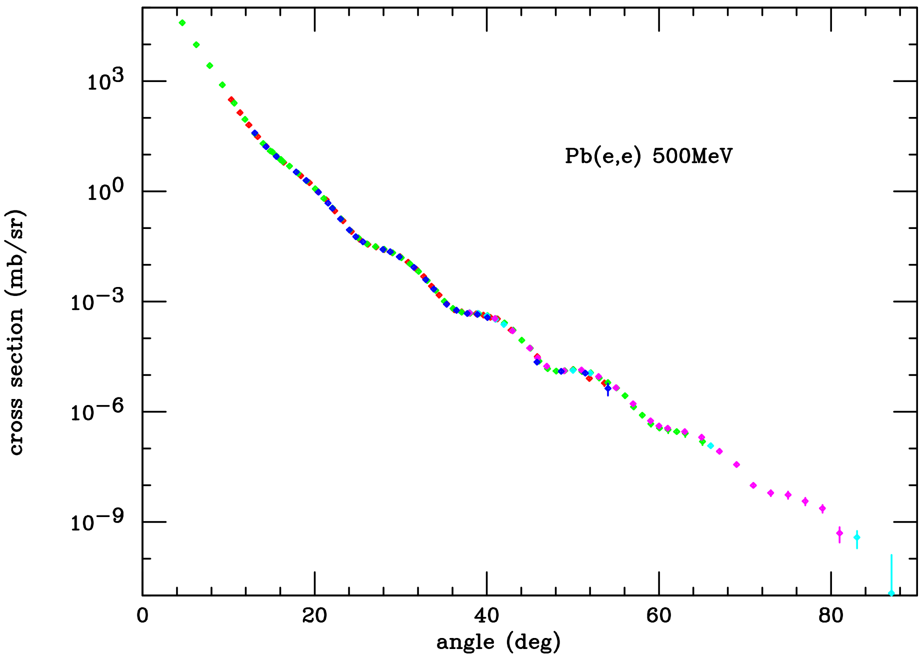
(e,e) + μX: densities + moments

Ingo Sick

Or: model-dependent vs. independent information on ρ

Electron scattering (I=0)

determines $\rho(r)$ with great detail
possible if q_{max} large
with best data can get ± model-independent ρ



Determination of $\rho(r)$: pedagogical example

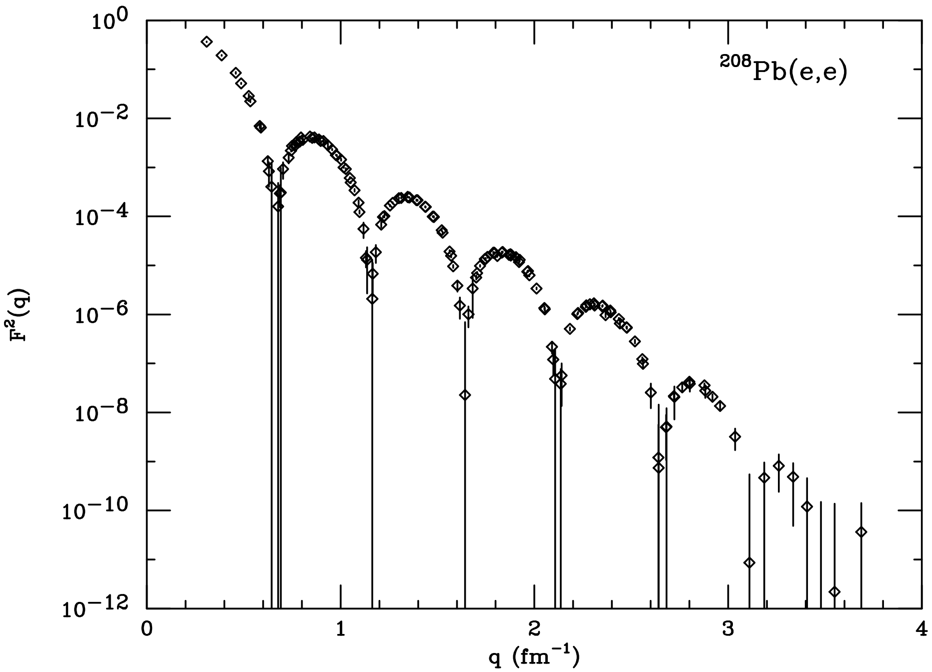
can convert σ's to F's

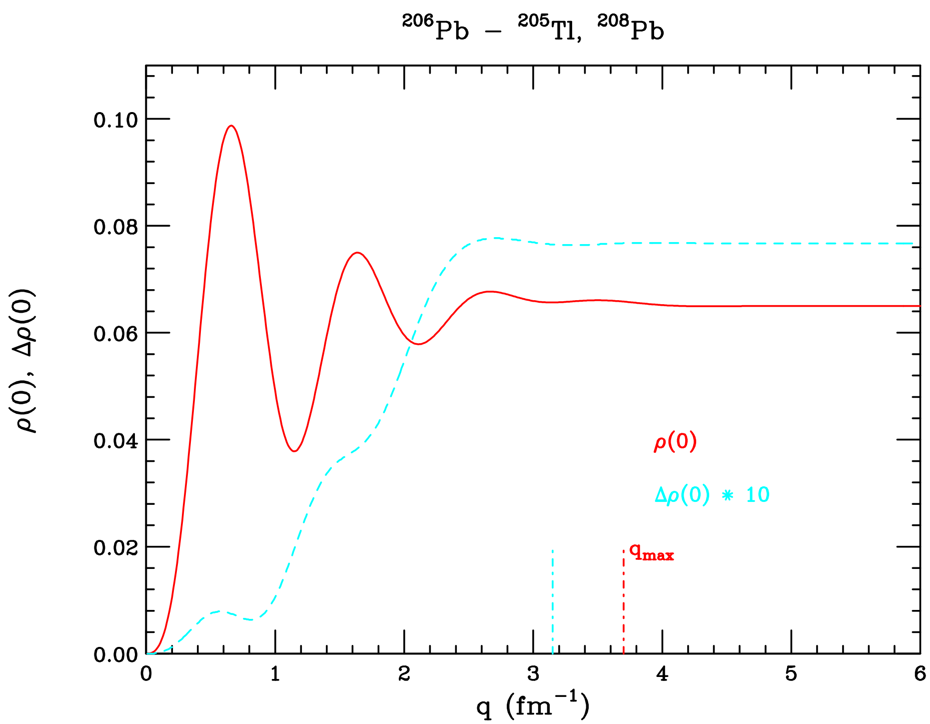
$$F^2_{exp} = F^2_{model}\frac{\sigma_{exp}}{\sigma^{DW}_{model}}, \qquad \delta F^2_{exp} = \delta\sigma_{exp}/\sigma_{point}$$

and use

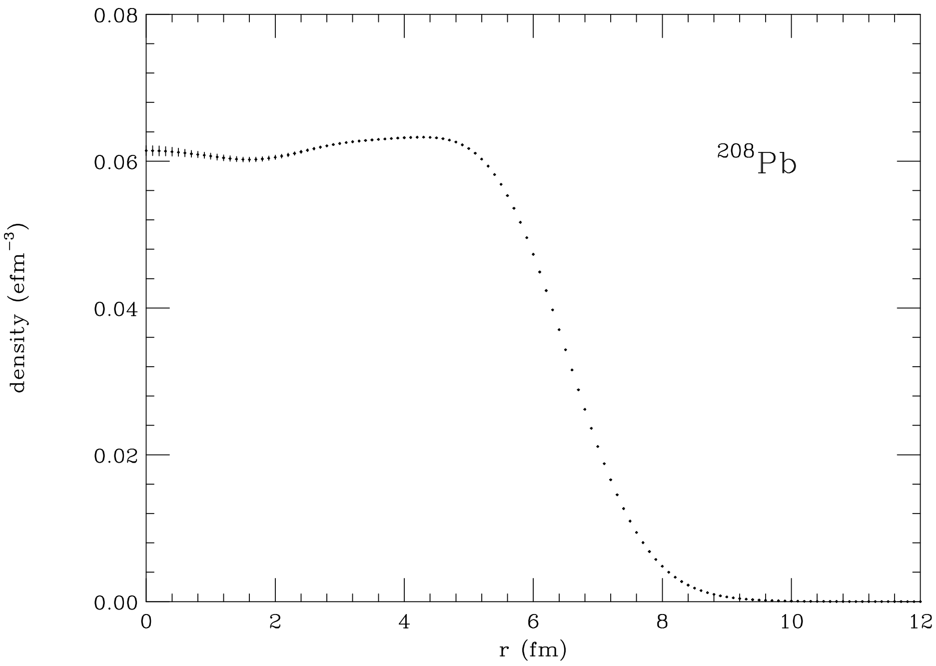
$$\rho(r, q_{max}) = ... \int_0^{q_{max}} F(q)\frac{sin(qr)}{qr}q^2 dq$$

