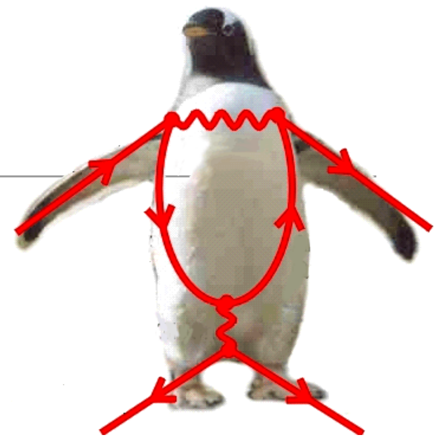
Flavor physics in the SM and beyond



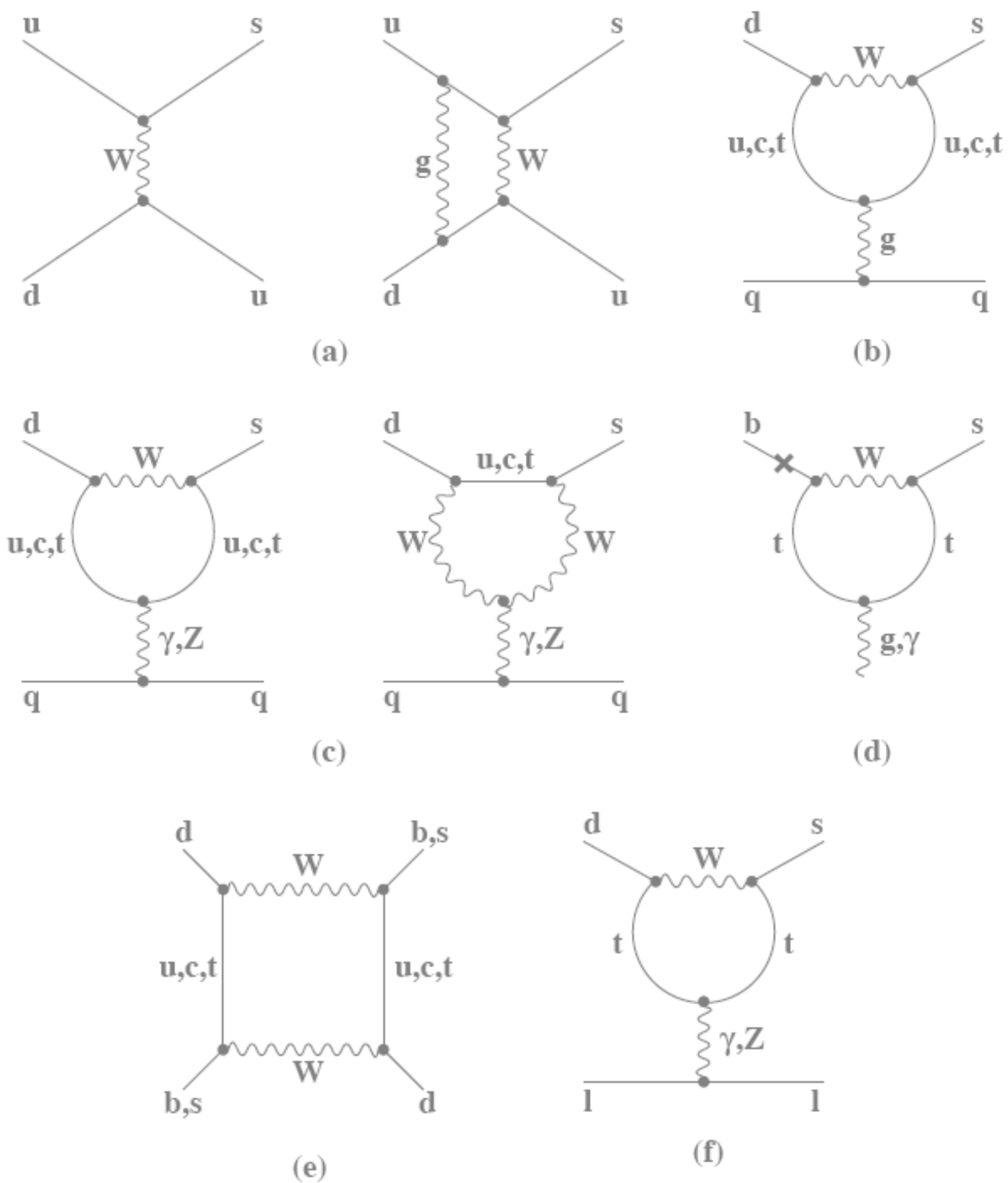
Lecture I: Concepts of Quark Flavor Physics

Lecture II: Effective Weak Hamiltonians

- Effective weak interactions at low energies
- Concepts of effective field theory

Lecture III: Connecting UV Physics to Experiments

Lecture II: Effective Weak Hamiltonians



Effective field theory (a first encounter)

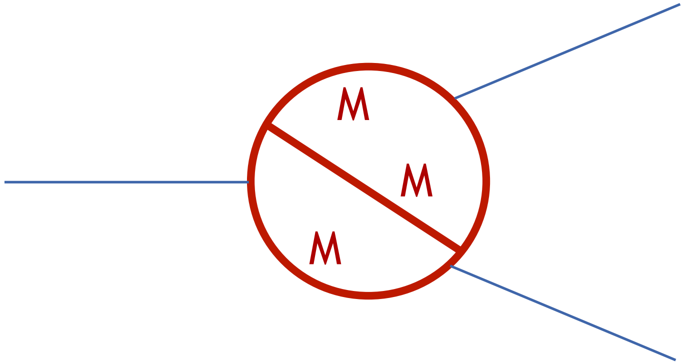
Weak-interaction processes are characterized by many different mass scales (m_t, m_W, m_Z, m_b, m_K, …), which make higher-order calculations exceedingly difficult

