The pion-nucleon (πN) interaction below 100 MeV

What has been learnt after 30 years of painstaking experimentation and analysis of the meson-factory measurements?

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What makes πN Physics interesting?

- The πN interaction is used as input in the description of more complex systems, e.g., of the NN system
- Determination of the πN coupling constant
    - Tests: Current Algebra, PCAC and Goldberger-Treiman relation, GMO sum rule, etc.
    - Properties of the deuteron, etc.
- Information extracted from the πN system is used in QCD tests (πN σ term)
- The πN system is the simplest hadronic system involving one baryon

Low-energy πN reactions and observables

- Reactions
  - $\pi^+ p \to \pi^+ p$ (elastic scattering, ES)
  - $\pi^- p \to \pi^- p$ (ES)
  - $\pi^- p \to \pi^0 n$ (charge exchange, CX)

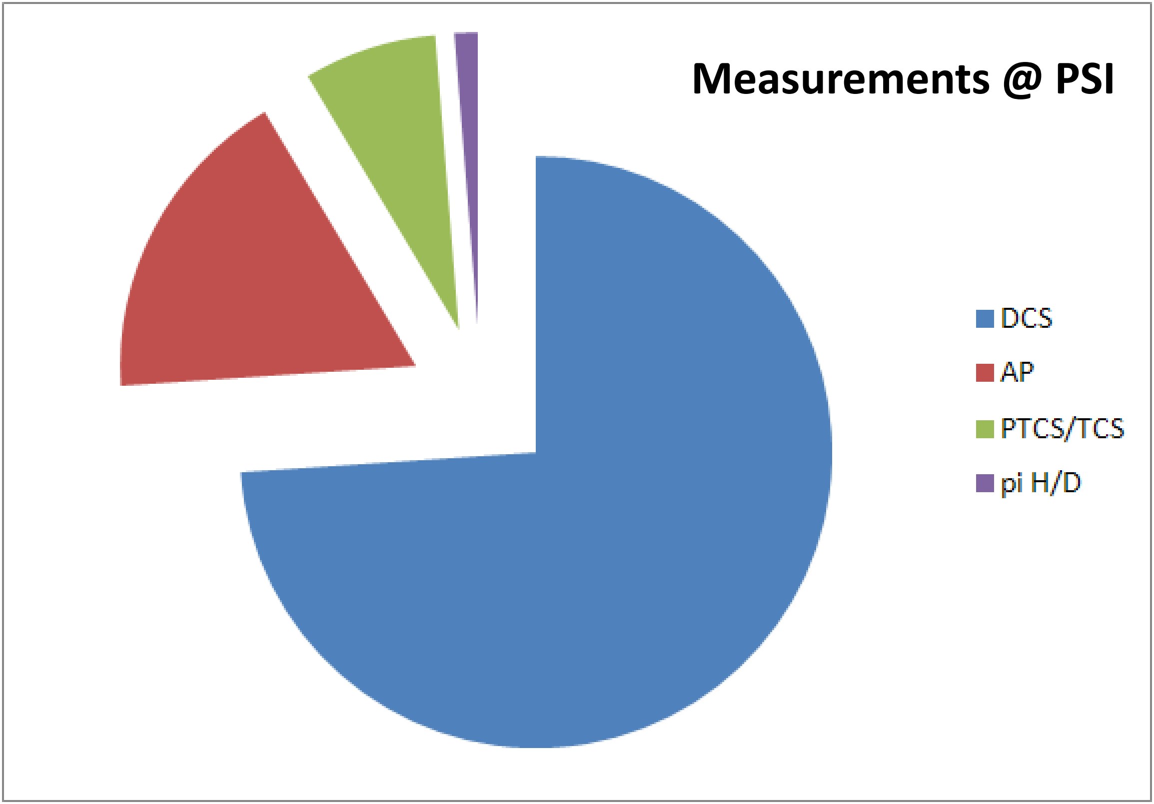
- Observables
  - Differential cross section (DCS)
  - Analysing power (AP)
  - Partial-total/total cross section (PTCS/TCS)
  - Measurements of the strong shift and width of pionic hydrogen/deuterium → πN scattering lengths

PSI

Meson-factory low-energy πN measurements

Los Alamos Meson Physics Facility

TRIUMF



Analyses of the πN measurements

Identifier	Input	Output	Method	Energy range
GWU (SAID)	DCSs, APs	δ_{IJ}, f_{IJ}	DRs	All
Jülich/Bonn	δ_{IJ} (DCSs, APs)	LECs	χPT (DRs)	Up to ≈ 100 MeV
Valencia	δ_{IJ}	LECs	CBχPT	≤ 100 MeV
Zurich	DCSs, APs, PTCSs, TCSs	Model parameters (LECs, δ_{IJ}, f_{IJ})	Phenomenological or hadronic model	≤ 100 MeV

Helsinki, KA/KH, London, Nijmegen, NMSU, etc.

R.A. Arndt, W.J. Briscoe, I.I. Strakovsky, R.L. Workman, *Extended partial-wave analysis of πN scattering data*, Phys. Rev. C 74 (2006) 045205; R.L. Workman, R.A. Arndt, W.J. Briscoe, M.W. Paris, I.I. Strakovsky, *Parameterization dependence of T-matrix poles and eigenphases from a fit to πN elastic scattering data*, Phys. Rev. C 86 (2012) 035202; SAID Analysis Program: gwdac.phys.gwu.edu
M. Hoferichter, J. Ruiz de Elvira, B. Kubis, Ulf-G. Meißner, *Roy-Steiner-equation analysis of pion-nucleon scattering*, Phys. Rep. 625 (2016) 1-88

DRs: Dispersion Relations; LECs: Low-Energy Constants; χPT: Chiral Perturbation Theory; CBχPT : Covariant Baryon χPT

The triangle identity

- **Isospin invariance** in the hadronic part of the πN interaction → only **two** (complex) amplitudes enter the physical description of the **three** πN reactions:
  - I = 3/2 amplitude (f_3) and
  - I = 1/2 amplitude (f_1).

π^+p: $\mathbf{f_3}$, π^-p ES: $\mathbf{(2f_1 + f_3)/3}$, π^-p CX: $\mathbf{\sqrt{2}(f_3 - f_1)/3}$

The **triangle identity** follows:

$$\boxed{f_{\pi^+ p} - f_{\pi^- p} = \sqrt{2}\, f_{CX}}$$

W.R. Gibbs, Li Ai, W.B. Kaufmann, *Isospin breaking in low-energy pion-nucleon scattering*, Phys. Rev. Lett. 74 (1995) 3740-3743
E. Matsinos, *Isospin violation in the πN system at low energy*, Phys. Rev. C 56 (1997) 3014-3025

The trouble with DR analyses

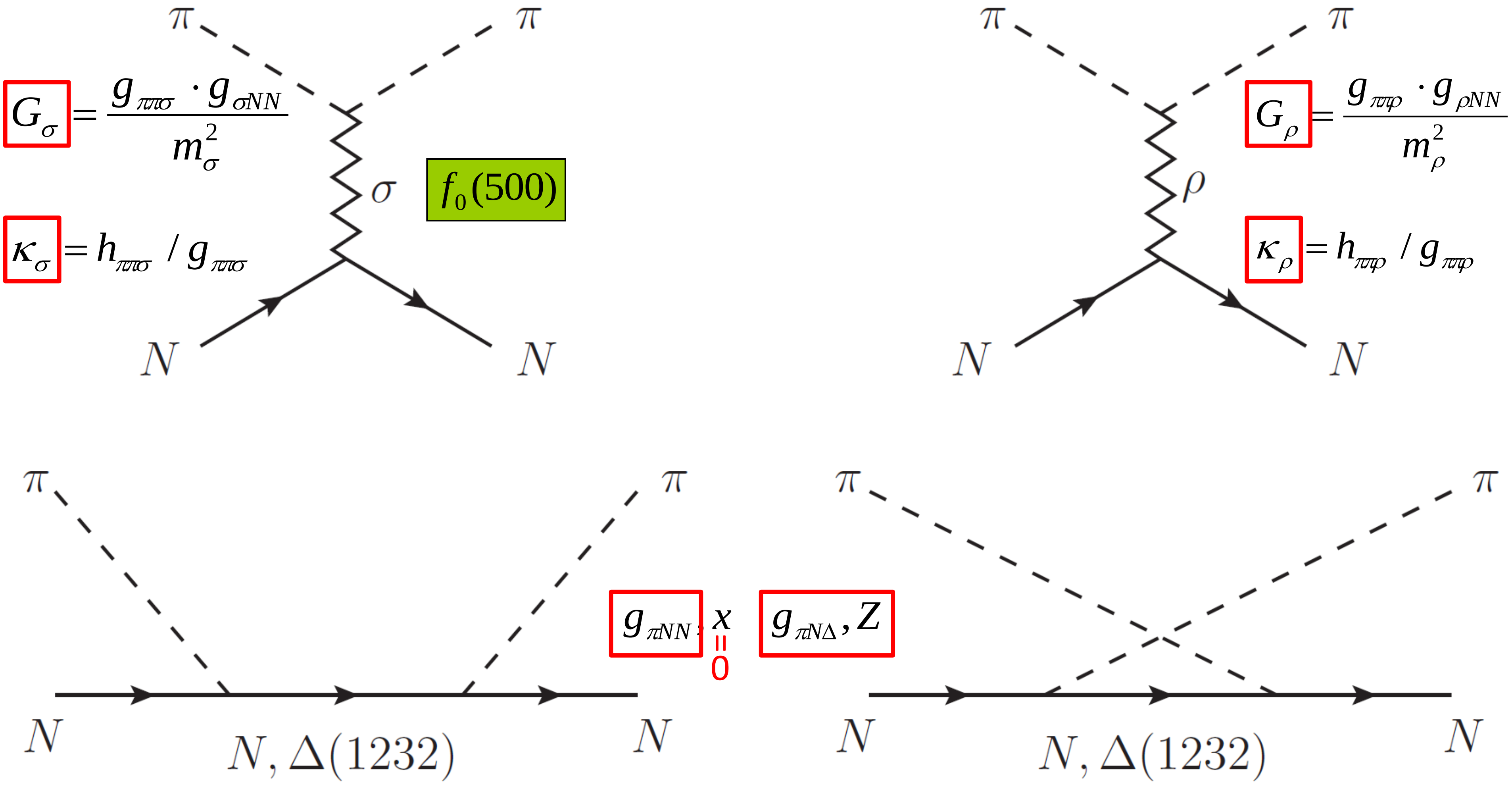
DR analyses are general, but...

- Not possible to analyse the low-energy data without influence from the measurements acquired at high energies: extensive databases (DBs)
- Not possible to analyse the measurements of the three reactions *separately*
- Not possible to obtain meaningful uncertainties in the output and predictions
- Theoretical constraints are imposed on the analysis *in the entire energy range* of the measurements

What makes the *low-energy* πN measurements interesting?

