μSR and Superconductors

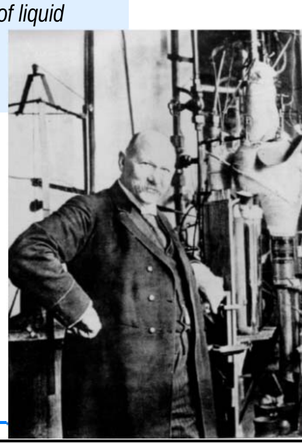
- μSR and Superconductors (SC)
  - Crash course on SC
  - Nanometer scale parameters
  - How to determine them by μSR
    - Abrikosov state (Bulk μSR and LEM)
    - Meissner state (LEM)
- Appendices
  - Ginzburg-Landau equation
  - Pairing symmetry
  - Two-gap superconductors

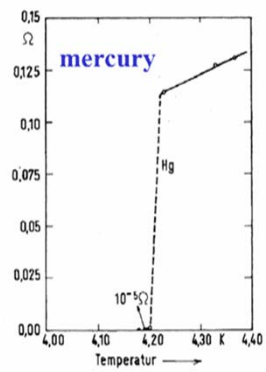
Crash course on superconductivity

Superconductivity -- Introduction

Discovery by Kamerlingh Onnes in 1911 in mercury

Received the Nobel Prize in 1913 for "*his investigations on the properties of matter at low temperatures which led, inter alia, to the production of liquid helium*".

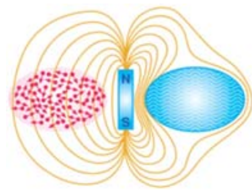




Temperature dependence of the resistance of a Hg sample

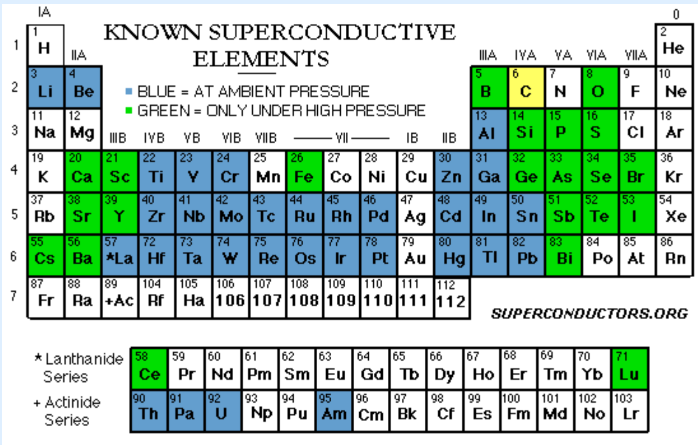
Superconductivity -- Introduction

Below the transition temperature a superconductor expels a magnetic field from its inner core (Meissner and Ochsenfeld effect)



Quelle: http://staff.ee.sun.ac.za/wjperold/Research/research.htm

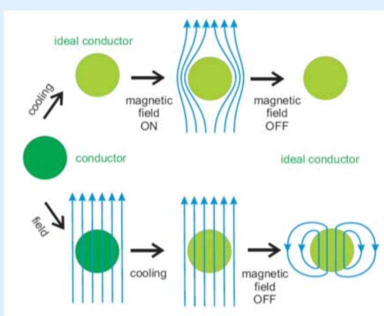
Superconductivity -- Introduction



Superconductivity -- Timeline

1908	Kammerling Onnes: production of liquid helium
1911	Kammerling Onnes: discovery of zero resistance
1933	Meissner and Ochsenfeld: superconductors expell applied magnetic fields (MOE)
1935	F. and H. London: MOE is a consequence of the minimization of the electromagnetic free energy carried by superconducting current
1950	Ginzburg and Landau: phenomenological theory of superconductors
1950	Maxwell and Reynolds et al.: isotope effect
1957	Abrikosov: 2 types of superconductors (magnetic flux)
1957	Bardeen, Cooper, and Schrieffer: BCS theory -- superconducting current as a superfluid of Cooper pairs
1962	Josephson: Joesephson effect
1986	Berdnorz and Müller: High-Tc's superconductors

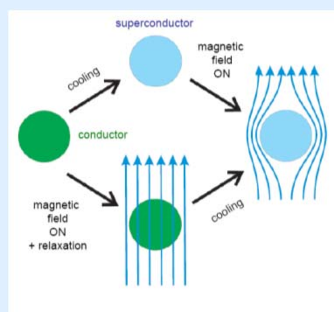
Ideal Conductor



An ideal conductor in magnetic field

Lenz-Faraday's law: currents to keep *B* constant inside of the sample

Superconductor

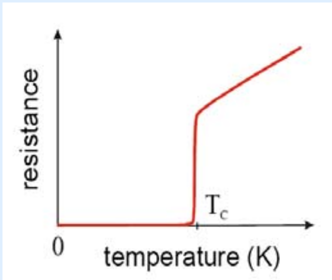


A superconductor in magnetic field

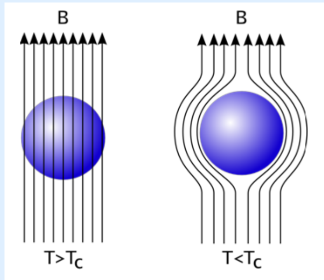
New thermodynamic state of matter

From: Lecture on Superconductivity, Alexey Ustinov, Uni. Erlangen, 2007

Main Characteristics:



Kamerlingh Onnes



Meissner and Ochsenfeld

~~Is a superconductor "just" an ideal conductor?~~

New thermodynamic state of matter!

Nanometer scale parameters

Magnetic penetration depth:	λ
Coherence length:	ξ

Phenomenological London's equations

F. and H. London,
Proc. Roy. Soc. A149, 71 (1935)

In a sample without resitance, the electrons will feel a force:

$$\mathbf{F} = -e\mathbf{E} = m\frac{\partial\langle\mathbf{v}\rangle}{\partial t}$$

Recalling that the current density is: $\mathbf{j} = -n_s e\langle\mathbf{v}\rangle$

one obtains the first London equation (*acceleration equation):*

$$\Lambda\frac{\partial\mathbf{j}}{\partial t} = \mathbf{E} \quad \text{with} \quad \Lambda = \frac{m}{n_s e^2}$$

Taking the curl of this equation

using the 3rd and 4th Maxwell equations

$$\nabla \times \mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0\mathbf{j} \qquad (\text{assuming } \frac{\partial\mathbf{E}}{\partial t} = 0)$$

one obtains:

$$\Delta\frac{\partial\mathbf{B}}{\partial t} = \frac{1}{\lambda^2}\frac{\partial\mathbf{B}}{\partial t}$$

$$\Delta\frac{\partial\mathbf{j}}{\partial t} = \frac{1}{\lambda^2}\frac{\partial\mathbf{j}}{\partial t}$$

$$\text{with: } \lambda^2 = \frac{\Lambda}{\mu_0} = \frac{m}{\mu_0 e^2 n_s}$$

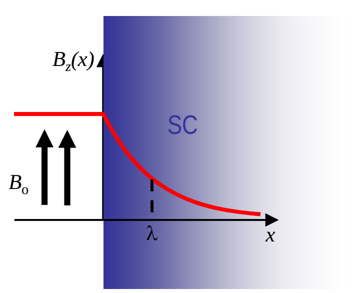
$$\Delta\mathbf{B} = \frac{1}{\lambda^2}\mathbf{B}$$

$$\Delta\mathbf{j} = \frac{1}{\lambda^2}\mathbf{j}$$

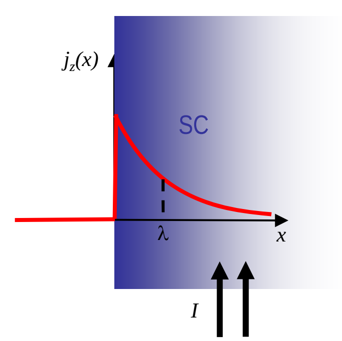
$$\text{with: } \lambda^2 = \frac{\Lambda}{\mu_0} = \frac{m}{\mu_0 e^2 n_s}$$

