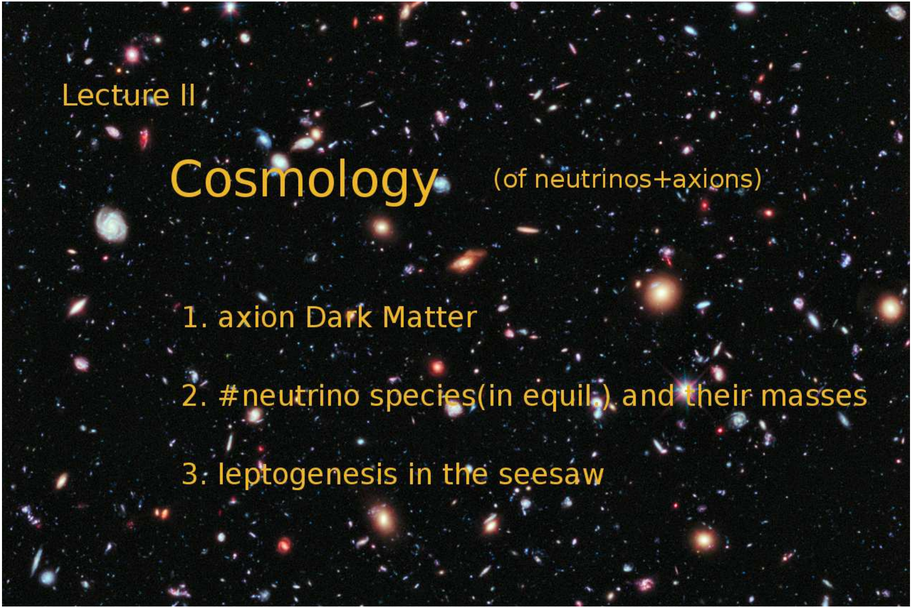

Lecture II
Cosmology (of neutrinos+axions)
1. axion Dark Matter
2. #neutrino species(in equil.) and their masses
3. leptogenesis in the seesaw

Cosmology

huge range of energy/density/timescales

observables(/fossils) :

1. baryon asymmetry (?$T \gtrsim m_W$) (BSM)
2. DM relic abundance (BSM)
3. abundance of light nuclei ($D,^3He,^4He,^7Li$ formed at $T \lesssim$ MeV) (SM)
4. CMB ($T_\gamma \sim$ eV) (SM+DM)
5. (structure formation (mat-rad equality $\rightarrow$ now)) (need DM)

exotics should be consistent with this data = explain/not-disturb (at creation+after) ...how to know how exotics affect these observables ?

idea 1 :

how many exotics present ? + what do they do ?

oops : thats for particles... axion DM = field ? what about phase transitions ?

ask the Path Integral (= black box for theorists that sums quantum mechanical amplitudes)

Reply : at Leading Order, use Einstein'sE for GR, Klein-Gordon for classical scalar, "Boltzmann" for particles

Ask the Delphic Oracle (= path integral)

Suppose add some feebly-interacting exotic to the Lagrangian...
What are relevant variables and equations to describe evolution ?

- variables = expectation values of n-pt functions

  $\langle\phi\rangle \equiv \phi_{cl} \leftrightarrow$ classical field for bosons(...misalignment axions)

  $\langle\phi(x_1)\phi(x_2)\rangle \leftrightarrow$ (propagator) + distribution of particles $f(x,p)$

  neglect 3+pt fns, because exotics feebly coupled
- get Eqns of motion for expectation values (in Closed Time Path formulation)

  feebly interacting $\Rightarrow$ at Leading Order, use classical saddle pt

  Einsteins Eqns with $T^{\mu\nu}(a_{cl}, f)$ (+ quantum corrections)

  Klein Gordon in curved space, (+ quantum corrections)

  Schwinger-Dyson in curved space for f ...$\approx$ Boltzmann Eqns (?)

$\Rightarrow$**leading order is simple :** Einsteins Eqns, Klein Gordon, and Boltzmann...

the QCD Axion as Cold Dark Matter

$m_a \lesssim m_\nu$, but *COLD* Dark Matter ?

Recall : KSVZ model $=$ a is phase if gauge-singlet complex scalar Φ

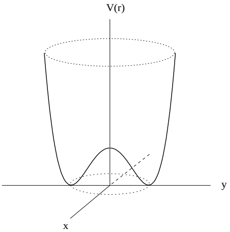
Lets compare axion vs WIMP as CDM ?

WIMP = *simple*
(particles = use Bolzmann)

in equilibrium at $T \gtrsim m_{WIMP}$
then "freezout"$\leftrightarrow$ relic density.
One epoch/calculation
$\simeq$ the WIMP miracle !
(can complexify...)
then LSS people do N-body...

(QCD) axion...relevant at many scales :
1-"PQ"PT : born massless when Φ gets a vev $\sim 10^{11}$ GeV : pre/post inflation ?
...classical/quantum field eqns,top.deffects...
2-QCDPT :acquires mass = becomes DM...what mass+ turn-on ? field or particles ? ?
3-$\rho_{mat} \sim \rho_{rad}$ ($\frac{\rho_{mat}}{\rho_{mat}}$ starts to grow, T ~3eV) : what is short-distance fluctuation spectrum of axion field/particles ?
4-growing fluctuations $t_{eq} \rightarrow t_0$:
Do axions grow stucture like WIMPs ?
And what does axion DM look like in our galaxy today ?

1) Which first : inflation or the birth of the axion ?



1. *IF* first the axion is born....

   $$\Phi \to f e^{ia/f} \quad (f \sim 10^{11} \text{ GeV})$$

   $|\Phi|$ and new quarks heavy, a massless
2. ...then inflation

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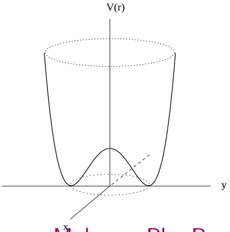

Mukanov PhysRep

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   a constant across U, develops classical fluctuations

   $$\frac{\delta a}{a} \sim \frac{H_I}{2\pi f}$$

   (WHAT ? quantum fluctuations, expanded beyond causally connected volume...are classical when re-enter causally connected V after inflation)

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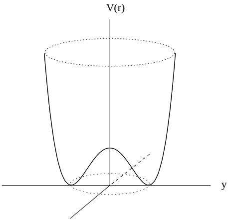

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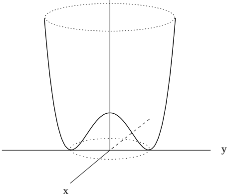
a constant across U, develops classical fluctuations
$$\frac{\delta a}{a} \sim \frac{H_I}{2\pi f}$$
different from inflaton $\Leftrightarrow$ isocurvature density fluctuations
Planck : $\Rightarrow H_I \lesssim 10^7 \sqrt{f/10^{12}}$ GeV
? or non-canonical kin.terms for a ? ...

WantzShellard
HanannHRW
FolkertsCristianoRedondo

...

3. Laaaater : QCD Phase Transition ($T \sim 200$ MeV)

1) If inflation first...



1. In the beginning, there was inflation

   avoids CMB bounds on isocurvature fluctuations :

2. Then the axion is born

$$\Phi \to f e^{ia/f}$$

   * a massless, random $-\pi f \leq a_0 \leq \pi f$ in each horizon

$$\langle a_0^2 \rangle_{U\ today} \sim \pi^2 f^2/3$$

   * ...one string/horizon

   ...