- The low-energy πN DB is extensive enough to enable exclusive analyses
- Important theoretical constraints, which are valid in the high-energy region (e.g., isospin invariance of the hadronic part of the πN interaction), might not hold at low energy
- The extrapolation of the partial-wave amplitudes into the unphysical region (e.g., to extract estimates of the πN Σ term) is expected to be more reliable when exclusively based on ‘close-by’ input
    - M.G. Olsson, *The nucleon sigma term from threshold parameters*, Phys. Lett. B 482 (2000) 50-56
    - J.M. Alarcón, J. Martin Camalich, J.A. Oller, *Chiral representation of the πN scattering amplitude and the pion-nucleon sigma term*, Phys. Rev. D 85 (2012) 051503
- The modelling of the hadronic part of the πN scattering amplitude is simple(r) at low energy, as
    - hadronic form factors are not needed,
    - up to ε^2 terms suffice in the K-matrix parameterisations, and
    - the contributions from the partial waves l>3 are negligible

The hadronic model (ETH model)



$\boxed{G_\sigma} = \frac{g_{\pi\pi\sigma} \cdot g_{\sigma NN}}{m_\sigma^2}$

$\boxed{\kappa_\sigma} = h_{\pi\pi\sigma} / g_{\pi\pi\sigma}$

$\boxed{f_0(500)}$

$\boxed{G_\rho} = \frac{g_{\pi\pi\rho} \cdot g_{\rho NN}}{m_\rho^2}$

$\boxed{\kappa_\rho} = h_{\pi\pi\rho} / g_{\pi\pi\rho}$

$\boxed{g_{\pi NN}}, \underset{\overset{\|}{0}}{x}$ $\boxed{g_{\pi N\Delta}, Z}$

+ all well-established (four-star) *s* and *p* higher baryon resonances (HBRs) with masses up to 2 GeV

The hadronic model: history

- P.F.A. Goudsmit, H.J. Leisi, E. Matsinos
    - *Pionic atoms, the relativistic mean-field theory and the pion-nucleon scattering lengths*, Phys. Lett. B 271 (1991) 290-294
    - *A pion-nucleon interaction model at the tree level*, πN Newsl. 6 (1992) 60
    - *Pionic atoms, the relativistic mean-field theory and the pion-nucleon scattering lengths*, Few-Body Syst., Suppl. 5 (1992) 416-422
    - *A new pion-nucleon interaction model*, Helv. Phys. Acta 65 (1992) 878-879
    - *A dynamical model for the pion-nucleon interaction*, πN Newsl. 8 (1993) 98
    - *A pion-nucleon interaction model*, Phys. Lett. B 299 (1993) 6-10
- **P.F.A. Goudsmit, H.J. Leisi, E. Matsinos, B.L. Birbrair, A.B. Gridnev, *The extended tree-level model of the pion-nucleon interaction*, Nucl. Phys. A 575 (1994) 673-706**
- P.F.A. Goudsmit, H.J. Leisi, E. Matsinos, *The low-energy pion-nucleon interaction*, Helv. Phys. Acta 67 (1994) 369-391
- **N. Fettes, E. Matsinos, *Analysis of recent π^+p low-energy differential cross-section measurements*, Phys. Rev. C 55 (1997) 464-473**
- E. Matsinos, *πN scattering below 100 MeV*, πN Newsl. 13 (1997) 132-137
- **E. Matsinos, *Isospin violation in the πN system at low energy*, Phys. Rev. C 56 (1997) 3014-3025**
- **E. Matsinos, G. Rasche, *Aspects of the ETH model of the pion-nucleon interaction*, Nucl. Phys. A 927 (2014) 147-194**

K-matrix parameterisations

π^+p

$$q \cot \delta_{0+}^{3/2} = (a_{0+}^{3/2})^{-1} + b_3\epsilon + c_3\epsilon^2$$

$$\tan \delta_{1-}^{3/2}/q = d_{31}\epsilon + e_{31}\epsilon^2$$

$$\tan \delta_{1+}^{3/2}/q = d_{33}\epsilon + e_{33}\epsilon^2 + \frac{\Gamma_\Delta M_\Delta}{2q_\Delta^3(p_{0\Delta} + m_p)} \frac{(p_0 + m_p)q^2}{W(M_\Delta - W)}$$

Seven parameters for *s* and *p* waves

π^-p ES and CX

Similar expressions for the I=1/2 amplitudes (no HBR in P_{13}, HBRs in P_{11})

$$\tan \delta_{1-}^{1/2}/q = d_{11}\epsilon + e_{11}\epsilon^2 + \sum_{i=1}^{2} \frac{(\Gamma_R)_i (M_R)_i \left((p_{0R})_i + m_p\right)}{2(q_R^3)_i \left((M_R)_i + m_p\right)^2} \frac{(W + m_p)^2 q^2}{W\left((M_R)_i - W\right)(p_0 + m_p)}$$

For each of the two reactions, fourteen parameters for *s* and *p* waves (seven for the I=3/2, seven for the I=1/2 amplitudes); the seven I=3/2 parameters are fixed from the π^+p reaction

Electromagnetic corrections

- The electromagnetic (EM) interaction induces distortions to the partial-wave amplitudes and phase shifts
  - A. Gashi, E. Matsinos, G.C. Oades, G. Rasche, W.S. Woolcock, *Electromagnetic corrections to the phase shifts in low energy π^+p elastic scattering*, Nucl. Phys. A 686 (2001) 447-462
  - A. Gashi, E. Matsinos, G.C. Oades, G. Rasche, W.S. Woolcock, *Electromagnetic corrections for the analysis of low energy π^-p scattering data*, Nucl. Phys. A 686 (2001) 463-477
  - G.C. Oades, G. Rasche, W.S. Woolcock, E. Matsinos, A. Gashi, *Determination of the s-wave pion-nucleon threshold scattering parameters from the results of experiments on pionic hydrogen*, Nucl. Phys. A 794 (2007) 73-86

H.-Ch. Schröder et al., *The pion-nucleon scattering lengths from pionic hydrogen and deuterium*, Eur. Phys. J. C 21 (2001) 473-488

$$a_{\pi^- p \to \pi^- p} = 0.0883 \pm 0.0008 \, m_c^{-1}$$

$$a_{\pi^- p \to \pi^- p} = 0.0858 \pm 0.0007 \, m_c^{-1}$$

From amplitudes to observables

No-spin-flip and spin-flip contributions
Expansion of the πN scattering amplitude in a Legendre series

$$M_{fi} = f + g\,\vec{\sigma}\cdot\vec{n}$$

$$\vec{n} = \frac{\vec{q}\,'\times\vec{q}}{\left|\vec{q}\,'\times\vec{q}\right|}$$

$$f = f_C + \sum_{l=0}^{\infty}\left((l+1)\,f_{l+} + l f_{l-}\right)P_l(\cos\theta)$$

$$g = g_C + i\sum_{l=1}^{\infty}\left(f_{l+} - f_{l-}\right)\sin\theta\,P_l'(\cos\theta)$$

$$l \le 3$$

The DCS and the AP are given by the expressions

$$\frac{d\sigma}{d\Omega} = \frac{q'}{q}\left(\left|f\right|^2 + \left|g\right|^2\right)$$

$$A = \frac{2\,\Re\left[f\,\bar{g}\right]}{\left|f\right|^2 + \left|g\right|^2}$$

Sensitivity to small partial waves

Strategy in the analysis

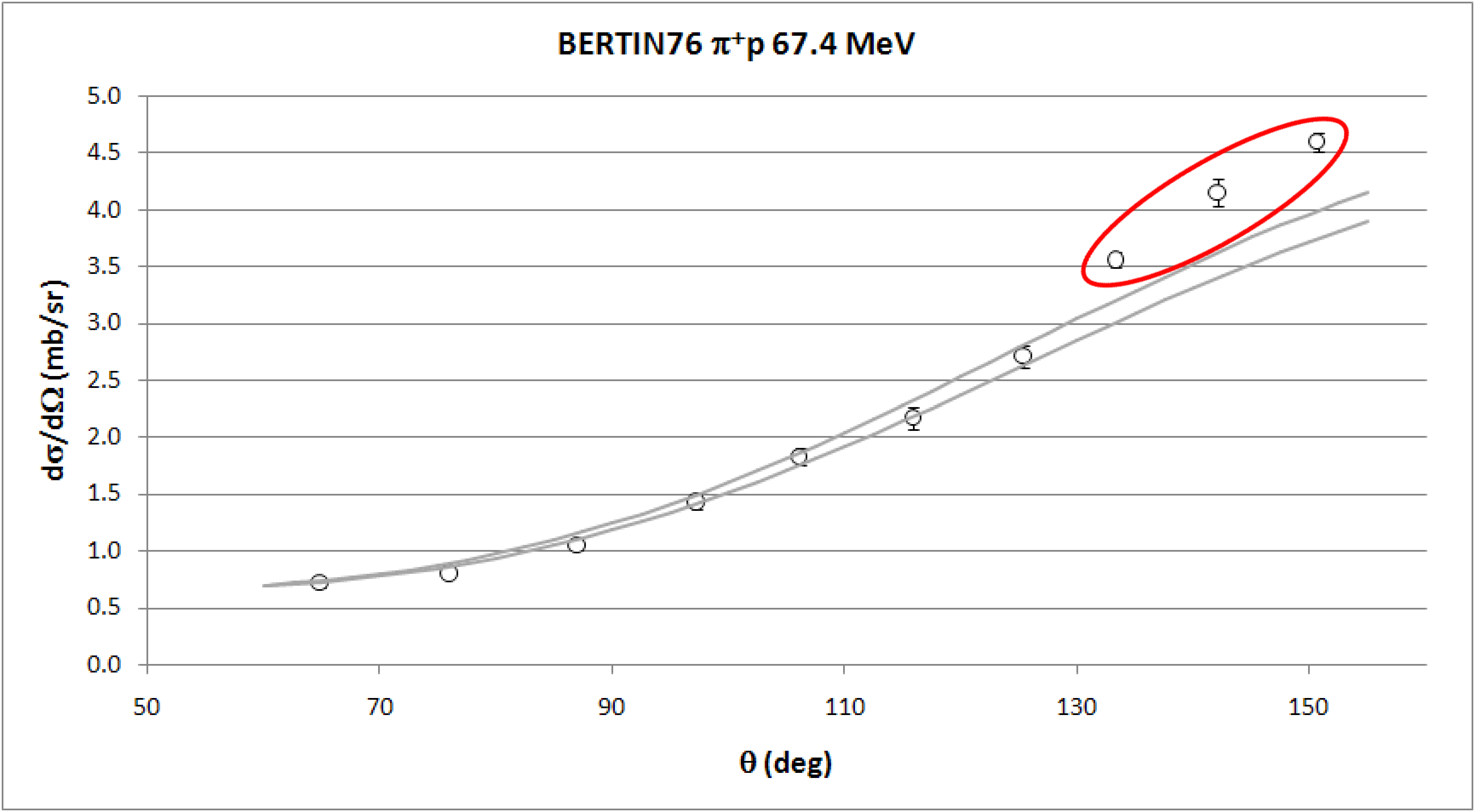
- Treat π^+p, exclude outliers, fix I=3/2 amplitudes (phenomenological model)
- Treat π^-p ES, exclude outliers (phenomenological model)
- Treat π^-p CX, exclude outliers (phenomenological model)