1st Nano-scale Param.: London Penetration Depth

$$\Delta\mathbf{B} = \frac{1}{\lambda^2}\mathbf{B}$$



$$\Delta\mathbf{j} = \frac{1}{\lambda^2}\mathbf{j}$$



$$B_z(x) = B(0)\exp(-x/\lambda)$$

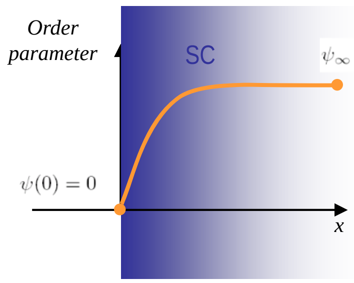
$$j_z(x) = \frac{I}{2\pi R\lambda}\exp(-x/\lambda)$$

$$\lambda = \sqrt{\frac{m^*}{\mu_0 e^2 n_s}}$$

2^{nd} Nano-scale Param.: Coherence Length

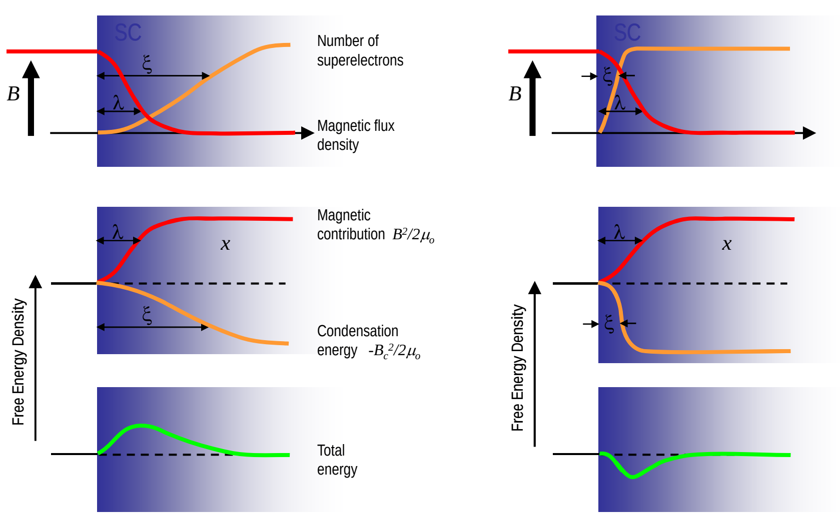
The coherence length describes the variation of the fact that the superconducting electron density cannot vary abruptly (see Appendix).

In some limiting cases, it can be considered as the typical length of the superconducting pair.



$$\xi = \frac{\hbar v_F}{\pi \Delta}$$

Type I and Type II Superconductors



Type I **Type II**

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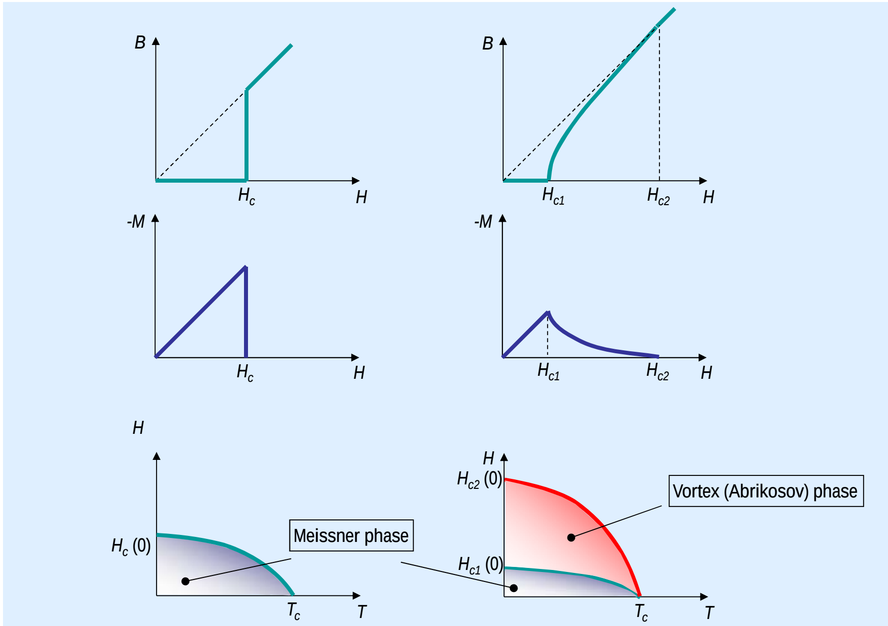
Type I ($\lambda < \xi$)



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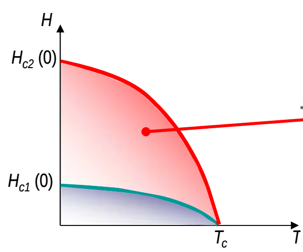
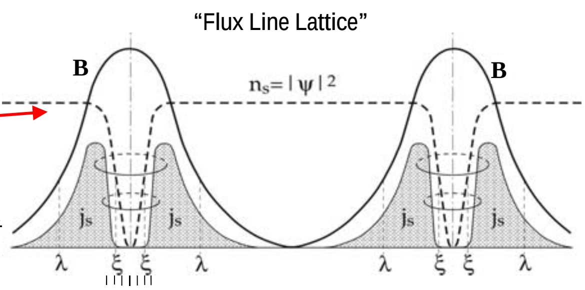
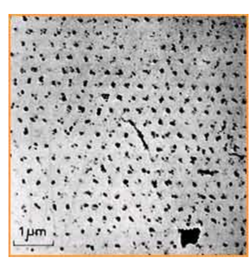
Vortex (Abrikosov) phase





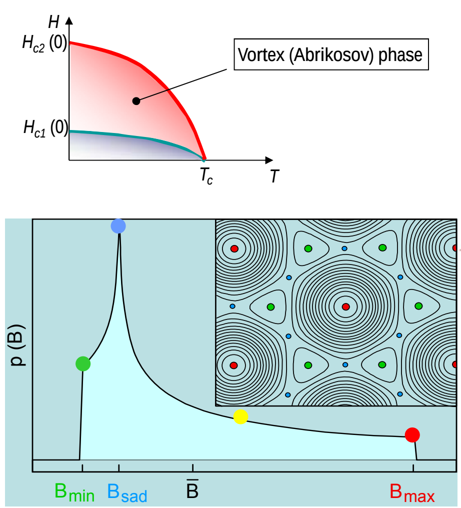
Magnetic flux quantum
$\phi_0 = h/2e = 2.067 \cdot 10^{-15}$ Wb

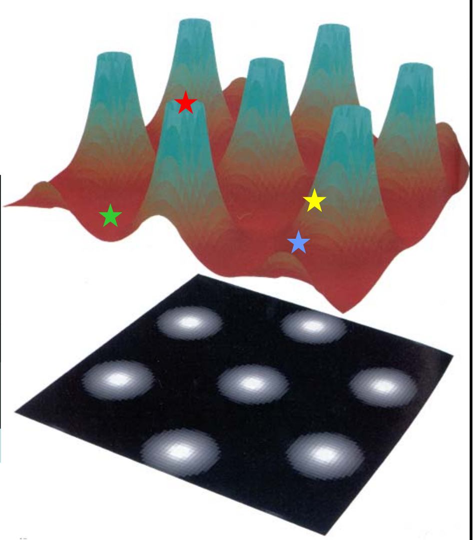


Bitter Decoration
Pb-4at%In rod, 1.1K, 195G
U. Essmann and H. Trauble, Phys. Lett 24A, 526 (1967)

Abrikosov state (Bulk μSR and LEM)

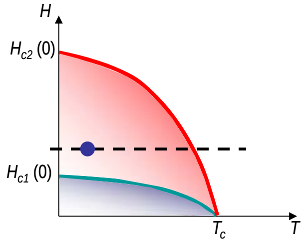
Muons and Field Distribution in Type II S.C.

