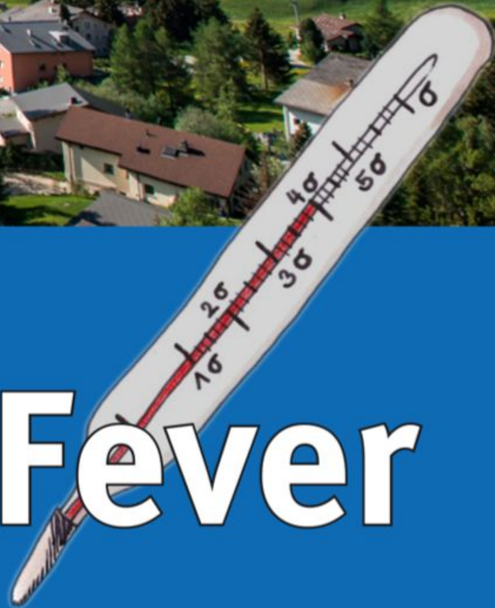
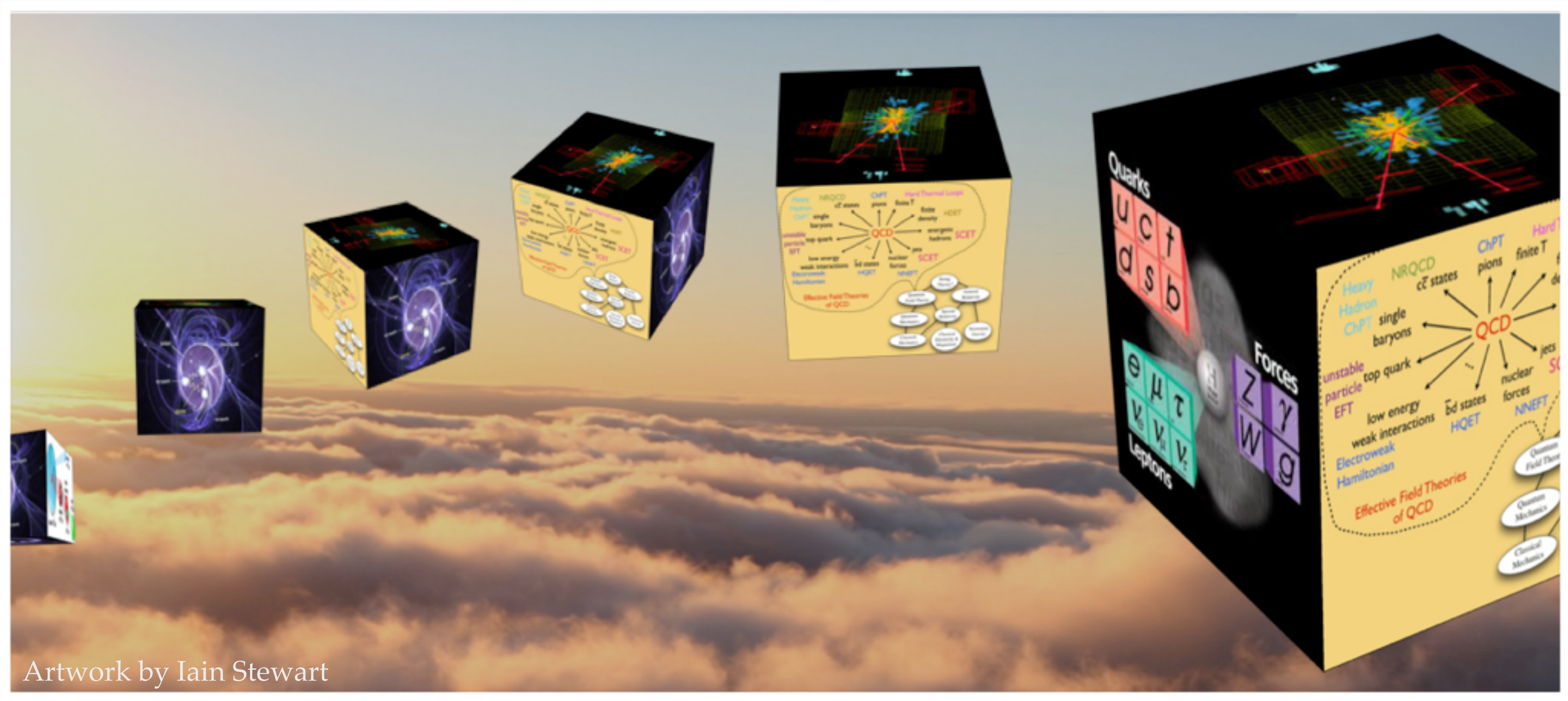
PSI Summer School

Particle Flavour Fever



Lyceum Alpinum, Zuoz, August 12–18, 2018



Artwork by Iain Stewart

2018 PSI Summer School — Particle Flavour Fever (Zuoz, Switzerland, 12-18 August 2018)

Flavor Physics in the Standard Model and Beyond

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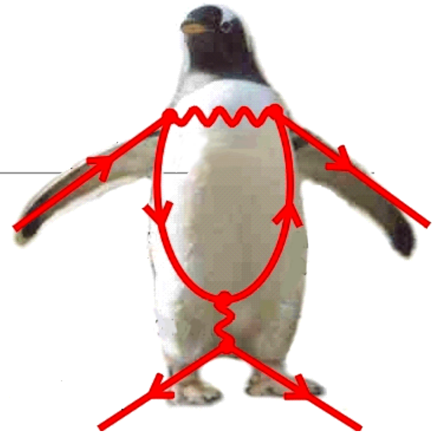
Quark flavor physics



Lecture I: Concepts of Quark Flavor Physics

- Introduction and motivation
- Yukawa couplings, CKM matrix, unitarity triangle (UT)
- Neutral meson mixing, some UT determinations
- CP violation in the interference of mixing and decay

Lecture II: Effective Weak Hamiltonians

Lecture III: Connecting UV Physics to Experiments

Lecture I: Concepts of Quark Flavor Physics

Flavor physics as an indirect BSM probe

The **hierarchy problem** (mechanism of EWSB) and the **origin of flavor** are two big mysteries of fundamental physics; connect to several deep questions:

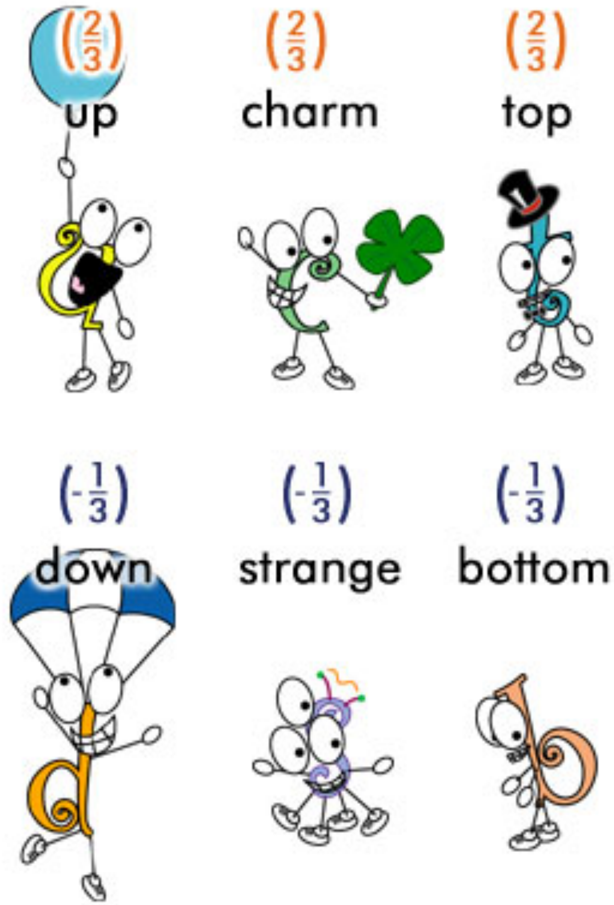
- **Origin of mass** of elementary particles?
- **Stability of the electroweak scale?**
- **Matter-antimatter asymmetry** in the Universe?
- **Origin of fermion generations** and the **hierarchies** in the spectrum of fermion masses and mixing angles?

In the SM, **flavor physics is connected to EWSB** via the Higgs Yukawa interactions

Higgs and flavor physics provide unique opportunities to probe the structure of electroweak interactions at the quantum level!

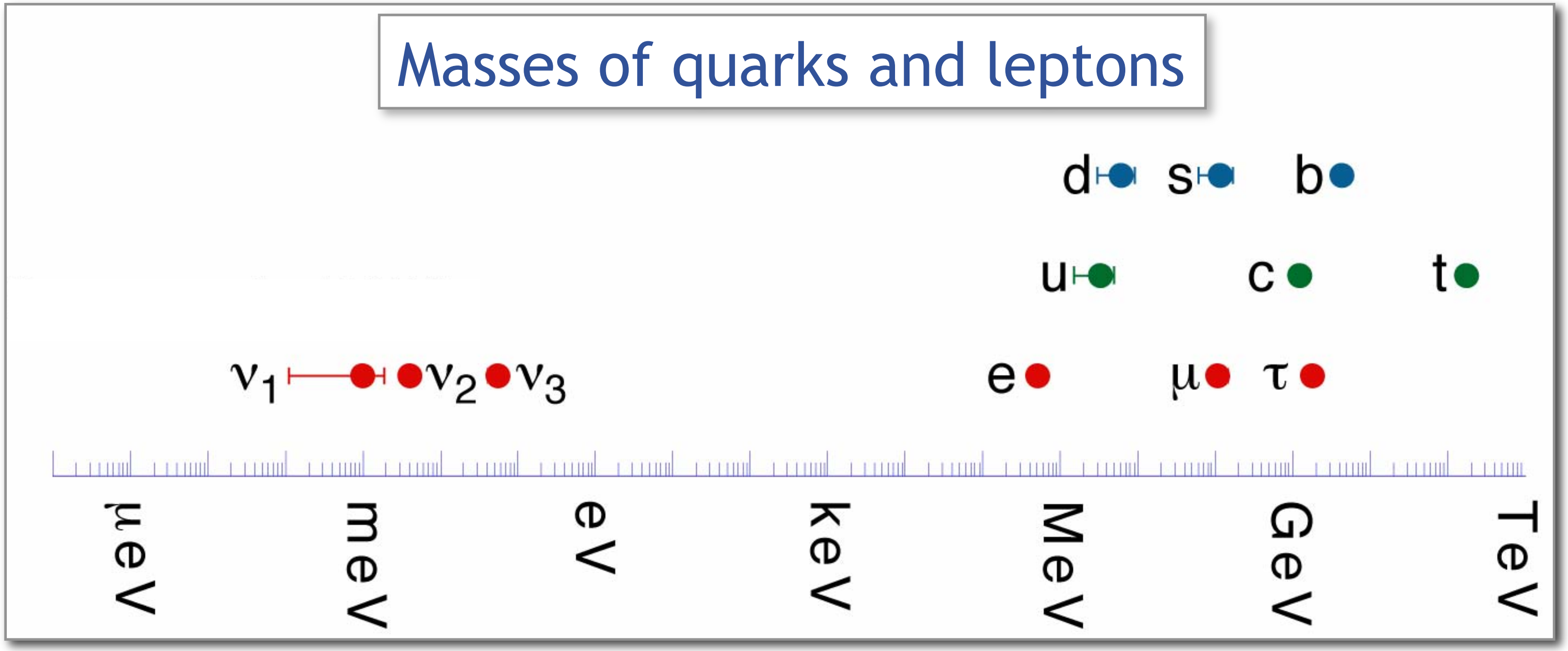
Flavor physics

- What is “flavor”?
- Generations: triplication of fermion spectrum without obvious necessity
- Dynamical explanation of flavor?
- Equally mysterious as dynamics of electroweak symmetry breaking
- Connection between two phenomena?



