Gamma_{\mathrm{real}} = \int d\Phi_n \int_0^{\omega_0'} d^3 k_\gamma |\mathcal{M}_{\mathrm{real}}|^2 + \int_{k_0 > \omega_0'} d\Phi_{n+1} |\mathcal{M}_{\mathrm{real}}|^2$$

- First photon PS integral can be solved analytically (with finite photon mass λ) in the soft photon approximation: $|\mathcal{M}_{\mathrm{real}}|^2 = f(k_\gamma)|\mathcal{M}_{\mathrm{LO}}|^2$

Real Emission

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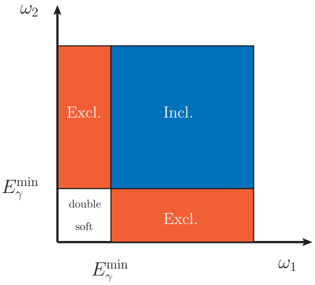
- Logarithmic IR singularity when photon energy $k_0 \to 0$.

$$\Gamma_{\text{real}} = \int d\Phi_n F_{\text{soft}}(\omega_0', \lambda)|\mathcal{M}_{\text{LO}}|^2 + \int_{\omega>\omega_0'} d\Phi_{n+1}|\mathcal{M}_{\text{real}}|^2$$

- First photon PS integral can be solved analytically (with finite photon mass λ) in the soft photon approximation: $|\mathcal{M}_{\text{real}}|^2 = f(k_\gamma)|\mathcal{M}_{\text{LO}}|^2$
- $F_{\text{soft}}|\mathcal{M}_{\text{LO}}|^2 + 2\text{Re}(\mathcal{M}_{\text{virt}}\mathcal{M}_{\text{LO}})$ is free of IR-divergences ($\ln\lambda$) but it is not adequate for real experiments since they do not provide a sufficiently small ω_0' ($\omega_0' \ll m_\mu$).
- Also other methods on the market: dipoles, FKS, antenna.

Real Emission in RMD

RMD branching ratio is defined for a minimum photon energy $E_\gamma^{\min}$.



Double bremsstrahlung: two photons in the final state. We distinguish "Inclusive" and "Exclusive" BRs:

- $\mathcal{B}^{\text{Exc}}(E_\gamma^{\min}) =$ ■,
- $\mathcal{B}^{\text{Inc}}(E_\gamma^{\min}) =$ ■ + ■.

NLO Branching Ratios

Results: BRs

	$\mu \to e\nu\bar{\nu}\gamma$ [$E_\gamma^{\min} = 10$ MeV]	$\mu \to e\nu\bar{\nu}\gamma$ [MEG]	$\mu \to e(e^+e^-)\nu\bar{\nu}$
$\mathcal{B}_{\rm LO}$	1.308×10^{-2}	6.204×10^{-8}	3.6054×10^{-5}
$\mathcal{B}_{\rm NLO}^{\rm Inc}$	$1.289\,(1)_{\rm th} \times 10^{-2}$	$5.84\,(2)_{\rm th} \times 10^{-8}$	$3.5987\,(8)_{\rm th} \times 10^{-5}$
$\mathcal{B}_{\rm NLO}^{\rm Exc}$	$1.286\,(1)_{\rm th} \times 10^{-2}$	—	—
K (Inc)	0.985	0.94	0.998
K (Exc)	0.983	—	—
$\mathcal{B}_{\rm EXP}$	$^{\dagger}1.4\,(4) \times 10^{-2}$	$^{\star}6.03\,(14)_{\rm st}\,(53)_{\rm sys} \times 10^{-8}$	$^{\ddagger}3.4\,(4) \times 10^{-5}$

† Crittenden et al - PR 121 (1961) 1823

⋆ MEG - EPJC 76 (2016) 108 $E_e > 45$ MeV & $E_\gamma > 40$ MeV

‡ SINDRUM - NPB 260 (1985) 1

(τ): experimental error of lifetimes.

K-factor: $\mathrm{K} = \mathcal{B}^{\rm NLO}/\mathcal{B}^{\rm LO}$.

(th): assigned th. error:

- RMD: $(\alpha/\pi)\ln(m_e/m_\mu)\ln(E_\gamma^{\min}/m_\mu)$,
- Rare: μ_R variation.