Elimination of the outliers

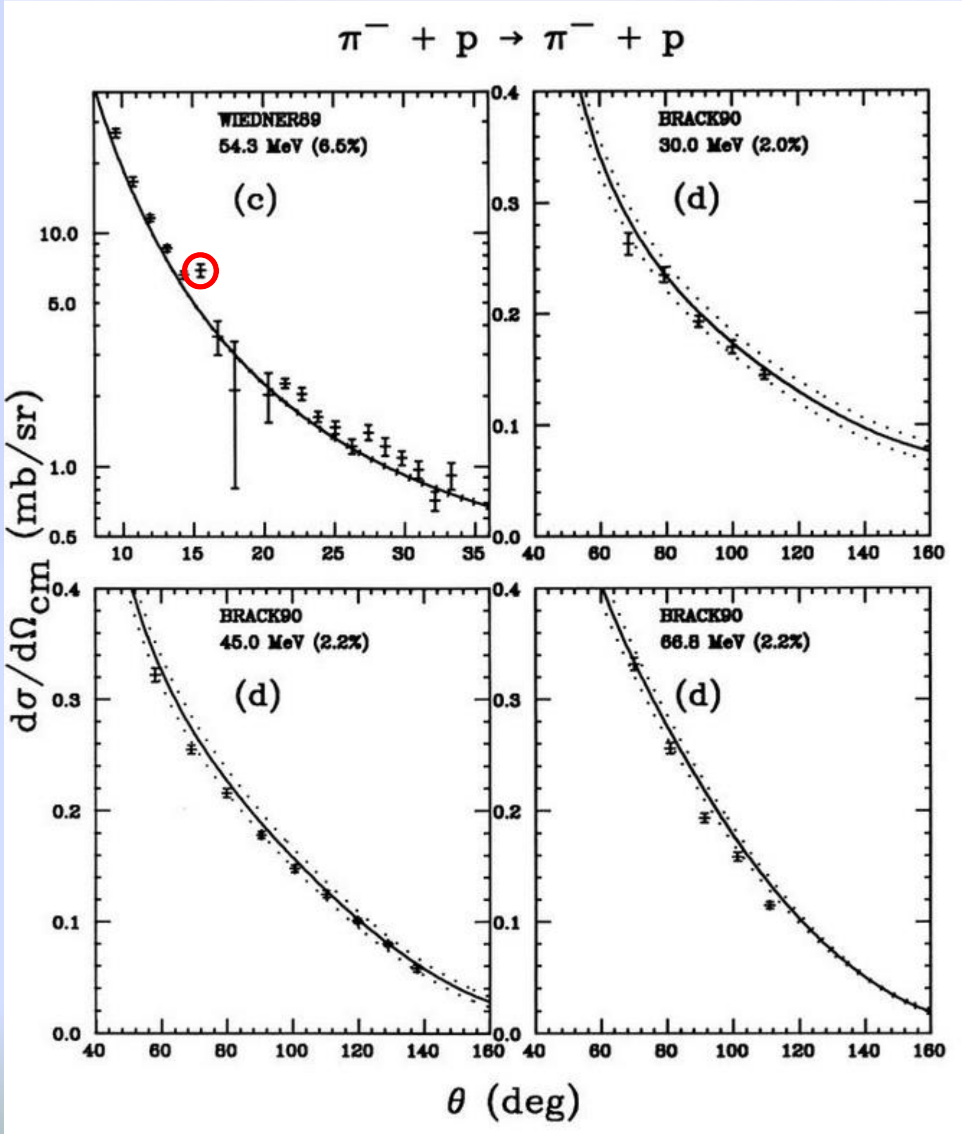
- Treat truncated π^+p and π^-p ES DBs (hadronic model), extract model parameters
- Treat truncated π^+p and π^-p CX DBs (hadronic model), extract model parameters
- Compare the results (model parameters, phase shifts, scattering lengths/volumes, predictions for the amplitudes and the observables) of the two analyses with the hadronic model

Main analysis

Example of obvious outliers



Example of obvious outliers



P.F.A. Goudsmit, H.J. Leisi, E. Matsinos, *The low-energy pion-nucleon interaction*, Helv. Phys. Acta 67 (1994) 369-391

Low-energy πN measurements not used

- H. Denz et al., *$\pi^{\pm}p$ differential cross sections at low energy*, Phys. Lett. B 633 (2006) 209-213; H. Denz, Ph.D. dissertation, Tübingen University, 2004; available from: tobias-lib.uni-tuebingen.de/dbt/volltexte/2004/1323/

Extensive analyses of the CHAOS $\pi^{\pm}$p DCSs

- E. Matsinos, G. Rasche, *Analysis of the low-energy $\pi^{\pm}p$ differential cross sections of the CHAOS Collaboration*, Nucl. Phys. A 903 (2013) 65-80
- E. Matsinos, G. Rasche, *New analysis of the low-energy $\pi^{\pm}p$ differential cross sections of the CHAOS Collaboration*, Int. J. Mod. Phys. E 24 (2015) 1550050

Minimisation function

$$\chi_j^2 = \sum_{i=1}^{N_j} \left(\frac{y_{ij}^{\mathrm{exp}} - z_j y_{ij}^{\mathrm{th}}}{\delta y_{ij}^{\mathrm{exp}}} \right)^2 + \left(\frac{z_j - 1}{\delta z_j} \right)^2$$

Scale factor

$$z_j = \frac{\sum_{i=1}^{N_j} y_{ij}^{\mathrm{exp}} y_{ij}^{\mathrm{th}} / (\delta y_{ij}^{\mathrm{exp}})^2 + (\delta z_j)^{-2}}{\sum_{i=1}^{N_j} (y_{ij}^{\mathrm{th}} / \delta y_{ij}^{\mathrm{exp}})^2 + (\delta z_j)^{-2}}$$

$$(\chi_j^2)_{\mathrm{min}} = \sum_{i=1}^{N_j} \frac{(y_{ij}^{\mathrm{exp}} - y_{ij}^{\mathrm{th}})^2}{(\delta y_{ij}^{\mathrm{exp}})^2} - \frac{\left(\sum_{i=1}^{N_j} (y_{ij}^{\mathrm{exp}} - y_{ij}^{\mathrm{th}}) y_{ij}^{\mathrm{th}} / (\delta y_{ij}^{\mathrm{exp}})^2 \right)^2}{\sum_{i=1}^{N_j} (y_{ij}^{\mathrm{th}} / \delta y_{ij}^{\mathrm{exp}})^2 + (\delta z_j)^{-2}}$$

R.A. Arndt, L.D. Roper, *The use of partial-wave representations in the planning of scattering measurements: Application to 330 MeV np scattering*, Nucl. Phys. B 50 (1972) 285-300

Analysis

Significance level in statistical tests is set to the equivalent of 2.5σ in the normal distribution ($\approx 1.24 \cdot 10^{-2}$)

Experiment with worst description is treated

Worst measurement of that experiment is removed (the absolute normalisation is also subject to the test)

Outliers are removed one by one and the optimisation with the phenomenological model is repeated

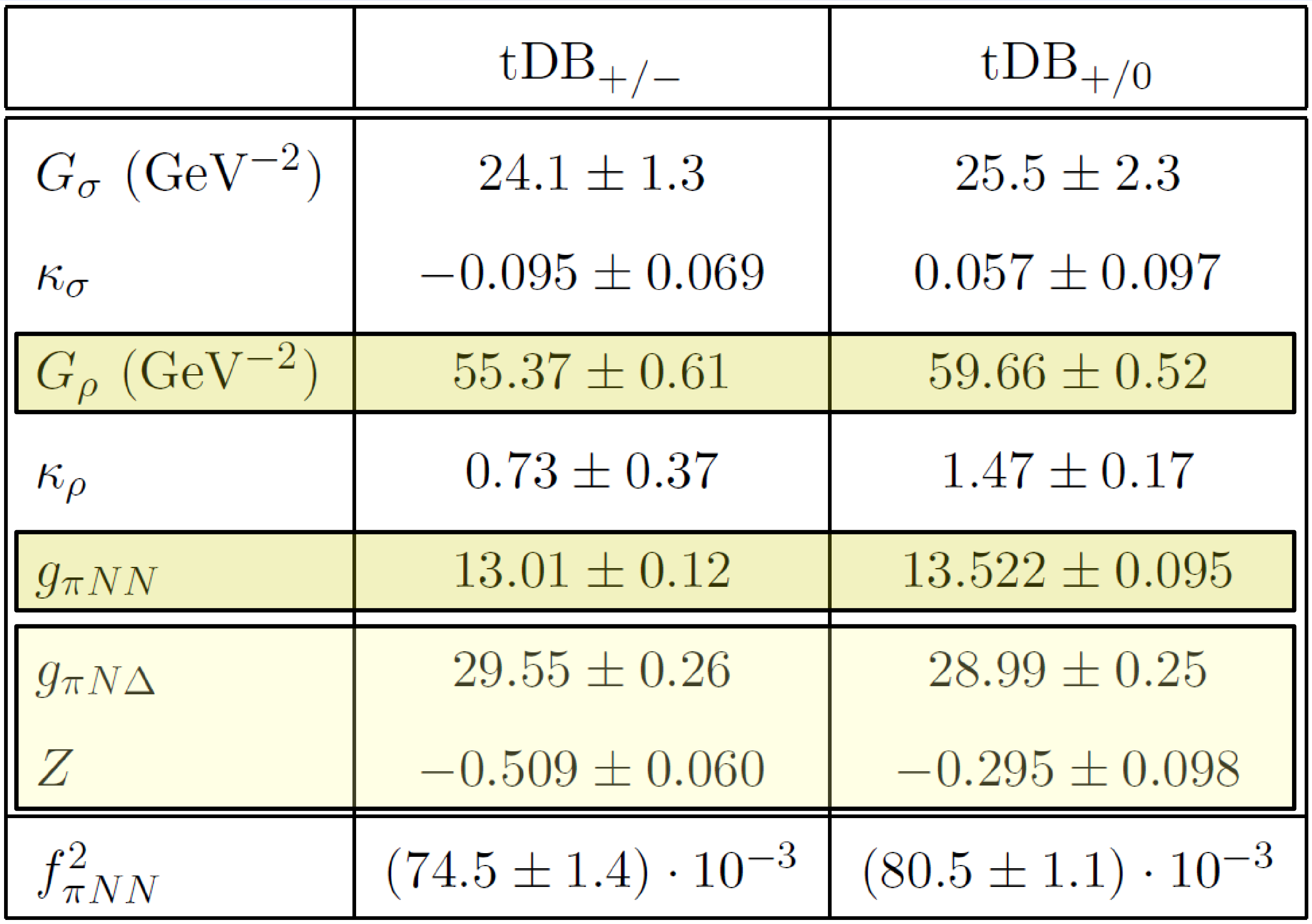
Elimination of the outliers

Reaction	Initial NDF	Initial χ^2	Final NDF	Final χ^2	Excluded DFs	Reduction in χ^2	χ^2 per excluded DF
π^+p	452	913.3	414	537.4	38	375.9	9.9
π^-p ES	332	519.6	324	369.1	8	150.5	18.8
π^-p CX	326	390.2	322	318.3	4	71.9	18.0