$\rho(r, q_{max})$ = damped oscillation, $\rho(r,\infty) = \rho(r)$, convergence yields $\delta\rho$





Resulting density for ^{208}Pb



very small error bars

even at $r = 0$ where largest

Benchmark for $\delta\rho$ for standard analyses

expansion of $\rho(r)$ in (truncated) basis
multi-parameter fit of σ via phase-shift code

Differences between ρ's of neighboring nuclei

isotopic differences

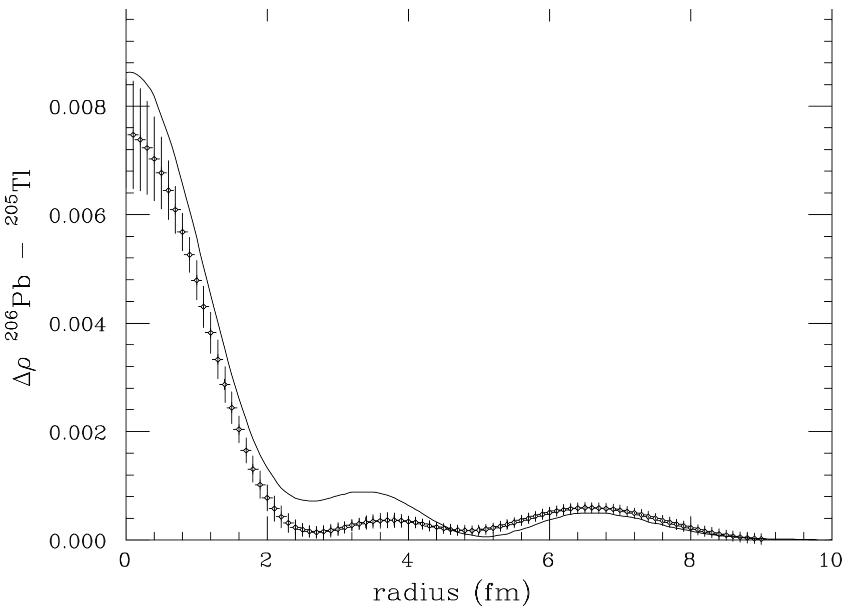
- **in general not particularly instructive**
- **not particularly accurate, mainly Δrms**

isotonic differences

- **more instructive**
- **if have *accurate* cross section ratios**

Prime example: $^{206}Pb - ^{205}Tl$

3s radial distribution, shell model orbit in nuclear interior



With not-so-good data base, $q_{max} < 2fm^{-1}$: must use model-densities

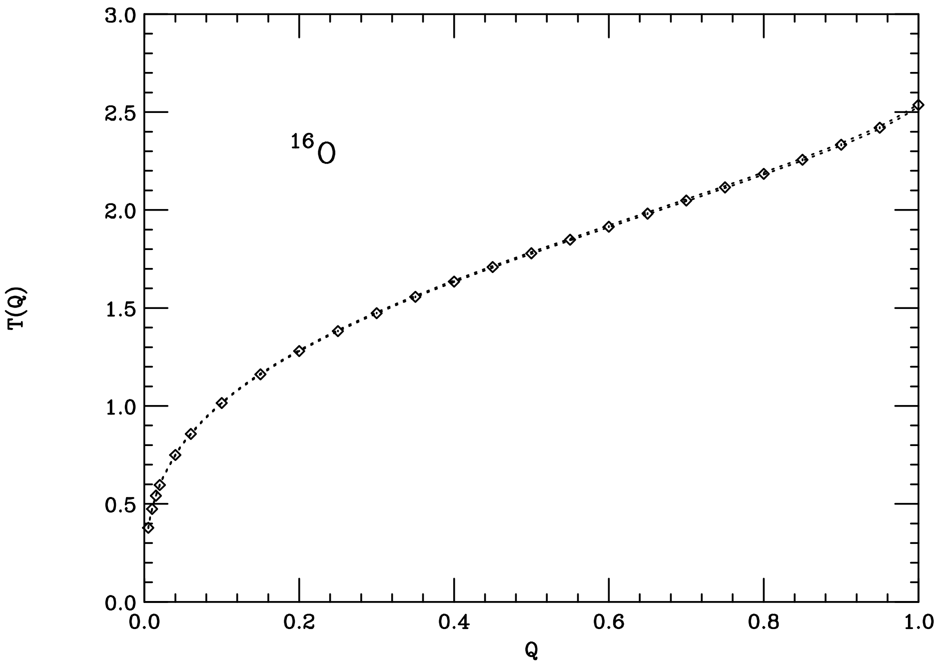
3PF $\rho(r) = (1 + w(r/c)^2)/(1 + e^{(r-c)/z})$

3PG $\rho(r) = (1 + w(r/c)^2)/(1 + e^{(r^2-c^2)/z^2})$

Model-independent information: **partial moment function**

$T(Q) = \int_0^Q r(Q')\, dQ'$ **with** Q **= fraction of charge out to radius** r

$T(Q)/Q$ **= linear moment averaged over charge** Q



Strictly model-independent, well-defined δT, not intuitive, never caught on

rms-radii from (e,e)

familiar from A≤2: $F(q) = 1 - q^2\langle r^2\rangle/6 + q^4\langle r^4\rangle/120 - ...$

Difficult to exploit

cannot reach $q = 0$, must extrapolate

problem with higher $\langle r^n\rangle$

at very small q the "1" dominates

PW not valid

Worse: extrapolation can generate nonsense

fit of proton data with Pade parameterization

$q_{max} = 2fm^{-1}$

can generate $R = 1.48fm$

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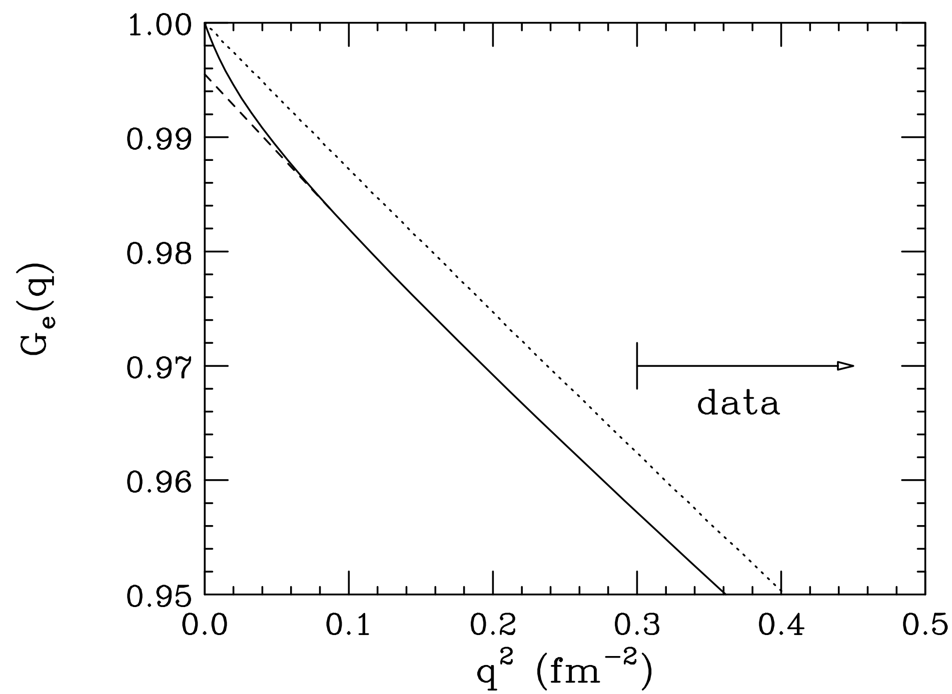