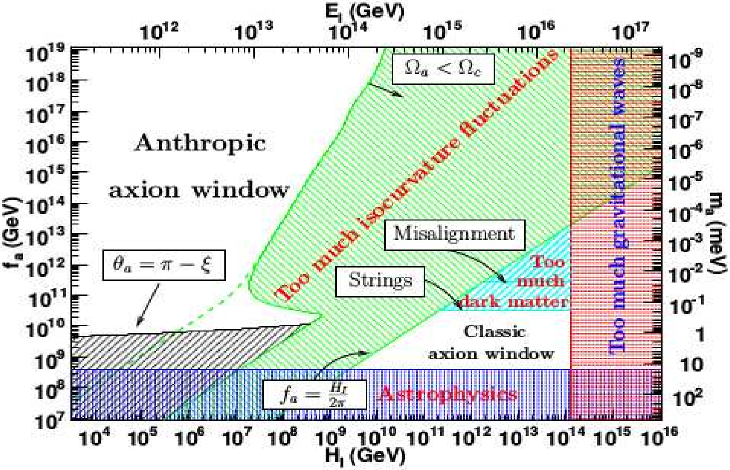
   Eqns of Motion for massless field in FRW smooth field on horizon scale

   string network "should" scale :
   confirm with string network on lattice, but need latticespacing $< 1/f$ and box $> 1/H$...

   Hiramatsu etal
   Klaer+Moore

3. Laaaater : QCD Phase Transition ($T \sim 200$ MeV)

PQ scale f for DM axions vs H_I (expansion rate during inflation)



Wantz thesis, with Shellard

2) At the QCD PT :the axion mass turns on

QCD Phase Transition ($T \sim 200$ MeV) : (tilt mexican hat)

$$m_a(t) : 0 \to f_\pi m_\pi / f \quad \Leftrightarrow \quad V(a) \approx f_{\rm PQ}^2 m_a^2 [1 - \cos(a/f_{\rm PQ})] \approx \frac{m_a^2}{2} a^2 - \frac{m_a^2}{8 f^2} a^4 + \dots$$

∗ ... at $H < m_a$, "misaligned" axion field starts oscillating around the minimum
∗ scalar field eqns in FRW : energy density $\simeq m_a^2 \langle a_0 \rangle^2 / R^3(t)$
(higher now, for smaller mass $\Rightarrow$ correct Ω for $m_a \gtrsim 10^{-5}$eV)
∗ strings go away (radiate cold axion particles, $\vec{p} \sim H \lesssim 10^{-6} m_a$)

Hiramatsu etal 1012.5502

"the axion mass turns on"

tour de force of
Borsanyi etal, Nature16
BudapestMarseilleWupertal lattice collab.

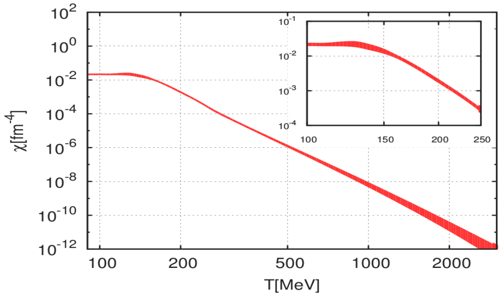
QCD Phase Transition ($T \sim 200$ MeV) :

in the thermal bath are "instantons" (have $\int d^4 x G\tilde{G} \in \boldsymbol{N}$).
$m_a^2(T) f_a^2 \approx \chi(T)$, topological susceptibility, where

$$\chi(T) \equiv \int d^4 x \langle G(x)\tilde{G}(x)G(0)\tilde{G}(0)\rangle_T$$

Calculate on lattice, for physical m_q, $T \neq 0$, at various T...
$\Rightarrow m_a(T)$...compare to $H(T)$ and know when axion starts to oscillate...

$\Rightarrow$ allows to predict misalignment-axion contribution to relic DM density, as fn of f (or m_a)



Pause : axion vs WIMP

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Pause

Have seen how the phase of a complex scalar gets a vev then a mass.

Could estimate analytically relic mass density ± 10 ? (do better with hard work)

Remains to see : why is the QCD axion a CDM candidate ?
$\Leftrightarrow$ *what is a successful Cold Dark Matter candidate* ?

1. CDM redshifts like matter $\propto 1/R^3(t)$, starting before the U is matter dominated already checked this
2. CDM grows small density fluctuations like WIMPs on Large Scale Structure/CMB scales
3. when density fluctuations become $\mathcal{O}(1)$ and collapse, any CDM candidate should reproduce current observations at least as well as WIMPs

What is a density fluctuation ?

Newtonian gravity (inside causally connected volume of U) :

$$\delta(\vec{x},t) \equiv \frac{\rho(\vec{x},t)-\overline{\rho}(t)}{\overline{\rho}(t)} \qquad \overline{\rho}(t) \equiv \frac{1}{V}\int_V d^3x\rho(\vec{x},t)$$

Then can take fourier transform :

$$\delta(\vec{k},t) = \int d^3x e^{i\vec{k}\cdot\vec{x}}\delta(\vec{x},t)$$

this is interesting to do for small fluctuations $|\delta| \ll 1$, because can drop δ^2,$\delta\vec{v}$ and get linear eqns !

Einstein gravity invariant under coordinate reparametrisation = can redefine time such that $\delta\rho = 0$? But physics hsould be same ? Bardeen constructed reparam-invar formalism for fluctuations, called "gauge invariant", these days everyone uses "Newtonian gauge". Summary : not worry.

Dynamics (from Eisteins Eqns) curious :
1) $\overline{\rho}(t)$ causes homogeneous U expansion.
2) $\delta(\vec{k},t)$ only feels gravitational attraction of fluctuations
3) expansion dilutes fluctuation growth...δ frozen in RadDom, grows in MatD.

3) Initial conditions for density fluctuations (after the QCD PT)

1 : inflaton's $\delta\rho/\rho$ on LargeScaleStructure scales imprinted on axion field(+part.)

...but what is short-distance spectrum ?

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2 : axion born after inflation :
field spatially random on QCDPT-horizon scale $\equiv$ **miniclusters**
$\frac{\delta\rho}{\rho} \sim \mathcal{O}(1)$ on comoving scale $1/H_{QCD}$: $M_{mini} \sim 10^{-12} M_{sol}$

Hogan,Rees
Tkachev+Kolb

collapse around mat-rad equality — if to dense objects,
these objects could behave like CDM ?

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2b. ? what fluctuations on QCD-horizon for axions particles from strings ? $\frac{\delta\rho_a}{\rho_a} \sim 1$ on scale H^{-1}_{QCDPT} ? ?

4) Eqns to grow $\delta\rho/\rho$: different for field vs particles ?

1)non-rel axion particles described by $f(x,p) \Rightarrow$ dust, like WIMPs : (so Boltzmann Eqns + N-body work)

$$T_{\mu\nu} = \rho v_\mu v_\nu = \begin{bmatrix} \rho & \rho\vec{v} \\ \rho\vec{v} & \rho v_i v_j \end{bmatrix}$$

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2)Classical field : $T_{\mu\nu} = \partial_\mu a \partial_\nu a - g_{\mu\nu}(\partial^\alpha a \partial_\alpha a - V(a))$

...in non-relativistic limit : $a = (\phi e^{-imt} + \phi^* e^{imt})/\sqrt{2}$

$$T_{\mu\nu} \to \begin{bmatrix} \rho & \rho\vec{v} \\ \rho\vec{v} & \rho v_i v_j + \Delta T_{ij} \end{bmatrix}$$

$$\Delta T^i_j \sim \partial^i \phi^* \partial_j \phi \ , \ \lambda|\phi|^4$$

Sikivie

$\Rightarrow$ classical field has different pressure, + self-interactions at $\mathcal{O}(\lambda)$

? extra pressures distinguish axion field from WIMPs in structure formation ?

(Parenthese : Klein Gordon or Einstein's Eqns to evolve axion field ?)

?its the same dynamics, so as choose what is convenient ?