Effective field theory is a powerful tool in quantum field theory:

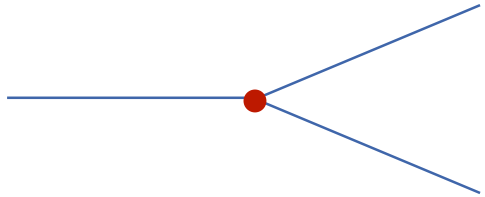
- systematic formalism for the analysis of **multi-scale problems**
- simplifies **practical calculations**, often makes them feasible
- basis of **factorization** (i.e. scale separation) and **resummation** of large logarithmic terms
- particularly important in QCD, where short-distance effects are calculable perturbatively but long-distance effects are not

Effective field theory (a first encounter)

- At low energies, the exchange of heavy, virtual particles (M»E) leads to local effective interactions



exchange of heavy, virtual particles between light SM particles

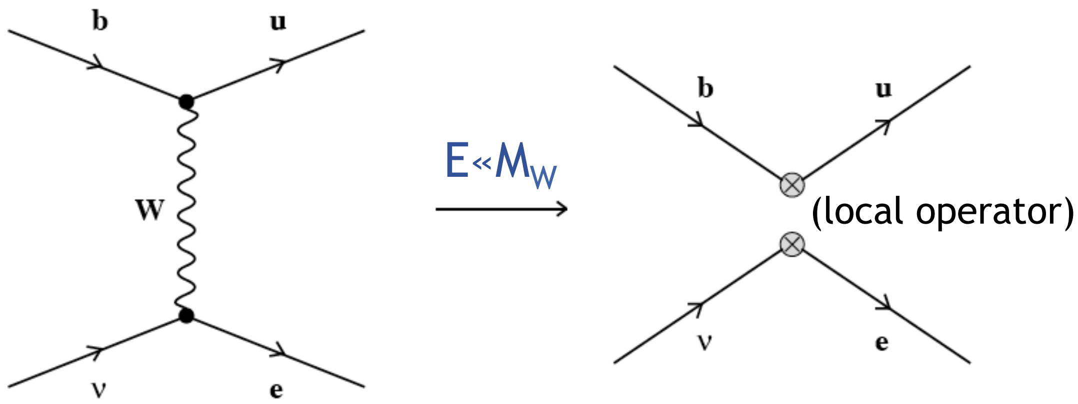


induced, effective local interactions at low energies

- Effective field theory offers systematic description of effects of modes with large virtualities through an expansion in local operators

W exchange at low energy (Fermi theory)

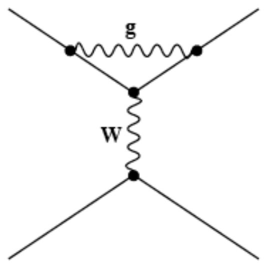
- Fermi theory of weak interactions describes W-boson exchange in terms of local 4-fermion couplings
- Consider:



- Fermi constant: $G_F/\sqrt{2} = g_2^2/8M_W^2$
  - determines scale of weak interactions

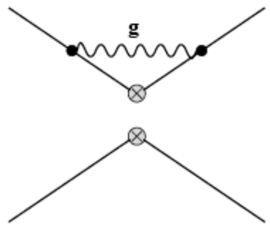
W exchange at low energy (Fermi theory)

- Semileptonic decay: QCD corrections influence both graphs in same way
- Resulting "effective" interaction for $E \ll M_W$:



$$\mathcal{L}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{ub}\, C_1(\mu)\, \bar{e}_L \gamma_\mu \nu_L\, \bar{u}_L \gamma^\mu b_L$$

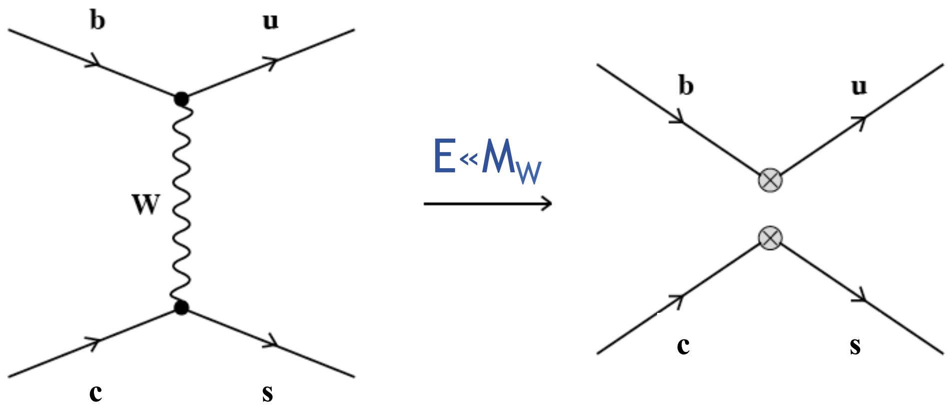
$C_1=1$



- Scaling $1/M_W^2$ for d=6 operators explains weakness of "weak" interactions

W exchange at low energy (Fermi theory)