——— Pade-fit

.... Fit with 'normal' rms-radius

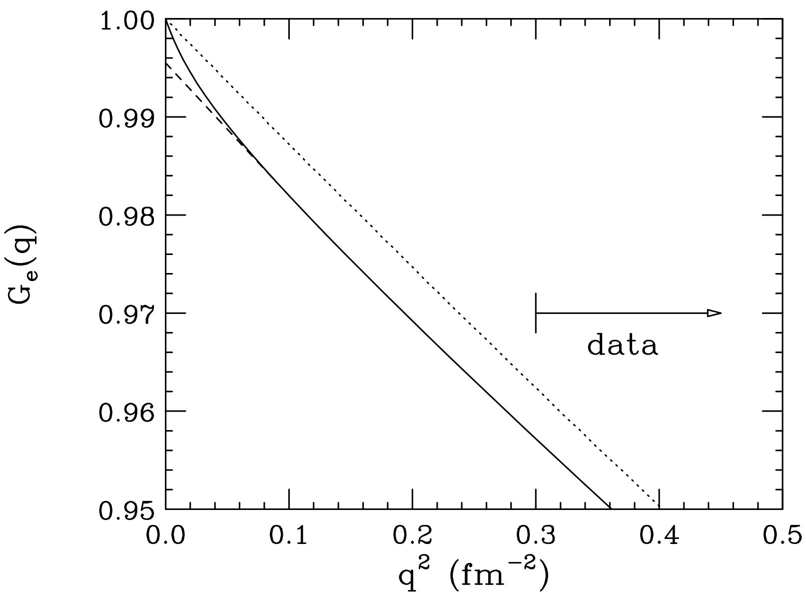
Note: data are floating

Understanding of $R = 1.48 fm$

split fit into two contributions $G = G_1 + G_2$:

G_1 = Pade for $q^2 > 0.06$ plus dashed line for $q^2 < 0.06$

G_2 = Pade − G_1



G_1 has 'normal' q=0 slope, norm of 0.995

$G_2 \sim e^{-q^2/(0.02 fm^2)}$

corresponds to $\rho \sim e^{-r^2/(200 fm^2)}$

G_2 leads to large rms-radius despite small norm of ~ 0.005

Choice of parameterization of $G(q)$ implies choice of $\rho(r)$

harmless-looking $G(q)$ (*e.g.* Pade) can correspond to outrageous $\rho(r)$

For sensible R *must* study behavior of $\rho(r)$ at large r

to avoid nonsense not visible in $G(q)$

Conclusion: parameterization of $F(q)$ is bad idea, for *all* A

Better: use procedure standard for $A > 2$

$$R^2 = \int \rho(r)\, r^4 \;\; 4\pi \; dr$$

parameterize $\rho(r)$ with model-density
fit $\sigma(E, \theta)$ with $\rho(r)$, using phase-shift code
impose (explicitly or with model) sensible behavior of $\rho(r \gg)$

Is also the correct way for $A \leq 2$!

For best results: constrain $\rho(r \gg)$ using *physics*

large r most important for R
add physics to fix *shape* of $\rho(r)$
fall-off of ρ dominated by least-bound constituent
p (nuclei), π^+ (proton)
Whittaker function, depending on separation energy
in general known

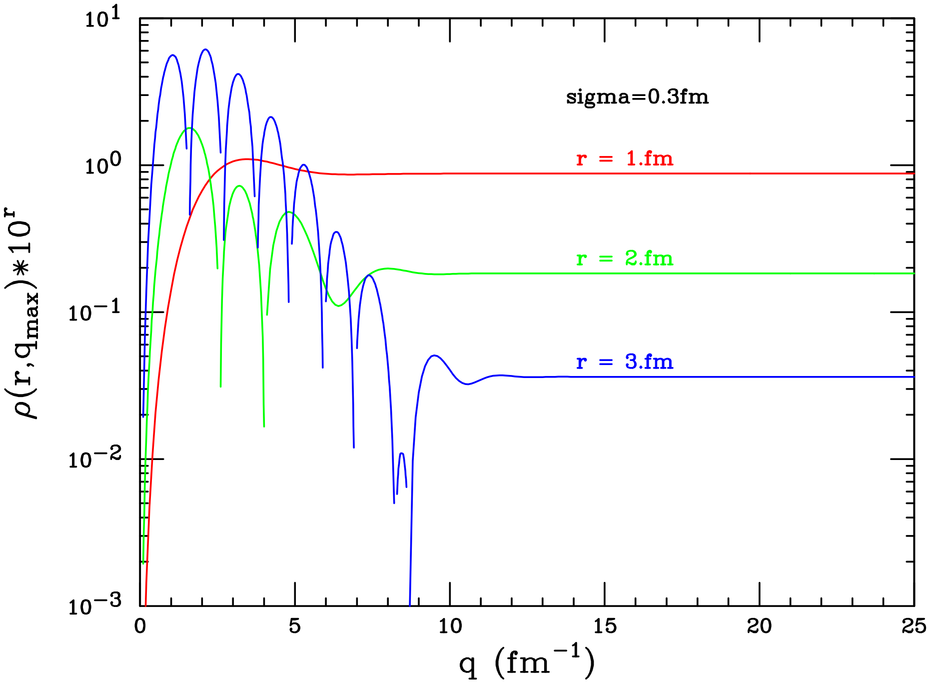
Counterintuitive, but true

for most accurate R helpful to use data up to *largest* q's
these σ fix *shape* of $\rho(r)$ including tail

For demonstration study

$$\rho(r, q_{max}) = ...\frac{1}{r}\int_0^{q_{max}} F(q)\ sin(qr)\ q\ dq$$

For accurate rms-radius need $\rho(r)$ at large r

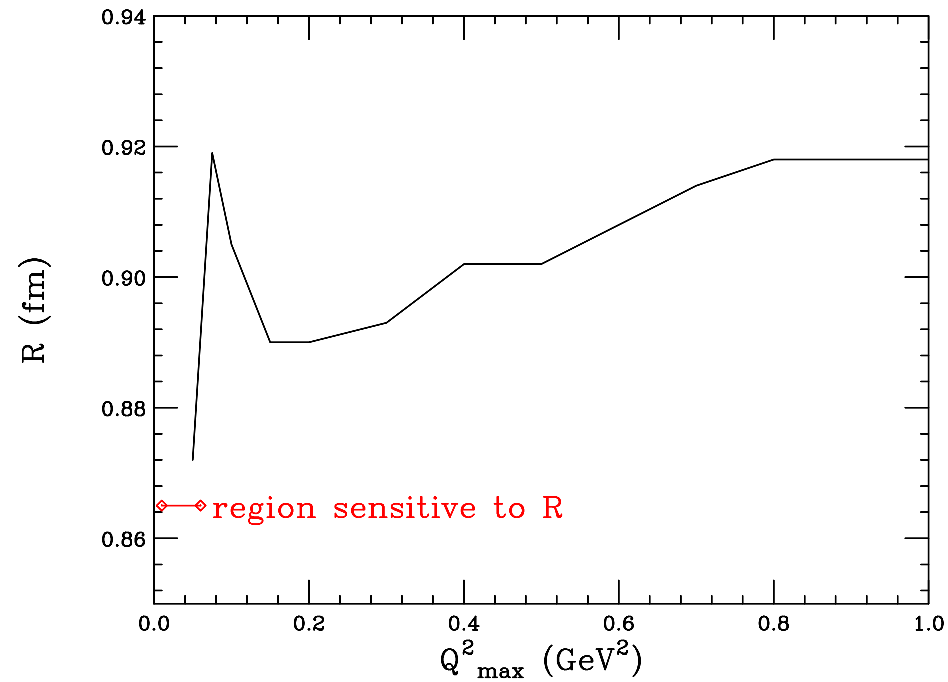


For accurate ρ at the larger r need data up to larger q's

For accurate rms-radius use all the high q data available

Data at large q do have effect!