Flavor physics

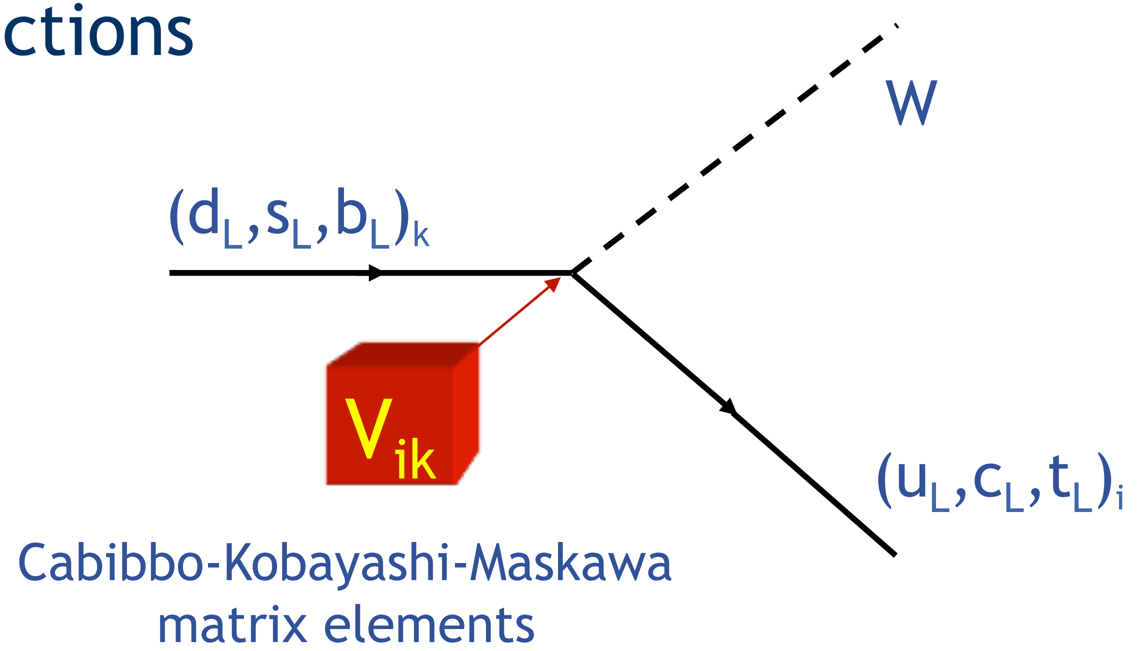
- Hierarchies in fermion mass spectrum:



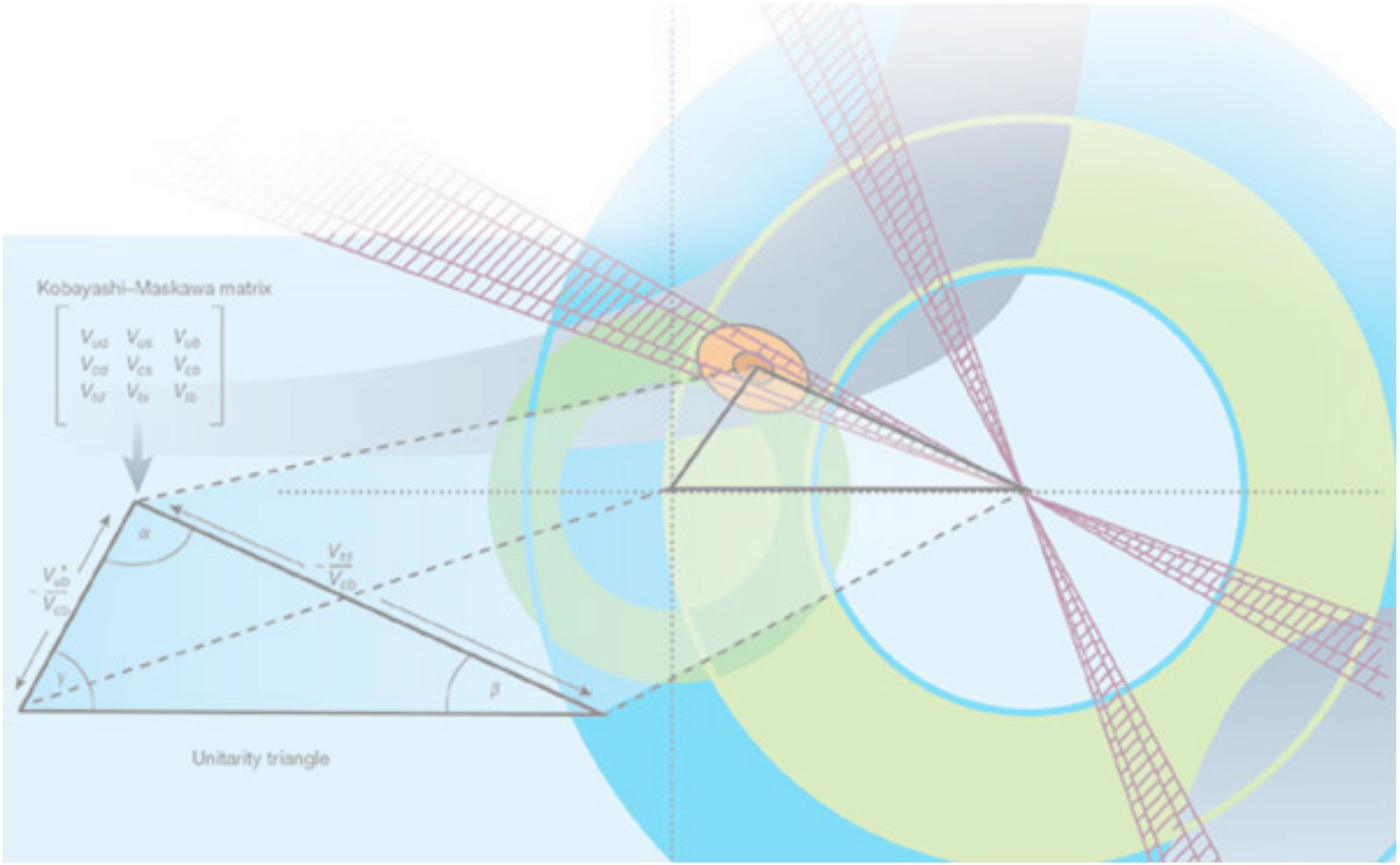
- Likewise, hierarchies in quark mixings

Flavor physics

- Flavor physics studies communication between different generations
- Standard Model: present only in charged-current interactions



Yukawa couplings and CKM matrix



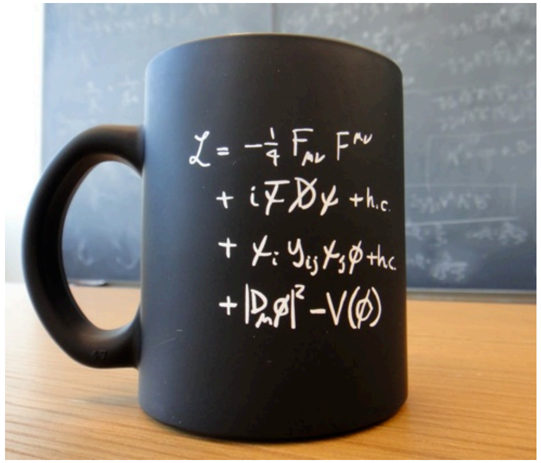
Yukawa couplings

$$\mathcal{L}_{\text{SM}} = -\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu,a} - \frac{1}{4}W^a_{\mu\nu}W^{\mu\nu,a} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

$$+\bar{Q}_L i\not{D}\, Q_L + \bar{u}_R i\not{D}\, u_R + \bar{d}_R i\not{D}\, d_R + \bar{L}_L i\not{D}\, L_L + \bar{e}_R i\not{D}\, e_R$$

$$+(D_\mu\phi)^\dagger D^\mu\phi - V(\phi)$$

$$-\bar{d}_R Y_d \phi^\dagger Q_L - \bar{u}_R Y_u \tilde{\phi}^\dagger Q_L - \bar{e}_R Y_e \phi^\dagger L_L + \text{h.c.}$$



SM Lagrangian is (almost) invariant under a huge global flavor symmetry $[U(3)]^5$,

$$U(3)_{Q_L} \otimes U(3)_{u_R} \otimes U(3)_{d_R} \otimes U(3)_{L_L} \otimes U(3)_{e_R}$$

which is broken only by the Yukawa interactions

Yukawa couplings

- Most general, gauge invariant and renormalizable interactions of Higgs and matter fields:

generation index

				$SU(2)_L$	$U(1)_Y$
L_L^i :	$\begin{pmatrix} \nu_e \\ e_L \end{pmatrix}$,	$\begin{pmatrix} \nu_\mu \\ \mu_L \end{pmatrix}$,	$\begin{pmatrix} \nu_\tau \\ \tau_L \end{pmatrix}$	2	-1/2
Q_L^i :	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$,	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$,	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$	2	+1/6
e_R^i :	e_R ,	μ_R ,	τ_R	1	-1
u_R^i :	u_R ,	c_R ,	t_R	1	+2/3
d_R^i :	d_R ,	s_R ,	b_R	1	-1/3

Yukawa couplings

$$\Phi : \begin{pmatrix} \phi_1^+ \\ \phi_2^0 \end{pmatrix}, \qquad \widetilde{\Phi} = i\sigma_2\Phi^* : \begin{pmatrix} \phi_2^{*0} \\ -\phi_1^{*-} \end{pmatrix}$$

	SU(2)$_L$	U(1)$_Y$
	2	±1/2

- Yukawa couplings:

$$\boxed{\mathcal{L}_Y = -\bar{e}_R^i\, Y_e^{ij}\, \Phi^\dagger\, L_L^j - \bar{d}_R^i\, Y_d^{ij}\, \Phi^\dagger\, Q_L^j - \bar{u}_R^i\, Y_u^{ij}\, \widetilde{\Phi}^\dagger\, Q_L^j + \text{h.c.}}$$