Possible analyses with the hadronic model

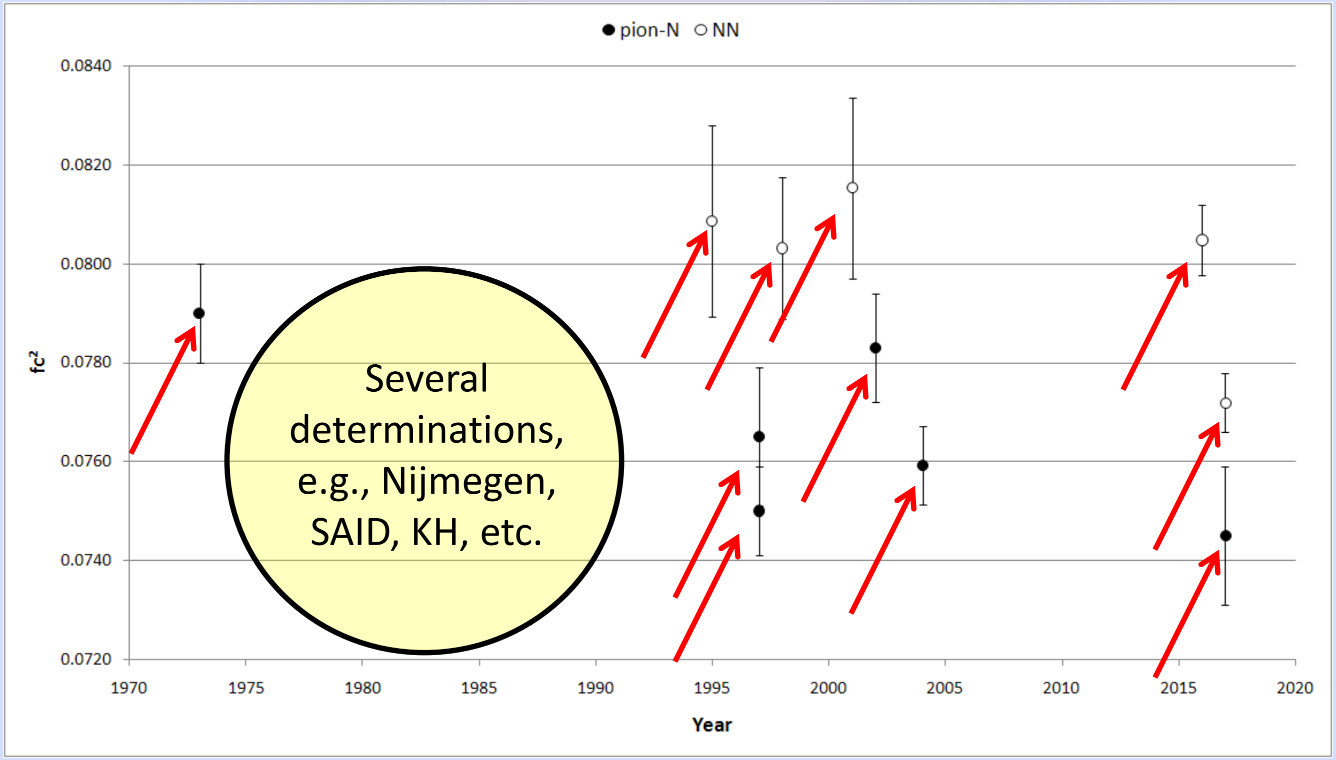
π^+p and π^-p ES
π^+p and π^-p CX
π^-p ES and CX
~~All eggs in one basket analysis~~

Results for the model parameters



	$\mathrm{tDB}_{+/-}$	$\mathrm{tDB}_{+/0}$
G_σ (GeV^{-2})	24.1 ± 1.3	25.5 ± 2.3
κ_σ	-0.095 ± 0.069	0.057 ± 0.097
G_ρ (GeV^{-2})	55.37 ± 0.61	59.66 ± 0.52
κ_ρ	0.73 ± 0.37	1.47 ± 0.17
$g_{\pi NN}$	13.01 ± 0.12	13.522 ± 0.095
$g_{\pi N\Delta}$	29.55 ± 0.26	28.99 ± 0.25
Z	-0.509 ± 0.060	-0.295 ± 0.098
$f^2_{\pi NN}$	$(74.5 \pm 1.4) \cdot 10^{-3}$	$(80.5 \pm 1.1) \cdot 10^{-3}$

The πN coupling constant



Several determinations, e.g., Nijmegen, SAID, KH, etc.

D.V. Bugg, A.A. Carter, J.R. Carter, *New values of pion-nucleon scattering lengths and f_2*, Phys. Lett. B 44 (1973) 278-280

T.E.O. Ericson et al., *πNN coupling from high precision np charge exchange at 162 MeV*, Phys. Rev. Lett. 75 (1995) 1046-1049

J.J. de Swart, M.C.M. Rentmeester, R.G.E. Timmermans, *The status of the pion-nucleon coupling constant*, πN Newslett. 13 (1997) 96-107; arxiv:nucl-th/9802084

E. Matsinos, *Isospin violation in the πN system at low energies*, Phys. Rev. C 56 (1997) 3014-3025

J. Rahm et al., *np scattering measurements at 162 MeV and the πNN coupling constant*, Phys. Rev. C 57 (1998) 1077-1096

J. Rahm et al., *np scattering measurements at 96 MeV*, Phys. Rev. C 63 (2001) 044001

T.E.O. Ericson, B. Loiseau, A.W. Thomas, *Determination of the pion-nucleon coupling constant and scattering lengths*, Phys. Rev. C 66 (2002) 014005

D.V. Bugg, *The pion nucleon coupling constant*, Eur. Phys. J. C 33 (2004) 505-509

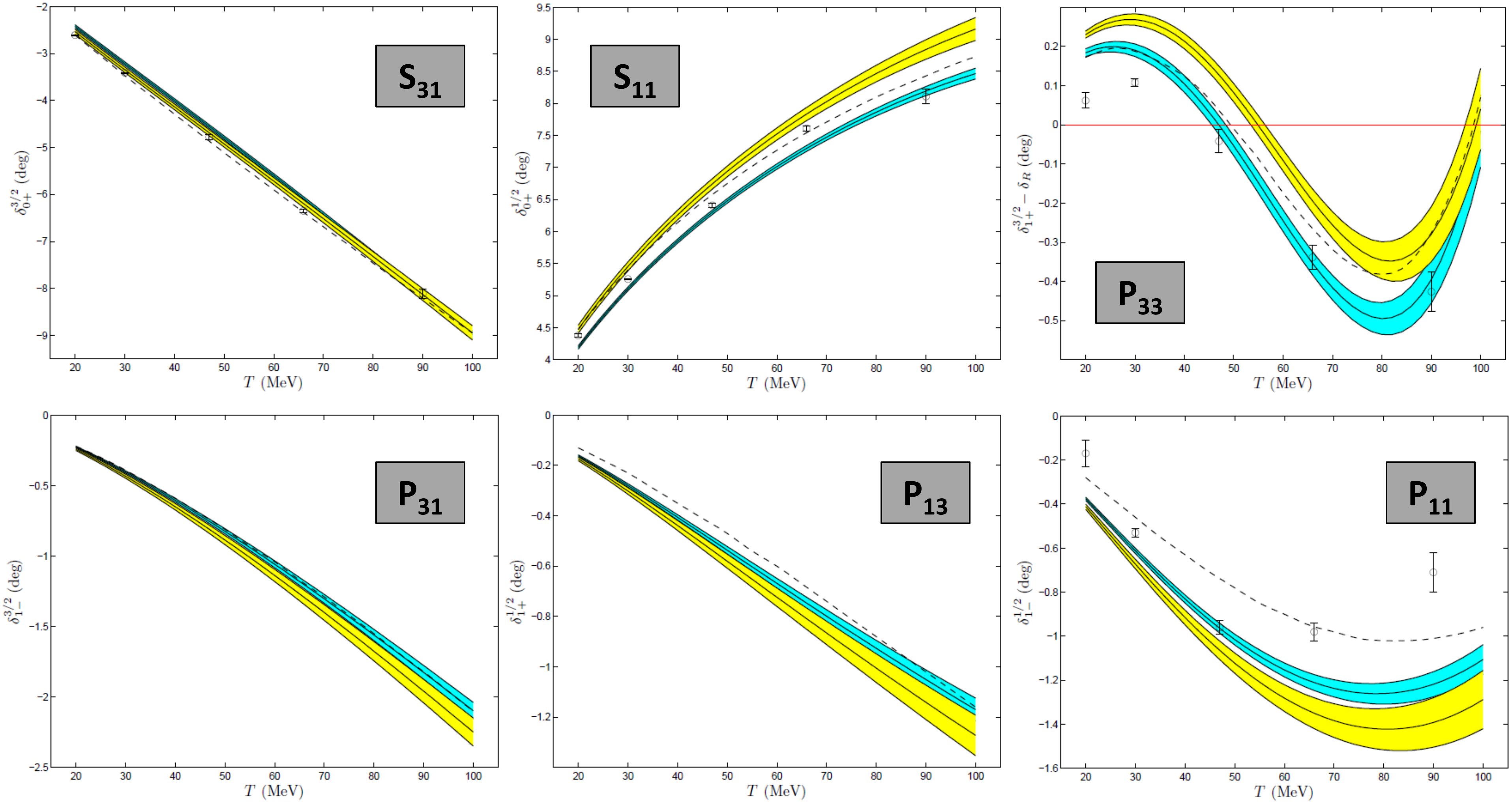
V.A. Babenko, N.M. Petrov, *Study of the charge dependence of the pion-nucleon coupling constant on the basis of data on low-energy nucleon-nucleon interactions*, Phys. Atomic Nuclei 79 (2016) 67-71

R. Navarro Pérez, J.E. Amaro, E. Ruiz Arriola, *Precise determination of charge dependent pion-nucleon-nucleon coupling constants*, Phys. Rev. C 95 (2017) 064001

E. Matsinos, G. Rasche, *Update of the phase-shift analysis of the low-energy πN data*, arxiv:1706.05524

“…A recent determination of f^2 by the Uppsala group from backward np cross sections is shown to be model dependent and inaccurate, and therefore completely uninteresting…”

πN *s*- and *p*-wave phase shifts



Blue: **tDB$_{+/-}$**, **Yellow**: **tDB$_{+/0}$**

Simplification at low energy

$$\mathscr{F}(\vec{q}\,',\vec{q}) = \underbrace{\boxed{b_0}+\boxed{b_1}\vec{\tau}\cdot\vec{t}}_{s\text{ wave}} + \underbrace{\left(\boxed{c_0}+\boxed{c_1}\vec{\tau}\cdot\vec{t}\right)\vec{q}\,'\cdot\vec{q}+i\left(\boxed{d_0}+\boxed{d_1}\vec{\tau}\cdot\vec{t}\right)\vec{\sigma}\cdot(\vec{q}\,'\times\vec{q})}_{p\text{ wave}}$$

Isoscalar

$$A^{+} = \frac{1}{3}(2A^{3/2}+A^{1/2})$$

Isovector

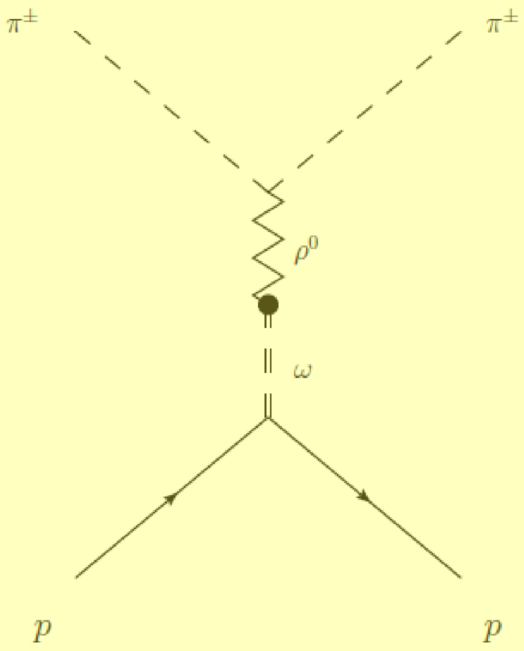
$$A^{-} = \frac{1}{3}(A^{3/2}-A^{1/2})$$