recent example: proton
find dependence of R on q_{max} of data
show example of Lee *et al.*

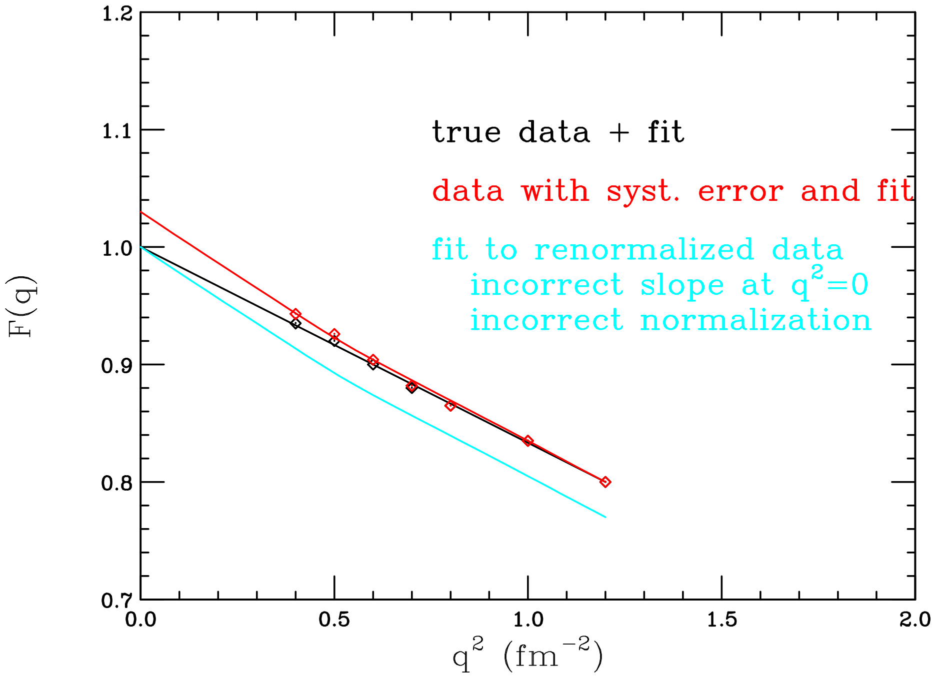


observed by several analyses, but not understood
$\rho(r, q_{max})$ explains it!

Lesson: data up to largest q fix $\rho(r)$ best $\rightarrow$ most accurate R

Uncertainties of R

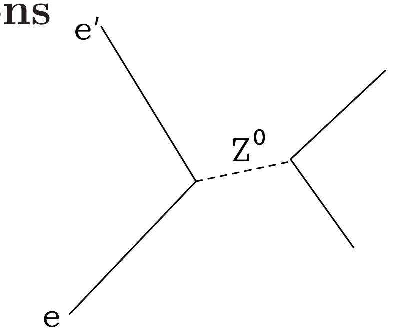
can reach $<1\%$
often dominated by knowledge of overall norm of σ
traditional floating of σ detrimental
does often more harm than good!



$\rightarrow$ high premium on accurate *absolute* normalization *rms*-radii of neutron distribution: parity-violating (e,e)

interference γ-Z_0 exchange

analyzing power for longitudinally-polarized electrons



Accident of Standard Model

PV coupling to n 10 times larger than to p!

$\Rightarrow$ non-PV (e,e) sees basically protons

$\Rightarrow$ PV (e,e) sees basically neutrons!

$$\frac{\widetilde{F}_{C0}(q)}{F_{C0}(q)} = \beta_V^p + \beta_V^n \frac{FT(\rho_n(r))}{FT(\rho_p(r))},$$

$$\boxed{\beta_V^p = 0.04, \quad \beta_V^n = -0.5}$$

Observations first made by Donnelly, Dubach, Sick, 1989:

- PV scattering can be used to determine R of ρ_n to $\pm 1\%$
- PV experiment on ^{208}Pb is practical, and is being done: PREX
  can possibly be extended to measure z

Not 'standard' experiment to be done for many nuclei

asymmetry $\sim 10^{-6}$

But:

accurate rms-radius of $\rho_n(r)$ for *e.g. Pb* worth (large) effort

strongly correlated with EOS of neutron matter *rms*-radii of *valence* nucleon orbits

magnetization measurable via magnetic form factors (back angle scattering)

dominated by unpaired valence nucleon (p or n)

Problem: configuration mixing

ruins simple relation $F(q) = \ldots \int R^2(r) j_{\Lambda-1}(qr) r^2 dr$

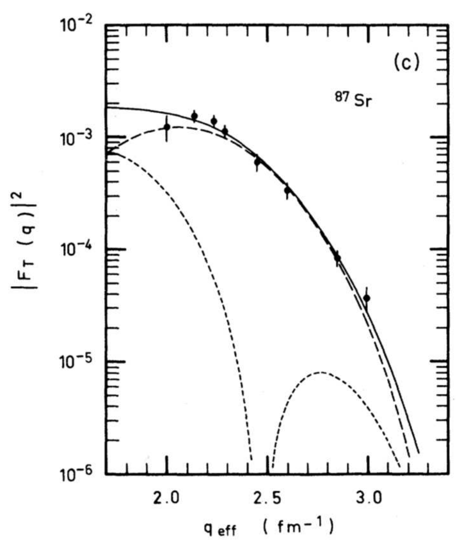
Most clean case

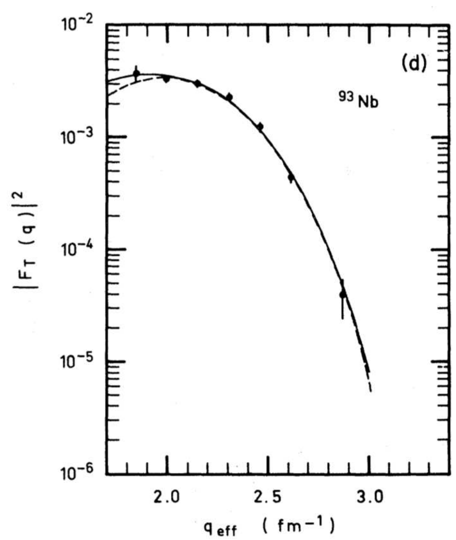
scattering from nucleon in shell with highest $j = j_m$ in nucleus