Y: 1 -1/2 -1/2 1/3 -1/2 +1/6 -2/3 +1/2 +1/6

- Y_e, Y_d, Y_u: arbitrary complex 3x3 matrices
- Electroweak symmetry breaking: $\langle\phi_2^0\rangle = v/\sqrt{2}$

Yukawa couplings

- Gauge principle allows arbitrary generation-changing interactions, since fermions of different generations have equal gauge charges!
- Usually such couplings are eliminated by field redefinitions:

$$\psi^i \rightarrow U^{ij}\,\psi^j$$

unitary (i.e., probability preserving) “rotation” in generation space

Yukawa couplings

- Diagonalize Yukawa matrices using biunitary transformations, e.g.:

$$Y_e = W_e \, \lambda_e \, U_e^\dagger \, ; \qquad \lambda_e = \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

- Then perform field redefinitions:

$$e_L \rightarrow U_e \, e_L \, , \quad e_R \rightarrow W_e \, e_R$$
$$u_L \rightarrow U_u \, u_L \, , \quad u_R \rightarrow W_u \, u_R$$
$$d_L \rightarrow U_d \, d_L \, , \quad d_R \rightarrow W_d \, d_R$$

- This diagonalizes the mass terms, giving masses $m_f = y_f \, (v/\sqrt{2})$ to all fermions

CKM matrix

- Effect of field redefinitions on weak interactions in the mass basis (QCD and QED invariant)
- Charged currents:

$$\mathcal{L}_{cc} = \frac{g_2}{\sqrt{2}} W^\mu \left(\bar{u}_L, \bar{c}_L, \bar{t}_L\right) \gamma_\mu V \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + \mathrm{h.c.}\,; \qquad V = U_u^\dagger U_d$$

  - generation changing couplings proportional to V_{ij}:

$$d_L^i \to u_L^j + W^- \quad \propto V_{ji} \qquad\qquad u_L^i \to d_L^j + W^+ \quad \propto V_{ij}^*$$

(Cabibbo-Kobayashi-Maskawa matrix)

CKM matrix

- Neutral currents:

$$\mathcal{L}_{\rm nc} = \frac{g_2}{\cos\theta_W} Z^\mu \sum_f \Big[\bar{f}_L U_f^\dagger \left(T_f^3 \frac{1-\gamma_5}{2} - Q_f \sin^2\theta_W \right) U_f f_L$$

$$+ \bar{f}_R W_f^\dagger \left(-Q_f \sin^2\theta_W \right) W_f f_R \Big]$$

cancel each other

  - no generation-changing interactions!
    (at level of elementary vertices)
  - GIM mechanism (Glashow-Iliopoulos-Maiani, 1970)
  - led to prediction of charm quark (K-$\bar{\text{K}}$ mixing)
- Likewise, Higgs couplings are flavor-diagonal in the fermion mass basis (only in SM)

CKM matrix

- For yet unknown reasons, the quark mixing matrix is strongly hierarchical
- This yields to the suppression of flavor-changing processes in the SM

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$|V_{\text{CKM}}| = \begin{pmatrix} 0.97434^{+0.00011}_{-0.00012} & 0.22506 \pm 0.00050 & 0.00357 \pm 0.00015 \\ 0.22492 \pm 0.00050 & 0.97351 \pm 0.00013 & 0.0411 \pm 0.0013 \\ 0.00875^{+0.00032}_{-0.00033} & 0.0403 \pm 0.0013 & 0.99915 \pm 0.00005 \end{pmatrix}$$

CKM matrix

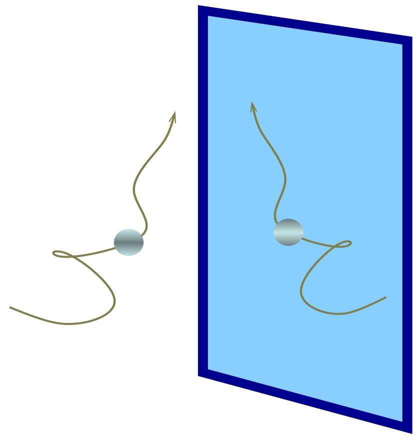
- Unitary 3x3 matrix V can by parameterized by 3 Euler angles und 6 phases
- Not all phases are observable, since under phase redefinitions $q \rightarrow e^{i\varphi_q} q$ of the quark fields:

$$V \rightarrow \begin{pmatrix} e^{-i\varphi_u} & 0 & 0 \\ 0 & e^{-i\varphi_c} & 0 \\ 0 & 0 & e^{-i\varphi_t} \end{pmatrix} V \begin{pmatrix} e^{i\varphi_d} & 0 & 0 \\ 0 & e^{i\varphi_s} & 0 \\ 0 & 0 & e^{i\varphi_b} \end{pmatrix}, \qquad V_{ij} \rightarrow e^{i(\varphi_d^i - \varphi_u^j)}\, V_{ij}$$

- 5 of 6 phases can be eliminated by suitable choices of phase differences!

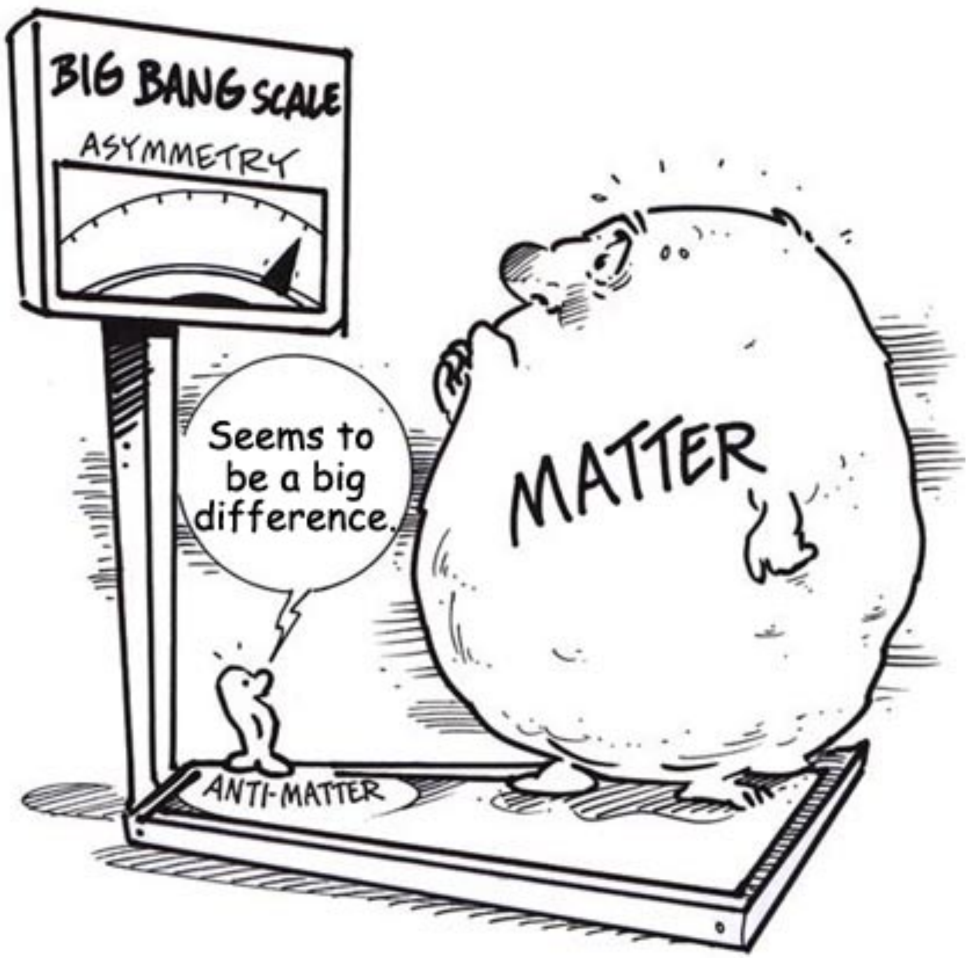
CKM matrix

- Remaining phase δ_{CKM} is source of all CP-violating effects in Standard Model (assuming θ_{QCD}=0)
  - weak interactions couple to left-handed fermions and right-handed antifermions
  - violate P and C maximally, but would be invariant under CP and T if all weak couplings were real
  - physical phase of CKM matrix breaks CP invariance



- Allows for an absolute distinction between matter and antimatter!

CKM matrix



... and we still do not understand this difference!

CKM matrix

- CP violation required to explain the different abundances of matter and antimatter in the universe (baryogenesis)
- CP violation in quark sector requires N≥3 fermion generations
- Model for explanation of CP violation led to prediction of the third generation!

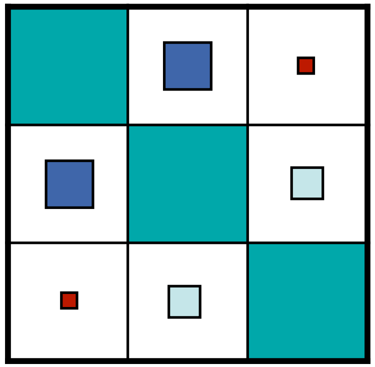
  Kobayashi, Maskawa (1973)

CKM matrix

- Form of V not unique (phase conventions)
- Several parameterizations used; a very useful one is due to Wolfenstein (1983):

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

- Hierarchical structure in $\lambda \approx 0.22$
- Remaining parameters O(1)
- Complex entries $O(\lambda^3)$



CKM matrix

- Jarlskog determinant:

  for arbitrary choice of i,j,k,l the quantity

$$\boxed{\mathrm{Im}(V_{ij}V_{kl}V_{il}^{*}V_{kj}^{*}) = J \sum_{m,n} \varepsilon_{ikm}\,\varepsilon_{jln}}$$

  is an invariant of the CKM matrix (independent of phase conventions)

- CP invariance is broken if and only if $J \neq 0$
- Wolfenstein parameterization:

$$J = O(\lambda^6) = O(10^{-4}) \quad \text{rather small}$$

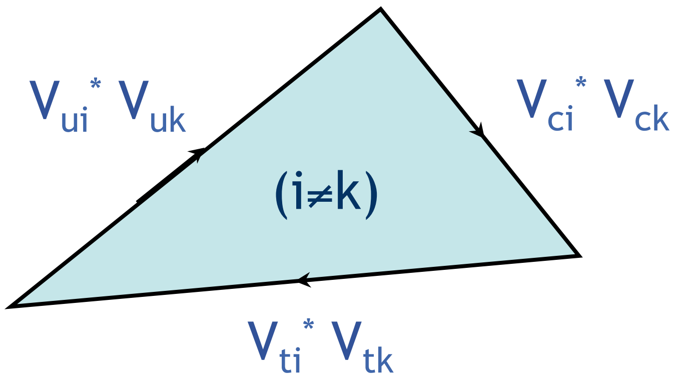
- From data: $J = (3.04^{+0.21}_{-0.20}) \times 10^{-5}$

Unitarity triangle

- Unitarity relation $V^\dagger V = V V^\dagger = 1$ implies:

$$\boxed{V_{ji}^* V_{jk} = \delta_{ik} \text{ and } V_{ij}^* V_{kj} = \delta_{ik}}$$