Analysis shows that the aforementioned effects predominantly originate in differences in the isovector part of the πN scattering amplitude

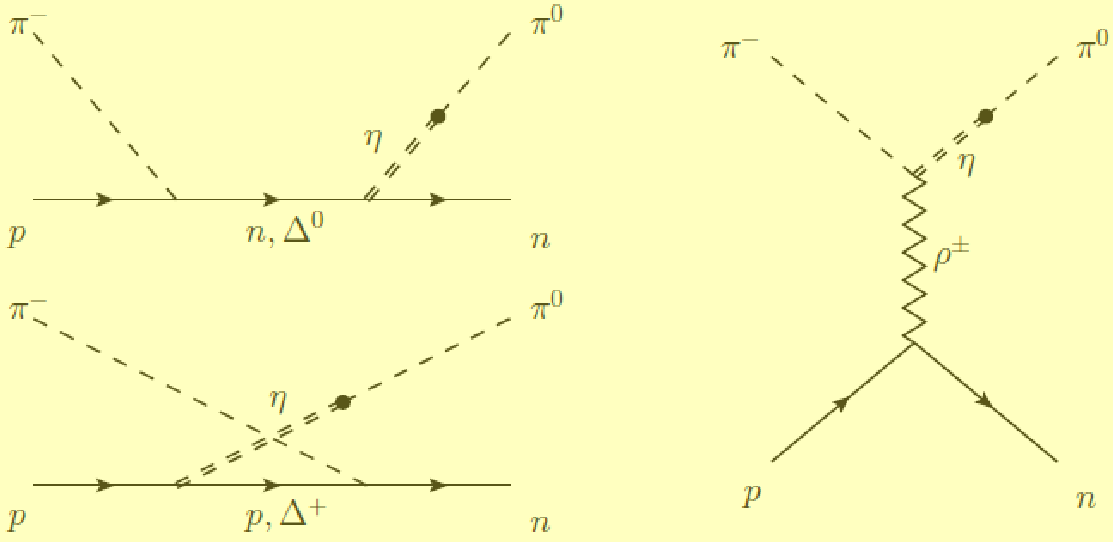
	b_0 (m_c^{-1})	b_1 (m_c^{-1})	c_0 (m_c^{-3})	c_1 (m_c^{-3})	d_0 (m_c^{-3})	d_1 (m_c^{-3})
σ	0.0710	-	0.0063	-	-0.0004	-
ρ	-	-0.07472	-	-0.0114	-	-0.01064
N	-0.0097	-0.00072	0.0016	0.1513	-0.1498	-0.00006
Δ(1232)	-0.0594	-0.00195	0.1904	0.0378	-0.0377	-0.05014
HBRs	0.0003	-0.00032	0.0082	-0.0022	0.0023	-0.00702
Sum	0.0021(12)	-0.07771(60)	0.2064(24)	0.1755(18)	-0.1856(19)	-0.06786(81)

Isospin-violating Feynman graphs

ρ^0-ω mixing



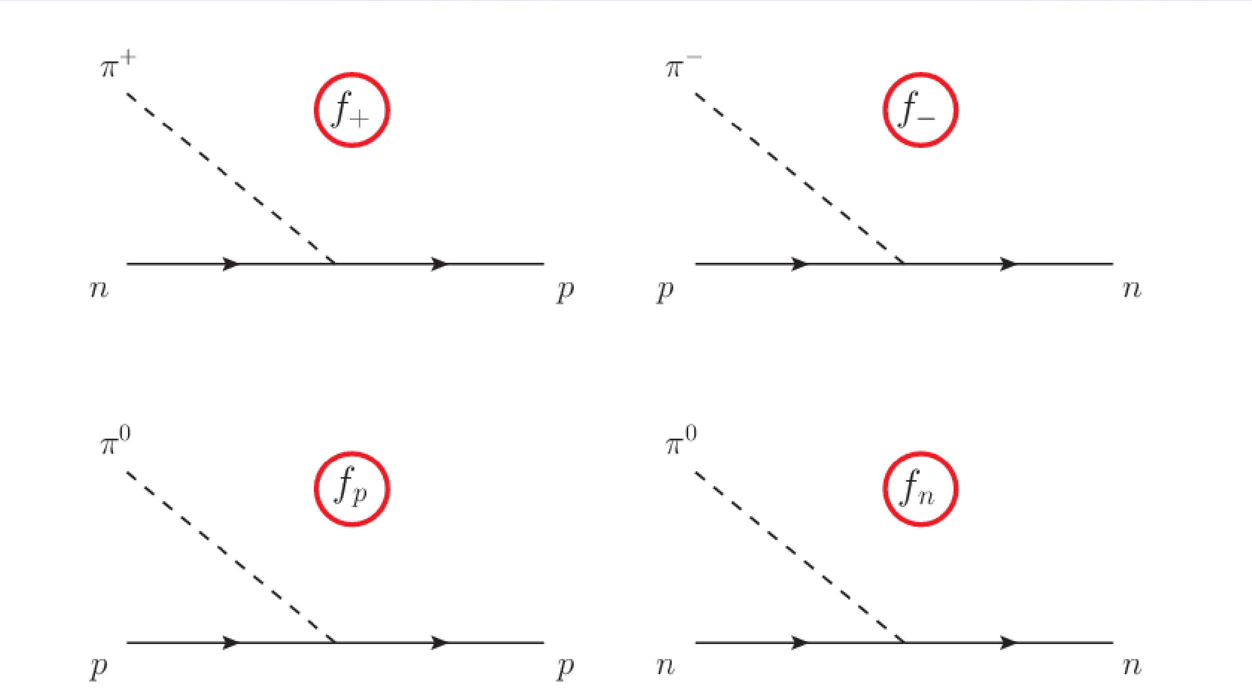
η-π^0 mixing



Affects $\pi^\pm p$ ES reactions

Affects $\pi^- p$ CX reaction

Splitting of the coupling constants



$$f_c^2 = f_+ f_-$$

$$f_0^2 = f_p f_n$$

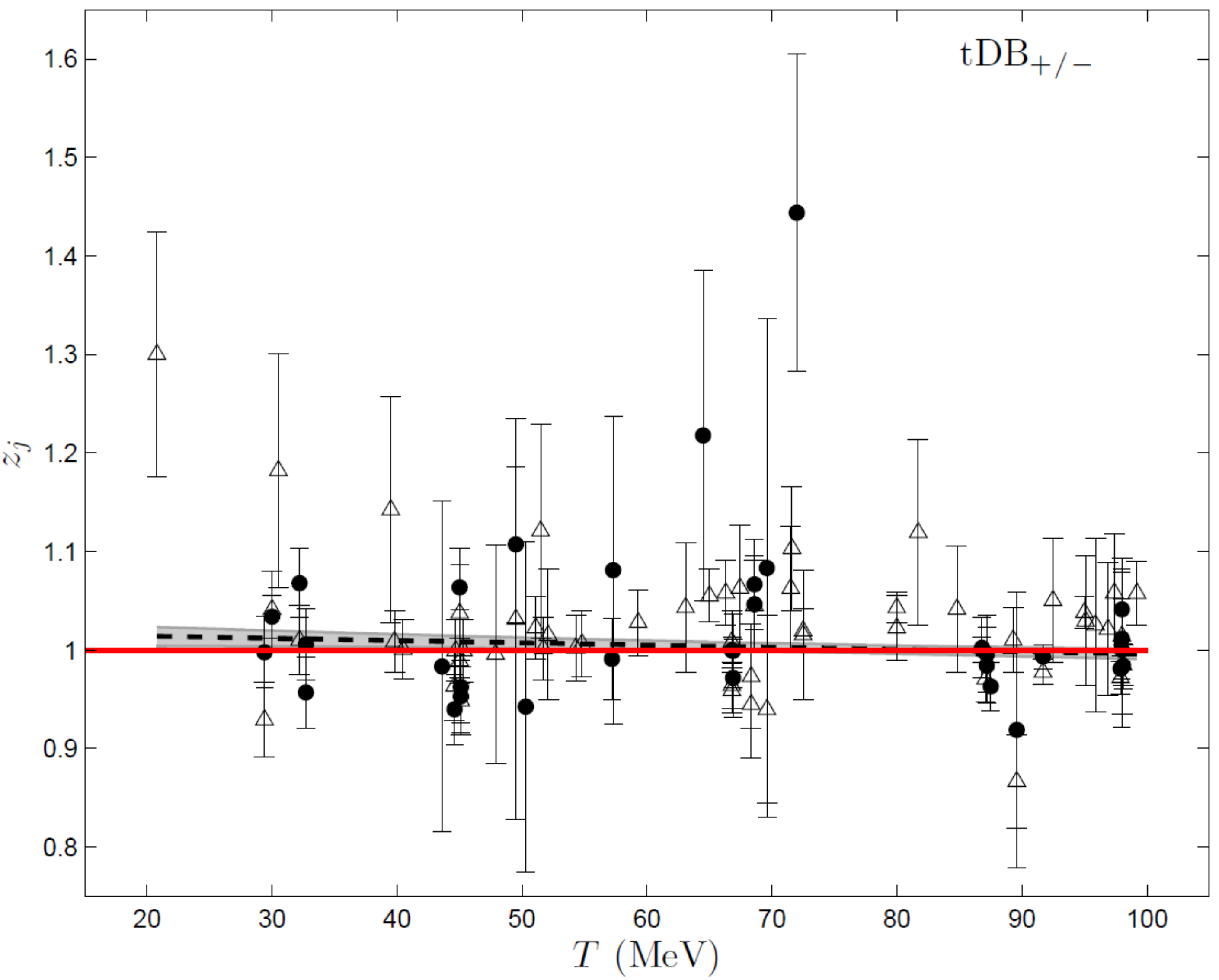
The splitting in $g_{\pi NN}$ has been predicted/extracted in a variety of studies, which generally agree that $f_c^2 \neq f_0^2$

No splitting: J.J. de Swart, M.C.M. Rentmeester, R.G.E. Timmermans, *The status of the pion-nucleon coupling constant*, πN Newslett. 13 (1997) 96-107; arxiv:nucl-th/9802084