restriction to highest multipolarity $\Lambda = 2j_m$

$\rightarrow$ shells with $j < j_m$ cannot contribute

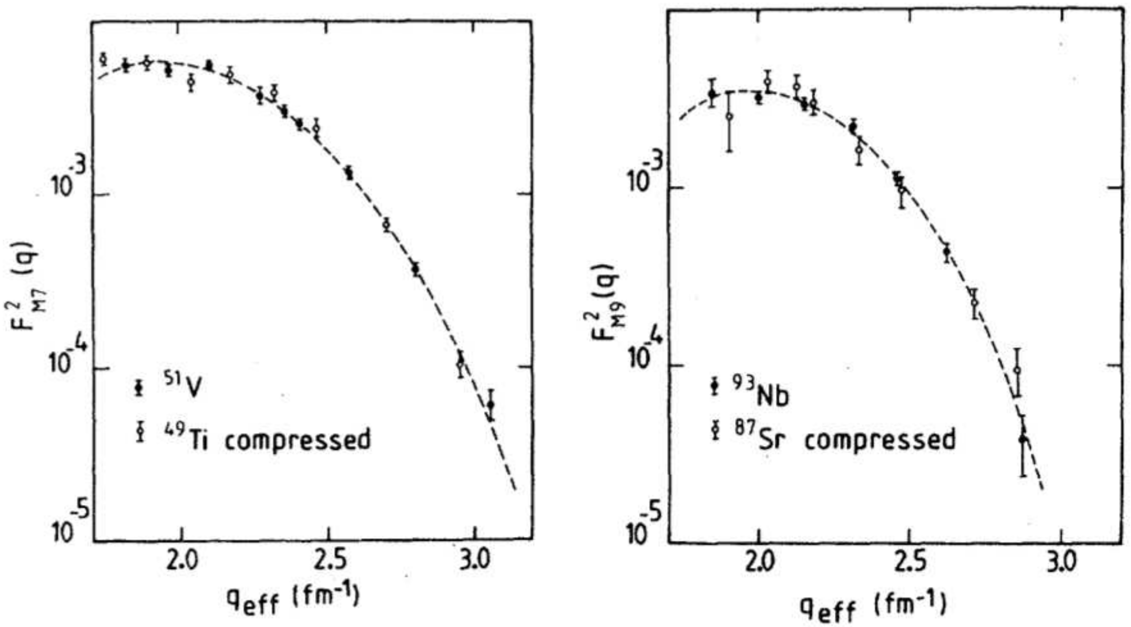
experiments done for nuclei such as ^{49}Ti, ^{51}V (M7), ^{87}Sr, ^{93}Nb (M9)





For neighboring pair with odd n/p

can make direct n-p comparison
can compress q-scale of, say, ^{87}Sr and compare data to data for ^{93}Nb
modelindependent



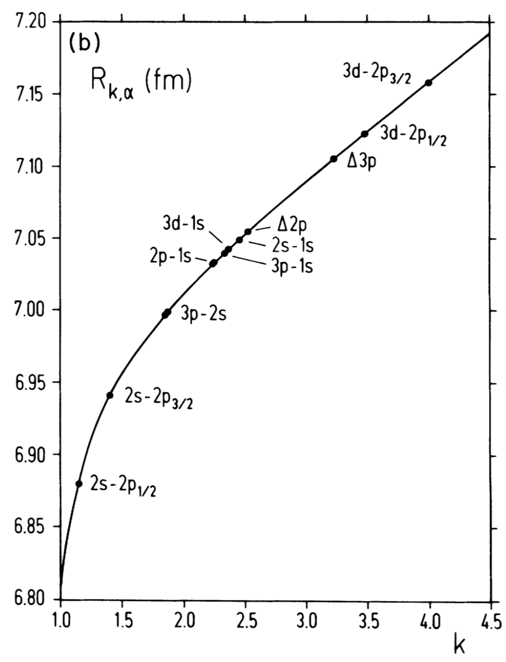
→ comparison with accuracy in valence-$R_{n,p}$ of ∼1%

Unfortunately not possible for many pairs
and not for *large* A where most interesting

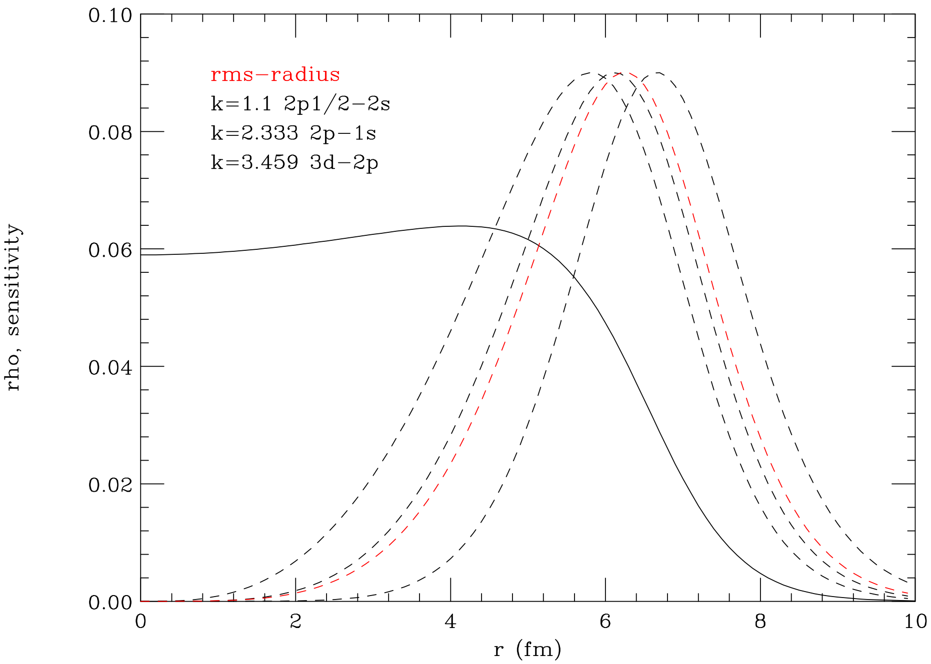
Radii from muonic X-rays

several transitions between 1s, 2p, 2s ... states
determine Barrett moments $M_k = \langle r^k e^{-\beta r} \rangle$
with k dependent on transition
with β dependent on Z
model-independent
usually discussed in terms of Barrett radii R_k: $M_k = \frac{3}{R_k^3} \int_0^{R_k} r^{k+2}\, e^{-\beta r}\, dr$
radius of uniform charged sphere with same M_k

Example: ^{208}Pb (Bergem *et al.*, PSI experiment)



Sampling of $\rho(r)$ by moments: weight functions



M_k: model-independent, but as non-intuitive as $T(Q)$

Non-optimal:

cannot easily get moments required elsewhere

optical shifts ($k \sim 1.9$ for $K\alpha$)

elastic electron scattering ($k = 2$ for rms-radius)

Solution: combine info from (e,e)+μX

Do these probes yield compatible results?

doubts since proton-radius puzzle!