To obtain Klein-Gordon in curved space from $T^{\mu\nu}_{\ ;\nu} = 0$:

$$
\begin{aligned}
T^{\mu\nu}_{\ ;\nu} &= \nabla_\nu[\nabla^\mu\phi\nabla^\nu\phi] - \nabla_\nu[g^{\mu\nu}\left(\frac{1}{2}\nabla^\alpha\phi\nabla_\alpha\phi - V(\phi)\right)] \\
&= (\nabla_\nu\nabla^\mu\phi)\nabla^\nu\phi + \nabla^\mu\phi(\nabla_\nu\nabla^\nu\phi) - g^{\mu\nu}\nabla_\nu\nabla^\alpha\phi\nabla_\alpha\phi + g^{\mu\nu}V'(\phi)\nabla_\nu\phi \\
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\end{aligned}
$$

1. if $\delta(\equiv \delta\rho(\vec{k},t)/\overline{\rho}(t)) \ll 1$ (at $z > 10$?), use Einsteins eqns for $T_{\mu\nu} \sim \phi^2$ because can be linearised = solvable.
   (compare axion field eqn cpled to gravity : $(\Box - m^2)\phi \sim G_N\phi^3$)

2. when density fluctuations are $\mathcal{O}(1)$, solve field eqns ?
   easier to impose phase continuity on NR field, than curl-free velocity in $T^{\mu\nu}$

Growing small fluctuations like WIMPs

- inside horizon, but conformal time, $T^{\mu}_{\nu;\mu} = 0$,

with $\rho(\vec{x},\tau) = \bar{\rho}(\tau)(1+\delta(\vec{x},\tau))$, $\theta = \nabla\cdot\vec{v}$ gives

$$\begin{aligned}
\partial_\tau\delta + \nabla\cdot\vec{v} &= 0 + -\nabla\cdot[\delta\vec{v}] && \text{continuity} \\
\partial_\tau\theta + \mathcal{H}\theta + \vec{v}\cdot\nabla\theta + \nabla\vec{v}\cdot\nabla\vec{v} &= -\nabla^2 V_N + \nabla^2\left(\frac{\nabla^2\sqrt{\rho}}{2m^2\sqrt{\rho}} + |\lambda|\frac{\rho}{m^4}\right) && \nabla \text{ of Euler} \quad ,
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- in fourier space (used Poisson : $\nabla^2 V_N = \frac{3\mathcal{H}^2}{2}\widetilde{\delta}, \Omega_{cdm} = 1$)

$$\begin{aligned}
\partial_\tau\widetilde{\delta}_{\vec{k}} + \widetilde{\theta}_{\vec{k}} &= -0 + \int \frac{d^3q}{(2\pi)^3}\alpha_{WIMP}(\vec{q},\vec{k})\widetilde{\delta}_{\vec{q}}\,\widetilde{\theta}_{\vec{k}-\vec{q}} \\
\partial_\tau\widetilde{\theta}_{\vec{k}} + \mathcal{H}\widetilde{\theta}_{\vec{k}} + \frac{3\mathcal{H}^2}{2}\widetilde{\delta}_{\vec{k}} &= 0 + \int \frac{d^3q}{(2\pi)^3}\beta_{WIMP}(\vec{q},\vec{k})\widetilde{\theta}_{\vec{q}}\,\widetilde{\theta}_{\vec{k}-\vec{q}} + \text{axions}
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- for small δ (small k/large dist.), physics/numerics says non-linearities negligeable :

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_N\bar{\rho}_a\delta + c_s^2\frac{k^2}{R^2(t)}\delta \simeq 0$$

($c_s^2 \sim \delta P/\delta\rho$) irrelevant because $k \to 0$

$\Rightarrow$ axion DM : grows linear/small density fluctuations like WIMPs

When $\delta\rho/\rho \sim 1$, how to axions grow structure ?

do extra pressures affect "non-linear" structure formation ?

1. Simple : are stable/stationary solutions different for axion-field vs dust ?
Rindler-Daller+Shapiro, Chavanis, ...
Stable solution for axion-field is the size/mass of an asteroid ($\sim 10^{-13} M_{sol}$, ok in galaxy)

2. Numerically solve field eqns with extra pressures and compare to N-body (= dust) ? EtalBroadhurt, Niemeyer etal, MoczVogelsangerEtal

Axion Asteroids : stable solution that could occur after collapse ?

1 look for *time-independent* solution to eqns (!NB : eqns for $\rho(x,t)$, NOT $\delta(x,t)$)

$$
\begin{aligned}
\partial_t \rho &= -\nabla \cdot \rho \vec{v} && \text{continuity} \\
\rho \partial_t \vec{v} + \rho \vec{v} \cdot \nabla \vec{v} &= \rho \nabla \left(\frac{\nabla^2 \sqrt{\rho}}{2m^2 \sqrt{\rho}} + |\lambda| \frac{\rho}{m^4} - V_N \right) && \text{Euler} \quad ,
\end{aligned}
$$

find (set $\vec{v}, \partial_t = 0$ and do dim analysis) :

$$
\left(\frac{1}{2m^2 R^2} - |\lambda| \frac{M}{m^4 R^3} - G_N \frac{M}{R} \right) \simeq 0 \quad \Rightarrow \quad R \sim \frac{m_{pl}^2}{4m^2 M} \left(1 \pm \sqrt{1 \mp \frac{4|\lambda| M^2}{m_{pl}^2}} \right)
$$

(allow breathing mode(Chavanis) + rotation(DavidsonSchwetz) for $m \sim 10^{-4}$ eV, $\lambda \sim$-10^{-45} of QCD axion born after inflation)

$$
\Rightarrow \quad R \sim \frac{m_{pl}^2}{4m^2 M} \lesssim 100 \text{ km} \quad , \quad M \lesssim \frac{m_{pl} f}{m} \sim 10^{-(13\pm1)} M_\odot \simeq \begin{cases} \text{asteroid!} \\ \lesssim \text{minicluster} \end{cases}
$$

3 ok as galactic DM (between pico→microlensing)

? dynamics ? do asteroids form ? survive ? numerical problem...

Neutrinos in cosmology

- leptogenesis : $T : 10^{12} \to 100$ GeV, generate a lepton asym in CPV dynamics, use SM B+L Violation to transform to baryons
- Big Bang Nucleosynthesis ($H, D,^3 He,^4 He,^7 Li$ at $T \sim$MeV)
  how many species of relativistic ν in the thermal soup ?
- decoupling of photons —e+p→H (CMB spectrum today)
  cares about radiation density $\leftrightarrow N_\nu, m_\nu$

...all about interaction rates of particles in the U...

an "EFT" for particle interactions in the early U ?

- EFT = recipe to study observables at scale ℓ
  1. choose *appropriate* variables to describe *relevant* dynamics
  2. 0th order interactions, by sending all parameters $\left\{ \begin{array}{ll} L \gg \ell & \to \infty \\ \delta \ll \ell & \to 0 \end{array} \right.$
  3. then perturb in ℓ/L and δ/ℓ

Example : interactions in the early Universe of age τ_U ($\tau_U \sim 10^{-24}$ sec)

$\star$ processes with $\tau_{int} \gg \tau_U$ ...neglect !

$\star$ processes with $\tau_{int} \ll \tau_U$ ...assume in thermal equilibrium !

$\star$ processes with $\tau_{int} \sim \tau_U$ ...calculate this dynamics

$\star$ can then do pert. theory in slow interactions and departures from thermal equil.