Results: RτD

	$\tau \to e\bar{\nu}\nu\gamma$	$\tau \to \mu\bar{\nu}\nu\gamma$
$\mathcal{B}_{\rm LO}$	1.834×10^{-2}	3.663×10^{-3}
$\mathcal{B}^{\rm Inc}_{\rm NLO}$	$1.728\,(10)_{\rm th}(3)_\tau \times 10^{-2}$	$3.605\,(2)_{\rm th}(6)_\tau \times 10^{-3}$
$\mathcal{B}^{\rm Exc}_{\rm NLO}$	$1.645\,(19)_{\rm th}(3)_\tau \times 10^{-2}$	$3.572\,(3)_{\rm th}(6)_\tau \times 10^{-3}$
K (Inc)	0.94	0.98
K (Exc)	0.90	0.97
$\mathcal{B}_{\rm EXP}$	$^\dagger 1.847\,(15)_{\rm st}(52)_{\rm sy} \times 10^{-2}$	$^\dagger 3.69\,(3)_{\rm st}(10)_{\rm sy} \times 10^{-3}$

† BABAR - PRD 91 (2015) 051103

Comparison with Babar exclusive measurements:

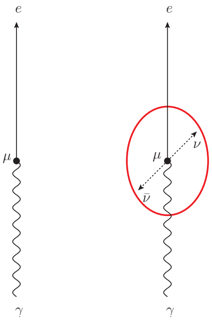
	$\tau \to e\bar{\nu}\nu\gamma$	$\tau \to \mu\bar{\nu}\nu\gamma$
$\Delta^{\rm Exc}$	$2.02\,(57) \times 10^{-3} \to 3.5\sigma$	$1.2\,(1.0) \times 10^{-4} \to 1.1\sigma$

Results: BRs dependence on $\not{E}_{\rm max}$

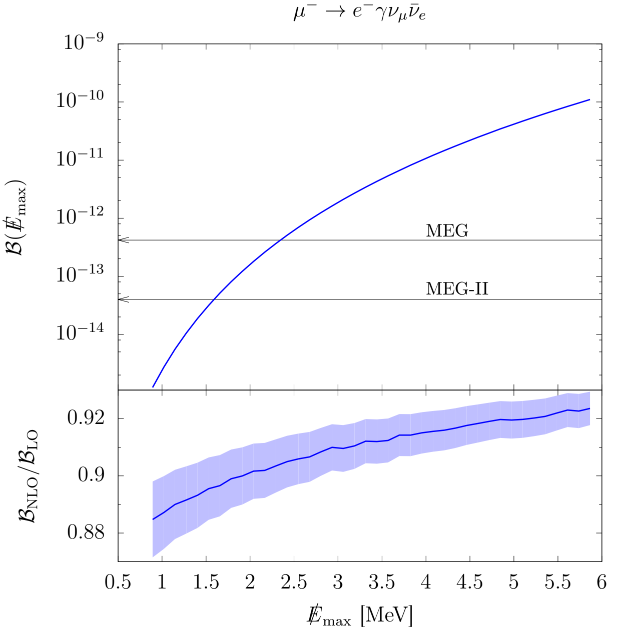
- $\mathcal{B}(\not{E}_{\rm max})$: branching ratio at the end point region

$$m_\mu - E_{\rm vis} = \not{E} \leq \not{E}_{\rm max} \to 0,$$

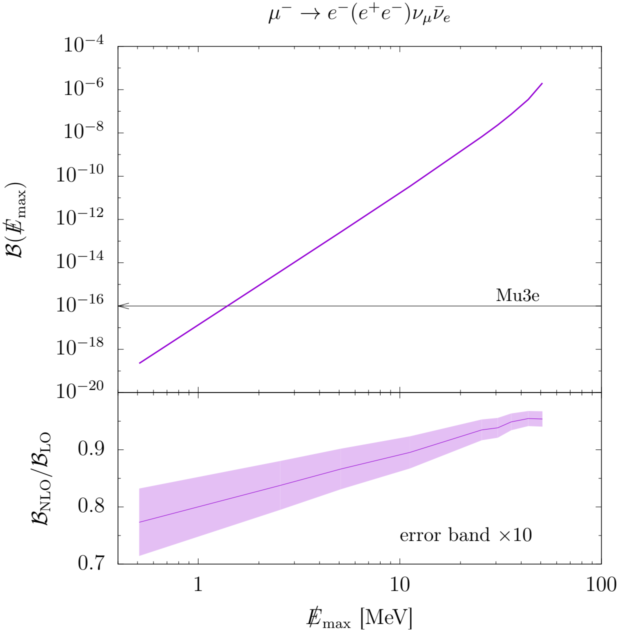
$$\text{(RMD)}: \; E_{\rm vis} = E_e + E_\gamma,$$
$$\text{(Rare)}: \; E_{\rm vis} = E_{e1} + E_{e2} + E_{e3}.$$