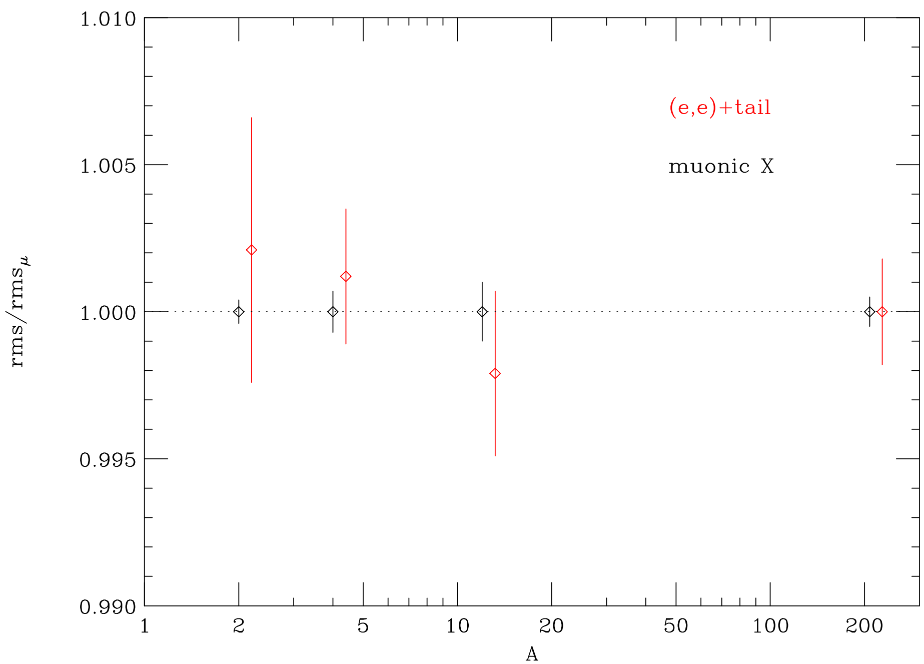
Compare radii from (e,e), μX

only for nuclei with best (e,e) data base

only for cases where (e,e) analyzed with tail-constraint

fixes *shape* of $\rho(r)$ at large r

produces much safer, more accurate radii

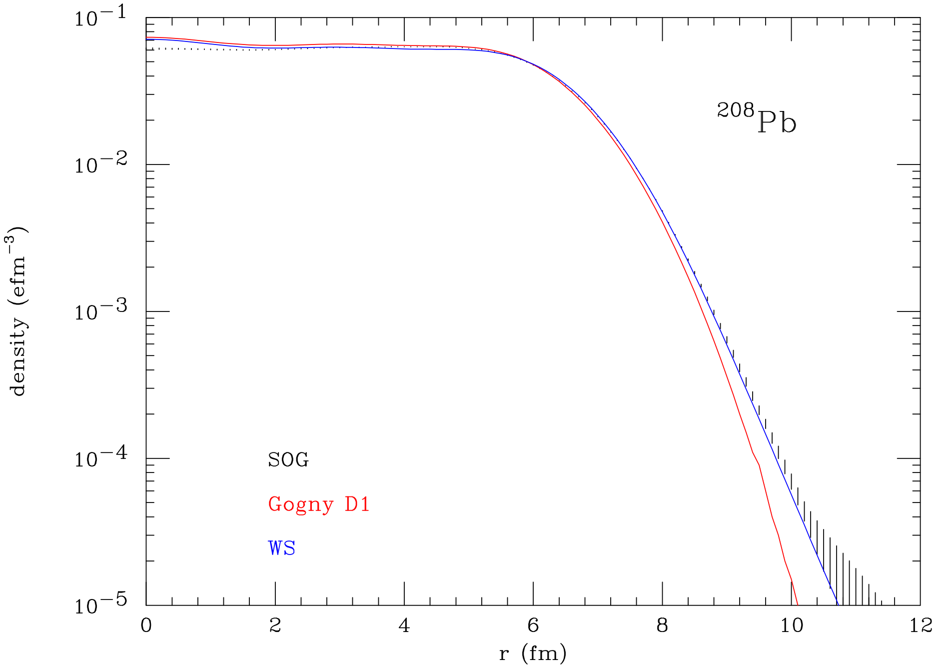


excellent agreement

problem with A=1 is *proton*-problem!

Perfect agreement also for other moments

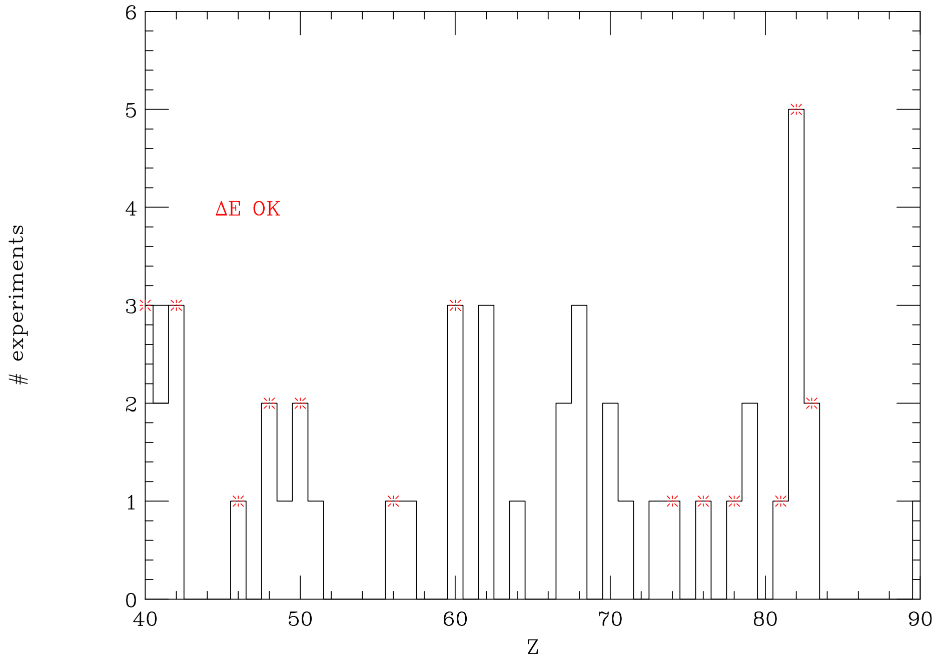
- ^{208}Pb world data on (e,e) analyzed together with 4 M_k's
  - available in 77
- perfect agreement with all M_k's
- M_k help in particular at large r
  - reduce error bars



optimal: combine (e,e)+μX ..... for nuclei where possible

Caveat: availability of good data

(e,e) available only for some nuclei
μX available for some
isotope shifts from atoms available for most



For many nuclei (e,e) had too large ΔE → better data on *rms* needed
here μX could provide *much* more accurate information

Combination (e,e)+μX discussed below for untypical case, ^{208}Pb

Combination (e,e)+μX

particularly useful to determine nuclear polarization NP corrections to μX

Bergem *et al.*: study NP corrections for ^{208}Pb

find deviations from theory

Predicted NP

1s $\sim$ 4keV

2s $\sim$ 1keV

2p $\sim$.4keV

much larger than exp. precision of $\sim$0.04keV

Fit Bergem *et al.*

Fit parameters of model-ρ + NP corrections to lowest transitions

many parameters, need all constraints possible

most helpful: ratio 2s-2p/2p-1s

ratio $R_{1.2}/R_{2.2}$

experimentally determined to 0.9786±0.0001

Ratio of R_k's from (model)-densities

(e,e) determines *shape* of $R(k)$

fixes ratio NP of 2s/1s

2PF, a=0.5	0.9845
2PF	0.9760
3PG, good fit	0.9793
SOG	0.9794

Need best $\rho(r)$ to predict accurate ratio of R_k's $\rightarrow$ better NP corrections

How much info on $\rho(r)$ can get from R_k alone?

information content of R_k's = ?
explore by analyzing sample 'data'

Study unavoidably model-dependent

must use models (3PG, 3PF, ..) to explore
can only hope to get ±integral properties
    radius, surface thickness, 'depression', ..

Data base used

R_k of Bergem *et al.*, $k = 1.1 \dots 4.0$
PSI experiment, 1988
    corrected for nuclear polarization
    = main focus of Bergem paper
add moments for $k = 5.4,\ 6.2$, from 3d and 5f-3d
    observed earlier by Ford *et al.*
    assign 'modern' uncertainties

Fit functions

2-parameter Fermi
2-parameter Gauss

Result

get perfect fit with 2PG
$\rightarrow$ information content of M_k's = 2 parameters

2PF slightly worse

c and z determined with very small error bars
2PG allows to go from R_k to $rms, ...$ with no loss in precision

More flexible fit function

3-parameter Fermi
3-parameter Gauss

Result

χ^2 does not significantly improve
$w \sim$ central depression *not* significant
sensitivity to $r < 4fm$ not big enough