Suppose the density of the U is dominated by relativistic particles in equilibrium ($\rho \propto T^4$)

$$H = \frac{\dot{a}}{a} = \sqrt{\frac{8\pi G}{3}\frac{g_{eff}\pi^2 T^4}{30}} \simeq \frac{1.7\sqrt{g_{eff}}}{m_{pl}} T^2 \quad , \quad g_{eff} \equiv \sum_{\overline{b},b} g_b + \frac{7}{8}\sum_{\overline{f},f} g_f$$

and $T(t) \sim 1/a(t) \Rightarrow a(t) = \sqrt{t/t_0}$, so

$$\tau_U(T) = \frac{1}{2H} \qquad \Rightarrow \qquad \tau_U(sec) \simeq 0.7\frac{MeV^2}{T^2}$$

Can estimate interaction rate of a particle in the plasma as

$$\Gamma_{int} \sim \frac{1}{\tau_{int}} \sim \beta \times n_{target} \times \sigma \sim \frac{gT^3}{\pi^2}\sigma$$

an example : QED

(lets forget IR divergences) For a e^- interacting with a bath of γs :

$$\beta\sigma(e\gamma \to e\gamma) = \frac{2\pi\alpha^2}{s} \ln \frac{s}{m_e^2}$$

For $s = (3T)^2$ (?or $s = T^2$) and $\sqrt{g_{eff}} \sim 10$:

$$\frac{\Gamma}{H} \quad \sim \quad \frac{g_\gamma T^3}{\pi^2} \frac{2\pi\alpha^2}{9T^2} \frac{1}{H} \sim \frac{m_{pl}}{3 \times 10^6 T}$$

$\Rightarrow e^-, \gamma$ in thermal equil for $T \lesssim 10^{13}$ GeV. Ditto e^+...
unbroken SU(N) : same scaling of $\Gamma/H(T)$, rate a bit bigger.

Another example : $(\nu e \to \nu e)$ at $T \ll m_W$

Interaction rate of a $\nu_{\mu,\tau}$ with $e^{\pm}$ (neglect rare n,p) :

$$\frac{\Gamma}{H} \sim \frac{g_{e^\pm} T^3}{\pi^2} \sigma \frac{1}{H} \quad \text{with} \quad \sigma \simeq \frac{G_F^2 s}{16\pi}$$

So $\Gamma \sim H$ when

$$\Gamma \sim \frac{G_F^2 T^5}{4\pi} \sim \frac{1.66\sqrt{g_{eff}}\, T^2}{m_{pl}}$$

$\Rightarrow$ neutrinos acquire equilibrium densities before $T \sim$ MeV.
$\nu_{\mu,\tau}, \overline{\nu}_{\mu,\tau}$ decouple from $e^{\pm}$ around $T \simeq 3.5$ MeV,
ν_e has also W exchange diagram = remain in equilibrium til $T \sim 2$ MeV.

Decouple at $T \gg m_\nu$, so *retain* relativistic number distribution 'til today
$\Rightarrow$ there is a Cosmic Neutrino BackGround.
(But $T_\nu = (4/11)^{1/3} T_\gamma$, because $e^{\pm}$ annihilation heats γ wrt ν)

(Exercise : *how to detect CNB ?*)

In the room, are $\sim 10^6$ WIMPS, $\sim 10^5$ Be ν, and $\sim 10^{10}$ Cosmic Background Neutrinos(CNB).

How to detect CNB ?

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Weinberg
Cocco Mangano
Messina

What about ν capture βdecay : $n + \nu_{CNB} \to p + e$?

To compare rates for ${}^3H \to {}^3 He + e + \bar{\nu}_e$ to $\nu_e + {}^3 H \to {}^3 He + e$:

$$\frac{n_{\nu CNB}}{\nu \text{ phase space}} \simeq \frac{T^3_{CNB}}{\pi^2}\frac{1}{Q^3} \sim \left(\frac{10^{-4}\mathrm{eV}}{20\mathrm{keV}}\right)^3 \sim 10^{-24}$$

But...$E_e = Q + m_\nu$
(recall for ${}^3H \to {}^3 He + e + \bar{\nu}_e$, $E_e \leq Q - m_\nu$)

So...if ever resolution better than m_ν...

What rate associated to neutrino masses $m_D \bar{\nu}_L \nu_R$?

1. below m_W/after EWPT(Elec.Weak PhaseTransition) : m^2-correction to gauge scattering

$$\frac{m_\nu^2 G_F^2}{4\pi} T^3 > \frac{1.7 g_{eff} T^2}{m_{pl}} \Leftrightarrow m_\nu \gtrsim 100 \text{ keV}$$

2. above m_t/before EWPT :
scattering via neutrino Yukawa : $\lambda \bar{\ell} H \nu_R$ (attach other end of Higgs to $t\bar{t}$)

$$\frac{\lambda^2}{4\pi} T > \frac{1.7 g_{eff} T^2}{m_{pl}} \Leftrightarrow \lambda \gtrsim 10^{-8}$$

($m_D \overline{\nu_L} \nu_R \sim$ few $\times$ 100 eV $\overline{\nu_L} \nu_R$)

Despite that there are six light chiral fermions in the model with Dirac ν-masses, only three are "in equilibrium" in the early U $\Leftrightarrow$ contribute to the radiation energy density.

BBN bounds on N_ν

$N_\nu \equiv$ number of 2-comp. relativistic νs with equilibrium energy density

1. (that cosmology measures $N_\nu \sim 3$ means neutrinos have gravitational interactions)
2. Big Bang Nucleosynthesis ($T \lesssim$ MeV, \$$\tau_U \sim$ few minutes) :
   - neutrons crucial to form $D,^{3,4}He, Li$
   - $n_n/n_p \propto exp\{-(m_n - m_p)/T\}$ in thermal equil at $T \gtrsim$ MeV
   - "freezes" when $\Gamma(n + \nu \to p + e) \lesssim H$, and $H^2 \simeq 3\rho_{rad}/m_{pl}^2$; $\rho_{rad} \supset \{\gamma, N_\nu \nu\}$

   $\Rightarrow$ "primordial" abundances of $D,^{3,4}He, Li$ constrain

$$N_\nu \lesssim 4.08$$

Mangano, Serpico

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CMB bounds on N_ν

3. Cosmic Microwave Background :(is a fit to a multi-parameter model...).
Roller coaster at $\ell > 150$ is a snapshot of sound waves in the plasma at recomb ; amplitude cares about ρ_b/ρ_γ. Is sensitive to time since mat-rad equality, which is sensitive to N_ν...but can compensate by changing other parameters !

PDB discussion of Verde-Lesgourges :
suppose other inputs cancel LO effect no N_ν ... what remains ?

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PDB discussion of Verde-Lesgourges :
suppose other inputs cancel LO effect no N_ν ... what remains ?
Argue that remaining effects cannot be cancelled by ajusting parmeters, so obtain :

$$N_\nu \lesssim 3.3 \pm 0.5$$

PLANCK 13
more restrictive with
other cosmo input

• **a late contribution to DM in cosmology :**
relic ν "free-stream" til they become non-rel. (after recomb. for $\Sigma \lesssim$ eV), then contribute to DM $\propto \sum_i |m_i| \equiv \Sigma$.

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• **Σ has effects on CMB :**
Relativistic $\rightarrow$ non-rel transition affects CMB *propagation*...parameter in cosmological fits :

Lesgourgues talk
CERN ν-platform kickoff

$$
\begin{array}{llll}
\sum & \lesssim & 0.1 \rightarrow .6\ eV & now:\ PLANCK, +LSS/Ly\alpha\ (in\ \Lambda CDM) \\
 & \lesssim & 0.6\ eV & now:\ PLANCK + BAO\ (in\ 12\ param\ \Lambda CDM) \\
 & \rightarrow & \lesssim 2m_{atm} & cosmo.indep.\ (Planck + EUCLID...) \\
 & & \sim m_{atm} & \Lambda CDM
\end{array}
$$

DiValentino etal
1507.06646

Fewer twiddles for precision cosmology ?

So far, compute on "back of envelope". Recall recipe :

To identify relevant interactions in the early Universe of age τ_U ($\tau_U \sim 10^{-24}$ sec)

1. processes with $\tau_{int} \gg \tau_U$ ...neglect !
2. processes with $\tau_{int} \ll \tau_U$ ...assume in thermal equilibrium !
3. processes with $\tau_{int} \sim \tau_U$ ...calculate this dynamics

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...sloppy is fine for 1,2 ; but if really want to calculate dynamics, need eqns for 3. ?