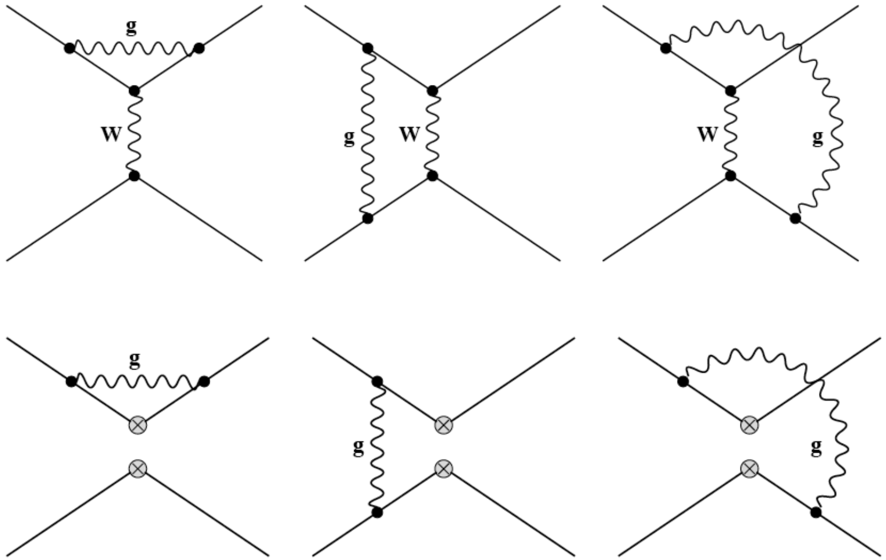
- W exchange between four different quark fields (nonleptonic decays):



- At tree level, analogous treatment as before

W exchange at low energy (Fermi theory)

- Complications for loop graphs:

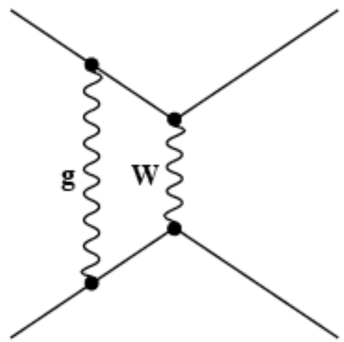


- Naïve Taylor expansion of W-boson propagator no longer justified!

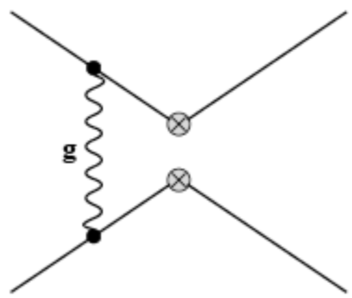
W exchange at low energy (Fermi theory)

- Problem with large loop momenta:

$$\int d^D p \, \frac{1}{M_W^2 - p^2} f(p) \neq \frac{1}{M_W^2} \int d^D p \left(1 + \frac{p^2}{M_W^2} + \ldots\right) f(p)$$



- But no differences at low loop momenta!
- Effect can be calculated and corrected for using perturbation theory, since effective coupling $\alpha_s(M_W)$ is small



W exchange at low energy (Fermi theory)

- Resulting effective interaction:

$$\mathcal{L}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{cs}^* V_{ub} \left[C_1(\mu)\, \bar{s}_L^j \gamma_\mu c_L^j \, \bar{u}_L^i \gamma^\mu b_L^i + C_2(\mu)\, \bar{s}_L^i \gamma_\mu c_L^j \, \bar{u}_L^j \gamma^\mu b_L^i\right]$$

with Wilson coefficients:

$$C_1(\mu) = 1 + \frac{3}{N_c} \frac{\alpha_s(\mu)}{4\pi} \left(\ln \frac{M_W^2}{\mu^2} - \frac{11}{6}\right) + O(\alpha_s^2)$$

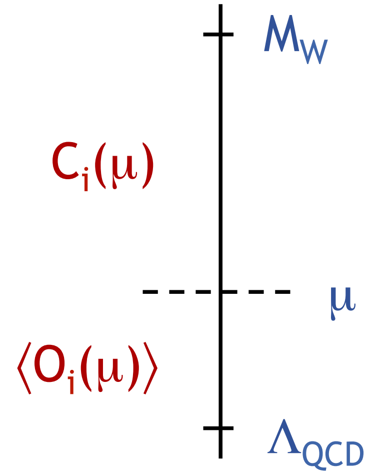
$$C_2(\mu) = -3 \frac{\alpha_s(\mu)}{4\pi} \left(\ln \frac{M_W^2}{\mu^2} - \frac{11}{6}\right) + O(\alpha_s^2)\,.$$

$\rightarrow$ accounts for effects of hard gluons ($p \sim M_W$)

Main idea of effective field theory

- Separation of short- and long-distance effects; schematically:

$$\int_{-p^2}^{M_W^2}\frac{dk^2}{k^2} = \int_{\mu^2}^{M_W^2}\frac{dk^2}{k^2} + \int_{-p^2}^{\mu^2}\frac{dk^2}{k^2}$$



- Short-distance effects ($p \sim M_W$) are perturbatively calculable
- Long-distance effects must be treated using nonperturbative methods
- Dependence on arbitrary separation scale μ controlled by RG equations