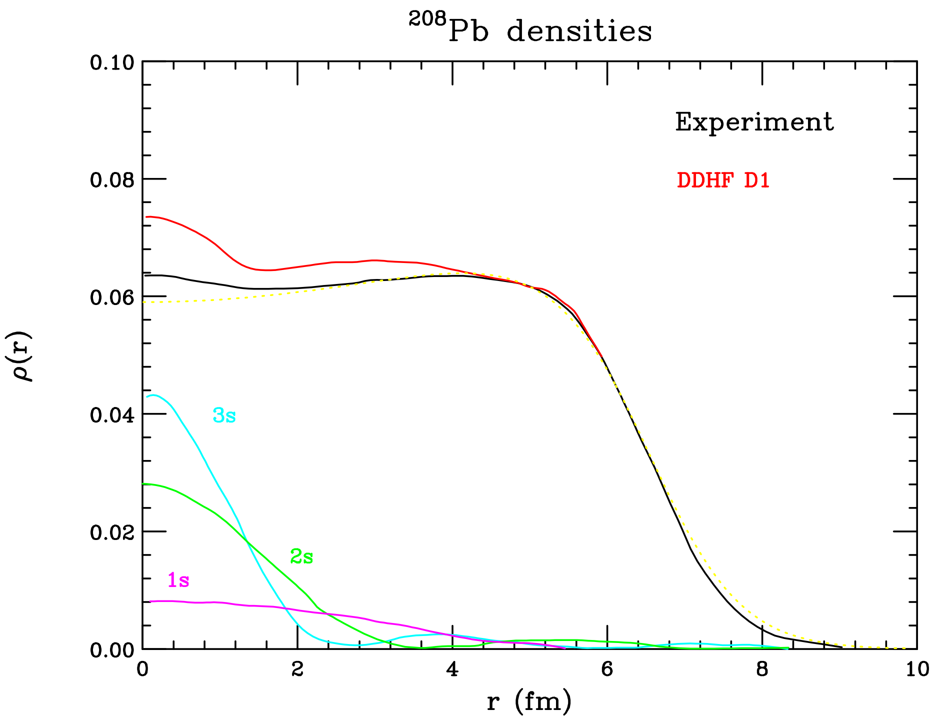
Central depression w

large for heavy nuclei away from valley? Bubble nuclei??
predicted for super-heavy, proton-rich nuclei

Lesson from 'early days'

not clear if w describes central depression
biggest effect of $(1 + wr^2/c^2)$ occurs in tail of $\rho(r)$ at *large* r

Example: ^{208}Pb: $w = 0.33 \rightarrow$ depression?? Hardly seen in modelindependent ρ



w essentially compensates too fast fall-off of Gaussian distribution

Better parameterization

$\rho(r) = 2PG + we^{-r^2/p^2}$ with w fit to data

decouples large r from small r

Result of fits

w not significant, δw too large

How could constrain extracted $\rho(r)$ more?

experience from analyzing (e,e):

very useful to fix shape of $\rho(r)$ at large r

fixes higher moments

Note: older R-determinations from (e,e) suffer from poor tails

What know on $\rho(r)$ at large r?

dominated by least-bound proton, radial wave function = Whittaker function

$R(r) \sim W_{-\eta,l+1/2}(2\kappa r)/r$ with $\kappa^2 = 2\mu E/\hbar^2$, $\eta = (Z-1)e^2/\hbar\sqrt{\mu/2E}$

depends only on removal energy E

presumably available from accurate mass-measurements

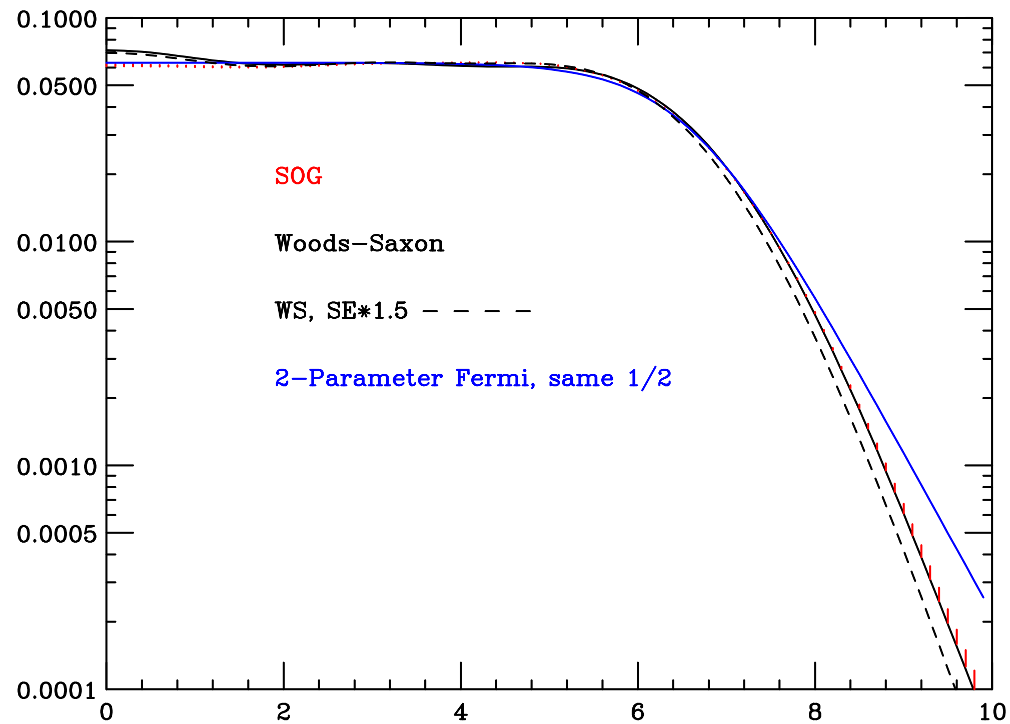
easy to calculate

overall normalization (asymptotic normalization) generally not known →shape

Shape automatically given if calculate $R(r)$ using Woods-Saxon potential

solution of Schrödinger equation for sensible potential
$\rightarrow$ physical shape of asymptotic $R(r)$

Add to M_k data: shape of $\rho(r \gg)$



Result of fits including tail-shape

2PF now gives fit with large χ^2
2PG still works well
has more realistic tail shape
preferable for large A
long known from (e,e)

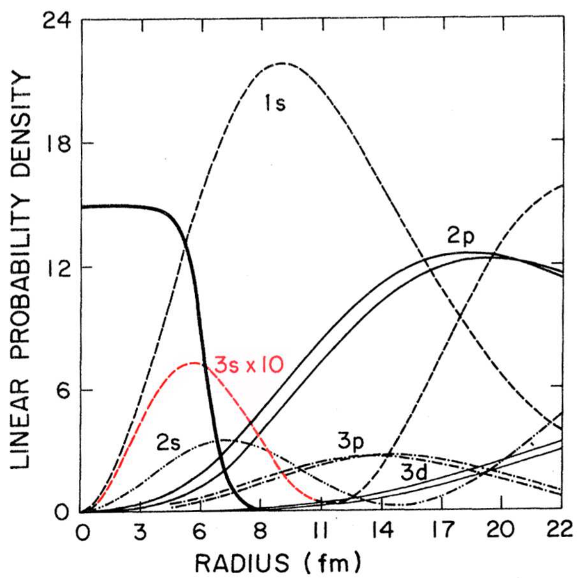
Compare tails for Pb

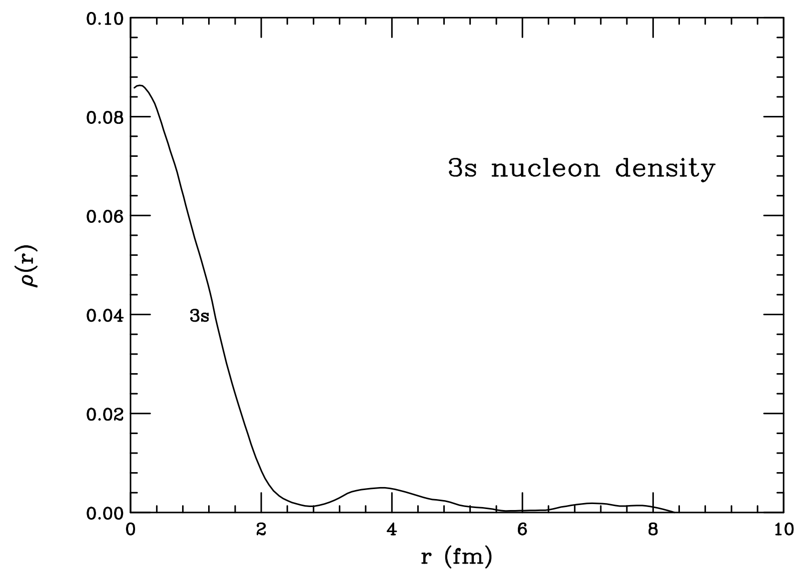
experimental SOG density
Woods-Saxon density with exp. separation energy
Woods-Saxon with increased separation energy
2-parameter Fermi with same 1/2-density radius

How to get more information on ρ at smaller r?

add other μX transitions
in particular transitions involving 3s!