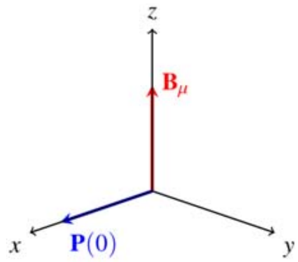




*Bishop et al., Scientific American **48** (1993)*

Vortex (Abrikosov) phase for µSR studies



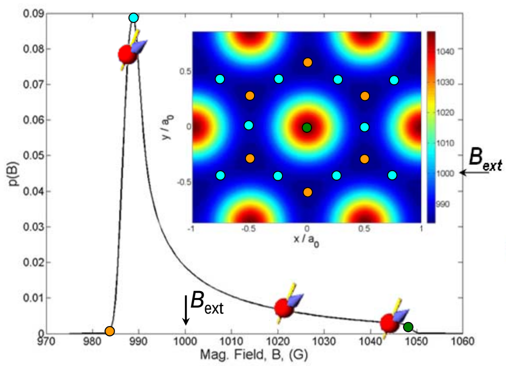


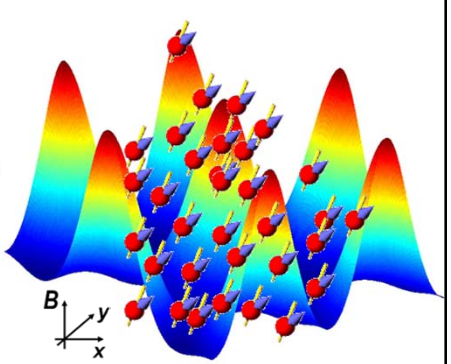
$$P_x(t) = \int f(\mathbf{B}_\mu)\left[\cos^2\theta + \sin^2\theta\cos(\gamma_\mu B_\mu t)\right] d\mathbf{B}_\mu$$

$$P_x(t) = \int f(\mathbf{B}_\mu)\cos(\gamma_\mu B_\mu t) d\mathbf{B}_\mu$$

"Transversal-Field µSR" (TF-µSR)

Vortex (Abrikosov) phase for µSR studies



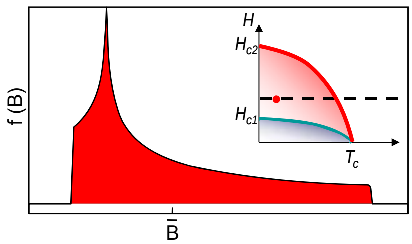


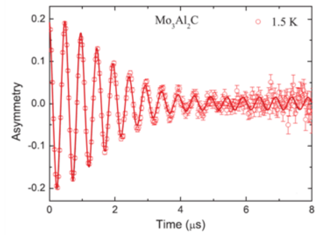
Since the muon is a local probe, the µSR relaxation function is given by the weighted sum of all oscillations:

$$P_x(t) = \int f(\mathbf{B}_\mu)\cos(\gamma_\mu B_\mu t) d\mathbf{B}_\mu$$

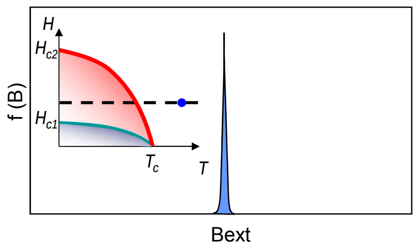
Mo_3Al_2C
Bauer et al., Phys. Rev. B 90, 054522 (2014)

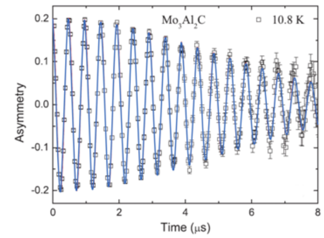
$T < T_c$





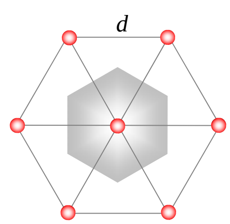
$T > T_c$





Field Distribution in "Extreme" Type II S.C.

- Ginburg-Landau parameter $\kappa \equiv \frac{\lambda}{\xi} \gg 1$
- Large range of fields (up to $B_{c2}/4$) where London model applies
- Vortex cores well separated and do not interact
- Vortex fields superimpose linearly



$$S = d^2 \frac{\sqrt{3}}{2}$$

$$\phi_0 = S \cdot B$$

$$\Rightarrow d = \sqrt{\frac{2\phi_0}{B\sqrt{3}}}$$

Reciprocal space vectors:

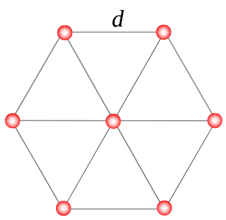
$$|\mathbf{a}^*| = |\mathbf{b}^*| = \frac{4\pi}{\sqrt{3}} \frac{1}{d}$$

$$\mathbf{K}_{m,n} = m \cdot \mathbf{a}^* + n \cdot \mathbf{b}^*$$

Field distribution: $\mathbf{B}(\mathbf{r})$?

$\mathbf{B}(\mathbf{r})$ must fulfill the modified London equation:

$$\mathbf{B}(\mathbf{r}) - \lambda^2 \Delta \mathbf{B}(\mathbf{r}) = \phi_0 \sum_n \delta(\mathbf{r} - \mathbf{r}_n)\hat{z}$$



We expect a periodic magnetic field and therefore can use:

$$\mathbf{B}(\mathbf{r}) = \sum_{\mathbf{K}} \mathbf{B}(\mathbf{K}) \exp(i\mathbf{K}\mathbf{r})$$

with Fourier components:

$$\mathbf{B}(\mathbf{K}) = \frac{1}{S} \int \mathbf{B}(\mathbf{r}) \exp(-i\mathbf{K}\mathbf{r}) d^2\mathbf{r}$$

The modified London equation becomes (fields only along $\hat{z}$):

$$\sum_{\mathbf{K}} (\mathbf{B}(\mathbf{K}) + \lambda^2 K^2 \mathbf{B}(\mathbf{K})) \exp(i\mathbf{K}\mathbf{r}) = N\phi_0 \sum_{\mathbf{K}} \exp(i\mathbf{K}\mathbf{r})$$

and one finds:

$$B_z(\mathbf{K}) = \frac{B}{1 + \lambda^2 K^2}$$

With: $B_z(\mathbf{r}) = \sum_{\mathbf{K}} \frac{B}{1 + \lambda^2 K^2} \exp(i\mathbf{K}\mathbf{r})$

The second moment $\langle \Delta B_z^2 \rangle = \langle B_z^2 \rangle - \langle B_z \rangle^2$ of the field distribution is given by:

$$\langle \Delta B_z^2 \rangle = \sum_{\mathbf{K} \neq 0} |B_z(\mathbf{K})|^2$$

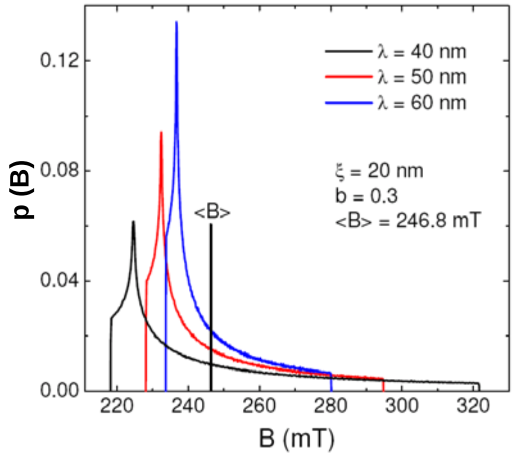
Taking into account the perfect triangular lattice where:

$$K^2 = K_{m,n}^2 = \frac{16\pi^2}{3d^2}(m^2 + mn + n^2) \quad \text{and that} \quad K^2\lambda^2 \gg 1$$

$$\langle \Delta B_z^2 \rangle = \frac{9\phi_0^2}{32\pi^4\lambda^4}(1 + \frac{1}{3^2} + \frac{1}{4^2} + \frac{2}{7^2} + ...)$$

$$\langle \Delta B_z^2 \rangle = 0.00371 \frac{\phi_0^2}{\lambda^4}$$

- By measuring the second moment of the field distribution (for example by μSR), we directly determine the London penetration



$$B_{min} - \langle B \rangle \propto \frac{1}{\lambda^2}$$

$$B_{max} - \langle B \rangle \propto \frac{1}{\lambda^2}$$

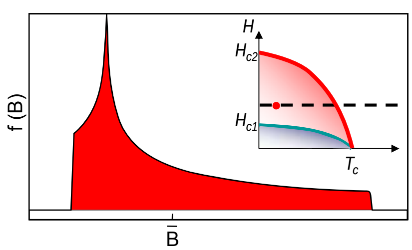
$$B_{sad} - \langle B \rangle \propto \frac{1}{\lambda^2}$$

Maisuradze et al.

Determination of λ by μSR

$$P_x(t) = \int f(\mathbf{B}_\mu) \cos(\gamma_\mu B_\mu t) d\mathbf{B}_\mu$$

$T < T_c$



$$P_x(t) = \underbrace{\exp\left[-\frac{\sigma^2 t^2}{2}\right]}_{\text{depolarization}} \times \underbrace{\cos(\gamma_\mu \langle B_{\mu,z} \rangle t)}_{\text{oscillations}}$$

where: $\sigma^2 = \gamma_\mu^2 \langle \Delta B_{\mu,z}^2 \rangle$

Assume a Gaussian distribution:

$$f(B_{\mu,z}) = \frac{1}{\sqrt{2\pi \langle \Delta B_{\mu,z}^2 \rangle}} \exp\left[-\frac{(B_{\mu,z} - \langle B_{\mu,z} \rangle)^2}{2 \langle \Delta B_{\mu,z}^2 \rangle}\right]$$

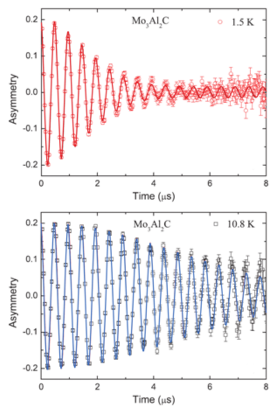
$$P_x(t) = \int \frac{1}{\sqrt{2\pi \langle \Delta B_{\mu,z}^2 \rangle}} \exp\left[-\frac{(B_{\mu,z} - \langle B_{\mu,z} \rangle)^2}{2 \langle \Delta B_{\mu,z}^2 \rangle}\right] \cos(\gamma_\mu B_{\mu,z} t) dB_{\mu,z}$$

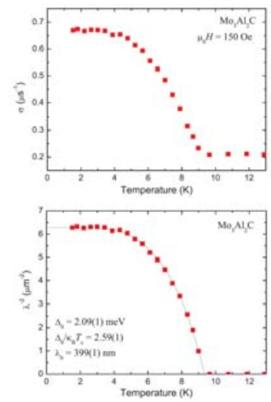
And since:

$$\langle \Delta B_z^2 \rangle = 0.00371 \frac{\phi_0^2}{\lambda^4}$$

One can determine the penetration depth by determining the depolarization rate !!

Mo_3Al_2C



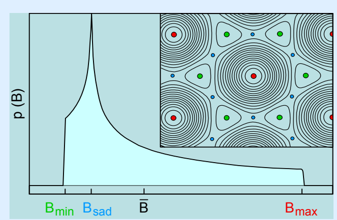


$$\langle \Delta B_z^2 \rangle = 0.00371 \frac{\phi_0^2}{\lambda^4}$$

$$P_x(t) = \underbrace{\exp\left[-\frac{\sigma^2 t^2}{2}\right]}_{\text{depolarization}} \times \underbrace{\cos(\gamma_\mu \langle B_{\mu,z} \rangle t)}_{\text{oscillations}}$$

where: $\sigma^2 = \gamma_\mu^2 \langle \Delta B_{\mu,z}^2 \rangle$

Weak Points of such Analysis

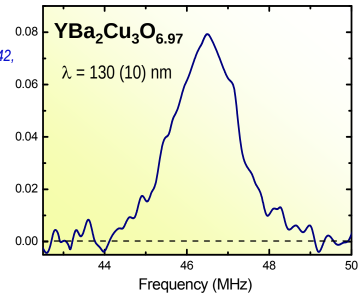


- One assumes that the muon depolarization (and therefore also that the field distribution) can be fitted by a Gaussian
- The field dependence of the second moment of the field distribution is neglected
- The size of the vortex core (2ξ) is neglected

Gaussian or not Gaussian.. that's the question!

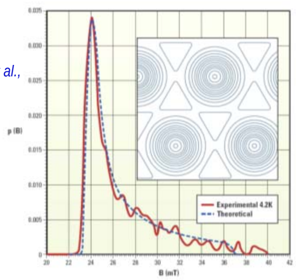
- In polycrystals or sintered samples: large density and disorder of pinning sites
  ➡ strong smearing of the field distribution
- The asymmetry of the field distribution appears in single crystals

Pümpin et al., Phys. Rev. B 42, 8019 (1990)

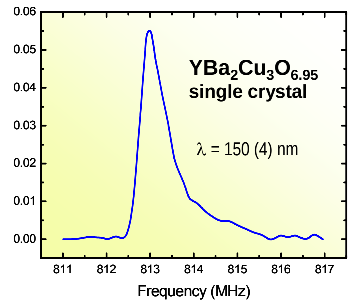


Pd-In alloy

Charalambous et al., Phys. Rev. B 66, 054506 (2002)



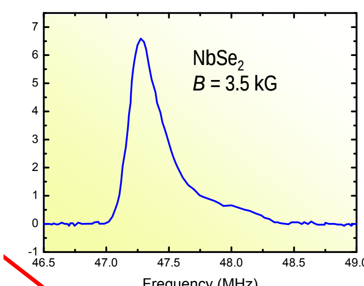
Sonier et al., PRL 83, 4156 (1999)

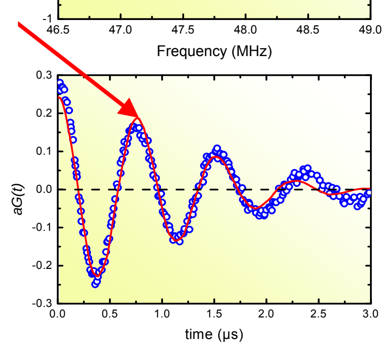


Gaussian or not Gaussian.. that's the question!