- For $i \neq k$ this gives 6 triangle relations, in which a sum of 3 complex numbers adds up to zero:



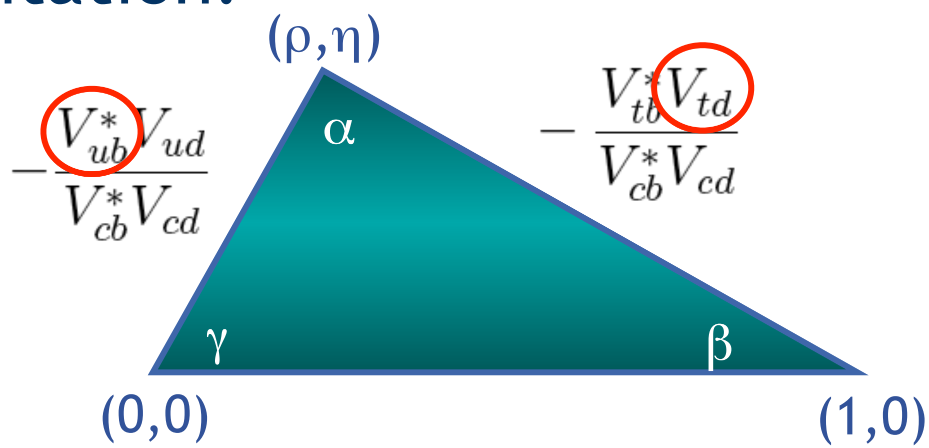
area = J/2

Unitarity triangle

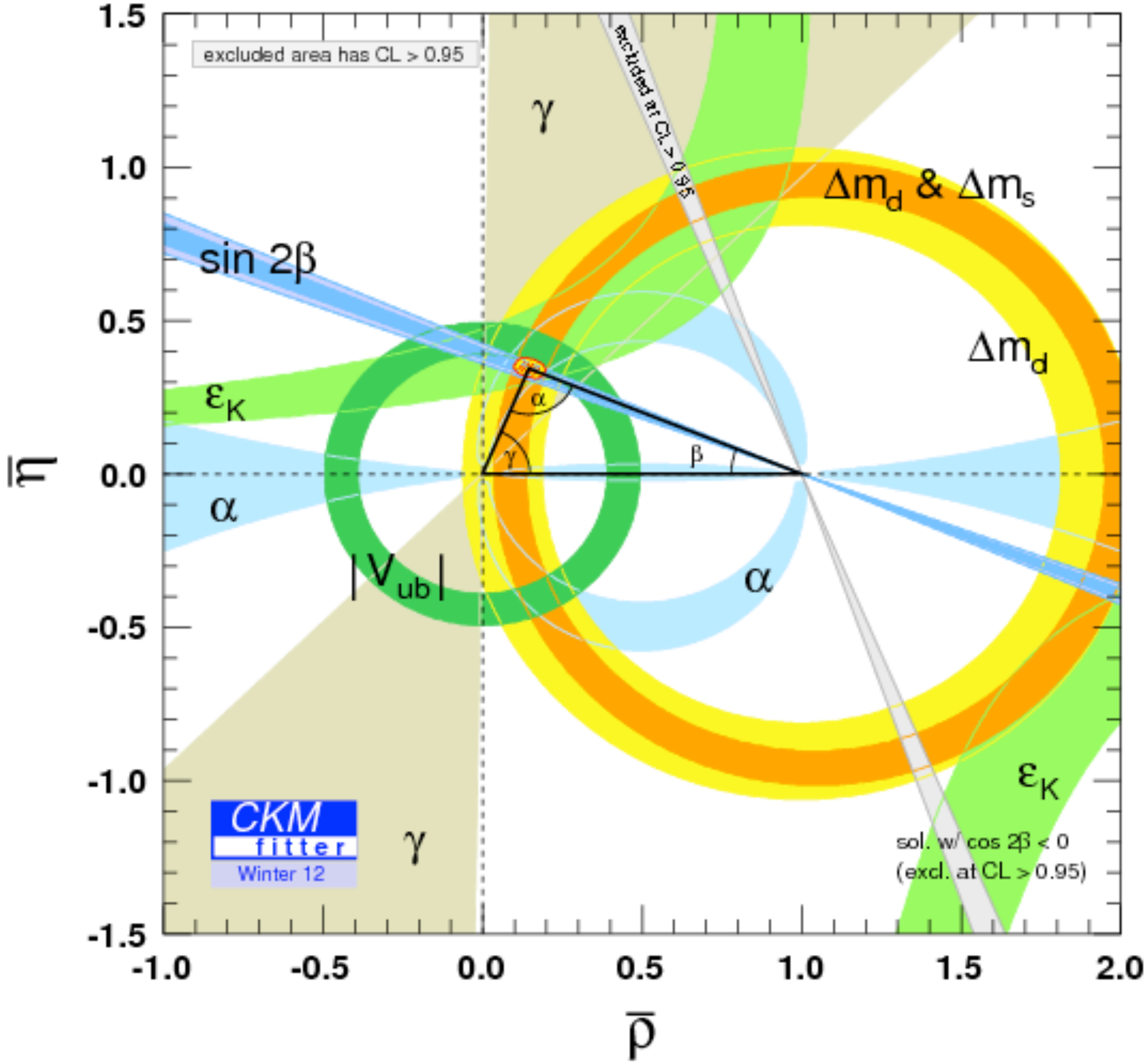
- Phase redefinitions turn triangles
- For two triangles, all sides are of same order in λ; *the* unitarity triangle is:

$$\boxed{V_{ub}^{*} V_{ud} + V_{cb}^{*} V_{cd} + V_{tb}^{*} V_{td} = 0}$$

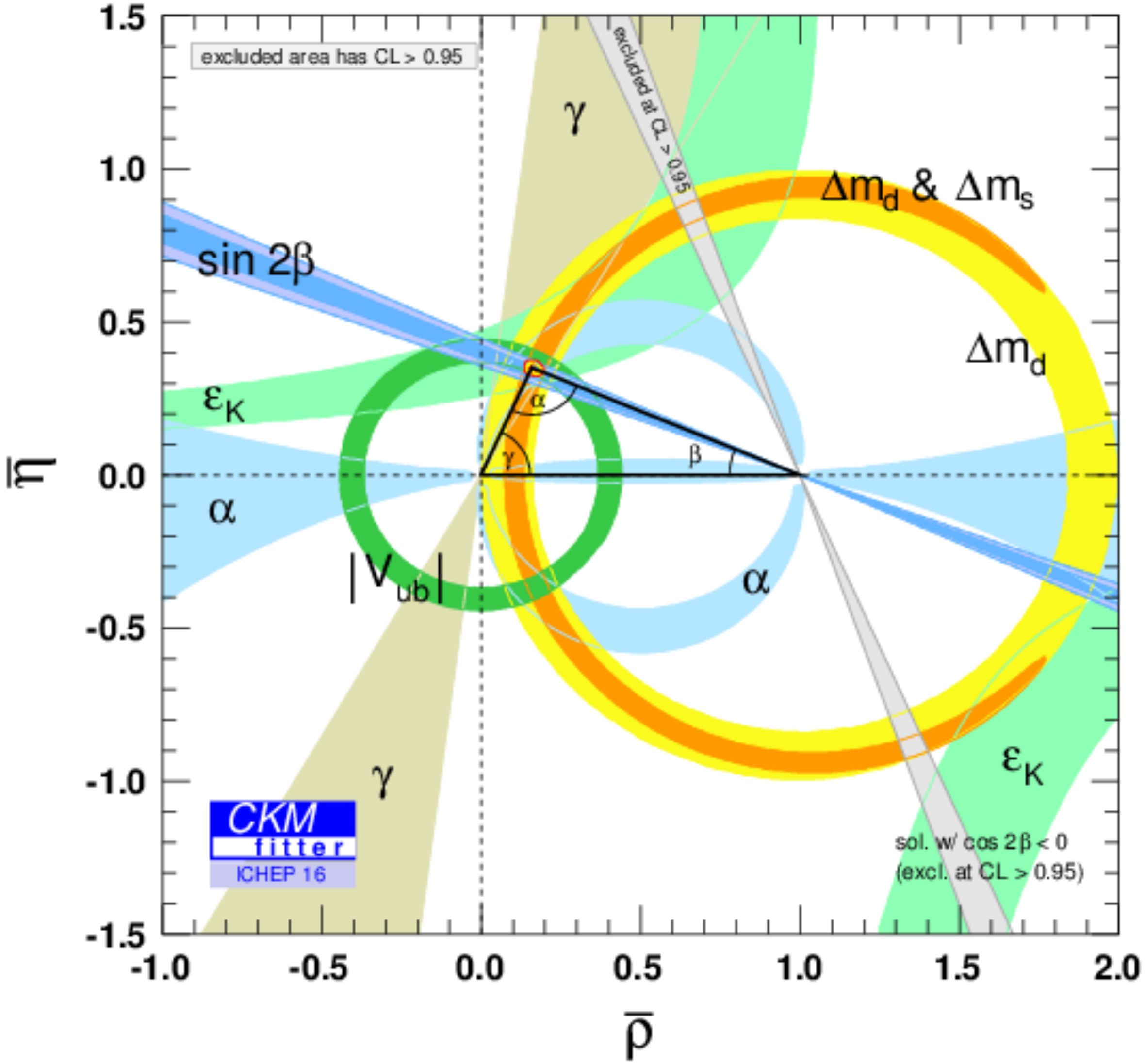
- Graphical representation:



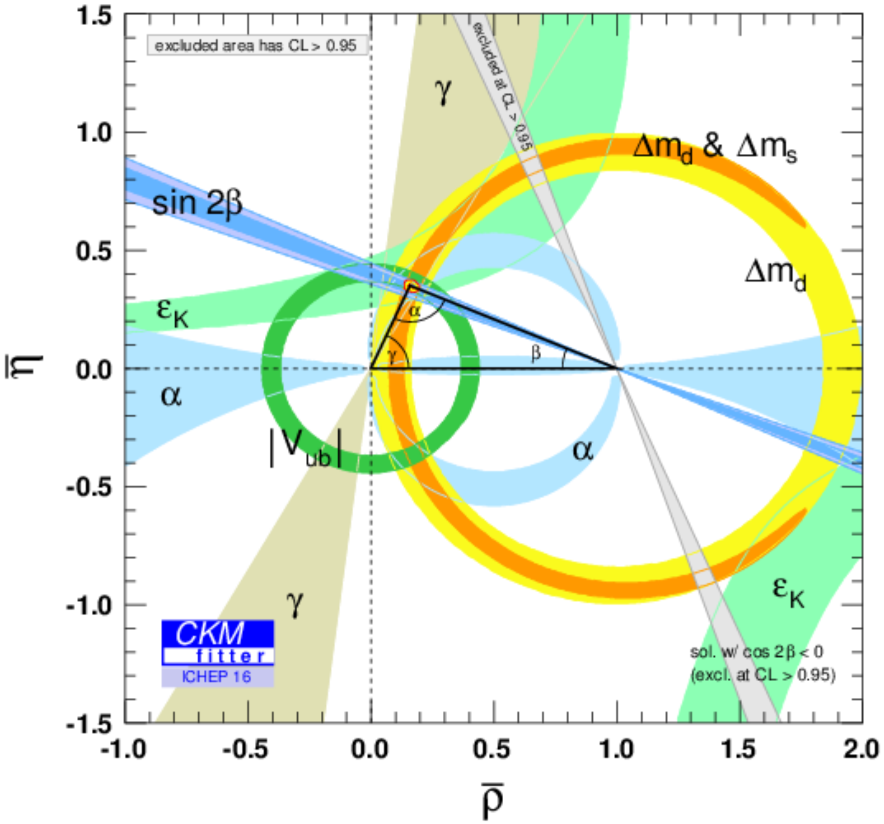
2012 knowledge of the unitarity triangle