Dynamical Eqns : can one use Boltzmann Eqns ? ? ?

Ludwig Boltzmann : 1844-1906 / Max Planck : 1858-1947 ($\hbar \sim 1900$)

early U : $\rho \propto T^4 >$ nucleus for $T > 100$ MeV
$\tau_U \sim$ nanosecond at $T \sim 100$ GeV

curiously, usually yes !

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Why is that? Ask the closed-time-path, finite-density Path Integral for Eqns of motion for the number operator...(Real-Time Finite-Temp Field Theory/ 2Particle-Irreducible Eqns/ Kadanov-Baym-Schwinger-Dyson Eqns)

$$\frac{d}{dt}\hat{n} = +i[\hat{H}_0, \hat{n}] - [\hat{H}_I, [\hat{H}_I, \hat{n}]] + ...$$

(2nd Quant.,Heisenberg rep, t-dep ops)

$\hat{H}_0 =$ free Hamiltonian (*Integral* of hamiltonian density). Interaction rates from second +... terms.

1) (anti)commutators give Bose-Einstein/FD phase space factors

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2) ...if a hierarchy of interaction rates, then in the propagation eigenstate basis, looks like Boltzmann ?

...lets suppose we can use Boltzmann... (there is an orange section at the end of lectures, about how to get credible constants in rates = calculate thermally averaged rates)

Can neutrinos make the Universe we see ?

Leptogenesis

Leptogenesis is a class of recipes, that use majorana neutrino mass models to generate the matter excess. The model generates a lepton asymmetry (before the Electroweak Phase Transition), and the non-perturbative SM B+L violn reprocesses it to a baryon excess.

Thanks to Gustave Doré

Preamble

1. about "What the stars (and us) are made of" (5% of U)
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$$Y_B \equiv \left.\frac{n_B - n_{\bar{B}}}{s}\right|_0 = 3.86 \times 10^{-9} \Omega_B h^2 \simeq (8.53 \pm 0.11) \times 10^{-11}$$

PLANCK

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PLANCK

$\Rightarrow$ Question : where did that excess come from ?

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   X no ! After birth of U, there was "inflation"
   - (only theory explaining coherent temperature fluctuations in microwave background that arrive from causally disconnected regions today...)
   - "60 e-folds" inflation $\equiv V_U \rightarrow > 10^{90} V_U$

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3. created/generated/cooked after inflation...

Three ingredients to prepare in the early U (old russian recipe)

Sakharov

1. B violation : if U$_{\text{niverse}}$ starts in state of $n_B - n_{\bar{B}} = 0$, need $\cancel{B}$ to evolve to $n_B - n_{\bar{B}} \neq 0$

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3. out-of-thermal-equilibrium ...equilibrium = static. "generation" = dynamical process
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   From end inflation → BBN, Universe is an expanding, cooling thermal bath, so non-equilibrium from :
   - slow interactions : $\tau_{int} \gg \tau_U =$ age of Universe ($\Gamma_{int} \ll H$)
   - phase transitions :

ingredient 1 : Does the SM conserve B ?

B, L are global symmetries of the SM Lagrangian (q, ℓ doublets, e, u, d singlets)

$$\mathcal{L}_{SM} \supset \overline{q} \not{D} q \ , \ \overline{\ell} \not{D} \ell \quad , \quad \overline{\ell} H e \quad , \quad \overline{q} \widetilde{H} u \quad , \quad \overline{q} H d$$

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But the SM *does not* conserve $B + L$...
In QFT, there is the axial anomaly...
...anomalously, the fermion current associated to a classical symmetry is not conserved.

see Polyakov,
"Gauge Fields + Strings,"
6.3=qualitative effects of instantons

ingredient 1 : the SM *does not* conserve $B + L$

$B + L$ is anomalous. Formally, for one generation(α colour) :

$$\sum_{\frac{SU(\mathbf{2})}{singlets}} \partial^\mu(\overline{\psi}\gamma_\mu\psi) + \partial^\mu(\overline{\ell}\gamma_\mu\ell) + \partial^\mu(\overline{q}^\alpha\gamma_\mu q_\alpha) \propto \frac{1}{64\pi^2} W^A_{\mu\nu}\widetilde{W}^{\mu\nu A}.$$

where integrating the RHS over space-time counts "winding number" of the SU(2) gauge field configuration.

$\Rightarrow$ Field configurations of non-zero winding number are sources of a doublet lepton and three (for colour) doublet quarks for each generation.

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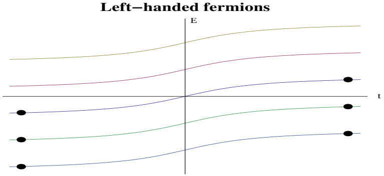

thanks to V Rubakov

SM B+L violation : rates

't Hooft
Kuzmin Rubakov+
Shaposhnikov

At $T = 0$ is tunneling process (from winding # to next, "instanton") : $\Gamma \propto e^{-8\pi/g^2}$

At $0 < T < m_W$, can climb over the barrier : $\Gamma_{\cancel{\mathrm{B+L}}} \sim \begin{array}{ll} e^{-m_W/T} & T < m_W \\ \alpha^5 T & T > m_W \end{array}$

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$\Rightarrow$ if produce a lepton asym, "sphalerons" partially transform to a baryon asym. ! !

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$\Rightarrow$ if produce a lepton asym, "sphalerons" partially transform to a baryon asym. ! !
$\star\star\star$ SM $\cancel{B+L}$ is $\Delta B = \Delta L = 3$ $(= N_f)$. No proton decay ! $\star\star\star$

• Want to make a baryon excess $\equiv Y_B$ after inflation, that corresponds today to ~ 1 baryon per 10^{10} γs.

• Three required ingredients : $\cancel{B}$, $\cancel{CP}$, $\cancel{TE}$.

Present in SM, but hard to combine to give big enough asym Y_B

Cold EW baryogen ? ? Tranberg et al ...

$\Rightarrow$ *evidence for physics Beyond the Standard Model (BSM)*

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$\Rightarrow$ evidence for physics Beyond the Standard Model (BSM)

One observation to fit, many new parameters...

$\Rightarrow$ prefer BSM motivated by other data $\Leftrightarrow m_\nu \Leftrightarrow$ seesaw ! (uses non-pert. SM $\cancel{B+L}$)

Minkowski, Yanagida
Gell-Mann Ramond Slansky

- add 3 singlet N to the SM in charged lepton and N mass bases, at scale $> M_i$:

add 18 parameters :
M_1, M_2, M_3
18 - 3 (ℓ phases) in λ

$$\mathcal{L} = \mathcal{L}_{SM} + \boldsymbol{\lambda}_{\alpha J} \overline{N}_J \ell_\alpha \cdot \phi - \frac{1}{2} \overline{N_J} M_J N_J^c$$

M_I unknown ($\not\propto v = \langle \phi^0 \rangle$), and Majorana ($\not L$). $\not{CP}$ in $\boldsymbol{\lambda}_{\alpha J} \in \boldsymbol{C}$.

The type I seesaw

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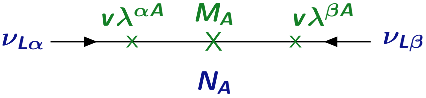
- at low scale, for $M \gg m_D = \lambda v$, light ν mass matrix



9 parameters :
m_1, m_2, m_3
6 in U_{MNS}

$$[m_\nu] = \boldsymbol{\lambda} M^{-1} \boldsymbol{\lambda}^T v^2$$

for $\lambda \sim h_t$, $M \sim 10^{15}$ GeV; $\lambda \sim 10^{-7}$, $M \sim 10$ GeV $\quad \sim .05$ eV

"natural" $m_\nu \ll m_f$: $m_\nu \propto \lambda^2$, and $M > v$ allowed.