Example: $NbSe_2$

Sonier et al., Rev. Mod. Phys. 72, 769 (2000)





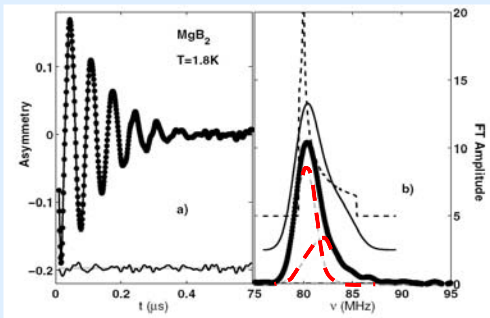
Masterschool PSI

Gaussian .. or even two Gaussians

Constructing the second moment of the field distribution by using two Gaussian components

Example MgB_2

S. Serventi et al., PRL 93, 217003 (2004)



$$\sigma_{tot}^2 = a_1\sigma_1^2 + a_2\sigma_2^2 + a_1 a_2(\omega_1 - \omega_2)^2$$

with:

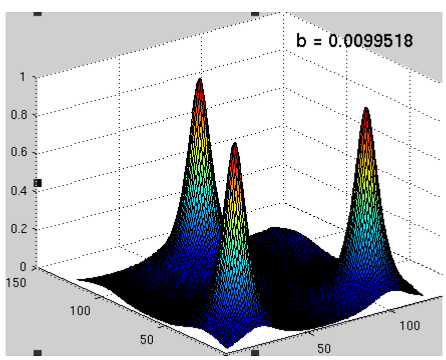
$$\sigma_i^2 = \gamma_\mu^2 \langle \Delta B^2 \rangle_i$$

$$\omega_i = \gamma_\mu \langle B \rangle_i$$

Field Dependence -- Numerical GL Solution

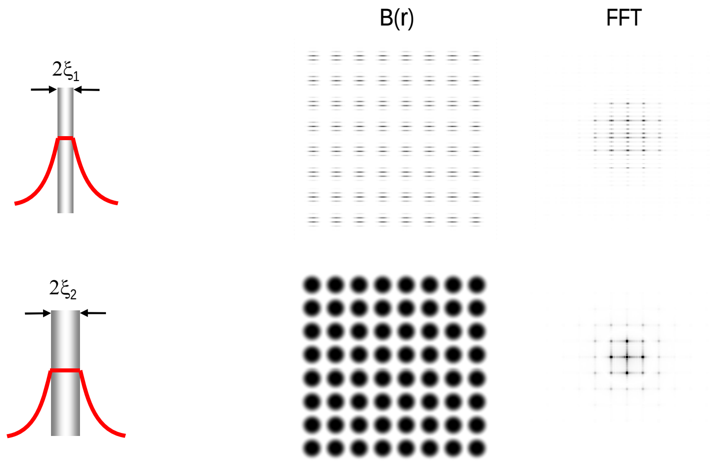
λ = 50 nm, ξ = 20 nm   ($b = B/B_{c2}$)

Field



Courtesy from A. Maisuradze

Effect of the coherence length



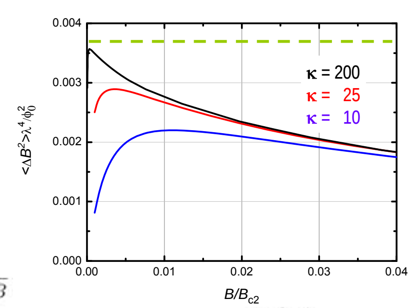
- The fact that the coherence length is finite leads to a “cutoff” of the high terms in the Fourier transform of B(r) when $K \sim \xi^{-1}$

More advanced Model

One model (among many others):
Modified London Model with Gaussian Cutoff
E.H. Brandt, J. Low Temp. Phys. 73, 355 (1988)

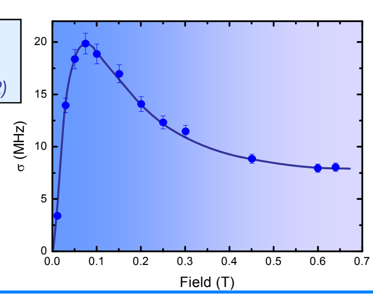
$$B_z(\mathbf{K}) = B\,\frac{1}{1+\lambda^2K^2}$$

$$\Rightarrow \quad \boxed{B_z(\mathbf{K}) = B\,\frac{e^{(-\xi^2K^2/2)}}{1+\lambda^2K^2}} \qquad K \propto \frac{1}{d} \propto \sqrt{B}$$



London limit: $\frac{\langle \Delta B_z^2 \rangle \lambda^4}{\phi_0^2} = 0.00371$

Example: MgB_2
Niedermayer et al., Phys. Rev. B 65, 094512 (2002)



By measuring the field dependence of the second moment of the field distribution by μSR

➡ determination of ξ

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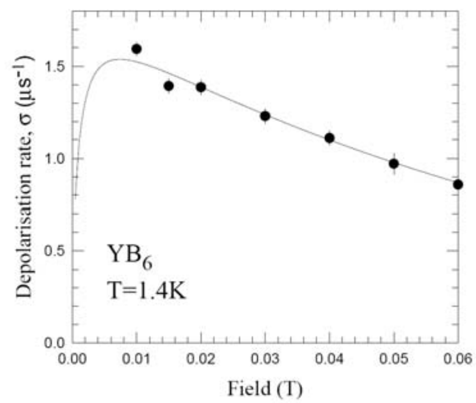
Example YB_6

Example: Cubic YB_6
From the field dependence of μSR depolarization rate (second moment)
➡ λ = 192 nm and ξ = 33 nm



Hillier et al., Phys. Rev. B 42, 8019 (1990)

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Other model:
Analytical solution of the Ginzburg-Landau equations considering a Lorentzian function for the order parameter $|\psi(r)|^2$ of an isolated vortex:

$$B(\mathbf{K}) = B(1-b^4)\frac{uK_1(u)}{\lambda^2 K^2}$$

where:

K_1 is a modified Bessel function of the second kind

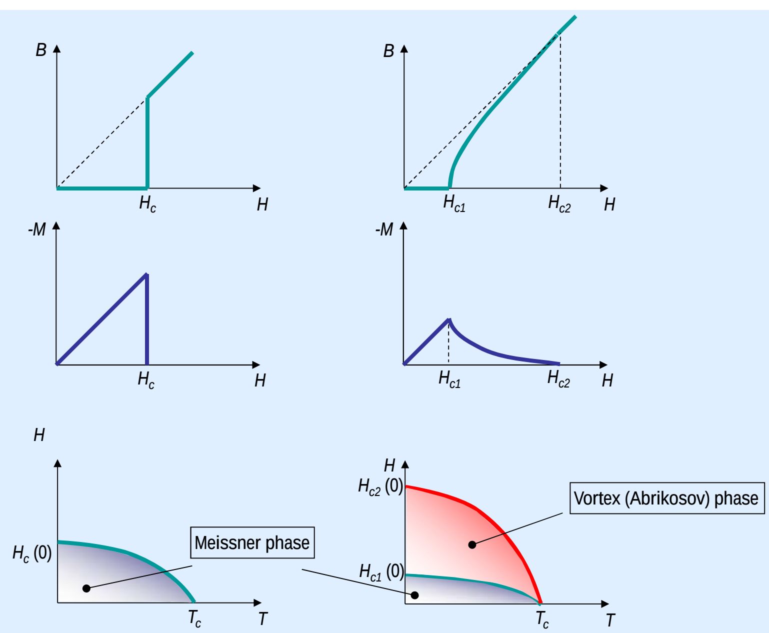
$$b \equiv B/B_{c2}$$

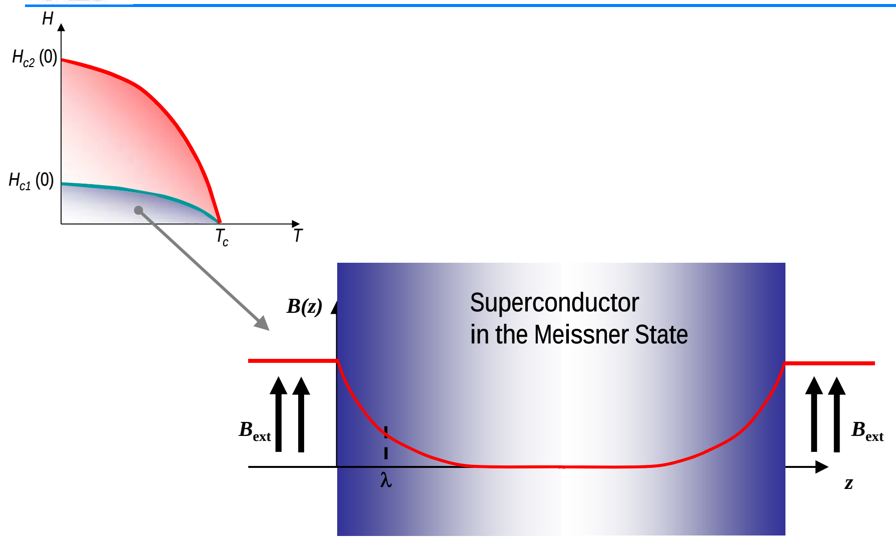
$$u^2 = 2\xi^2 K^2 (1+b)^4 [1 - 2b(1-b)^2]$$