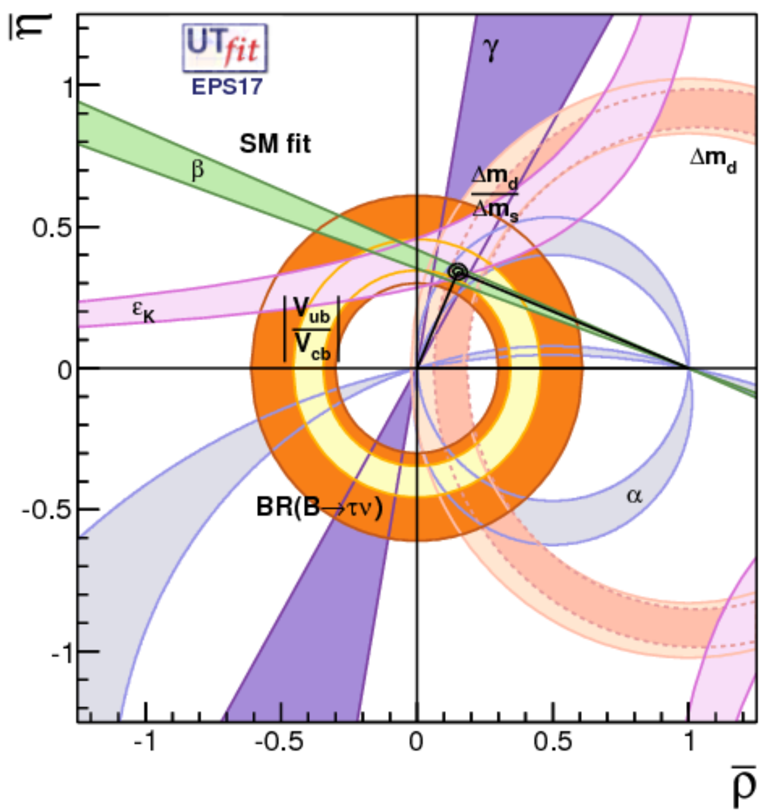


Present knowledge of the unitarity triangle



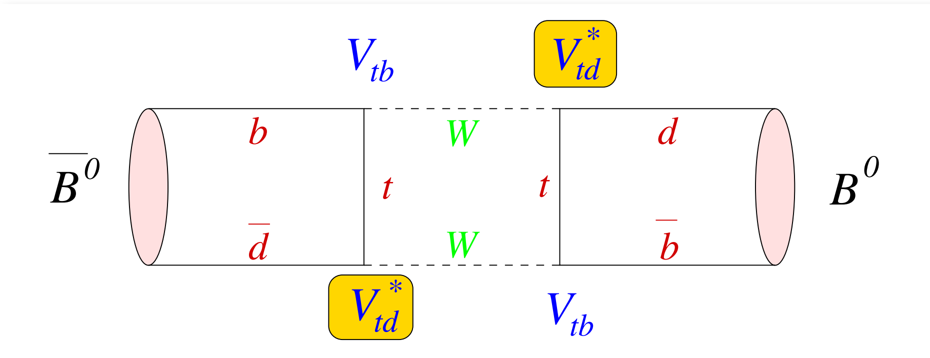
Present knowledge of the unitarity triangle

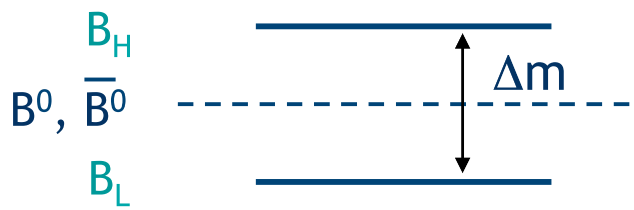




Oscillations of neutral mesons

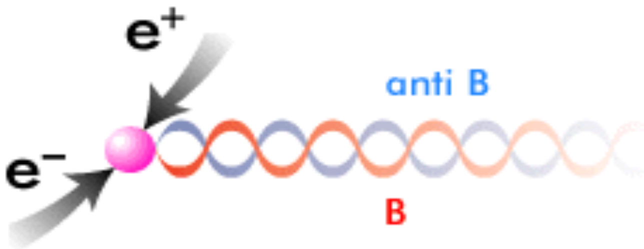
- Neutral mesons can be transformed into their antiparticles by second-order weak processes
- Analogy with quantum-mechanical system of coupled pendulums: state B^0 at t=0 develops into a superposition of states B^0 and $\overline{B}^0$ with time-oscillating amplitudes

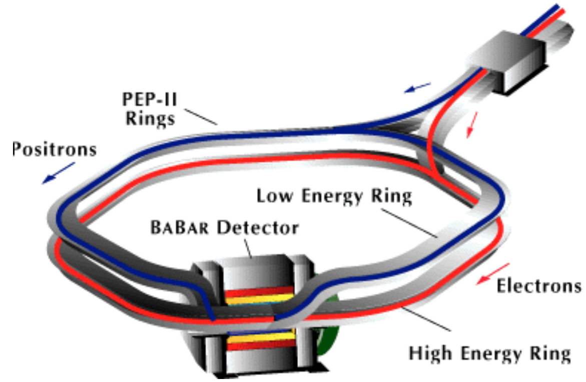




Oscillations of neutral mesons

- B-factories produce pairs of B^0 and $\overline{B}^0$ mesons in coherent quantum states





- Decay of one meson (with reconstruction of its flavor) initiates time measurement for the other meson

Quantum-mechanical treatment

(neglect exponential decay for simplicity)

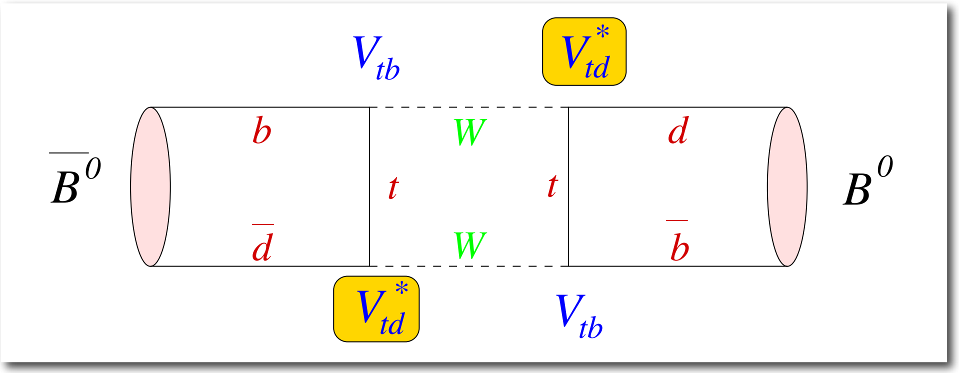
- Schrödinger equation for B^0 and $\bar{B}^0$:

$$i\frac{d}{dt}\begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix} = \begin{pmatrix} M & \frac{1}{2}e^{-2i\beta}\Delta m \\ \frac{1}{2}e^{2i\beta}\Delta m & M \end{pmatrix}\begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix},$$

mass eigenvalues:

$$M_{\pm} = M \pm \frac{\Delta m}{2}$$

- Non-diagonal entry due to box diagram:



$\propto (V_{tb}V_{td}^*)^2 \propto e^{2i\beta}$

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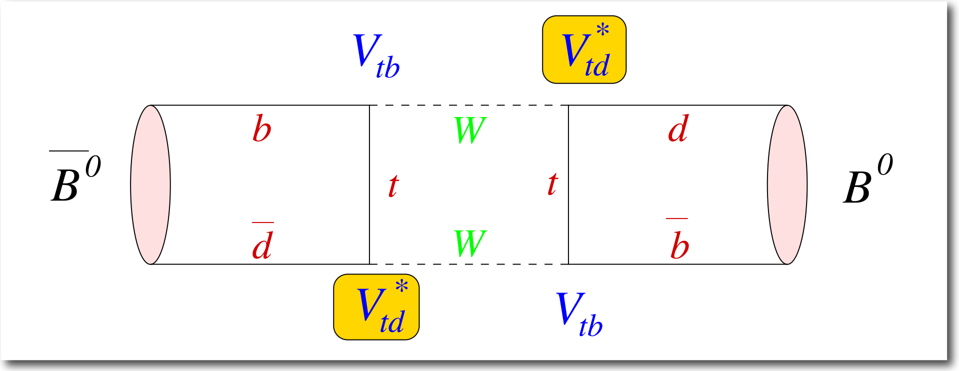