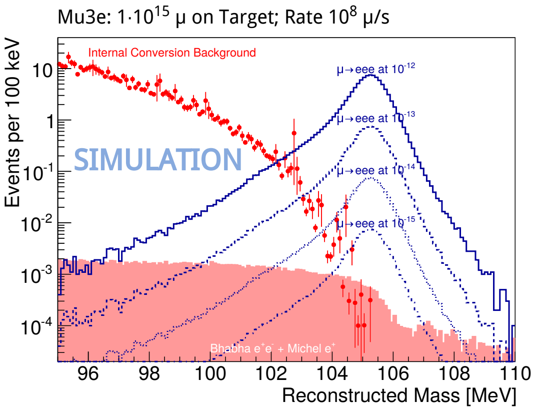
- Additional photon radiation is assumed to be "invisible".
- The $\not{E}_{\rm max}$ cut can be implemented as a restricted integration boundaries (no need of a veto function).



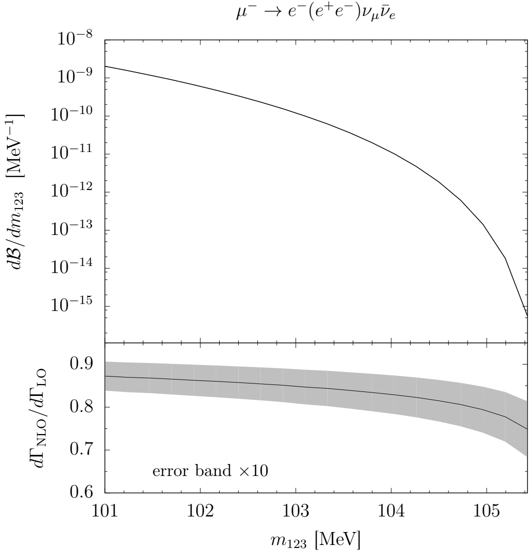
$\mu^- \to e^-\gamma\nu_\mu\bar{\nu}_e$
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
$\mathcal{B}(\not{E}_{\rm max})$
MEG
MEG-II
0.92
0.9
0.88
$\mathcal{B}_{\rm NLO}/\mathcal{B}_{\rm LO}$
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
$\not{E}_{\rm max}$ [MeV]



$\mu^- \to e^-(e^+e^-)\nu_\mu\bar{\nu}_e$
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-18}
10^{-20}
$\mathcal{B}(\not{E}_{\rm max})$
Mu3e
0.9
0.8
0.7
$\mathcal{B}_{\rm NLO}/\mathcal{B}_{\rm LO}$
error band $\times 10$
1
10
100
$\not{E}_{\rm max}$ [MeV]



Mu3e: 1·10^15 μ on Target; Rate 10^8 μ/s
Internal Conversion Background
μ→eee at 10^-12
μ→eee at 10^-13
μ→eee at 10^-14
μ→eee at 10^-15
SIMULATION
Bhabha e+e- + Michel e+
Events per 100 keV
Reconstructed Mass [MeV]



m_{123}: invariant mass of the three electrons.