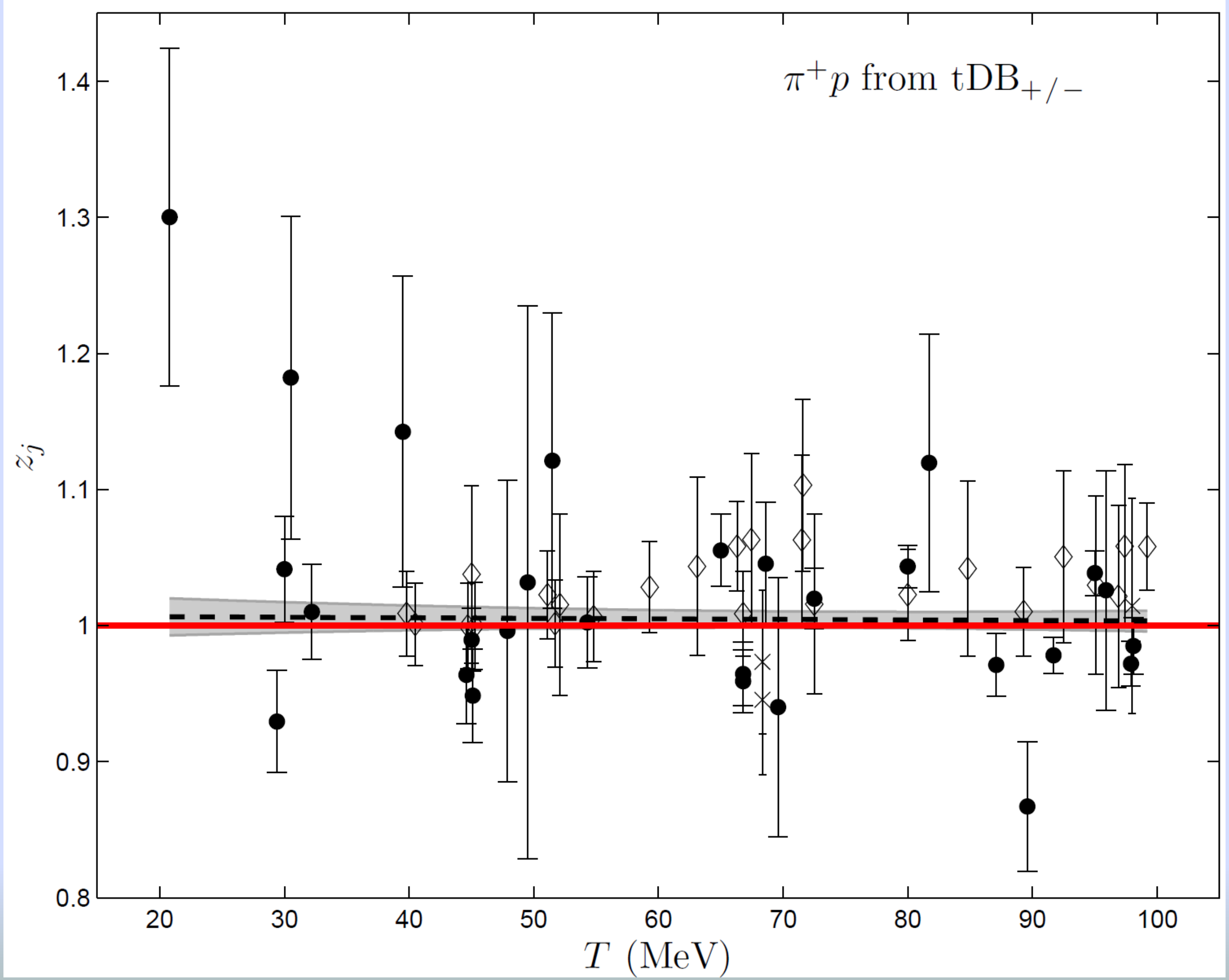
Analysis of the scale factors

- When analysed in terms of dependences on quantities relating to the data acquisition, the distribution of the scale factors must not exhibit systematic effects, e.g.,
  - No significant energy dependence
  - No significant dependence on the time, place, detector, technique, experimental group, etc., involved in the data acquisition