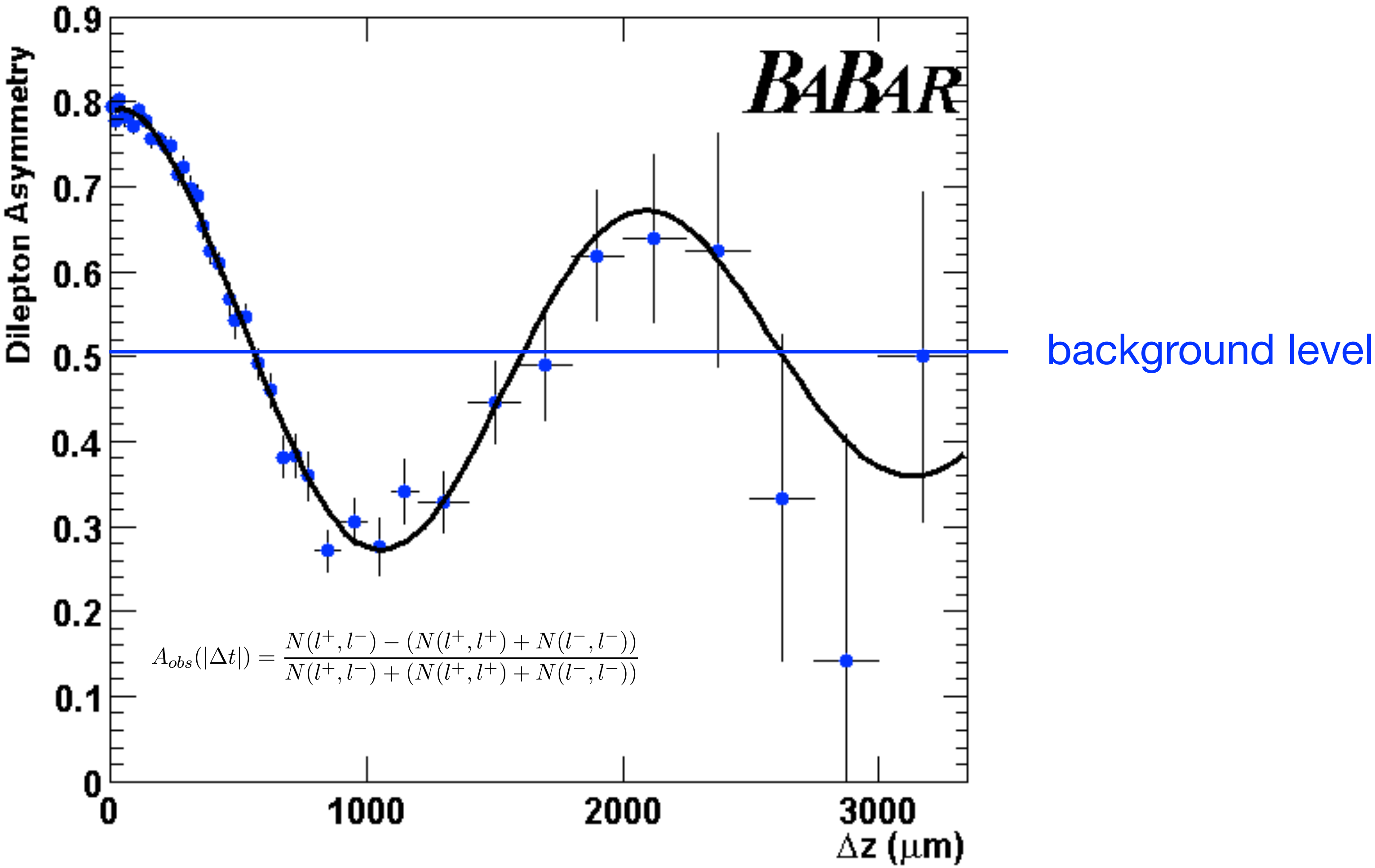
$\propto (V_{tb}V_{td}^*)^2 \propto e^{2i\beta}$

- Time evolution of an initial (at t=0) $\bar{B}^0$ state:

$$\boxed{|\psi(t)\rangle = e^{-iMt}\left[\cos(\tfrac{1}{2}\Delta m t)\,|\overline{B^0}\rangle + ie^{2i\beta}\sin(\tfrac{1}{2}\Delta m t)\,|B^0\rangle\right]}$$

Oscillations of neutral mesons

$$\frac{N(B^0\overline{B}^0)(\Delta t) - (N(B^0B^0)(\Delta t) + N(\overline{B}^0\overline{B}^0)(\Delta t))}{N(B^0\overline{B}^0)(\Delta t) + (N(B^0B^0)(\Delta t) + N(\overline{B}^0\overline{B}^0)(\Delta t))} = \cos(\Delta m_{B^0} \cdot \Delta t)$$



Determination of $|V_{td}|$ from Δm

- Master formula:

$$\Delta m = \frac{G_F^2 M_W^2}{16\pi^2}\,|V_{tb}V_{td}^*|^2\, S_0\!\left(\frac{m_t^2}{M_W^2}\right)\eta_{\rm QCD}\,\frac{1}{m_B}\,\langle B^0|\,(\bar{d}b)_{V-A}(\bar{d}b)_{V-A}\,|\bar{B}^0\rangle$$

$$S_0(x_t) \approx 0.784\, x_t^{0.76}$$

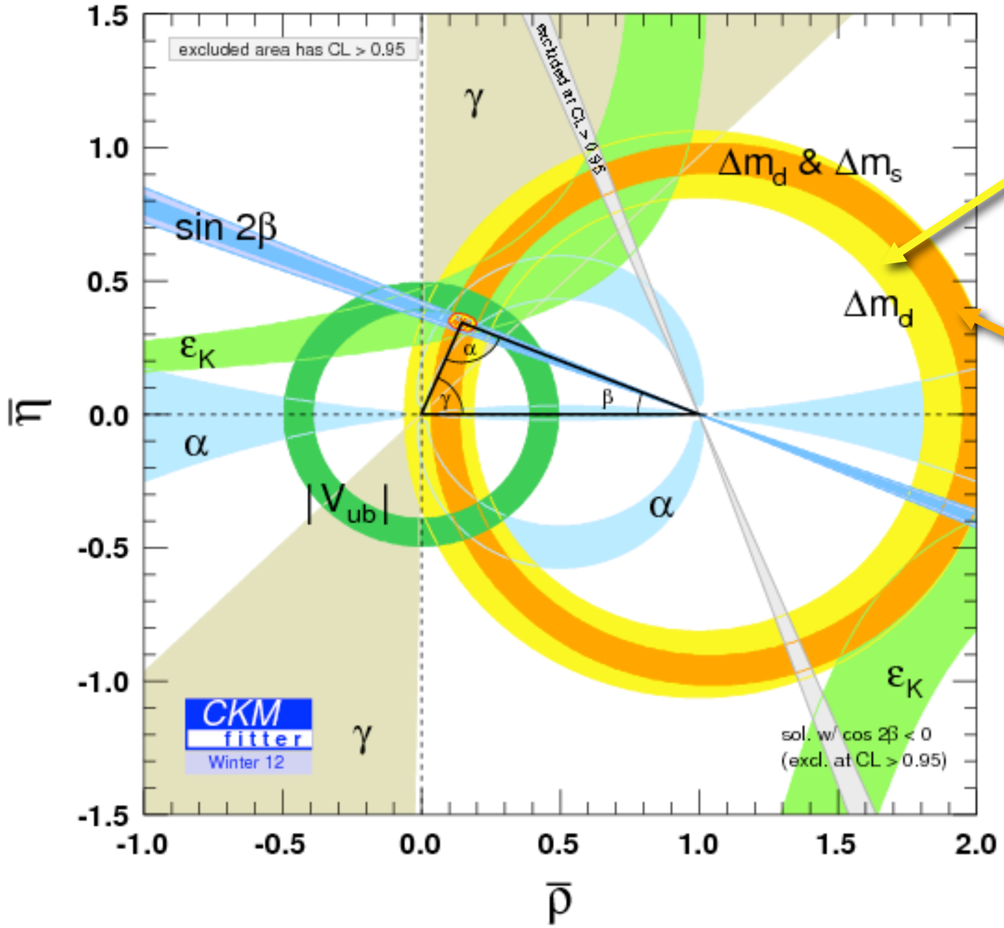
perturbative QCD correction

$$\equiv \frac{8}{3} B_B f_B^2 m_B^2$$

(from lattice QCD)

- Discovery of B-$\bar{\text{B}}$ mixing (ARGUS experiment, 1987) pointed to a very heavy top quark!

Determination of $|V_{td}|$ from Δm

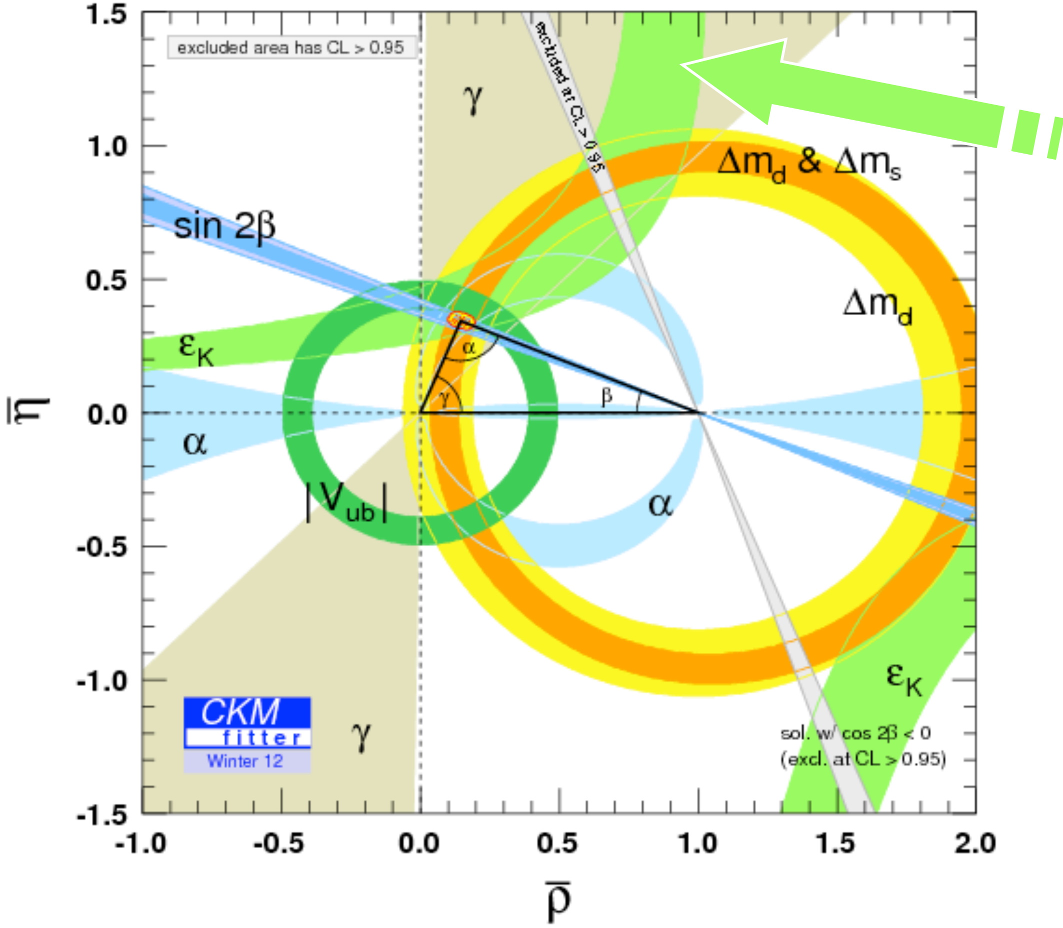


result derived from B_d mixing alone (large theoretical uncertainties)

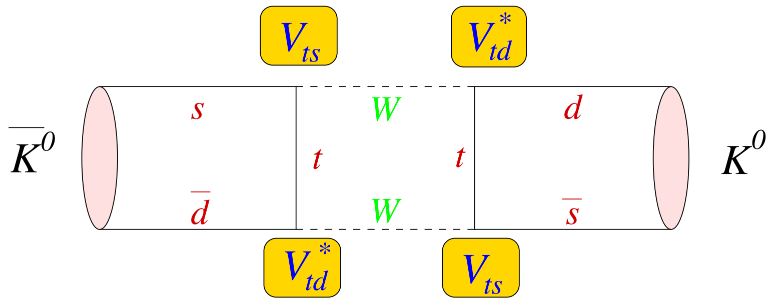
result derived from ratio of B_d and B_s mixing frequencies (reduced theoretical uncertainties)

$$\frac{\Delta m(B_d)}{\Delta m(B_s)} = \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{B_{B_d} f_{B_d}^2 m_{B_d}}{B_{B_s} f_{B_s}^2 m_{B_s}}$$