Main idea of effective field theory

- Why useful?
- Any sensitivity to high scales (including to physics beyond the Standard Model) can be treated using perturbative methods:

$$\boxed{C_i(\mu) = C_i^{SM}(M_W, m_t, \mu) + C_i^{NP}(M_{NP}, g_{NP}, \mu)}$$

- Nonperturbative methods (operator product expansion, lattice gauge theory, …) usually only work at low scales (typically μ~few GeV)

FCNC processes

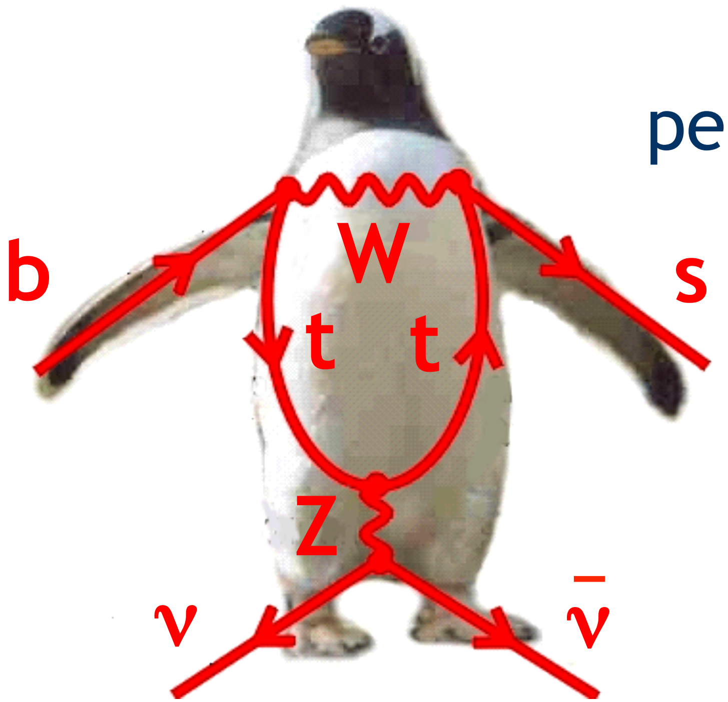
- While generation-changing couplings of W bosons to quarks exist, flavor-changing neutral currents such as

  $b \rightarrow s\gamma$, $b \rightarrow sZ^0$, $b \rightarrow s\nu\bar{\nu}$, $b \rightarrow sd\bar{d}$, $b\bar{d} \rightarrow d\bar{b}$, etc.
  (and others, also for light quarks)

  do not exist as elementary vertices in the Standard Model (GIM mechanism)

FCNC processes

- But such processes can be induced at loop level, e.g.:

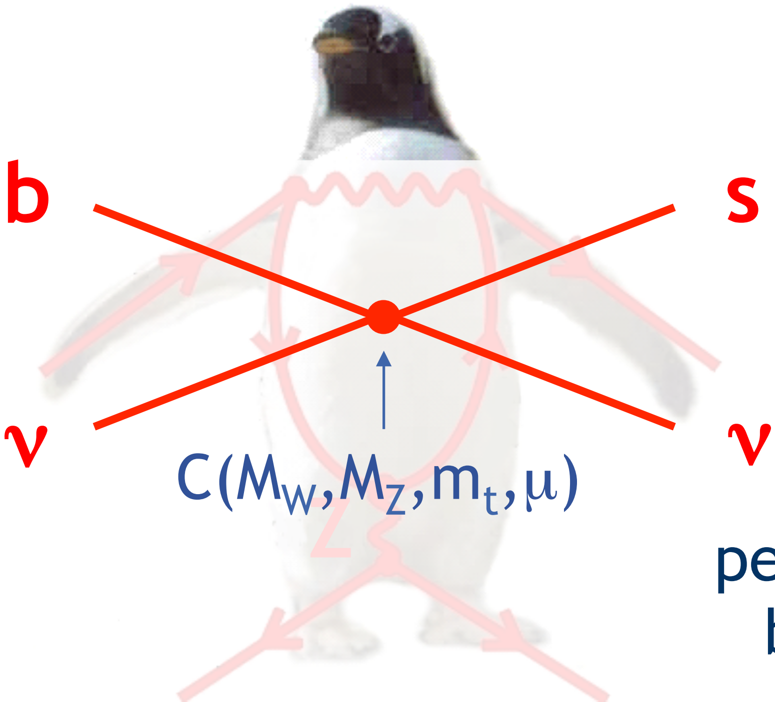


penguin diagram

loop-induced decay $b \rightarrow s\nu\bar{\nu}$

FCNC processes

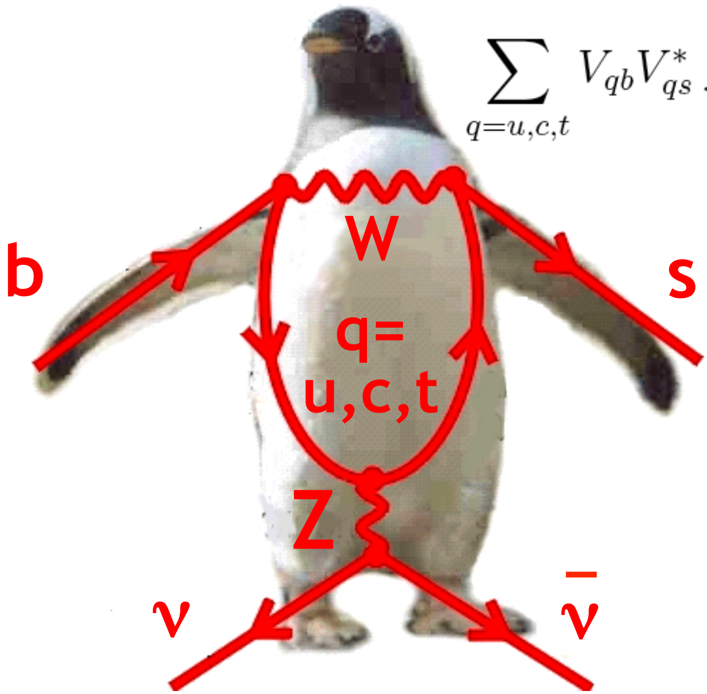
- Effective interaction at low energies ($E \ll M_W, M_Z, m_t$):



penguin diagram approximated
by local 4-fermion operator

FCNC processes

- Detailed analysis (penguin autopsy) exhibits that GIM mechanism is “incomplete” in this case:



$$\sum_{q=u,c,t} V_{qb}V_{qs}^* f\left(\frac{m_q^2}{M_W^2},\ldots\right) = V_{tb}V_{ts}^*\left[f\left(\frac{m_t^2}{M_W^2},\ldots\right) - f\left(\frac{m_u^2}{M_W^2},\ldots\right)\right]$$
$$+ V_{cb}V_{cs}^*\left[f\left(\frac{m_c^2}{M_W^2},\ldots\right) - f\left(\frac{m_u^2}{M_W^2},\ldots\right)\right]$$

Unitarity relation:

$$V_{tb}V_{ts}^* + V_{cb}V_{cs}^* + V_{ub}V_{us}^* = 0$$

$\rightarrow$ residual effect due to nontrivial mass dependence, often $\propto (m_t/M_W)^2$ or $\ln(m_t/\mu)$

FCNC processes

- Rich structure of couplings of Z^0,g,γ lead to a plethora of effective local d=6 operators
- Consider, e.g., decays of type b→s+X (or b→d+X, s→d+X), where X is flavor neutral:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[\sum_{q=u,c} V_{qb} V_{qs}^* \left(C_1\, Q_1^{(q)} + C_2\, Q_2^{(q)} \right) - V_{tb} V_{ts}^* \sum_{i=3,\ldots,10,7\gamma,8g} C_i\, Q_i \right]$$

W-boson exchange

penguin and box graphs

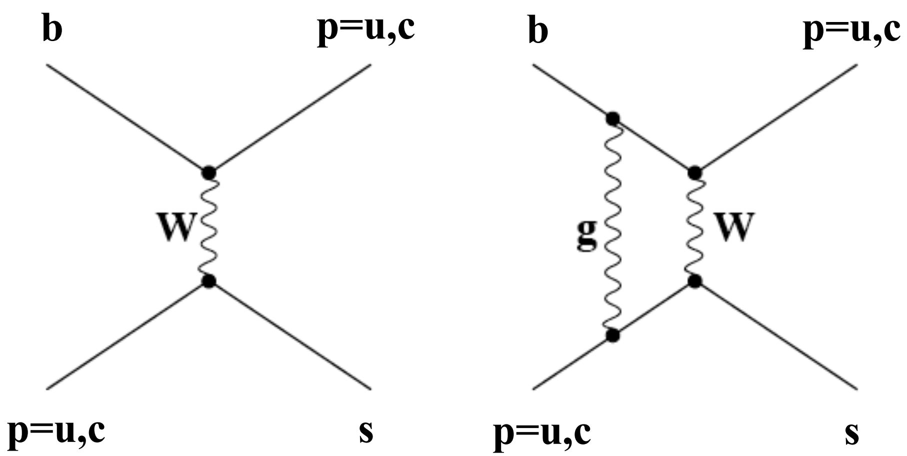
Operator basis

- Current-current operators (W exchange):

$$Q_1^{(p)} = (\bar{s}_i p_i)_{V-A}\,(\bar{p}_j b_j)_{V-A}$$

$$Q_2^{(p)} = (\bar{s}_i p_j)_{V-A}\,(\bar{p}_j b_i)_{V-A}$$

$$\boxed{(\bar{q}_1 q_2)_{V\pm A} \equiv \bar{q}_1 \gamma^\mu (1 \pm \gamma_5) q_2}$$



- Results analogous to earlier discussion):

$$C_1(M_W) = 1 - \frac{11}{6}\frac{\alpha_s(M_W)}{4\pi}$$

$$C_2(M_W) = \frac{11}{2}\frac{\alpha_s(M_W)}{4\pi}\,,$$

← results quoted at $\mu = M_W$ for simplicity

Operator basis

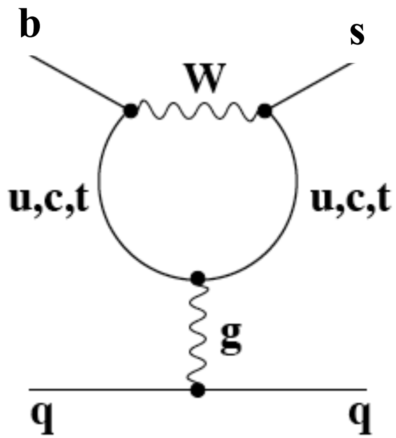
- QCD penguin operators:

$$Q_3 = (\bar{s}_i b_i)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}_j q_j)_{V-A}$$

$$Q_4 = (\bar{s}_i b_j)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}_j q_i)_{V-A}$$

$$Q_5 = (\bar{s}_i b_i)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}_j q_j)_{V+A}$$

$$Q_6 = (\bar{s}_i b_j)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}_j q_i)_{V+A}$$