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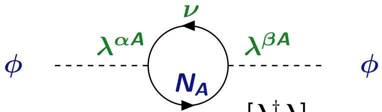
- at low scale, Higgs mass contribution



$$\delta m_\phi^2 \simeq -\sum_I \frac{[\boldsymbol{\lambda}^\dagger \boldsymbol{\lambda}]_{II}}{8\pi^2} M_I^2 \sim \frac{m_\nu M_I^3}{8\pi^2 v^4} v^2$$

for $M \gtrsim 10^7$ GeV $> v^2$ tuning problem

(? adding particles to cancel 1 loop...but higher loop ? Need symmetry to cancel ≥ 2 loop ?)
$\Rightarrow$ do seesaw with $M_I \lesssim 10^8$ GeV ?
(NB, in this talk, ϕ = Higgs, H = Hubble)

Leptogenesis in the type 1 seesaw : usually a Fairy Tale

Fukugita Yanagida
Buchmuller et al
Covi et al
Branco et al
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Once upon a time, a Universe was born.
At the christening of the Universe,the fairies give the Standard Model and the Seesaw (heavy sterile N_j with $\not{L}$ masses and $\not{CP}$ interactions) to the Universe.

Leptogenesis in the type 1 seesaw : usually a Fairy Tale

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If this asymmetry can escape the big bad wolf of thermal equilibrium...

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4 If asymmetry escapes the wolf of thermal equilibrium...

5 the lepton asym gets partially reprocessed to a baryon asym by non-perturbative $B + L$ -violating SM processes (“sphalerons”)

And the Universe lived happily ever after, containing many photons. And for every 10^{10} photons, there were 6 extra baryons (wrt anti-baryons).

Does it work ? Calculate something ?

Recipe : calculate suppression factor for each Sakharov condition, multiply together to get Y_B :

$$\frac{n_B - n_{\bar{B}}}{s} \sim \frac{1}{3g_*}\epsilon_{L,CP}\eta_{TE} \sim 10^{-3}\epsilon\eta \qquad (\text{want } 10^{-10})$$

$s \sim g_* n_\gamma$, ϵ = lepton asym in decay, $\eta = \cancel{\mathrm{TE}}$ process/γ

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Suppose at $T \gtrsim M_1$, a density $\sim T^3$ is produced.

Later, Lepton asym produced in $\not{C}\!\!\not{P}$ N decays, survives if not washed out by Inverse Decays = survives after ID out of equil :

$$\Gamma_{ID}(\phi\ell \to N) \simeq \Gamma_{decay} e^{-M_1/T} = \frac{[\lambda\lambda^\dagger]_{11} M_1}{8\pi} e^{-M_1/T} < \frac{10\, T^2}{m_{pl}}$$

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Fraction N remaining at T_{ID} when ID turn off :

$$\frac{n_N}{n_\gamma}(T_{ID}) \simeq e^{-M_1/T_\alpha} \simeq \frac{H}{\Gamma(N \to \ell_\alpha \phi)} \equiv \eta$$

Estimate ϵ, the CP asymmetry in decays

Kolb+Wolfram, NPB '80, Appendix

Recall (in **S**-matrix) $CP : \langle \phi\ell|\boldsymbol{S}|N\rangle \to \langle\overline{\phi\ell}|\boldsymbol{S}|\overline{N}\rangle = \langle\overline{\phi\ell}|\boldsymbol{S}|N\rangle$,($\overline{\eta}$ =anti-η)

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In leptogenesis, need $\cancel{CP}$, $\cancel{L}$ interactions of N_I...for instance :

$$\epsilon_I^\alpha = \frac{\Gamma(N_I \rightarrow \phi\ell_\alpha) - \Gamma(\bar{N}_I \rightarrow \bar{\phi}\bar{\ell}_\alpha)}{\Gamma(N_I \rightarrow \phi\ell) + \Gamma(\bar{N}_I \rightarrow \bar{\phi}\bar{\ell})} \qquad (\text{recall } N_I = \bar{N}_I)$$

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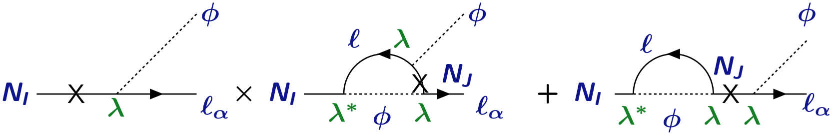

Just try to calculate ϵ_1 ?

• asym at tree × loop, *if* $\not{CP}$ from complex cpling *and* on-shell particles in the loop (divergences cancel in diff, need Im part of Feynman param integrtn)

Can use unitarity and CPT invariance of S-matrix to estimate ϵ from tree amplitudes.
Consider $M_1 \ll M_{2,3}$, asym from $\not{CP}$, $\not{L}$ decays of N_1 :

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loops, unitarity and all that...(estimate ϵ, no loop caln)

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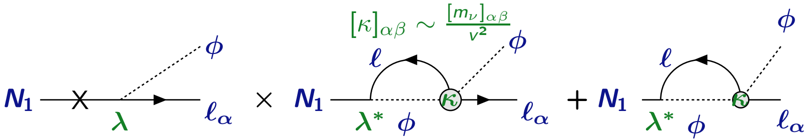

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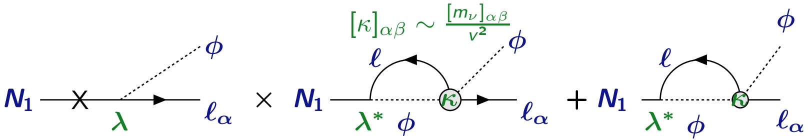

$$\epsilon_1 \sim \frac{1}{8\pi}\frac{\lambda^2 \kappa}{\lambda^2} M \quad < \quad \frac{3}{8\pi}\frac{m_\nu^{max} M_1}{v^2} \quad \sim 10^{-6}\frac{M_1}{10^9 \mathrm{GeV}}$$

Estimate Y_B

Recall($s \sim g_* n_\gamma$, $\epsilon =$ lepton asym in decay $\eta = \cancel{TE}$ process$/\gamma$) :

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$$\sim 10^{-3}\frac{H}{\Gamma}10^{-6}\frac{M_1}{10^9\text{GeV}}$$

for $M_1 \ll M_{2,3}$, need $M_1 \gtrsim 10^9$ GeV to obtain sufficient ϵ

?but give $\delta m_H^2 \gg m_H^2$?

do leptogenesis with $M_K < 10^7$ GeV ?

For $M_I \sim M_J \Leftrightarrow$ resonantly enhance ϵ ... up to $\epsilon \lesssim 1/8\pi$!
but need decays before Electroweak PT (to profit from sphalerons)... and ID out-of-equil :

$$\Gamma_{ID} \sim e^{-M/T}\Gamma(N \to \phi\ell) < H \quad \Rightarrow \quad M \gtrsim 10\,T_c$$

Fairy tale works for degen N_I for $M_I \gtrsim$ TeV
(but are $M_I \sim$ TeV any more detectable than $M_I \sim 10^9$ GeV ?)

νMSM : type 1 seesaw below 100 GeV gives BAU and DM

Asaka + Shaposhnikov
thesis Canetti
...

ingredients : SM +

$N_{2,3}$: 100 MeV $\lesssim M_{2,3} \lesssim$ 10 GeV, $\Delta M \lesssim \begin{cases} 10^{-6}\ \mathrm{eV} & Y_B, \Omega_{DM} \\ \mathrm{keV} & Y_B, NOT\ \Omega_{DM} \end{cases}$

Yukawas $\ni$ give 2 light SM neutrinos via seesaw

N_1 : $M_1 \sim$ keV. WDM candidate.

feebly coupled (negligeable contribution $m_{\nu,SM}$)

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Produce $\Delta L \to Y_B$ via oscillations of $N_{2,3}, \nu_{SM}$ before EPT
Produce $\Delta L \gtrsim 10^{-5}$ via osc. and decay of $N_{2,3}$ after EPT
Can produce sufficient distribution of N_1 via osc.