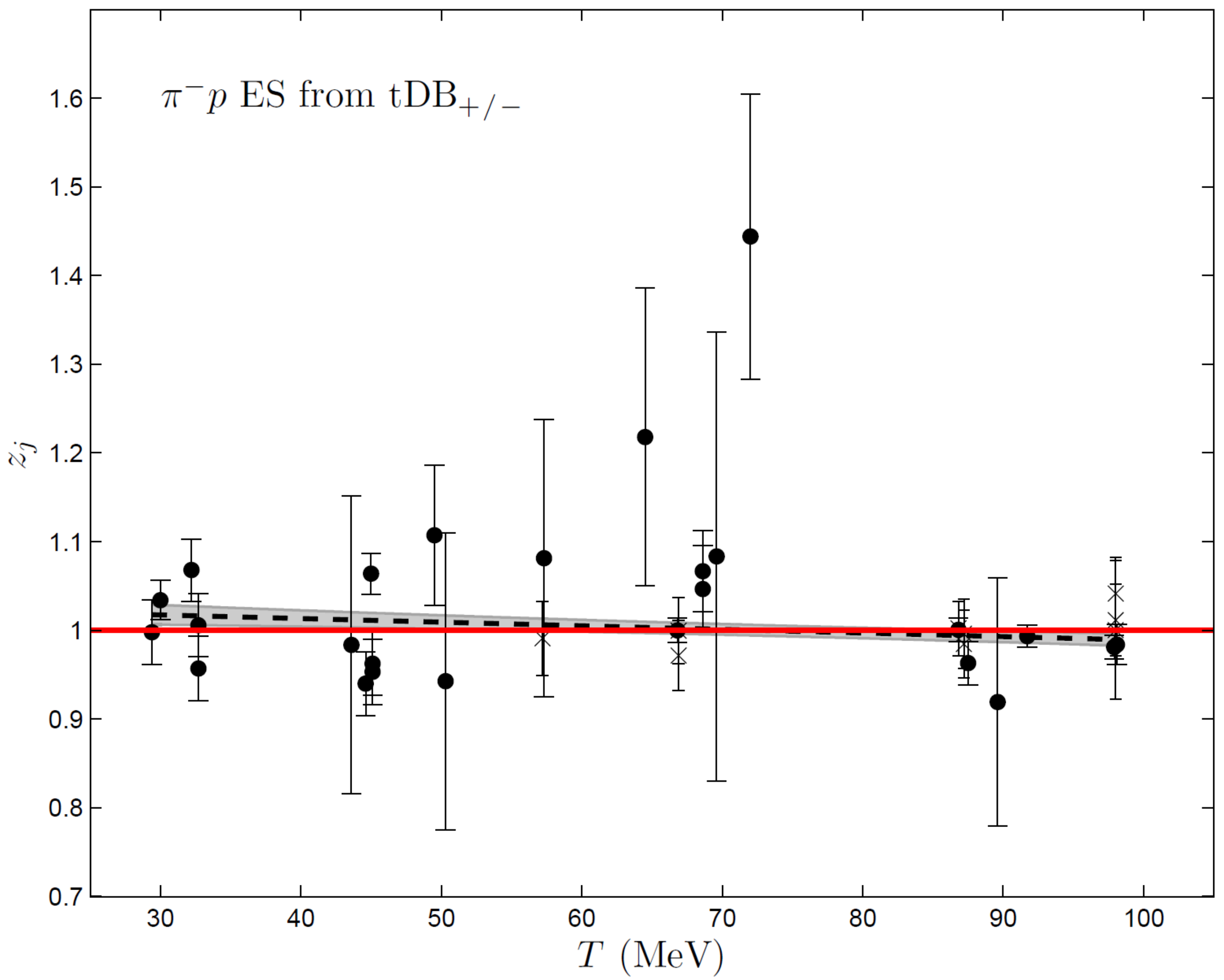
Analysis of the scale factors



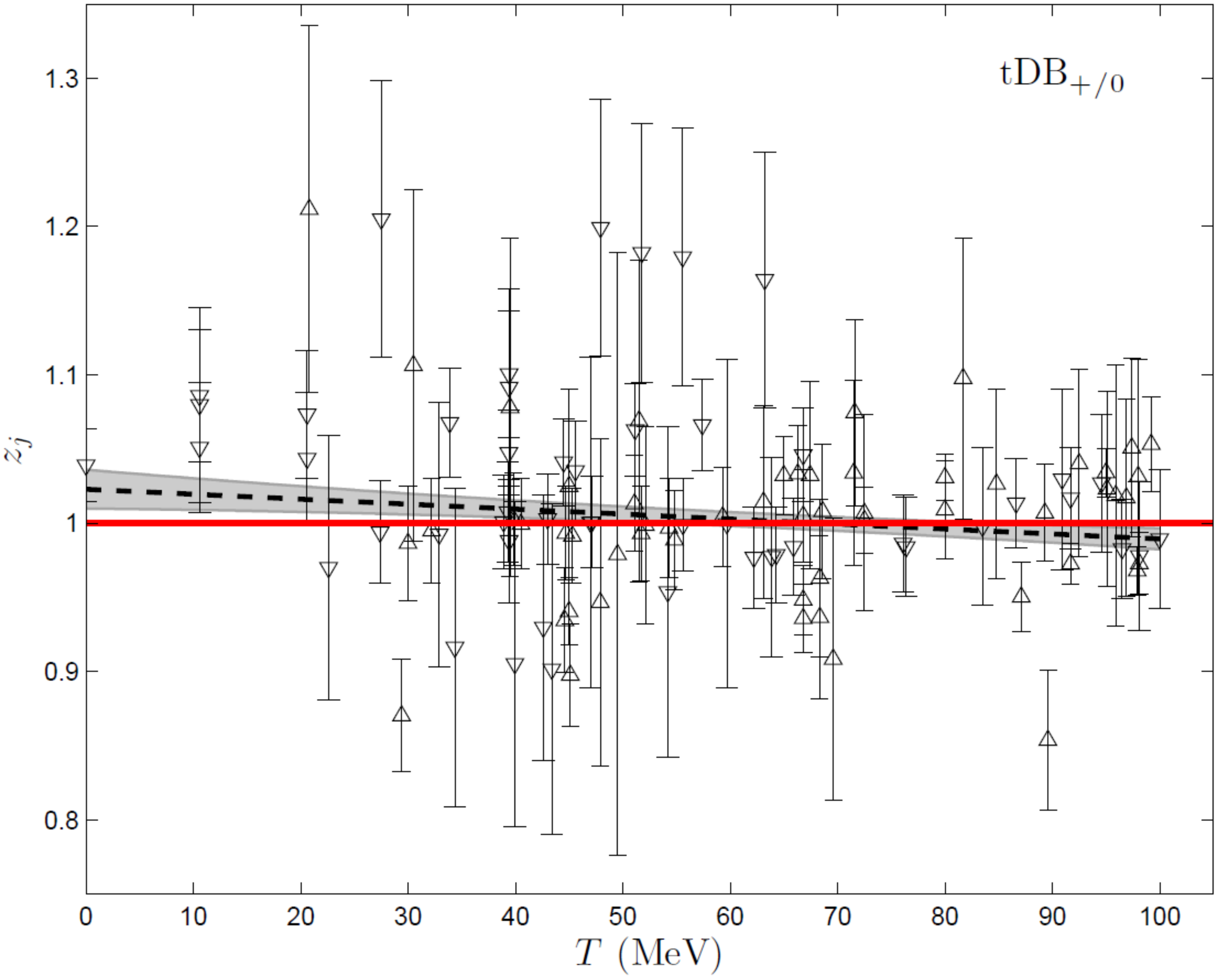
Analysis of the scale factors



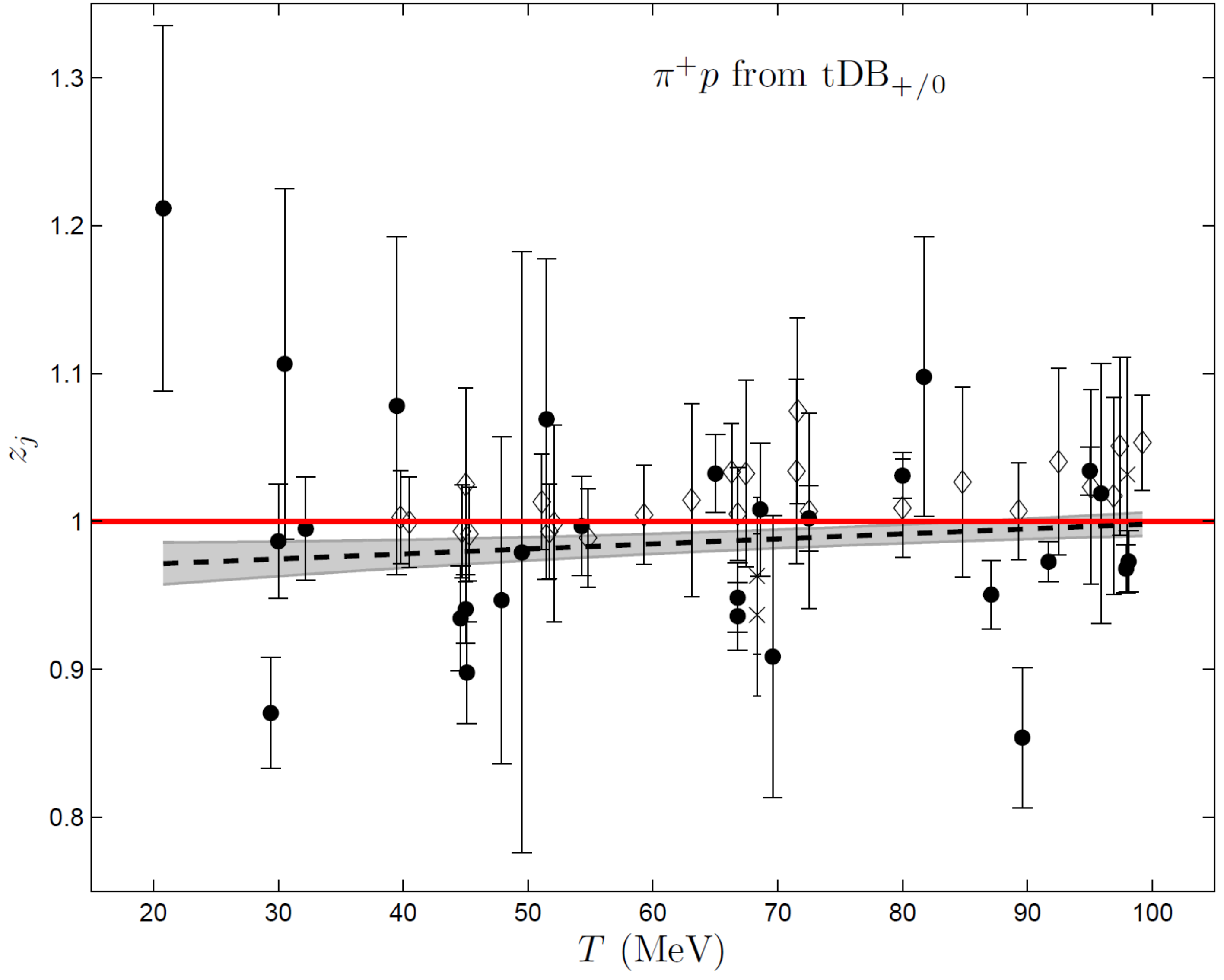
Analysis of the scale factors



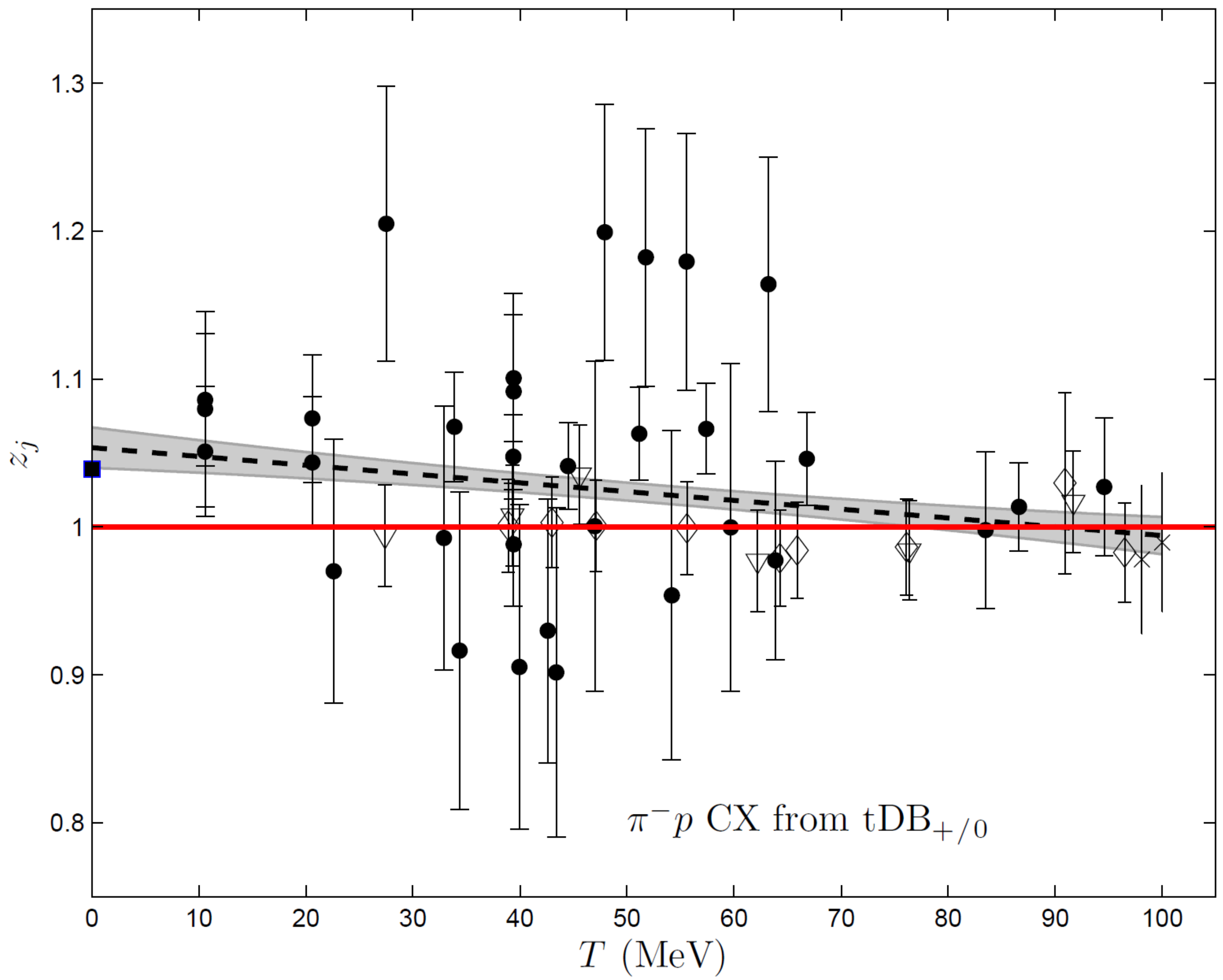
Analysis of the scale factors



Analysis of the scale factors



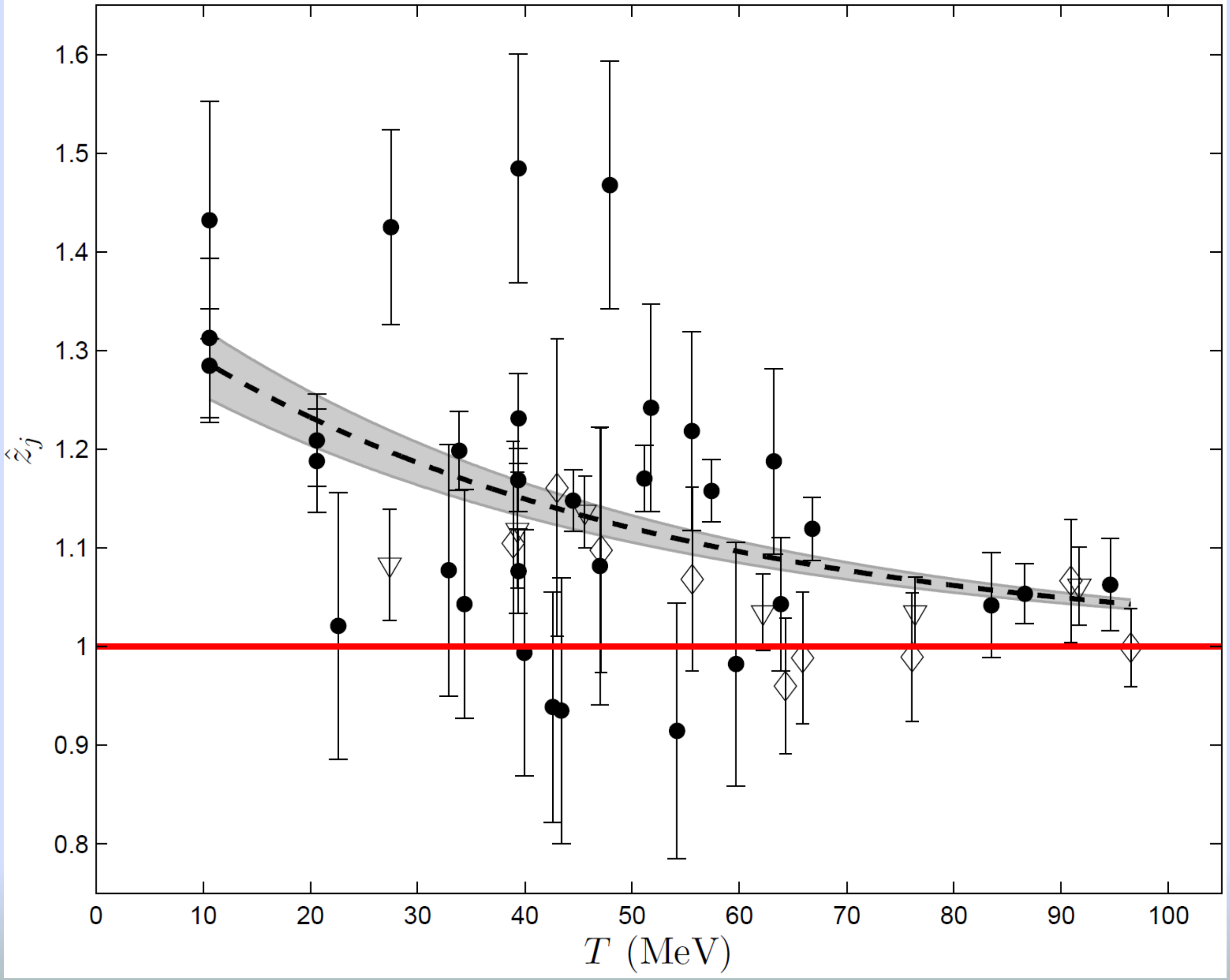
Analysis of the scale factors



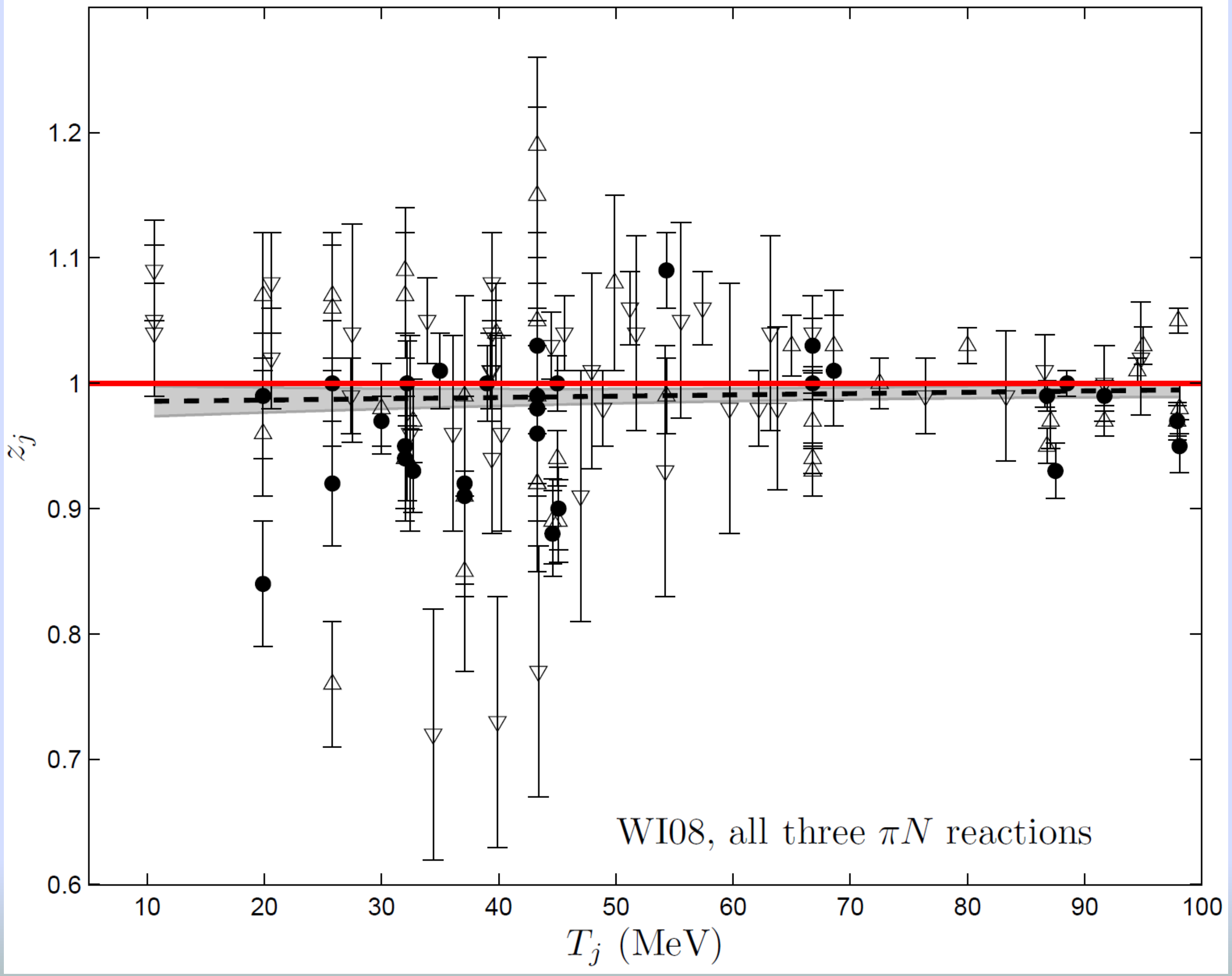
Analysis of the scale factors

- The analysis of the scale factors of the joint fits of the hadronic model to the π^+p and π^-p CX measurements reveals that the overall tendency