- Results:

$$C_3(M_W) = C_5(M_W) = -\frac{1}{6}\,\widetilde{E}_0\left(\frac{m_t^2}{M_W^2}\right)\frac{\alpha_s(M_W)}{4\pi}$$

$$C_4(M_W) = C_6(M_W) = \frac{1}{2}\,\widetilde{E}_0\left(\frac{m_t^2}{M_W^2}\right)\frac{\alpha_s(M_W)}{4\pi}$$

Loop function:

$$\widetilde{E}_0(x) = -\frac{7}{12} + O(1/x)$$

Operator basis

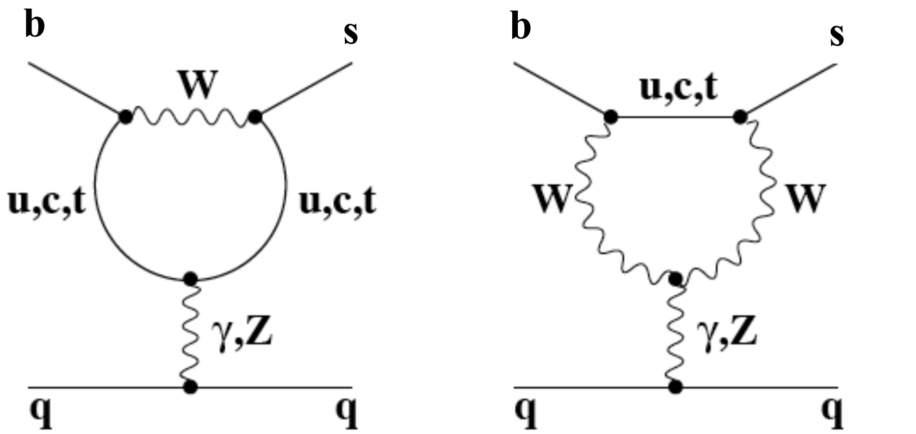
- Electroweak penguin operators:

$$Q_7 = (\bar{s}_i b_i)_{V-A} \sum_{q=u,d,s,c,b} \frac{3}{2} e_q \, (\bar{q}_j q_j)_{V+A}$$

$$Q_8 = (\bar{s}_i b_j)_{V-A} \sum_{q=u,d,s,c,b} \frac{3}{2} e_q \, (\bar{q}_j q_i)_{V+A}$$

$$Q_9 = (\bar{s}_i b_i)_{V-A} \sum_{q=u,d,s,c,b} \frac{3}{2} e_q \, (\bar{q}_j q_j)_{V-A}$$

$$Q_{10} = (\bar{s}_i b_j)_{V-A} \sum_{q=u,d,s,c,b} \frac{3}{2} e_q \, (\bar{q}_j q_i)_{V-A}$$



- Results:

$$C_7(M_W) = f\left(\frac{m_t^2}{M_W^2}\right) \frac{\alpha(M_W)}{6\pi}, \qquad C_8(M_W) = C_{10}(M_W) = 0$$

$$C_9(M_W) = \left[f\left(\frac{m_t^2}{M_W^2}\right) + \frac{1}{\sin^2\theta_W} g\left(\frac{m_t^2}{M_W^2}\right)\right] \frac{\alpha(M_W)}{4\pi}$$

Loop functions:

$$f(x) = \frac{x}{2} + \frac{4}{3}\ln x - \frac{125}{36} + O(1/x)$$

$$g(x) = -\frac{x}{2} - \frac{3}{2}\ln x + O(1/x)$$

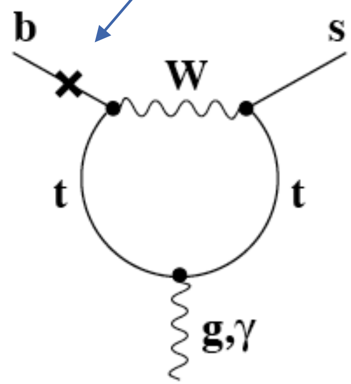
Operator basis

- Dipol operators:

$$Q_{7\gamma} = -\frac{e m_b}{8\pi^2}\,\bar{s}\,\sigma_{\mu\nu}\,(1+\gamma_5)\,F^{\mu\nu}\,b$$

$$Q_{8g} = -\frac{g_s m_b}{8\pi^2}\,\bar{s}\,\sigma_{\mu\nu}\,(1+\gamma_5)\,G_a^{\mu\nu} t_a\,b$$

chirality flip $\propto m_b$



- Results ($x=m_t^2/M_W^2$) :

$$C_{7\gamma}(M_W) = -\frac{1}{3} + O(1/x)$$

$$C_{8g}(M_W) = -\frac{1}{8} + O(1/x)$$

That's it !
(apart from operators containing leptons ...)

but these are most relevant for the $R_{K^{(*)}}$ anomalies

Operator basis for B→K*l+l- transitions

- In complete analogy to the case of four quarks, one finds that the relevant operators are:

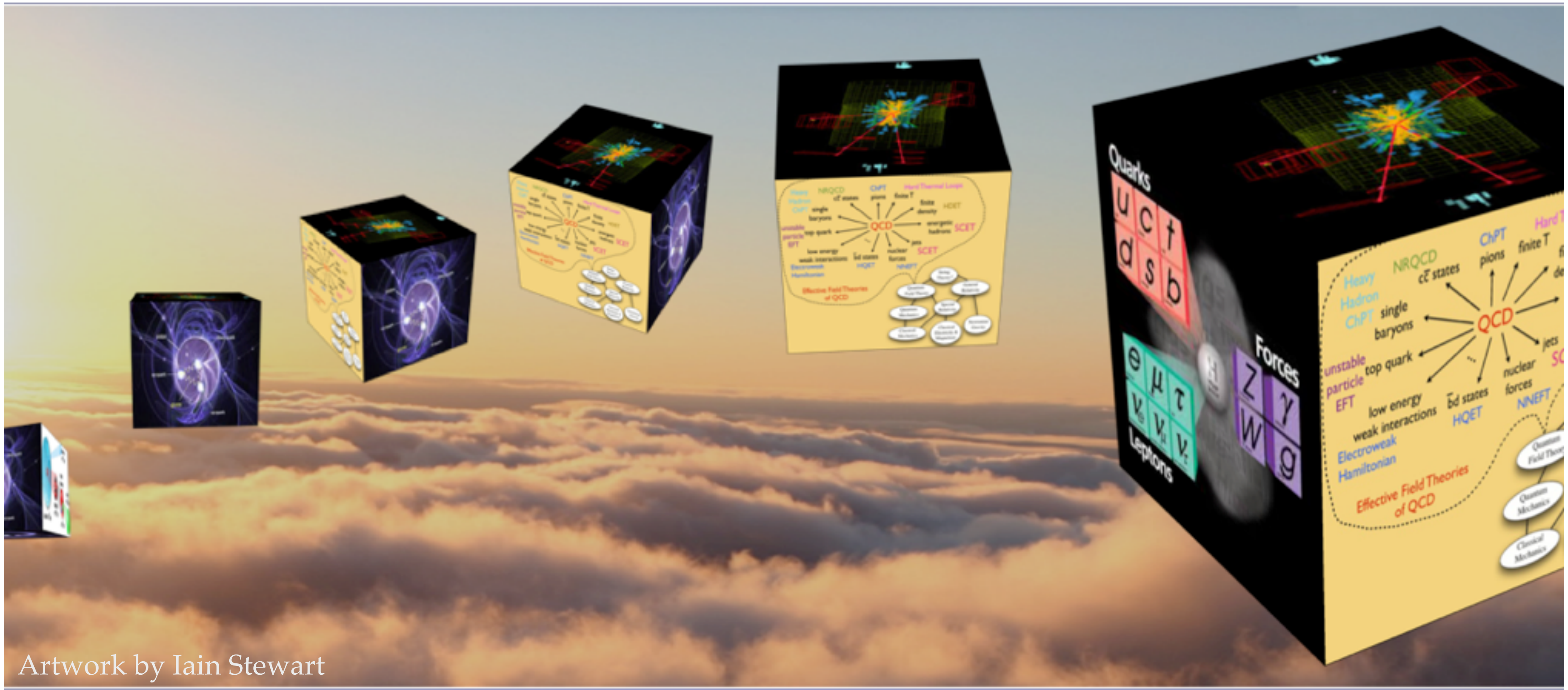
$$\mathcal{H}_{\text{eff}} = -\frac{4\,G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \sum_i (C_i O_i + C_i' O_i') + \text{h.c.}$$

with:

$$O_7 = \frac{m_b}{e} (\bar{s}\sigma_{\mu\nu} P_R b) F^{\mu\nu}, \qquad O_7' = \frac{m_b}{e} (\bar{s}\sigma_{\mu\nu} P_L b) F^{\mu\nu}$$

$$O_9 = (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell)\,, \qquad O_9' = (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \ell)$$

$$O_{10} = (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)\,, \qquad O_{10}' = (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$



Concepts of Effective Field Theory

Derivation of the effective Lagrangian

Consider a QFT with a characteristic high-energy scale M

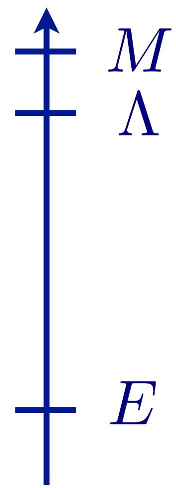
We are interested in performing experiments at energies $E \ll M$

<u>Step 1:</u> Choose a cutoff $\Lambda < M$ and divide all quantum fields into high- and low-frequency components ($\omega > \Lambda$ and $\omega < \Lambda$):

$$\boxed{\phi = \phi_L + \phi_H}$$

Recall:

$$\phi(x) = \int \frac{d^3k}{(2\pi)^3\, 2E_k} \left(a_k\, e^{-ik\cdot x} + a_k^\dagger\, e^{ik\cdot x} \right)$$



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$$\boxed{\phi = \phi_L + \phi_H}$$

Physics at low energies $E \ll \Lambda$ is entirely described in terms of the fields ϕ_L; Green functions of these fields can be derived from the generating functional:

$$Z[J_L] = \int \mathcal{D}\phi_L \, \mathcal{D}\phi_H \, e^{iS(\phi_L,\phi_H) + i\int d^Dx \, J_L(x)\, \phi_L(x)}$$