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tests :

$N_{2,3}$: beam dump, SHIP

N_1 as DM : X-rays from DM decay, WDM bounds (depend on momentum distribution)

How does asym generation work? (very simplified!)

1 at $T \lesssim TeV$ (recall $\lambda \lesssim 10^{-7}$) , produce N_2, N_3 via Yukawa interaction $\lambda \overline{N} \ell \cdot \phi$

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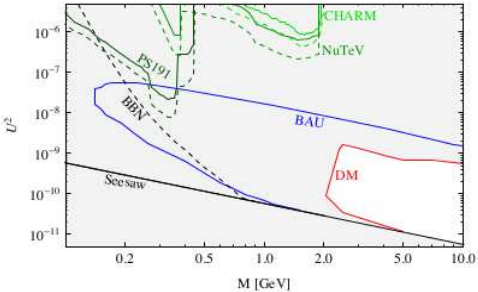
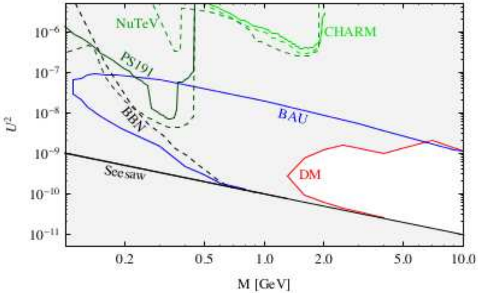
3 back to ν_L via λ
at $\tau_U \sim \tau_{osc}$, 1,2,3 are *coherent*, so CPV from λ-ΔM^2-λ gives flavour asyms in $\nu_{L\alpha}$ (to small)
lepton number in $\ell_L + N_R$ is conserved (actually, $L_{SM}+$ helicity of N_I)
from $\tau_{osc} \to \tau_{EWPT}$, asyms in $\nu_{L\alpha}$ seed asyms in $N \longrightarrow$ asyms in $\nu_{L\alpha}$ (enough asym)
...works also in detailed calculations with all available technology...
(eg also include lepton number violating interactions)

Teresi Hambye
Eijima + Shaposhnikov
Ghiglieri+ Laine





$$U^2 = \mathrm{Tr}[\lambda M^{-2} \lambda^\dagger]$$

Leptogenesis is a class of recipes, that use majorana neutrino mass models to generate the matter excess. The model generates a lepton asymmetry (before the Electroweak Phase Transition), and the non-perturbative SM B+L violn reprocesses it to a baryon excess.

⋆ efficient, to use the BSM for m_ν to generate the Baryon Asym.

⋆ using SM B+L violn ($\Delta B = \Delta L = 3$) avoids proton lifetime bound

⋆ *it works* ...rather well, for a wide range of parameters

Appendices

To compute credible constants in rates

In particle physics, compute decay rates in the particle rest frame... but... for a thermal population in FRW ? Expect "boost" of decay time for relativistic particles ? Consider decay $N \to \phi\ell$:

$$|\mathcal{M}(N \to \ell\phi)|^2 = 2|\lambda|^2 p_N \cdot p_\ell = |\lambda|^2(M_1^2 + m_\ell^2 - m_\phi^2)$$

N- rest-frame calculation, massless final state particules :

$$\Gamma(N \to \ell\phi) = \frac{1}{2M}\int |\mathcal{M}(N \to \ell\phi)|^2(2\pi)^4\delta^4(...)d\Pi_p d\Pi_q = \frac{|\lambda|^2 M_1}{16\pi}.$$

Not-rest-frame 2-particle phase space ? : with ($d\Pi_p = \frac{d^3p}{2E(2\pi)^3}$)

$$\begin{aligned}\int(2\pi)^4\delta^4(\sum p_i - \sum p_f)d\Pi_p d\Pi_q &= \int \frac{|\vec{p}_p|}{16\pi^2\sqrt{s}}d\Omega_p \qquad \text{rest frame} \\ = \int \tfrac{|\vec{p}_p - \vec{p}_q|}{32\pi^2\sqrt{s}}d\Omega_p &= \int \frac{\sqrt{(p_p \cdot p_q)^2 - m_p^2 m_q^2}}{16\pi^2 s}d\Omega_p,\end{aligned}$$

But to calculate in a thermal bath ?

1. finite density — can contribute "thermal masses" to 2-pt function
2. momentum distribution : calculate "rate density" $\gamma = \langle n_N \Gamma \rangle$:

$$\begin{aligned}\gamma(N \to \ell\phi) &= g_N \int \frac{d^3 p_N}{2E_N(2\pi)^3} e^{-E_N/T} \, 2|\lambda|^2 M_1^2 \int \tilde{\delta} \, d\Pi_\phi d\Pi_\ell \\ &= \frac{g_N T^3}{2\pi^2} z^2 K_1(z) \Gamma(N_1 \to \ell\phi),\end{aligned}$$

where K_1 = Bessel fn. Taking limits of Bessel fns gives time dilation as expected :

$$\begin{aligned}&\text{pour } z \ll 1 && \gamma \simeq n_N^{eq} \Gamma \frac{M}{T} \\ &\text{pour } z \gg 1 && \gamma \simeq n_N^{eq} \Gamma\end{aligned}$$

For a $2 \to 2$ process : rate density γ^{ij}_{mn}

$$
\begin{aligned}
\gamma^{ij}_{mn} &= \langle n_i n_j \sigma(i+j \to ..)\rangle \\
&= \int d\Pi_i d\Pi_j f_i^{eq} f_j^{eq} \int |\mathcal{M}(i+j \to m+n)|^2 \; \tilde{\delta} \; d\Pi_m d\Pi_n \\
&= g_i g_j \int d\Pi_i d\Pi_j e^{-(E_i+E_j)/T} \sqrt{(p_i \cdot p_j)^2 - m_i^2 m_j^2} \; \sigma((p_i+p_j)^2),
\end{aligned}
$$

$g_i g_j$ because cross-sections averaged on spins. Used $f_{Boltzman}$

Can do integrals on initial state momenta, indep of $|\mathcal{M}|^2$, by putting $1 = \int d^4 Q \delta^4(Q - p_i - p_j)$ dans l'intégral : initial distributions are 2bdy phase-space—see before).

$$
\begin{aligned}
\gamma^{ij}_{mn} &= g_i g_j \int \frac{d^4 Q}{(2\pi)^4} e^{-Q_0/T} \frac{\sqrt{(p_i \cdot p_j)^2 - m_i^2 m_j^2}}{4\pi s} \\
&\qquad \times \sqrt{(p_i \cdot p_j)^2 - m_i^2 m_j^2} \sigma((p_i+p_j)^2),
\end{aligned}
$$

do some algebra

define $s = Q_0^2 - \vec{Q}^2$ and $[(p_i \cdot p_j)^2 - m_i^2 m_j^2] = s^2 \lambda\left(1, \frac{m_i^2}{s}, \frac{m_j^2}{s}\right)/4$,
with $\lambda(a, b, c) = (a - b - c)^2 - 4bc$:

$$\gamma(i + j \to m + n) \quad = \quad g_i g_j \int \frac{dQ_0 d^3 Q}{(2\pi)^4} \frac{e^{-Q_0/T}}{4\pi s} [s^2 \lambda \left(1, \frac{m_i^2}{s}, \frac{m_j^2}{s}\right)] \sigma(Q^2)$$

then use $d^3|\vec{Q}| = \sqrt{Q_0^2 - s}\, ds\, d\Omega/2$ and keep $\int dQ_0$:

$$\begin{aligned} \gamma(i + j \to m + n) &= \frac{g_i g_j}{128\pi^5} \int s ds\, d\Omega \int_{\sqrt{s}} dQ_0 e^{-Q_0/T} \sqrt{Q_0^2 - s}\, \lambda(..)\sigma(s) \\ &= \frac{g_i g_j T}{32\pi^4} \int ds s^{3/2} K_1\left(\frac{\sqrt{s}}{T}\right) \lambda\left(1, \frac{m_i^2}{s}, \frac{m_j^2}{s}\right) \sigma(s), \end{aligned}$$

Then in the massless limit, $\lambda(1, x, y) \to 1$, so using

$$\int_0^\infty x^n K_1(x) dx = 2^{n-1} \Gamma(1 + n/2) \Gamma(n/2)$$

get same scaling with T as back-of-envelope, different coefficient.