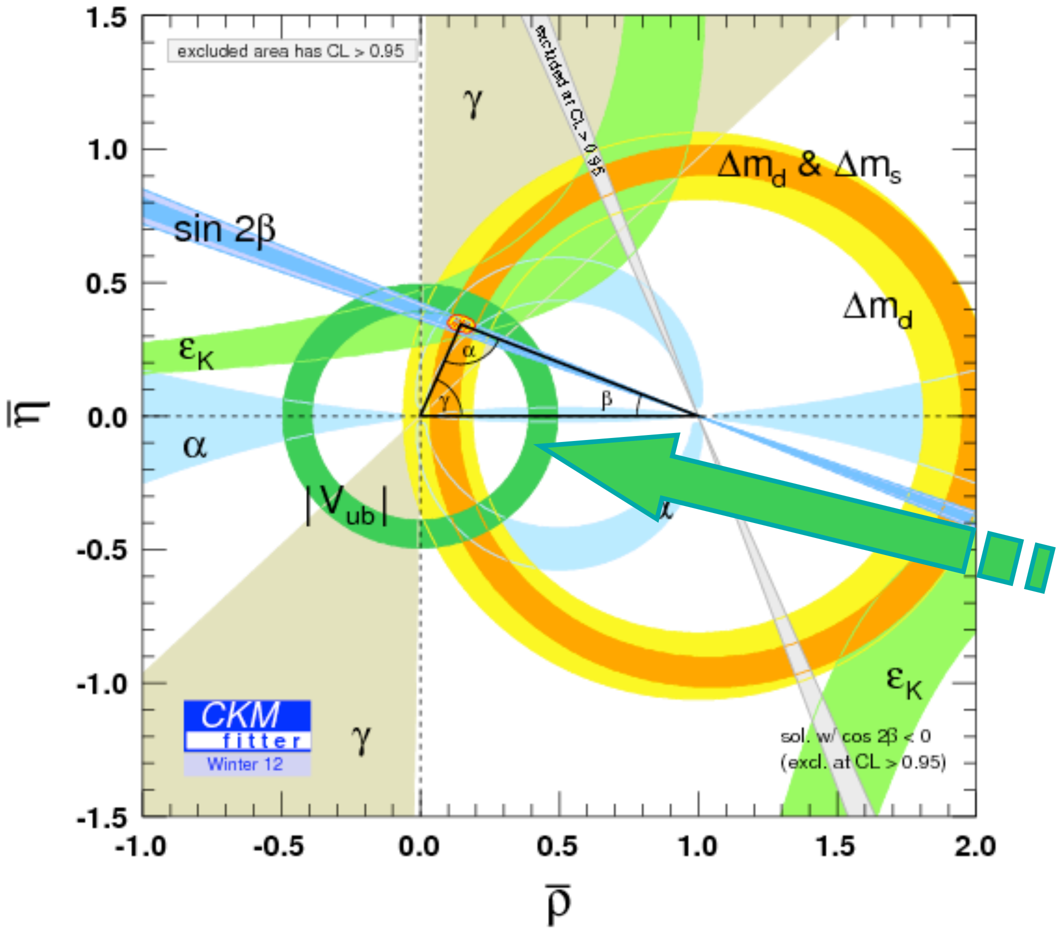
Determination of Im(V_{td}^2) from kaon mixing



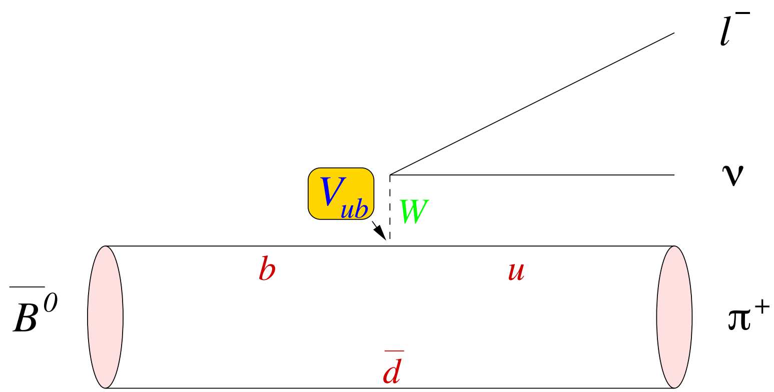
- Determination of Im(V_{td}^2) from CP violation in K^0-$\bar{K}^0$ mixing
- Large hadronic uncertainties (lattice QCD)



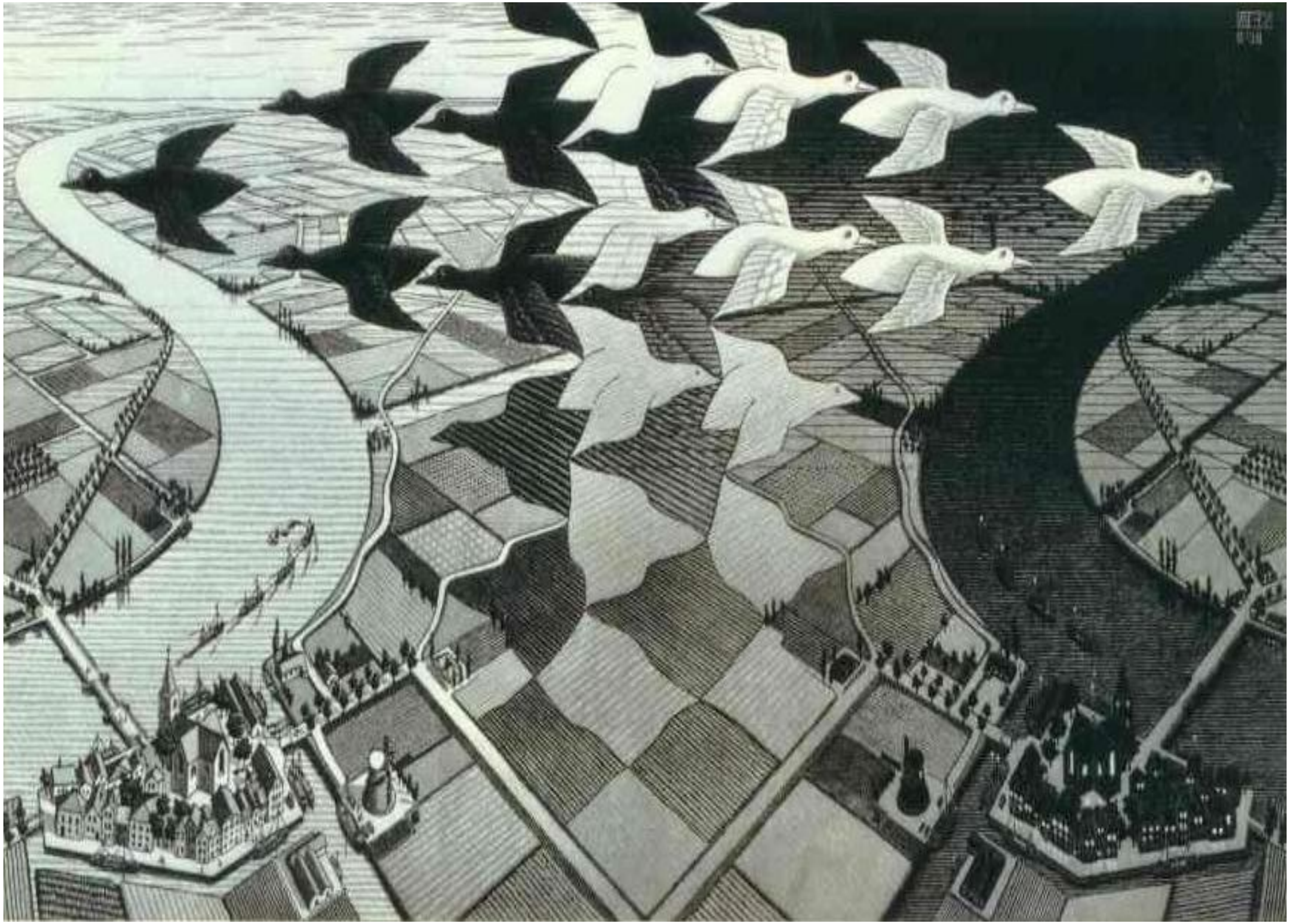
Determination of $|V_{ub}|$ from semileptonic decays



- Determination of $|V_{ub}|$ in inclusive and exclusive semileptonic B decays



CP violation in interference of mixing and decay



A more subtle quantum-mechanical effect:

Interference of mixing and decay in neutral B-meson decays into CP eigenstates

Time-dependent CP asymmetry provides **direct access to angles** of the unitarity triangle

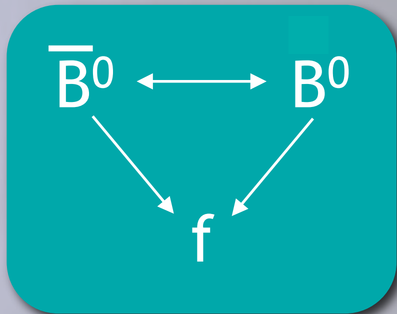
To see how this works, use our previous result for the time dependence of an initial $\bar{B}^0$ state (at t=0)

CP violation in interference of mixing and decay

- Time evolution of an initial (at t=0) $\overline{B}^0$ state:

$$|\psi(t)\rangle = e^{-iMt}\left[\cos(\tfrac{1}{2}\Delta mt)\,|\overline{B}^0\rangle + ie^{2i\beta}\sin(\tfrac{1}{2}\Delta mt)\,|B^0\rangle\right]$$

- Consider decay of a CP eigenstate f, with decay amplitudes A for $\overline{B}^0 \to f$ and $\bar{A}$ for $B^0 \to f$



- Amplitude for this decay at time t>0:

$$\mathcal{A}_{\bar{B}^0}(t) = e^{-iMt}\left[A\cos(\tfrac{1}{2}\Delta mt) + i\bar{A}\,e^{2i\beta}\sin(\tfrac{1}{2}\Delta mt)\right]$$

direct decay

indirect decay via mixing

CP violation in interference of mixing and decay

- Time dependence of decay rate:

$$\Gamma_{\bar{B}^0\to f}(t) \propto |A|^2\cos^2\frac{\Delta m t}{2} + |\bar{A}|^2\sin^2\frac{\Delta m t}{2} - \mathrm{Im}(A^*\bar{A}\,e^{2i\beta})\sin\Delta m t$$

$$\propto 1 + \frac{|A|^2-|\bar{A}|^2}{|A|^2+|\bar{A}|^2}\cos\Delta m t - \frac{2\mathrm{Im}(A^*\bar{A}\,e^{2i\beta})}{|A|^2+|\bar{A}|^2}\sin\Delta m t$$

