Monte Carlo simulations for the neutron EDM experiment at TRIUMF

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Background

TRIUMF

- TRIUMF is Canada's national laboratory for particle and nuclear physics. It is located in Vancouver, BC.
- An ultra-cold neutron (UCN) source and neutron electric dipole moment (nEDM) experiment are currently under construction at TRIUMF.

The nEDM measurement with Ramsey's method of separated oscillatory fields

- Neutrons enter a cylindrical storage cell polarized in the positive z direction (vertically).
- Two static fields are present; A magnetic field acting in the positive z direction (B_0), and an electric filed acting parallel or anti-parallel to B_0 ($E_\uparrow$ or $E_\downarrow$). Since we know $\omega\hbar = -2\mu_n B - 2d_n E$, by running the simulation with $E_\uparrow$, then with $E_\downarrow$, we can determine the EDM.
- A rotating field, B_1, is applied in the xy plane at frequency ω_i for a time τ. This will flip the spin by some amount.
- The neutrons are then allowed to precess freely for a time T. After this, the B_1 field is applied again for a time τ.
- The number of spin up and spin down neutrons are counted.
- This is repeated for 4 different ω_i values (as in the graph below), and done for both $E_\uparrow$ and $E_\downarrow$.
- $\triangle\omega$ is calculated between $E_\uparrow$ and $E_\downarrow$. The EDM can then be determined; $d_n = \frac{\hbar\triangle\omega}{4E}$
- A non-zero value of the nEDM has not yet been discovered. Currently we know $|d_n| < 2.9 \times 10^{-26}$ ecm [2].

PENTrack

- PENTrack is a free Monte Carlo program for tracking Protons, Electrons, and Neutrons.
- Recent changes make it possible to run full simulations of Ramsey's method of separated oscillatory fields, and track the spin of Neutrons, Xenon, and Mercury.
- EDM simulator Features:
  - Analytical and/or numerical B_0, B_1, and E fields.
  - Spin tracking, Larmor frequency tracking, simultaneous $E_\uparrow$ and $E_\downarrow$ spin tracking.
  - Compensation for ExV effect.
  - Geometry import from .stl file (Ability to design with most 3D CAD programs).