Derivation of the effective Lagrangian

Step 2: Since the high-frequency fields ϕ_H do not appear in the generating functional, we can **"integrate them out"** in the path integral:

$$Z[J_L] \equiv \int \mathcal{D}\phi_L \, e^{iS_\Lambda(\phi_L)+i\int d^Dx \, J_L(x)\,\phi_L(x)}$$

where

$$\boxed{e^{iS_\Lambda(\phi_L)} = \int \mathcal{D}\phi_H \, e^{iS(\phi_L,\phi_H)}}$$

and $S_\Lambda(\phi_L)$ is called the **Wilsonian effective action**

Dependence on the cutoff Λ enters via the condition on the frequencies of the fields

Derivation of the effective Lagrangian

Step 3: Effective action is **non-local** on the scale $\Delta t \sim 1/\omega$, corresponding to the propagation of high-energy modes that have been removed from the Lagrangian

Since the remaining fields have energies $\omega < \Lambda$, the non-local effective action can be expanded in an **infinite series of local operators:**

$$S_\Lambda(\phi_L) = \int d^D x \, \mathcal{L}_\Lambda^{\text{eff}}(x)$$

where:

$$\boxed{\mathcal{L}_\Lambda^{\text{eff}}(x) = \sum_i g_i \, Q_i(\phi_L(x))}$$

coupling constants (Wilson coefficients)

basis of local operators built out of fields ϕ_L and their derivatives

Dimensional analysis

Does a Lagrangian consisting of an infinite number of interactions and hence an infinite number of (renormalized) coupling constants give any predictive power?

- Not if one adopts an old-fashioned view about renormalization and renormalizable QFTs, but not all is lost!

We can use **naive dimensional analysis** to estimate the size of individual terms in the infinite sum to any given matrix element

Adopt units where $\hbar = c = 1$, hence $[m] = [E] = [p] = [x^{-1}] = [t^{-1}]$ are all measured in the same units (mass units)

Dimensional analysis

Denote by $[g_i] = -\gamma_i$ the mass dimension of the coupling constants in the effective Lagrangian

Since by assumption the theory has only a single fundamental scale M, it follows that:

$$g_i = C_i\, M^{-\gamma_i}$$

where by **naturalness** we expect that $C_i = O(1)$

At low energy, it follows that the contribution of a given term $g_i\, Q_i$ to an observable scales like:

$$C_i \left(\frac{E}{M}\right)^{\gamma_i} = \begin{cases} O(1)\,; & \text{if } \gamma_i = 0 \\ \ll 1\,; & \text{if } \gamma_i > 0 \\ \gg 1\,; & \text{if } \gamma_i < 0 \end{cases}$$

Dimensional analysis

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Therefore, operators with small γ_i are most important for $E \ll M$ and there is a finite number of such operators

This is what makes the effective Lagrangian useful !

Depending on the precision goal, one can truncate the infinite sum over interactions by only retaining operators whose γ_i values are smaller than a certain value

Dimensional analysis

Since the Lagrangian has mass dimension D = 4, it follows that the operators have mass dimension:

$$\delta_i = [Q_i] = D + \gamma_i$$

Hence we can summarize:

Dimension	Importance for $E \to 0$	Terminology
$\delta_i < D,\ \gamma_i < 0$	grows	relevant operators (super-renormalizable)
$\delta_i = D,\ \gamma_i = 0$	constant	marginal operators (renormalizable)
$\delta_i > D,\ \gamma_i > 0$	falls	irrelevant operators (non-renormalizable)

Only a **finite number** of relevant and marginal operators exist !

Dimensional analysis

Comments:

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$\delta_i < D,\ \gamma_i < 0$	grows	relevant operators (super-renormalizable)
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$\delta_i > D,\ \gamma_i > 0$	falls	irrelevant operators (non-renormalizable)

- **“relevant” operators** are usually unimportant, since they are forbidden by some symmetry (else “hierarchy problem”)
- **“marginal” operators** are all there is in renormalizable QFTs
- **“irrelevant” operators** are the most interesting ones, since they tell us something about the fundamental scale M

Comments

Examples of effective field theories:

High-energy theory	Fundamental scale	Low-energy theory
Standard Model	$M_W \sim 80\,\mathrm{GeV}$	Fermi theory
GUT	$M_{\mathrm{GUT}} \sim 10^{16}\,\mathrm{GeV}$	Standard Model
String theory	$M_S \sim 10^{18}\,\mathrm{GeV}$	QFT
11-dim. M theory	$\ldots$	String theory
$\ldots$	$\ldots$	$\ldots$
QCD	$m_b \sim 5$ GeV	HQET, NRQCD
	$M_{ChSM} \sim 1$ GeV	ChPT

- SM and GUTs are perturbative QFTs
- Fermi theory contains only irrelevant operators (4 fermions)
- String/M theory: fundamental theory is non-local and even spacetime breaks down at short distances

Comments

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$\ldots$	$\ldots$	$\ldots$
QCD	$m_b \sim 5$ GeV	HQET, NRQCD
	$M_{\mathrm{ChSM}} \sim 1$ GeV	χPT

- QCD at low energy: example with strong coupling, where the relevant degrees of freedom at low energy (hadrons) are different from the degrees of freedom of QCD
- Low-energy theory is strongly coupled, yet χPT is useful