J.R. Clem , J. Low Temp. Phys. 18, 427 (1975)
Z. Hao et al., Phys. Rev. B 43, 2844 (1991)
A. Yaouanc et al., Phys. Rev. B 55, 11107 (1997)

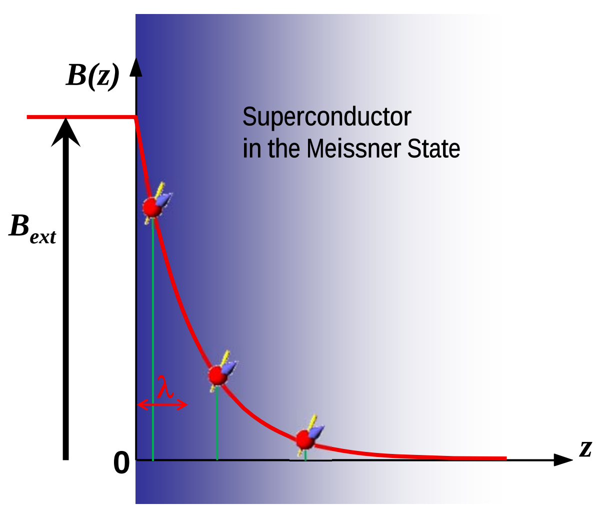
Meissner state (LEM)

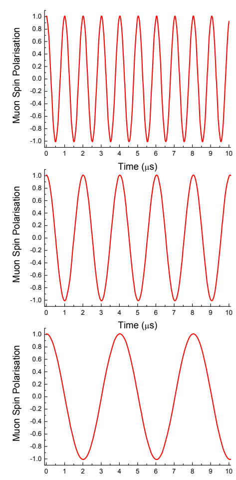
Type I ($\lambda < \xi$)





Meissner State Measurements





$$\omega_\mu(z) = \gamma_\mu B_{\text{loc}}(z)$$

Penetration depth

VOLUME 84, NUMBER 21 PHYSICAL REVIEW LETTERS 22 MAY 2000

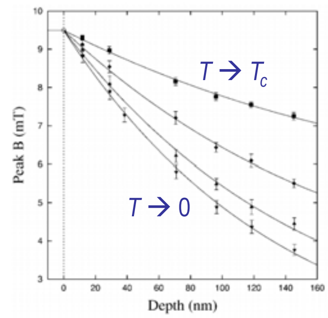
Depth-Resolved Profile of the Magnetic Field beneath the Surface of a Superconductor with a Few nm Resolution

T. J. Jackson,[1] T. M. Riseman,[1] E. M. Forgan,[1] H. Glückler,[2] T. Prokscha,[2] E. Morenzoni,[2] M. Pleines,[2,3] Ch. Niedermayer,[3] G. Schatz,[3] H. Luetkens,[2,4] and J. Litterst[4]

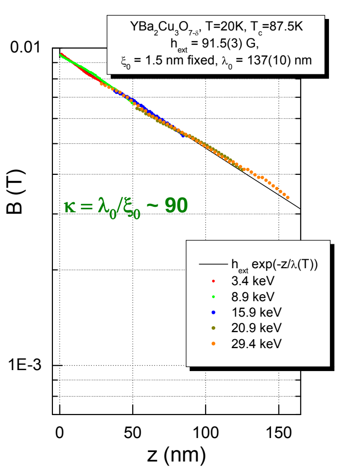
[1]*School of Physics and Astronomy, University of Birmingham, Birmingham B15 2TT, United Kingdom*
[2]*Labor für Myonspinspektroskopie, Paul Scherrer Institut, CH-5232 Villigen PSI, Switzerland*
[3]*Universität Konstanz, Fakultät für Physik, D-78434 Konstanz, Germany*
[4]*Technische Universität Braunschweig, D-38106 Braunschweig, Germany*

(Received 7 February 2000)

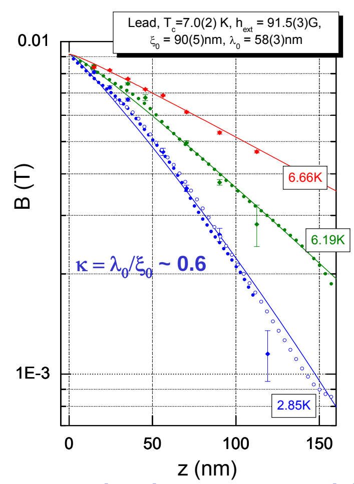
The variation of a magnetic field as a function of depth beneath the surface of an $YBa_2Cu_3O_{7-\delta}$ thin film in the Meissner state has been measured using low energy muons. The depth of implantation was varied from 20–150 nm by tuning the energy of the implanted muons from 3–30 keV. These are direct measurements of the penetration of a magnetic field beneath a superconducting surface which illustrate the power of low energy muons for near surface studies in superconductivity and magnetism.



Magnetic field profiles in $YBa_2Cu_3O_{7-\delta}$ and Pb



local: exponential



Non-local: non-exponential

T.J. Jackson et al., Phys. Rev. Lett. 84, 4958 (2000).
A. Suter et al., Phys. Rev. Lett. 92, 087001 (2004).
A. Suter et al., Phys. Rev. B72, 024506 (2005).

1st experimental determination of x_0 in a non-local superconductor; confirmation of some predictions of BCS (~50 years after theory!!)

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Appendix
Ginzburg-Landau Equations
Coherence Length

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Ginzburg-Landau Equations (1950)

Powerful phenomenological theory, based on the Landau theory of second order transition.

Pseudowave-function ψ acting as order parameter (in the normal phase $= 0$, in the superconducting phase $\neq 0$).

ψ describes the superconducting electrons and their density

$$n_s = |\psi(\mathbf{r})|^2$$

The free energy density f_s can be expanded in a series:

$$f_s = f_n + \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m}|(-i\hbar\nabla - 2e\mathbf{A})\psi|^2 + \frac{|\mathbf{B}|^2}{2\mu_0}$$

The order parameter and the vector potential are obtained by minimizing the Ginzburg-Landau formula with respect to ψ and $\mathbf{A}$.

2^{nd} Nano-scale Param.: Coherence Length

$$f_s = f_n + \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m}|(-i\hbar\nabla - 2e\mathbf{A})\psi|^2 + \frac{|\mathbf{B}|^2}{2\mu_0}$$

Let assume a situation without field and at an interface vaccuum/superconductor.