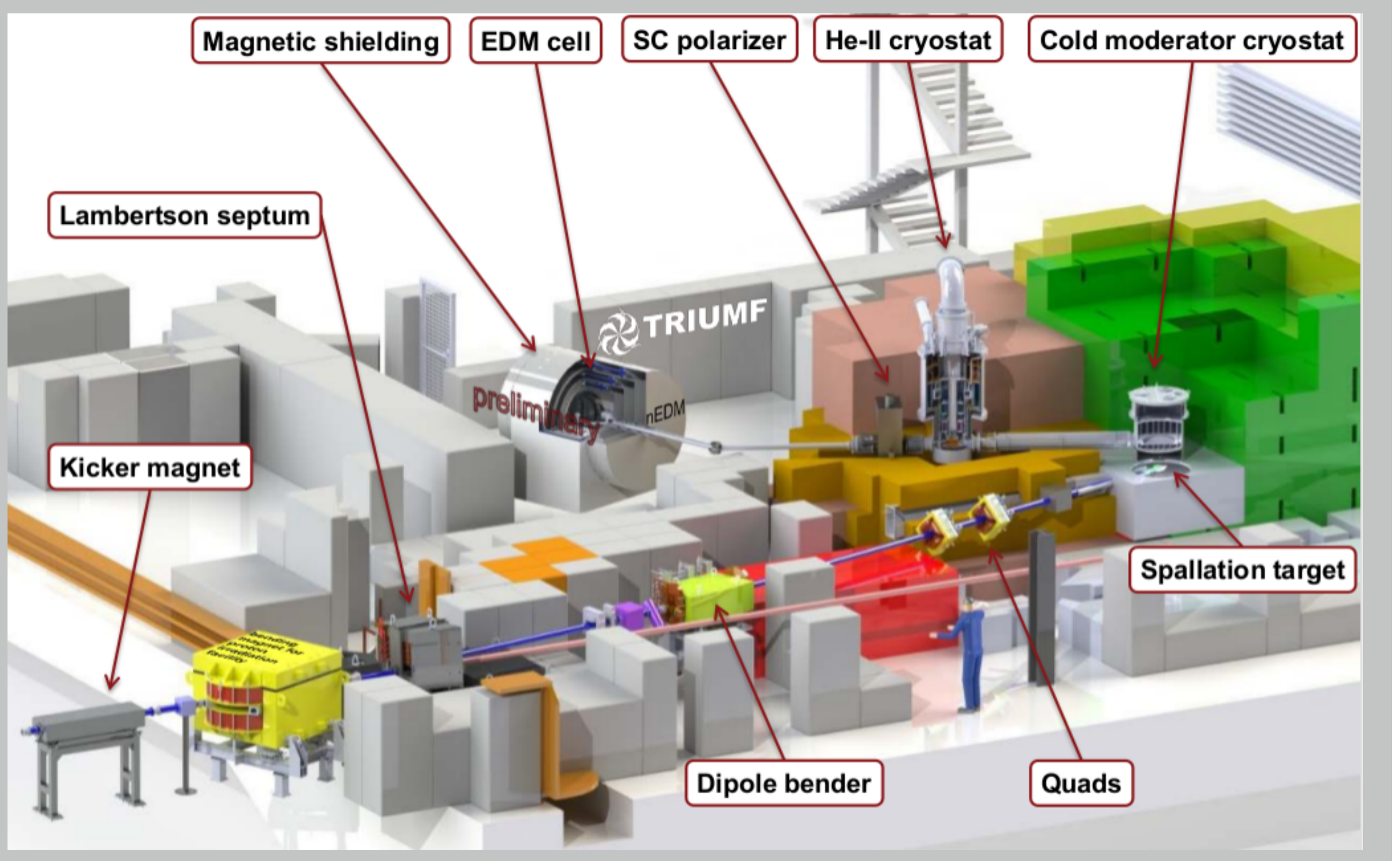

Figure: UCN beamline layout [1]