Units+useful relations

For $\hbar = c = k = 1$ (so $\hbar c = 197.3$ MeV fm, $(\hbar c)^2 = .3894$ GeV^2 mb), et $G_N = 1/m_{pl}^2$, $m_{pl} \simeq 1.2 \times 10^{19}$ GeV :

$$\begin{aligned} 1\mathrm{GeV} &\simeq 1.6 \times 10^{-3} erg \simeq 1.16 \times 10^{13}\ {}^oK \simeq 1.8 \times 10^{-24} gr \\ 1\mathrm{GeV}^{-1} &\simeq 2.0 \times 10^{-14} cm \\ &\simeq 6.6 \times 10^{-25} sec \end{aligned}$$

$$\begin{aligned} \mathrm{kiloparsec} &= 3 \times 10^{21} cm = 10^3 pc = 10^{-3} Mpc \\ keV &= 1.37 \times 10^{39} \mathrm{ergs/gr/sec} \end{aligned}$$

galaxie $\sim 10^{11}$ stars, $M \sim 10^{45}$ grammes, $R \sim 10^{23}$ cm ~ 100 kpc
sun(star) : $M \sim 2 \times 10^{33}$ grammes, $R \sim 7 \times 10^{10}$ cm

equilibrium distributions

Suppose particles in thermal/chemical equilibrium in early U (energy/conserved quantum numbers)
FRW has privileged coordinates, homog.+ isotropic, lets sit there.
FermiDirac (+) , BoseEinstein (-) phase space distributions are :

$$f_{i,\pm}^{\mathrm{eq}}(p) = \frac{1}{e^{(E_i-\mu_i)/T} \pm 1},$$

which gives equilibrium number densities ($m_i, \mu_i \ll T$,)

$$\begin{aligned} n_i^{\mathrm{eq}} &= \frac{g_i}{(2\pi)^3}\int d^3p f_i^{\mathrm{eq}}(p) \quad (g_i = \#\ \text{internal d.o.f. of part.}) \\ &\to \frac{g_i T^3}{\pi^2}\zeta(3) \times \left[1 + \frac{\mu_i \zeta(2)}{T\zeta(3)} + ...\right] \ \ \text{(bosons)} \\ &\to \frac{g_i T^3}{\pi^2}\zeta(3) \times \left[\frac{3}{4} + \frac{\mu_i \zeta(2)}{2T\zeta(3)} + ...\right] \ \ \text{(fermions)} \end{aligned}$$

$$\zeta(x+1) = \frac{1}{\Gamma(x+1)}\int_0^\infty \frac{t^x}{e^t - 1}dt \ \ , \ \ \frac{1}{e^t+1} = \frac{1}{e^t-1} - \frac{2}{e^{2t}-1}$$

where $\Gamma(n+1) = n!$, and $\zeta(2) = \pi^2/6$, $\zeta(3) = 1.202$, et $\zeta(4) = \pi^4/90$.

Is $f(x, p)$ appropriate in the early U at energies beyond the LHC and densities higher than the nucleus ?

T'is OK... $f(x, p)$ appears in the 2-pt function :

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T'is OK... $f(x, p)$ appears in the 2-pt function : for complex scalar ϕ :

$$\hat{\phi}(t, \vec{x}) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2k_0}} \left\{ \hat{a}_{\vec{k}} e^{-ik\cdot x} + \hat{b}^\dagger_{\vec{k}} e^{ik\cdot x} \right\}$$

With conserved Noether current :

$$\phi^\dagger(\partial_t \phi) - (\partial_t \phi^\dagger)\phi$$

Write 0-component (# op) as 2-pt function, where
$X \sim$ scale of system $\gg \delta \sim$ size of particles

$$\hat{N}(X - \frac{\delta}{2}, X + \frac{\delta}{2}) = \hat{\phi}^\dagger(X - \frac{\delta}{2})\partial_t \hat{\phi}(X + \frac{\delta}{2}) + \partial_t \hat{\phi}^\dagger(X - \frac{\delta}{2})\hat{\phi}(X + \frac{\delta}{2})$$

Then take fourier trans wrt δ :

$$\hat{N}(X,k) = \int \frac{d^4\delta}{(2\pi)^4} e^{ik\cdot\delta} \hat{N}(X-\delta/2, X+\delta/2)$$

Imagine to work with $\hat{a}(X)^\dagger, \hat{a}(X)$ X-dep, $\Leftrightarrow$ quantise in boxes $|\vec{\delta}|^3$ at pts X (discrete var).

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Imagine to work with $\hat{a}(X)^\dagger, \hat{a}(X)$ X-dep, $\Leftrightarrow$ quantise in boxes $|\vec{\delta}|^3$ at pts X (discrete var). Then For state $|n\rangle$, of particules of momenta k in the box at X :

$$\langle n|\hat{a}_k^\dagger(X)\hat{a}_p(X)|n\rangle = f(X,k)\delta^3(\vec{k}-\vec{p})(2\pi)^3$$

equilibrium energy density

$$\begin{aligned}
\rho_i^{\rm eq} &= \frac{4\pi g_i}{(2\pi)^3}\int dp\, p^2 \omega\, f_i^{\rm eq}(p) \\
&\to \quad m_i n_i = m_i g_i e^{-m_i/T}\left(\frac{m_i T}{2\pi}\right)^{3/2} \qquad (\mathrm{non-rel.}) \\
&\to \quad \frac{g_i T^4}{2\pi^2} 6\,\zeta(4) \times \left[1 + ...\right] \quad (m_i/T, \mu_i/T \to 0, \mathrm{bosons}) \\
&\to \quad \frac{g_i T^4}{2\pi^2} 6\,\zeta(4) \left[\frac{7}{8} + ...\right] \quad (m_i/T, \mu_i/T \to 0, \mathrm{fermions})
\end{aligned}$$

where $6\zeta(4) = \pi^4/15$.
So energy density of relativistic plasma is

$$\rho_{rad} = \frac{g_{eff}}{2}\frac{\pi^2 T^4}{15} = \frac{g_{eff}}{2}\rho_\gamma \qquad \mathrm{avec} \qquad g_{eff} \equiv \sum_{\overline{b},b} g_b + \frac{7}{8}\sum_{\overline{f},f} g_f$$

Bessel Fns

K_1 = Bessel fn (Gradshteyn and Ryzhik eqn 8.432.3) :

$$zK_1(z) = \int_z^\infty e^{-x}\sqrt{x^2 - z^2}dx \to \begin{cases} 1 & z \ll 1 \\ \sqrt{\frac{\pi z}{2}}e^{-z} & z \gg 1 \end{cases}$$

Recall :

$$n_{i,\mathrm{MB}}^{\mathrm{eq}} = \frac{g_i}{(2\pi)^3}\int d^3p f_{i,\mathrm{MB}}^{\mathrm{eq}}(p) = \frac{g_i T^3}{2\pi^2} z_i^2 K_2(z_i) \qquad z_i = \frac{m_i}{T} \neq 0, \ \mu = 0,$$

where $K_2(z)$ est fonction de Bessel :

$$z^2K_2(z) = \int_z^\infty x e^{-x}\sqrt{x^2 - z^2}dx \to \begin{cases} 2 & z \ll 1 \\ z\sqrt{\frac{\pi z}{2}}e^{-z} & z \gg 1 \end{cases}$$