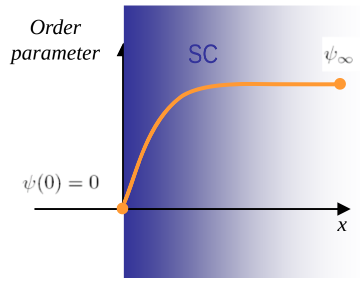
Minimizing the free energy with respect to ψ

$$\alpha\psi + \beta|\psi|^2\psi - \frac{\hbar^2}{2m}\nabla^2\psi = 0$$

Taking into account that $\psi(0) = 0$ and that $\psi(x \gg 0) = \psi_\infty$

$$\psi(x) = \psi_\infty \tanh\left(\frac{x}{\sqrt{2}\,\xi}\right)$$

with: $\xi = \sqrt{\frac{\hbar^2}{2m|\alpha|}}$ $\qquad \psi_\infty^2 = -\frac{\alpha}{\beta}$



$$\xi = \frac{\hbar v_F}{\pi\Delta}$$

Appendix
Pairing symmetry

T-Dependence of the SC carrier density

From μSR:

$$\sigma_\mu = \gamma_\mu \sqrt{\langle \Delta B^2 \rangle} \propto \frac{1}{\lambda^2}$$

$$\lambda = \sqrt{\frac{m}{\mu_0 e^2 n_s}}$$

$$\Rightarrow \quad \sigma_\mu \propto \frac{\mu_0 e^2}{m} n_s$$

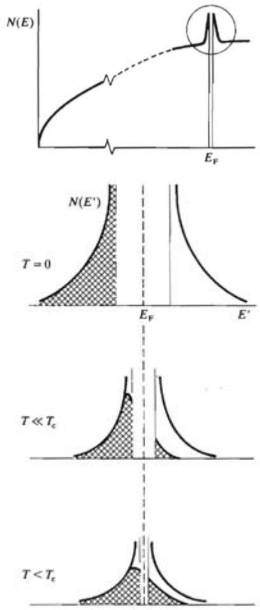
indications on the SC gap

By taking into account the thermal population of the quasiparticles excitations of the Cooper pairs (Bogoliubov quasiparticles):

$$n_s(T) = n_s(0)\left(1 - \frac{2}{k_B T}\int_0^\infty f(\epsilon, T)\,[1 - f(\epsilon, T)]\, d\epsilon\right)$$

with:

$$f(\epsilon, T) = \left(1 + \exp\left[\sqrt{\epsilon^2 + \Delta(T)^2}/k_B T\right]\right)^{-1}$$



- BCS conventional pairing: isotropic *s*-wave pairing

From μSR:

$$\sigma_\mu \propto \frac{1}{\lambda^2} = \frac{\mu_0 e^2}{m} n_s$$

$$n_s(T) = n_s(0)\left(1 - \frac{2}{k_B T}\int_0^\infty f(\epsilon, T)\,[1 - f(\epsilon, T)]\, d\epsilon\right)$$

If isotropic energy gap (s-wave):

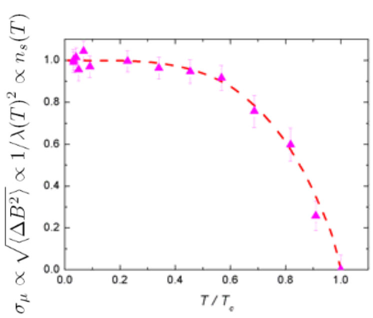
$$n_s(T) \propto ns(0)\left(1 - \sqrt{\frac{2\pi\Delta(0)}{k_B T}}\exp[-\Delta(0)/k_B T]\right)$$

and

$$\lambda(T) \propto \lambda(0)\left(1 + \sqrt{\frac{\pi\Delta(0)}{2k_B T}}\exp[-\Delta(0)/k_B T]\right)$$

B. Mühlschlegel, Z. Phys. 155, 313 (1959)

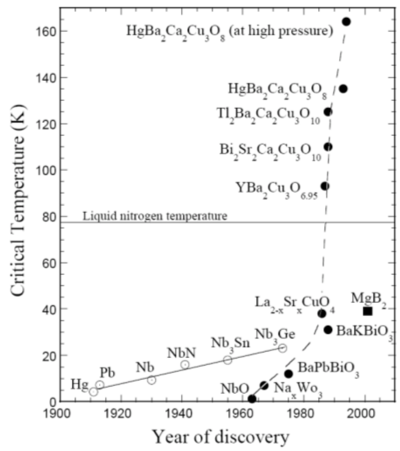
- The temperature dependence of the penetration depth provides information on the SC gap function.

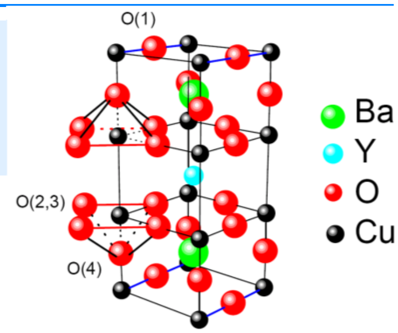


Mo_3Sb_7
R. Khasanov et al,
Phys. Rev. B 82, 016501 (2009)

High-Tc's Cuprates

Example: $YBa_2Cu_3O_{7-\delta}$
$2\ CuO_2$ planes
CuO chains: charge reservoirs





From A. Mourachkine *Room –Temperature Superconductivity*
Cambridge Int. Science Publ., 2004

High-Tc's Cuprates

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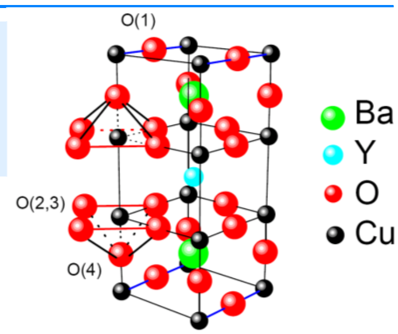
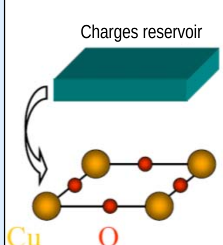


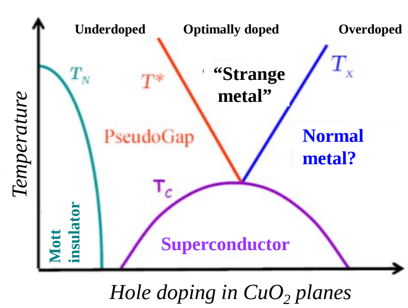
Without doping:
Band calculation predicts metallic state

BUT:

Strong on-site Coulomb repulsion (large *U*): blocks the motion of the $3d^9$ Cu-electrons in the CuO_2 plane ("Mott insulator")

AFM via superexchange





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Pairing Symmetry in Cuprates

Wave function of two electrons:

$\Psi(\mathbf{r}_1, s_1; \mathbf{r}_2, s_2) = \psi(\mathbf{r}_1, \mathbf{r}_2)\chi(s_1, s_2)$

where:

$\psi(\mathbf{r}_1, \mathbf{r}_2)$: space part

$\chi(s_1, s_2)$: spin part

spin singlet: $(\chi = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

$\Rightarrow S = 0 \rightarrow$ space part must be even.

$\Rightarrow$ s-wave (l = 0), d-wave (l = 2), etc...

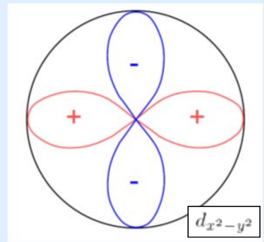
BCS High-Tc's

spin triplet: $(\chi = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle, ...)$

$\Rightarrow S = 1 \rightarrow$ space part must be odd.

$\Rightarrow$ p-wave (l = 1), f-wave (l = 3), etc...