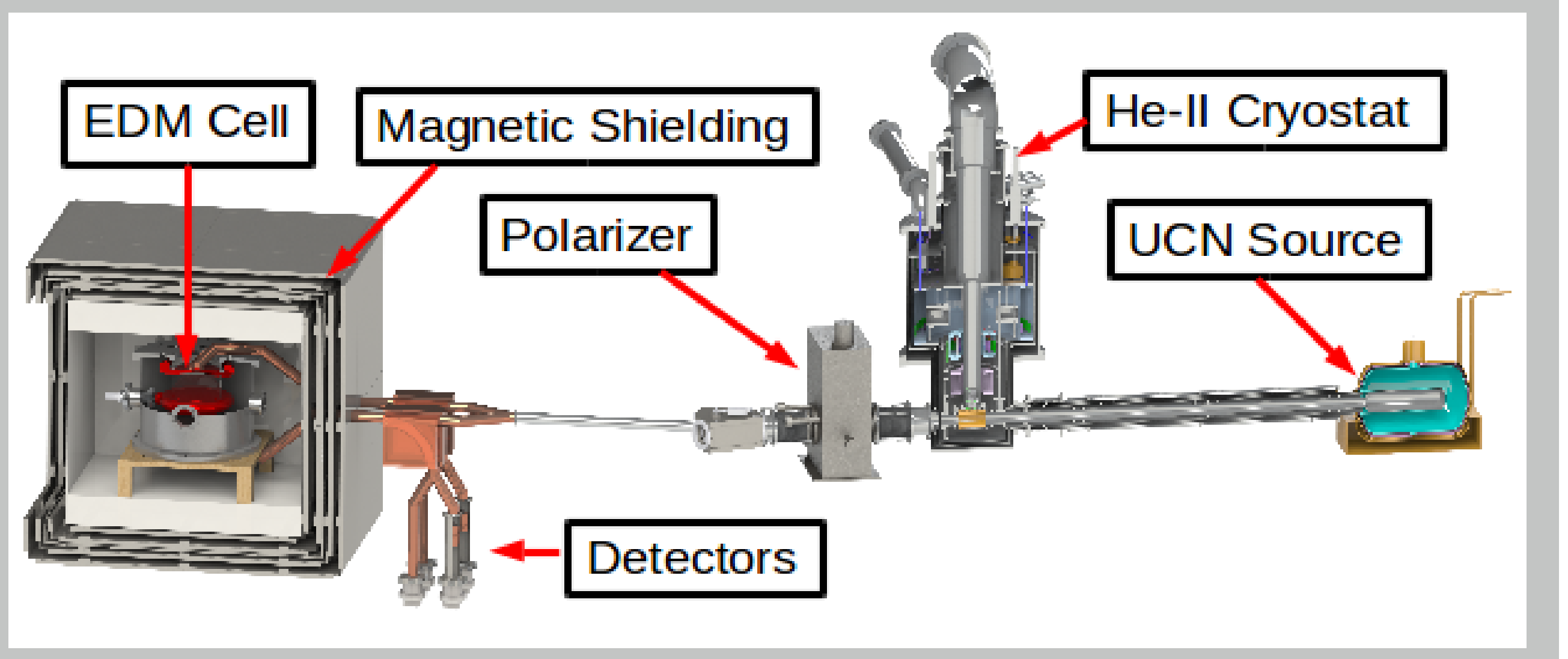

Figure: TRIUMF's Phase 2 nEDM layout

Effect of Magnetic Field Gradients on Larmor frequency

- The Larmor frequency is close to $\omega_0 = -\gamma B_0$, however there are some factors that will shift it from this value. These shifts need to be accounted for in order to achieve the high precision required in the nEDM experiment.
- Magnetic field inhomogeneities account for a significant portion of the systematic error in the most accurate nEDM experiment [2].
- Here, the effect of magnetic field inhomogeneities on the Larmor frequency are simulated by running Ramsey cycles similar to the real experiment.
- Parameters used: $B_0 = 1\,\mu$T, $B_{1,1D}\gamma\tau = \pi$, $B_{1,2D} = B_{1,1D}/2$, $T = 50$ s, $\tau = 1$ s, $E = 0$.
- $B_{1,1D}$ denotes a linearly oscillating B_1 field, and $B_{1,2D}$ denotes a circularly oscillating B_1 field.
- The neutron energy distribution is taken from a filling simulation for TRIUMF's phase 2 nEDM experiment.
- EDM cell height $H = 14$ cm, radius $R = 18.1$ cm.
- The ecm values shown are calculated assuming $E = 1 \times 10^6$ V/m (Proposed electric field strength of TRIUMF's Phase 2 nEDM experiment).

Table: Larmor frequency fit results

$\partial B_{0z}/\partial z$	Larmor Frequency (Hz)	error (Hz)	error (ecm)
0 ($\omega_0 = -\gamma B_0$)	29.164695300	N/A	N/A
0 (PENTrack, $B_{1,2D}$)	29.164695302	5.5×10^{-11}	5.7×10^{-30}
1nT/m (PENTrack, $B_{1,2D}$)	29.164776924	5.6×10^{-7}	5.8×10^{-26}
5nT/m (PENTrack, $B_{1,2D}$)	29.165092779	3.2×10^{-6}	3.3×10^{-25}
0 (PENTrack, $B_{1,1D}$)	29.164704336	7×10^{-12}	7×10^{-31}