Conclusions

- We studied the differential rates and BRs of radiative decay $\mu \to e\gamma\nu\bar{\nu}$ and the rare decay $\mu \to e(e^+e^-)\nu\bar{\nu}$ in the SM at NLO in α.
- QED RC were computed taking into account full mass dependence m_e/m_μ, needed for the correct determination of the BRs.
- $2\text{Re}(\mathcal{M}_{\text{virt}}\mathcal{M}^\star_{\text{LO}})$ and $|\mathcal{M}_{\text{real}}|^2$ are available as Fortran code.
- BRS: our predictions agree with the experimental value for $\mathcal{B}(\mu \to e\gamma\nu\bar{\nu})$, $\mathcal{B}(\mu \to eee\nu\bar{\nu})$ and Babar's measurement of $\mathcal{B}(\tau \to \mu\gamma\nu\bar{\nu})$.
- On the contrary, Babar's precise measurement of $\mathcal{B}(\tau \to e\gamma\nu\bar{\nu})$ differs from our prediction by 3.5σ.
- Search of CLFV: QED RC in the PS region where $m_\mu - E_{\text{vis}} \to 0$ can yield a $O(10\%)$ (negative) contribution to the width.

Backup slides

$\mathcal{B}(\not{E}_{\max})$

$\not{E}_{\max}$	$\mathcal{B}_{\mathrm{LO}}$	$\delta\mathcal{B}_{\mathrm{NLO}}$	$\mathcal{B}_{\mathrm{NLO}}$	K
no cut	$3.6054\,(1)_n \times 10^{-5}$	$-6.69\,(5)_n \times 10^{-8}$	$3.5987\,(1)_n\,(8)_{\mathrm{th}} \times 10^{-5}$	0.998
$1\,m_e$	$2.8979\,(6)_n \times 10^{-19}$	$-6.56\,(2)_n \times 10^{-20}$	$2.242\,(2)_n\,(17)_{\mathrm{th}} \times 10^{-19}$	0.77
$5\,m_e$	$4.641\,(1)_n \times 10^{-15}$	$-7.41\,(3)_n \times 10^{-16}$	$3.900\,(3)_n\,(20)_{\mathrm{th}} \times 10^{-15}$	0.83
$10\,m_e$	$3.0704\,(7)_n \times 10^{-13}$	$-4.04\,(2)_n \times 10^{-14}$	$2.666\,(2)_n\,(11)_{\mathrm{th}} \times 10^{-13}$	0.87
$20\,m_e$	$2.1186\,(5)_n \times 10^{-11}$	$-2.17\,(1)_n \times 10^{-12}$	$1.902\,(1)_n\,(6)_{\mathrm{th}} \times 10^{-11}$	0.90
$50\,m_e$	$7.151\,(1)_n \times 10^{-9}$	$-4.55\,(3)_n \times 10^{-10}$	$6.696\,(3)_n\,(13)_{\mathrm{th}} \times 10^{-9}$	0.93
$100\,m_e$	$2.1214\,(4)_n \times 10^{-6}$	$-9.47\,(6)_n \times 10^{-8}$	$2.027\,(1)_n\,(3)_{\mathrm{th}} \times 10^{-6}$	0.96

The total differential decay for a polarized μ or τ lepton in the tau r.f. is

$$\frac{d^6\Gamma^{\rm NLO}}{dx\, dy\, d\Omega_l\, d\Omega_\gamma} = \frac{\alpha\, G_F^2 m_\tau^5}{(4\pi)^6} \frac{x\beta}{1+\delta_{\rm W}(m_\mu, m_e)} \Bigg[G(x,y,c)$$

$$+ \; x\beta\, \hat{n}\cdot\hat{p}_l\, J(x,y,c) + y\, \hat{n}\cdot\hat{p}_\gamma\, K(x,y,c) + y\, x\beta\, \hat{n}\cdot(\hat{p}_l \times \hat{p}_\gamma)\; L(x,y,c) \Bigg]$$

where $x = 2E_l/m_\tau$, $y = 2E_\gamma/m_\tau$, $c = \cos\theta_{l\gamma}$. The polarization vector $n = (0, \vec{n})$ satisfies $n^2 = -1$ and $n \cdot p_\tau = 0$.
The function $G(x,y,c)$, and similarly for J and K, is given by

$$G(x,y,c) = \frac{4}{3yz^2} \left[g_{\rm LO}(x,y,z) + \frac{\alpha}{\pi}\, g_{\rm NLO}(x,y,z;y_{\rm min}) + \left(\frac{m_\tau}{M_W}\right)^2 g_{\rm W}(x,y,z) \right]$$

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Compared with previous work A. B. Arbuzov PLB 597 (2004) 285