Gap function: $\Delta(\mathbf{k})$ has a lower symmetry than the Fermi surface



As the gap disappears along some directions of the Fermi surface ("nodes"), extremely-low-energy quasiparticles excitations (and therefore significant pair-breaking) may occur at very low temperature

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Pairing Symmetry in High-Tc's

- BCS conventional pairing: isotropic *s*-wave pairing

From μSR:

$$\sigma_\mu \propto \frac{1}{\lambda^2} = \frac{\mu_0 e^2}{m} n_s$$

$$n_s(T) = n_s(0)\left(1 - \frac{2}{k_B T}\int_0^\infty f(\epsilon, T)\left[1 - f(\epsilon, T)\right] d\epsilon\right)$$

If isotropic energy gap (s-wave):

$$n_s(T) \propto n_s(0)\left(1 - \sqrt{\frac{2\pi\Delta(0)}{k_B T}} \exp[-\Delta(0)/k_B T]\right)$$

and

$$\lambda(T) \propto \lambda(0)\left(1 + \sqrt{\frac{\pi\Delta(0)}{2k_B T}} \exp[-\Delta(0)/k_B T]\right)$$

B. Mühlschlegel, Z. Phys. 155, 313 (1959)

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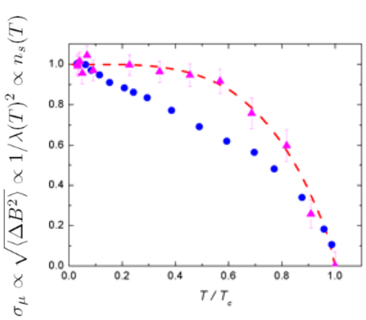

Single crystal $YBa_2Cu_3O_{6.95}$

J.E. Sonier et al, PRL 72, 744 (1994)

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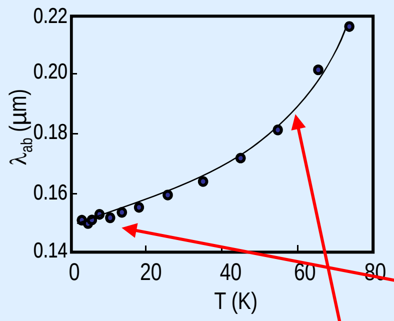
Pairing Symmetry in High-Tc's

µSR penetration depth on single crystal $YBa_2Cu_3O_{6.95}$

J.E. Sonier et al, PRL 72, 744 (1994)



Microwave measurements
W.N. Hardy et al., PRL 70, 3999 (1993)

s-wave SC state:

$$n_s(T) = n_s(0)\left(1 - \frac{2}{k_B T}\int_0^\infty f(\epsilon, T)\left[1 - f(\epsilon, T)\right] d\epsilon\right)$$

with:

$$f(\epsilon, T) = \left(1 + \exp\left[\sqrt{\epsilon^2 + \Delta(T)^2}/k_B T\right]\right)^{-1}$$

d-wave SC state:

$$n_s(T) = n_s(0)\left(1 - \frac{1}{\pi k_B T}\int_0^{2\pi}\int_0^\infty f(\epsilon, T)\left[1 - f(\epsilon, T)\right] d\varphi\, d\epsilon\right)$$

with:

$$f(\epsilon, T) = \left(1 + \exp\left[\sqrt{\epsilon^2 + [\Delta_s(T)\cos(2\varphi)]^2}/k_B T\right]\right)^{-1}$$

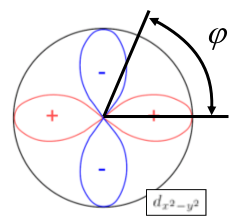
remembering that:

$$\lambda = \sqrt{\frac{m}{\mu_0 e^2 n_s}}$$

one gets: (for $T \ll T_c$)

$$\lambda(T) \propto \lambda(0)\left(1 + C\frac{T}{\Delta_s(0)}\right)$$

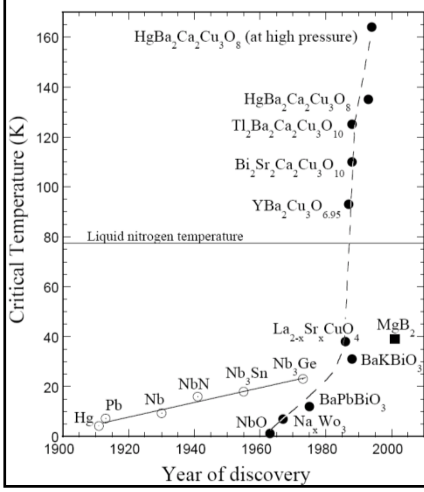
Hirschfeld & Goldenfeld, PRB 48, 4219 (1993)



Appendix
Two-gap superconductivity

Two Gaps Superconductivity – MgB_2

- Metallic SC with highest T_c
  J. Nagamatsu et al., Nature 410 (2001) 63
- SC mediated by phonon-coupling
- two-gap SC (Δ_σ and Δ_π)



From A. Mourachkine *Room –Temperature Superconductivity*
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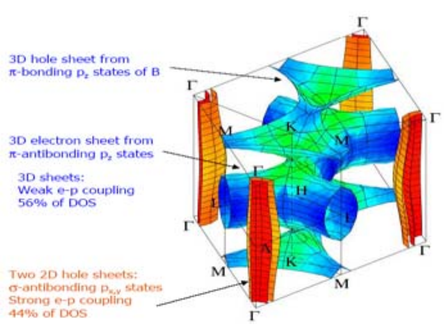
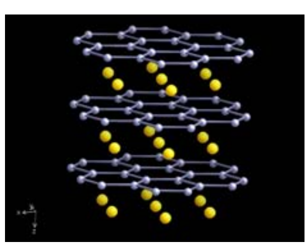
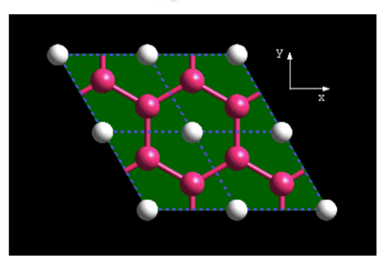






In-plane E_{2g} Boron mode strongly couples to the boron σ-band

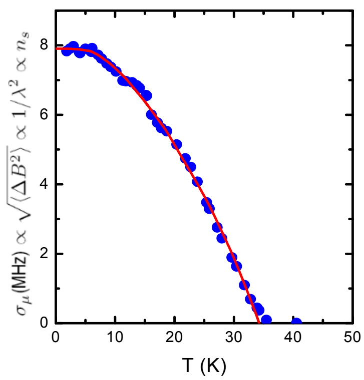
Two Gaps Superconductivity – MgB_2

Niedermayer et al., Phys. Rev. B 65, 094512 (2002)

Two SC gaps ⇒ Two kinds of Cooper pairs (weak interband processes)

$$n_s(T) = n_s(0)\left(1 - \frac{2w_\sigma}{k_B T}\int_0^\infty f_\sigma(\epsilon,T)\,[1-f_\sigma(\epsilon,T)]\,d\epsilon - \frac{2(1-w_\sigma)}{k_B T}\int_0^\infty f_\pi(\epsilon,T)\,[1-f_\pi(\epsilon,T)]\,d\epsilon\right)$$

$$\Rightarrow \quad \Delta_\sigma = 6.0(3)\ \text{meV} \quad \text{and} \quad \Delta_\pi = 2.6(2)\ \text{meV}$$

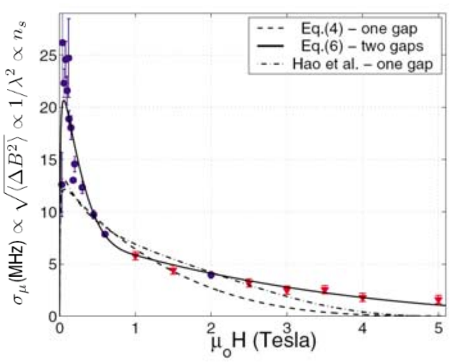


Similar results found recently by µSR on the "Sequicarbides" (Ln_2C_3 with Ln = La, Y)
Kuroiwa et al., Phys. Rev. Lett. 100, 097002 (2008)

Two Gaps Superconductivity – MgB_2

Evidence of two coherence lengths from field dependence of the second moment of the FLL

S. Serventi et al., PRL 93, 217003 (2004)



$$\xi = \frac{\hbar v_F}{\pi \Delta}$$

$$\langle \Delta B^2 \rangle = \sum_{\mathbf{K}\neq 0} |B(\mathbf{K})|^2$$

$$B(\mathbf{K}) = B\left[w_\sigma \frac{e^{(-\xi_\sigma^2 K^2/2)}}{1+\lambda^2 K^2} + (1-w_\sigma)\frac{e^{(-\xi_\pi^2 K^2/2)}}{1+\lambda^2 K^2}\right]$$

where:

$w_i = \dfrac{n_i}{n_\sigma + n_\pi}$ is the weigth of the two bands

$\xi_i =$ the coherence lengths of the two bands