Figure from Martin *et al.* (for ^{138}Ba)





rarely studied in literature
overlap with nucleus $\sim$ overlap of $l > 1$ states
population 3s in 1/2% region ($\sim$20% of 2s)
much more useful than $l \geq 3$

3s = level most sensitive next to 2s

3s should be observable at PSI

table from Martin *et al.* for ^{140}Ce

TABLE I. Relative intensities and level populations for all states with $n<4$ in ^{140}Ce. Numbers represent the percentage of all captured muons making the given transition, accounting for electric dipole, electric quadrupole, and Auger emission. An omitted entry implies no transition allowed or a transition probability <0.01.

Initial state	Population (%)	Final state $3s_{1/2}$	$3p_{3/2}$	$3p_{1/2}$	$3d_{5/2}$	$3d_{3/2}$	$2s_{1/2}$	$2p_{3/2}$	$2p_{1/2}$	$1s_{1/2}$
$n>4$		0.21	1.55	0.76	9.33	6.28	0.21	3.70	1.88	1.99
$4f_{7/2}$	35.91		0.05		35.33			0.54		
$4f_{5/2}$	27.01			0.03	1.69	24.88		0.09	0.32	
$4d_{5/2}$	5.64		1.4					4.0		0.18
$4d_{3/2}$	3.8		0.16	0.78				0.4	2.3	0.1
$4p_{3/2}$	0.7	0.07					0.14			0.5
$4p_{1/2}$	0.37	0.04					0.08			0.24
$4s_{1/2}$	0.16		0.09					0.07		
$3s_{1/2}$	0.37		0.2	0.15				0.02		
$3d_{5/2}$	46.4						0.04	45.0		1.3
$3d_{3/2}$	31.2						0.02	4.7	25.6	0.8
$3p_{3/2}$	3.5						0.8			2.6
$3p_{1/2}$	1.6						0.45			1.15
$2s_{1/2}$	2.1							1.2	0.9	
$2p_{3/2}$	59.7									59.7
$2p_{1/2}$	31.2									31.2

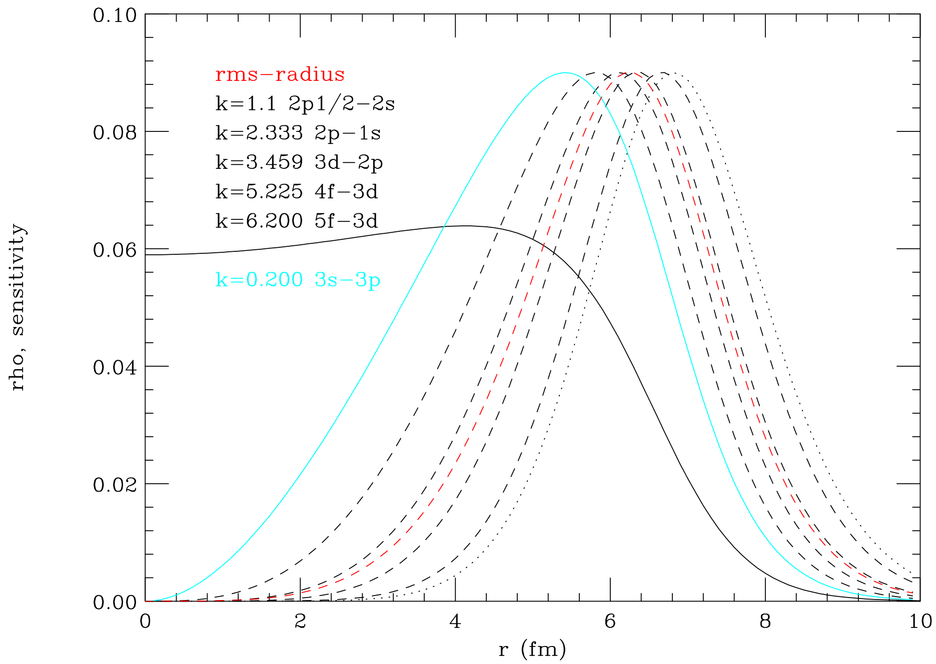
intensity 3s-3p $\sim$ 20% of 2s-2p

several peaks in 3s-2p region for Ba observed, not identified

for *e.g.* Pb energy predictable from (e,e) $\rightarrow$ identification straightforward

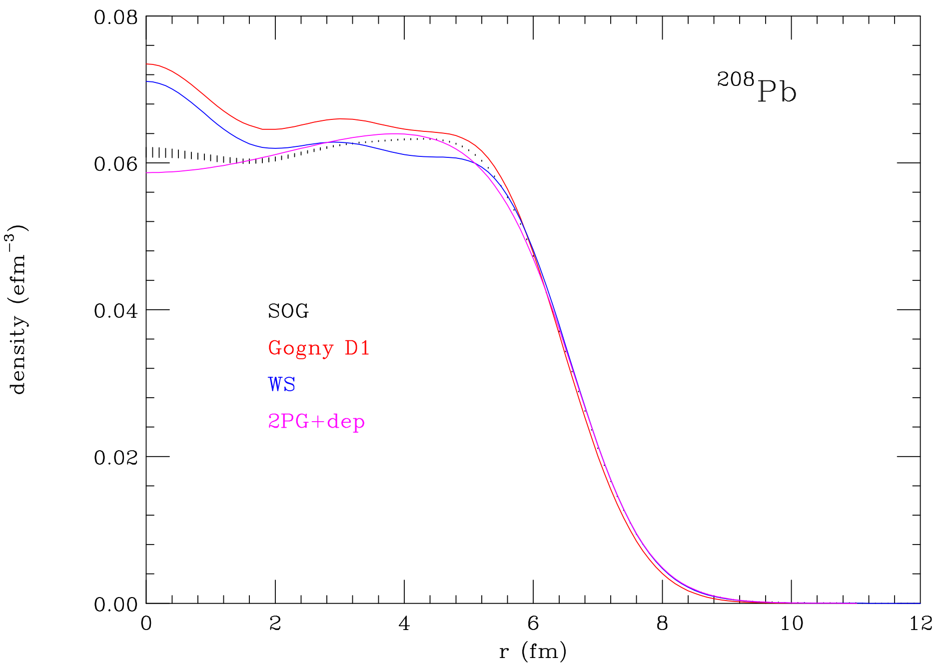
Crude determination of Barrett-k

from figure have $\rho(r)$, muon wave function
can calculate sensitivity as function of r
can compare to usual Barrett $\langle r^k e^{-\beta r} \rangle$ sensitivity
find $k \sim 0.2$
invent $R_{0.2}$ with sensible error bar
reanalyze data



Fits including 3s

2PF + dip: poor χ^2
2PG + dip: good χ^2, $\delta c/c \sim 0.002$, $\delta z/z \sim 0.005$
dip determined to $\pm 15\%$, ρ reproduces SOG in 2-5fm region



Lesson:

can learn significant information on ρ away from surface
high interest to observe a transition involving 3s state
with 3s information content μX $\sim$ (e,e) with $q_{max} = 2fm^{-1}$
3s would be very valuable to complement existing ^{208}Pb data

Conclusions

Electron scattering $\leftrightarrow$ muonic X-rays

detailed shape $\leftrightarrow$ precise moments

in many ways complementary
can learn much by analyzing together

For cases where not-so-good (e,e) data available

can get more info on $\rho(r)$ from μX if:
- **add physics knowledge on tail of density**
- **include transitions sensitive to *smaller* r, *i.e.* 3s**

Use of modern μ beams $\rightarrow$ most promising info on $\rho(r)$!