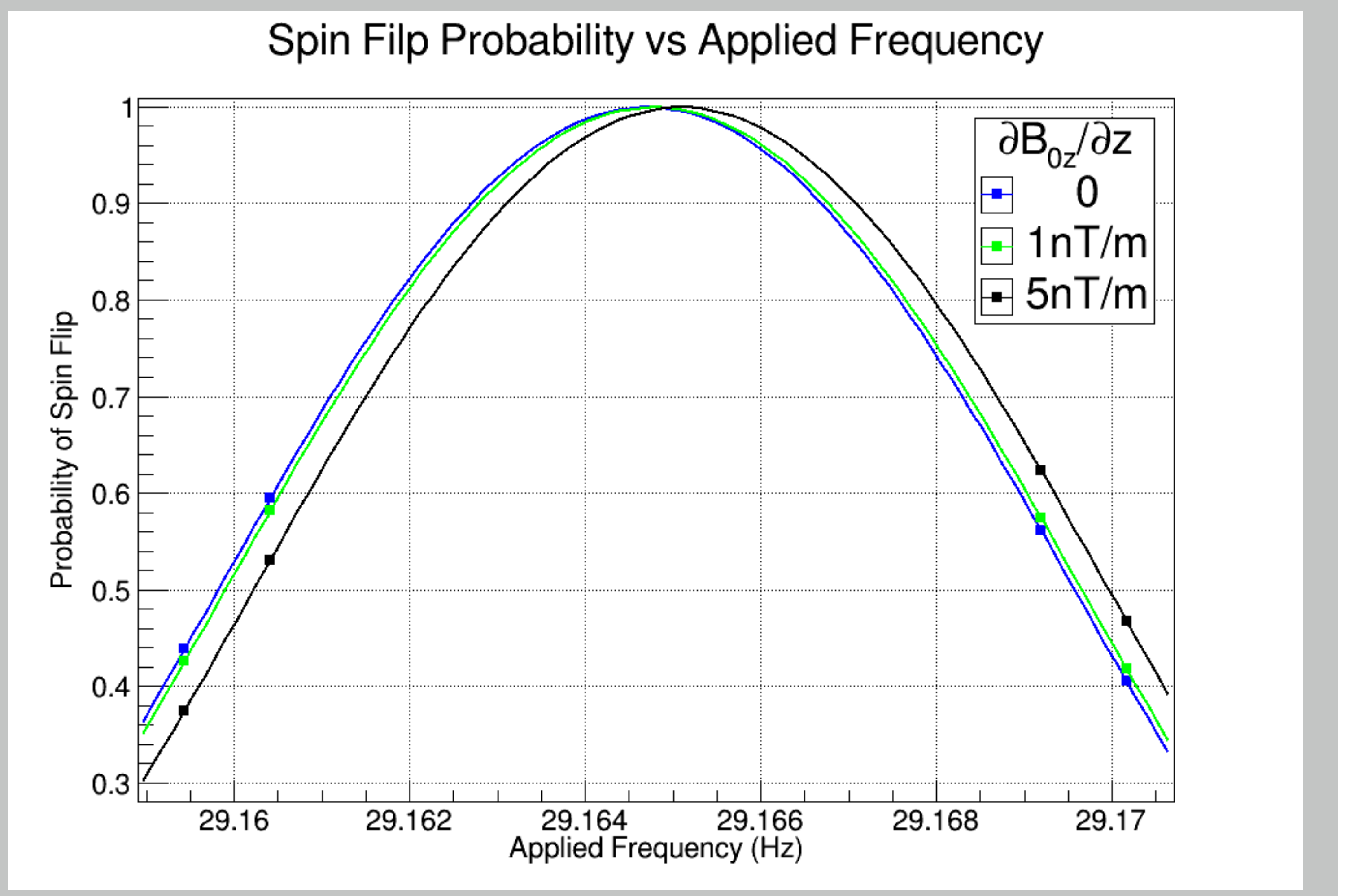

Figure: Ramsey cycle results for various field gradient strengths (fits done in the same way as the real experiment). These results are for a circularly rotating B_1 field ($B_{1,2D}$).

Table: Larmor frequency shifts. Comparison with theoretical predictions

$\partial B_{0z}/\partial z$	0 ($B_{1,2D}$)	1nT/m ($B_{1,2D}$)	5nT/m ($B_{1,2D}$)	0 ($B_{1,1D}$)
ω_L shift (from PENTrack simulation)	$1.62(6) \times 10^{-28}$ ecm	$8.44(6) \times 10^{-24}$ ecm	$4.11(3) \times 10^{-23}$ ecm	$9.342260(7) \times 10^{-25}$ ecm
Theoretical ω_L shift	0	8.43×10^{-24} ecm	4.12×10^{-23} ecm	$1.3234137 \times 10^{-24}$ ecm
$\omega_{L,\mathrm{Theory}} - \omega_{L,\mathrm{PENTrack}}$	$1.62(6) \times 10^{-28}$ ecm	$-1(6) \times 10^{-26}$ ecm	$1(3) \times 10^{-25}$ ecm	$4.079663(7) \times 10^{-25}$ ecm