- Rate for CP-conjugate process $B^0 \to \bar{f}$ given by same expression with $A \leftrightarrow \bar{A}$ and $\beta \to -\beta$

CP violation in interference of mixing and decay

- Time-dependent CP asymmetry:

$$A_{\mathrm{CP}}(t) \equiv \frac{\Gamma_{\bar{B}^0\to f}(t) - \Gamma_{B^0\to f}(t)}{\Gamma_{\bar{B}^0\to f}(t) + \Gamma_{B^0\to f}(t)} = \mathcal{C}\cos(\Delta m\, t) - \mathcal{S}\sin(\Delta m\, t)$$

$$\mathcal{C} = \boxed{\frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2}} \qquad \mathcal{S} = \boxed{\frac{2\mathrm{Im}(A^*\bar{A}\, e^{2i\beta})}{|A|^2 + |\bar{A}|^2}}$$

(direct CP asymmetry)

- Special case: decay amplitude dominated by a single partial amplitude with weak phase φ_A

$\Rightarrow$ $\mathcal{C} = 0$ and $\mathcal{S} = \sin[2(\beta-\varphi_A)]$

CP violation in interference of mixing and decay

- Allows determination of a weak phase (almost) free of hadronic uncertainties!
- 2 possibilities in SM:

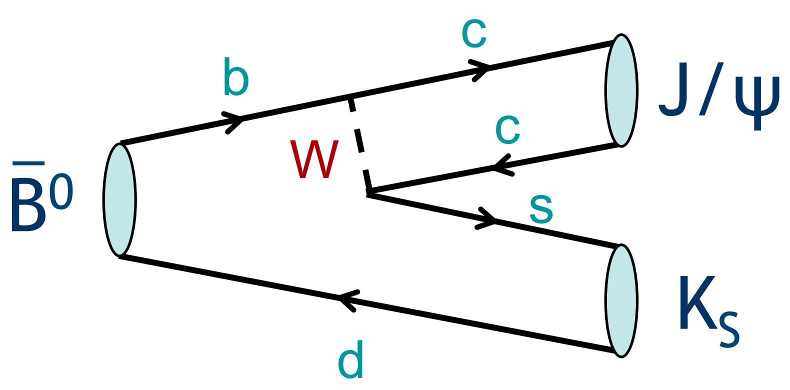
$\varphi_A = 0 \quad \Rightarrow \quad S = \sin(2\beta)$ (e.g. $B \to J/\psi K_S$, ϕK_S)

$\varphi_A = -\gamma \quad \Rightarrow \quad S = \sin[2(\beta + \gamma)] = -\sin(2\alpha)$ (e.g. $B \to \pi\pi, \rho\rho$)

- Comparing $\sin 2\beta$ values extracted from tree-dominated vs. loop-dominated processes is a sensitive probe for **New Physics**

CP violation in interference of mixing and decay

- “Golden” decay $B \rightarrow J/\psi\ K_S$:

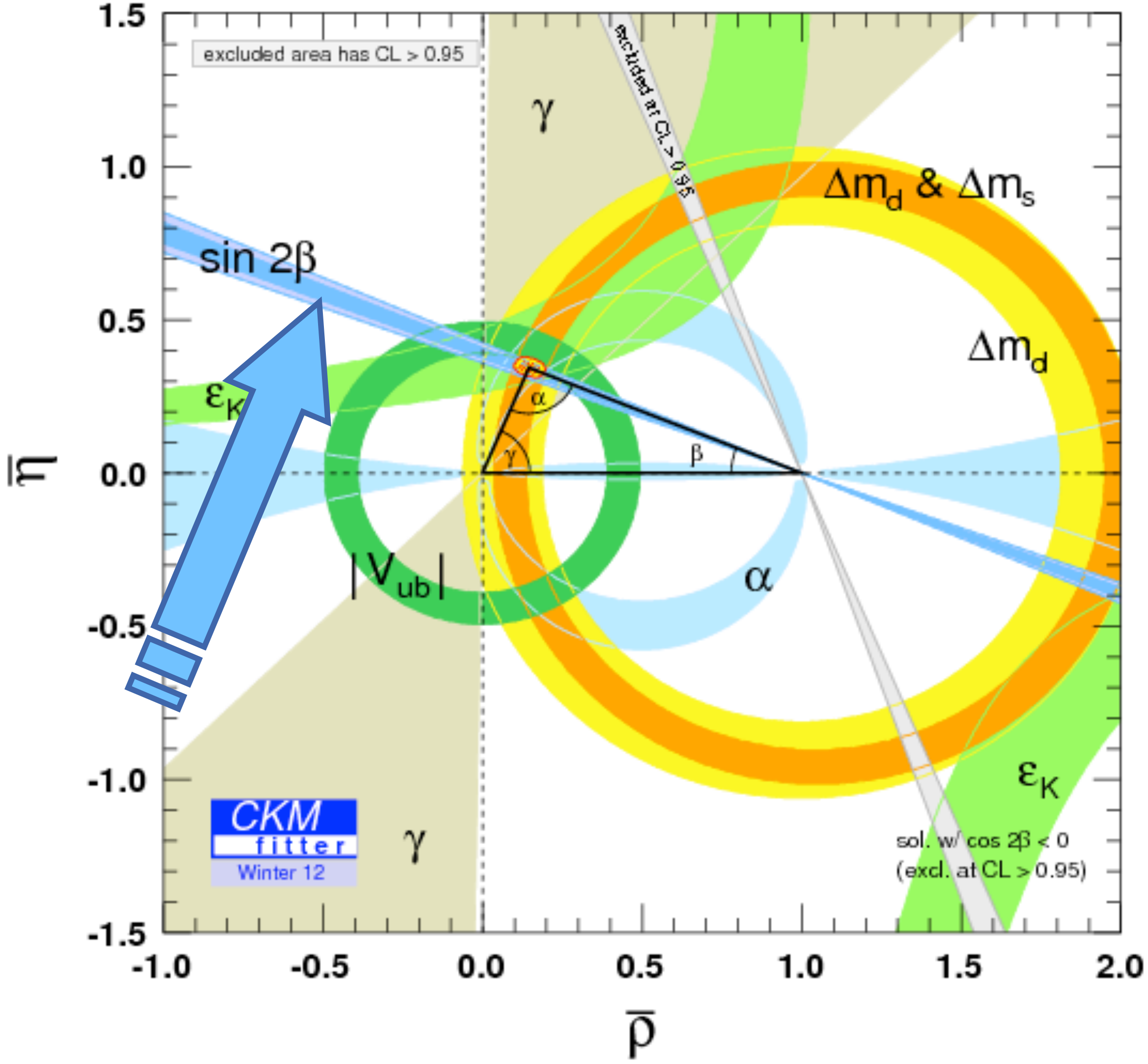


- Amplitude is real to very good approximation, $\varphi_A = 0$

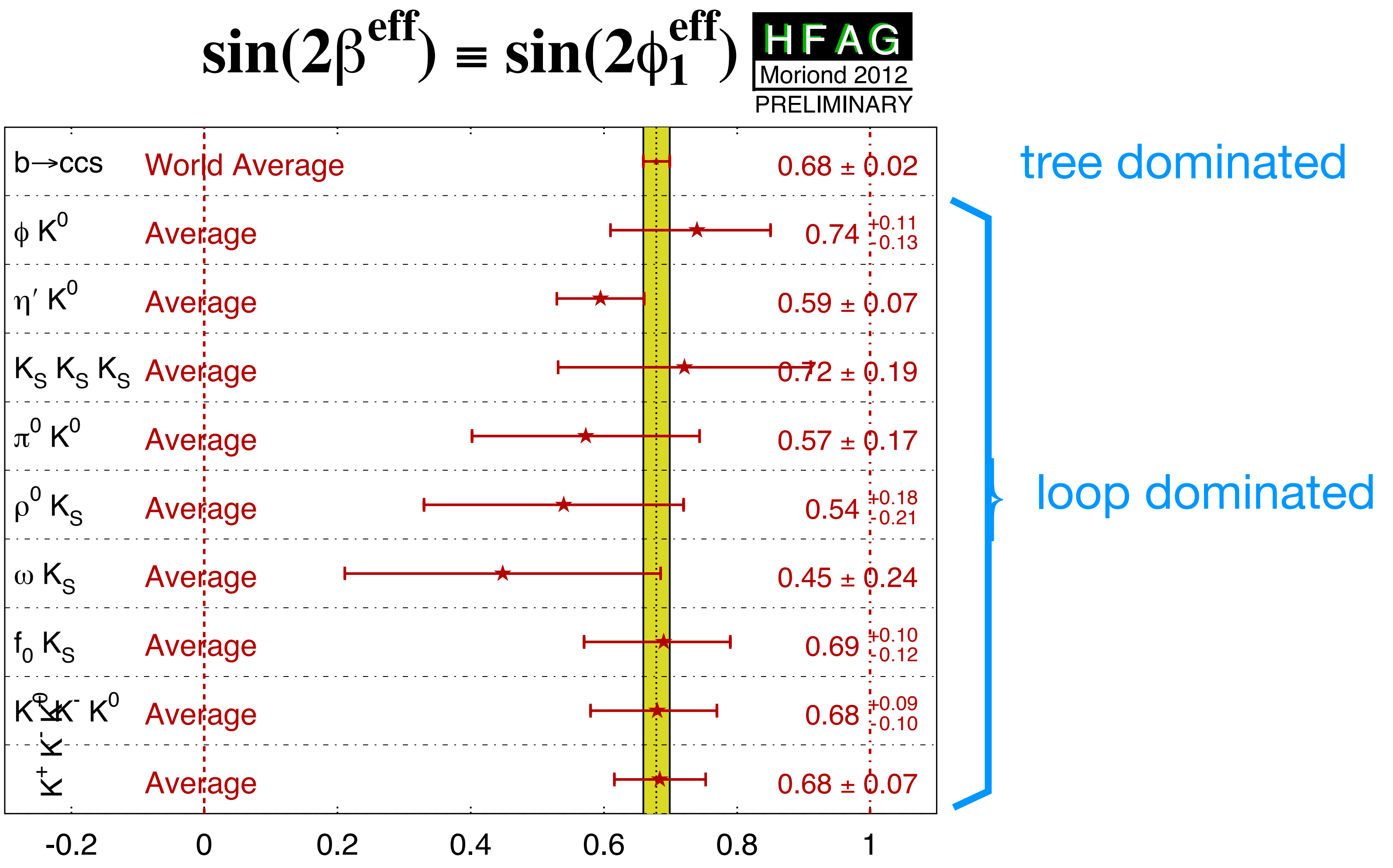
- CP asymmetry $S(f)=\sin 2\beta$ determines CP-violating phase β without knowledge of decay amplitude!
- Theoretical uncertainty only ~1%
- Very precise measurement of an angle of the unitarity triangle:

$$\sin 2\beta = 0.691 \pm 0.020$$

A very precise constraint on the unitarity triangle



sin2β from tree- and loop-dominated processes



No hint for New Physics (yet) !