Mathematical Section

Larmor frequency shift due to Average UCN height in the EDM cell [3]

$$\omega_L = \omega_0 - \gamma\triangle\overline{B}_z \quad (1)$$

$$\triangle\overline{B}_z = \frac{\partial B_{0z}}{\partial z}\overline{Z} = \frac{\partial B_{0z}}{\partial z}\frac{1}{6g}\left(-v_{0\uparrow}^2 - Hg + v_{0\uparrow}\sqrt{v_{0\uparrow}^2 + 2gH}\right) \quad (2)$$

γ is the neutron's gyromagnetic ratio, $\overline{Z}$ is the average UCN height, $v_{0\uparrow}$ is the velocity the neutron has at the bottom of the cell, g is gravity, and H is the height of the cell. The RHS of Equation 2 applies for neutrons that can reach the top of the cell (and assuming specular reflection).

The Bloch-Siegert (BS) shift [3]

$$\omega_L \simeq \omega_0 + \frac{\omega_1 \tan(\frac{\omega_1\tau}{4})}{4\omega_0 T(1 + \frac{8}{\omega_1 T}\tan(\frac{\omega_1\tau}{4}))} \quad (3)$$

$$\omega_1 = -\gamma B_{1,1D} \quad (4)$$

The BS shift applies if the B_1 field is oscillating in 1 dimension only ($B_{1,1D}$).

The Geometric Phase Effect (GPE) [4]

$$(\triangle\omega_{\uparrow\uparrow} - \triangle\omega_{\uparrow\downarrow}) = E\left(\frac{\frac{\partial B_{0z}}{\partial z}}{B_{0z}^2}\right)\frac{v_{xy}^2}{c^2}\left[1 - \frac{\omega_r^{*2}}{\omega_0^2}\right]^{-1} \quad (5)$$

$$\omega_r^{*2} = \frac{\pi^2}{6}\left(\frac{v_{xy}}{R}\right)^2 \quad (6)$$

$$(\triangle\omega_{\uparrow\uparrow} - \triangle\omega_{\uparrow\downarrow}) = E\left(\frac{\frac{\partial B_{0z}}{\partial z}}{B_{0z}^2}\right)\frac{v_{xy}^2}{c^2}\left[1 + \frac{sin^2(\alpha)sin(2\delta)}{2\delta sin(\delta - \alpha)sin(\delta + \alpha)}\right] \quad (7)$$

c is the speed of light, v_{xy} is the velocity in the xy plane, R is the radius of the storage cell. α and δ are related to the path of the neutron in the cell. Equation 5 applies when there is partly diffuse reflection, and Equation 7 applies when there is specular reflection.

Geometric Phase Effect

- The GPE will create a false EDM signal in the experiment.
- The parameters and procedure of this simulation are similar to the simulation above.
- Simultaneous $E_\uparrow$ and $E_\downarrow$ spin tracking is used (Therefore, no is error shown).
- Only one neutron velocity is used.
- $E = 1 \times 10^8$ V/m (to magnify effect).
- Some possible reasons for discrepancies are resolution/accuracy issues in PENTrack, or that Equations 5 and 7 are not valid for this situation.

Table: GPE simulation results. $\partial B_{0z}/\partial z = 1$ nT/m

Simulation	Specular reflection ($v_{xy} \simeq 3$ m/s)	Partly diffuse reflection ($v_{xy} \simeq 3.5$ m/s)
Simulated GPE	1.87×10^{-27} ecm	2.78×10^{-27} ecm
Theoretical GPE	1.58×10^{-27} ecm	2.37×10^{-27} ecm
Difference (Simulation - Theory)	2.9×10^{-28} ecm	4.1×10^{-28} ecm

References

[1] R. Picker, "The UCN facility and EDM experiment at TRIUMF". TRIUMF International Peer Review 2013. Nov 2013

[2] Baker CA, Doyle DD, Geltenbort P, et al. "Improved Experimental Limit on the Electric Dipole Moment of the Neutron". Phys. Rev. Lett. 97:131801. Sep 2006

[3] D.J.R. May. "A High Precision Comparison Of The Gyromagnetic Ratios Of The 199Hg Atom And The Neutron". PhD thesis, The University of Sussex, May 1998.

[4] Pendlebury JM, Heil W, Sobolev Y, et al. "Geometric-phase-induced false electric dipole moment signals for particles in traps". Physical Review A, 70:032102. Sep 2004

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