of the modelling is to generate **overestimated** fitted DCSs for the π^+p reaction and **underestimated** ones for the π^-p CX reaction at low energy
- There is a difficulty to account for the π^-p CX DB in an isospin-invariant framework

Reproduction of the π^{-}p CX measurements

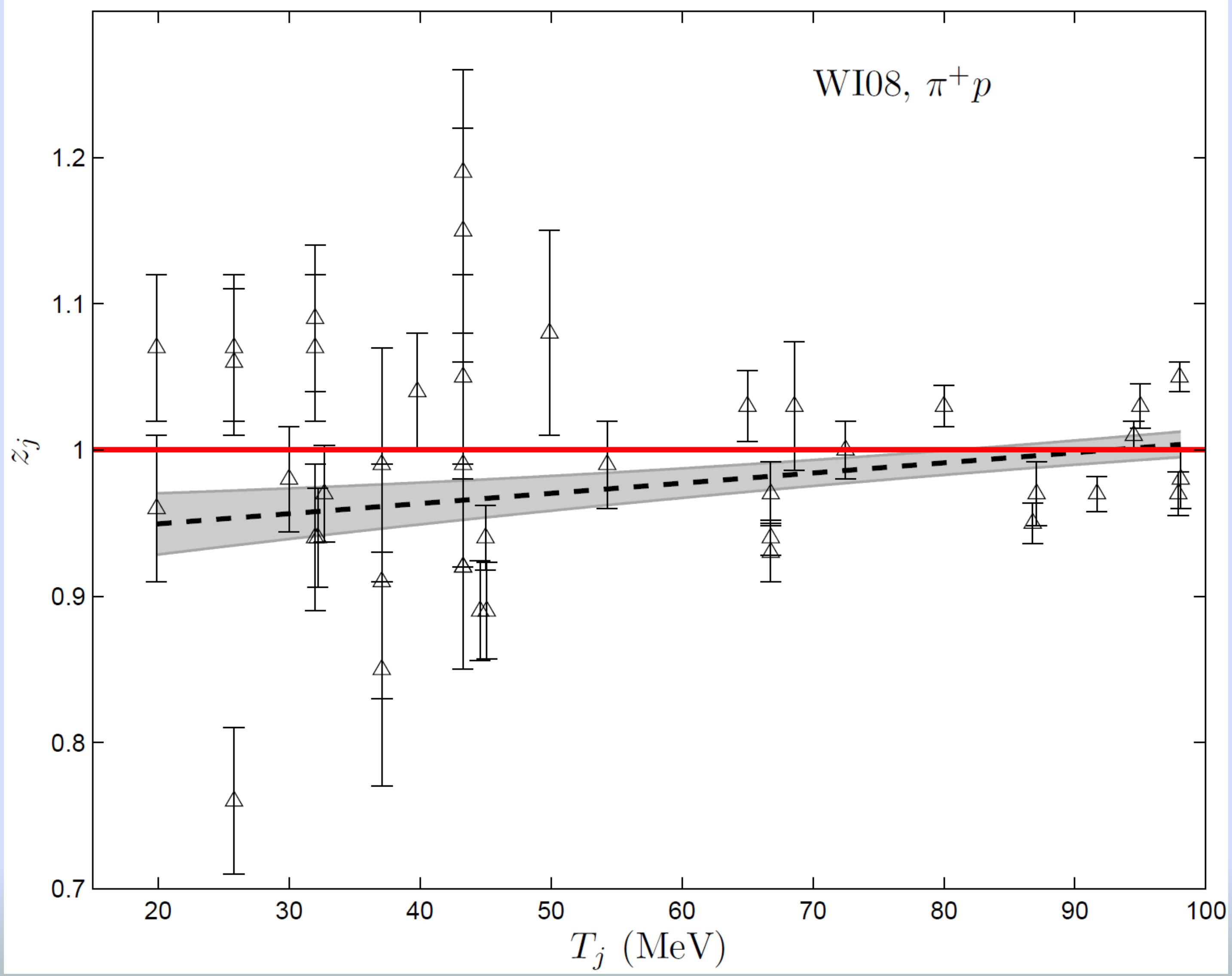


Analysis of the scale factors of WI08



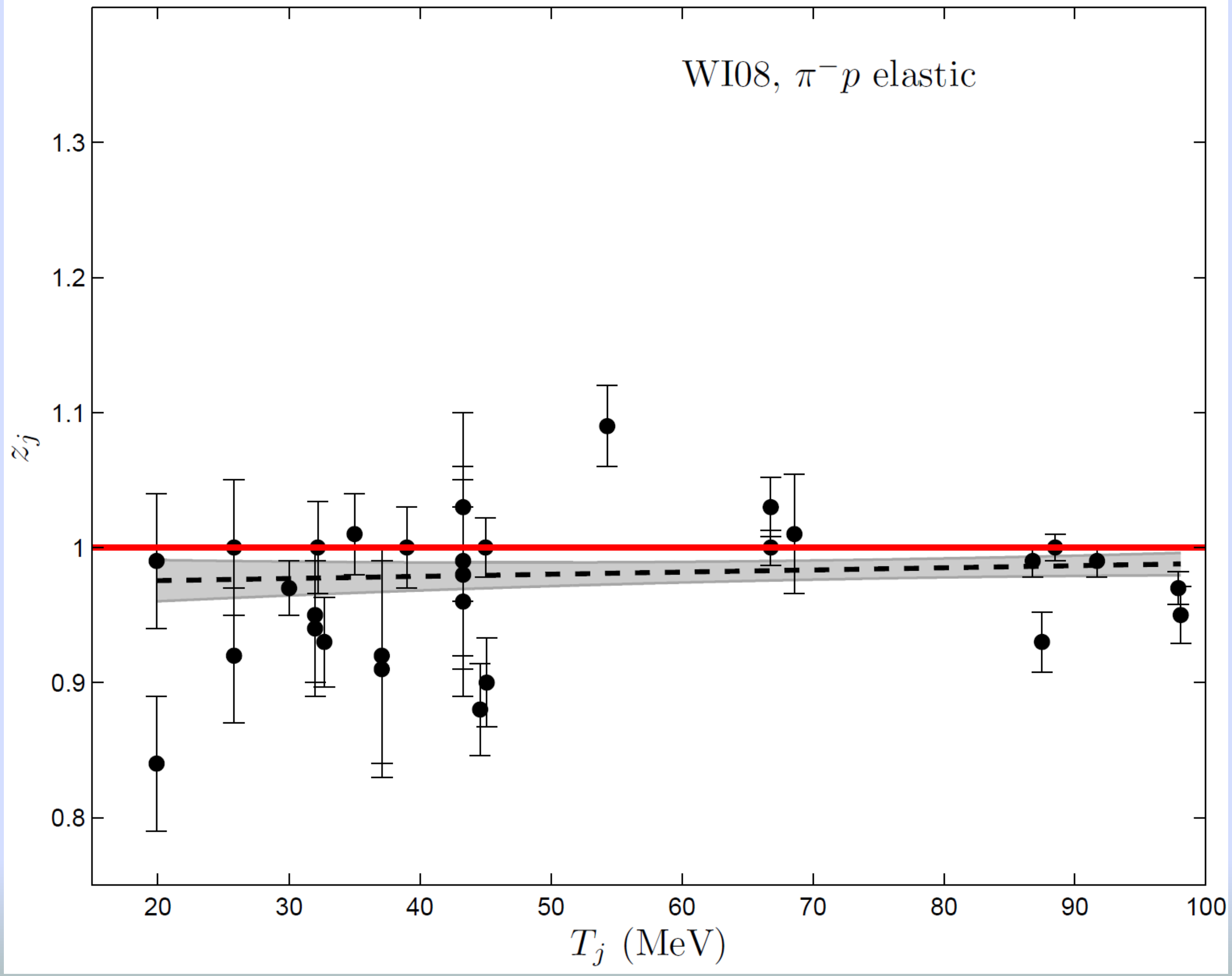
E. Matsinos, G. Rasche, *Systematic effects in the low-energy behavior of the current SAID solution for the pion-nucleon system*, Int. J. Mod. Phys. E 26 (2017) 1750002

Analysis of the scale factors of WI08



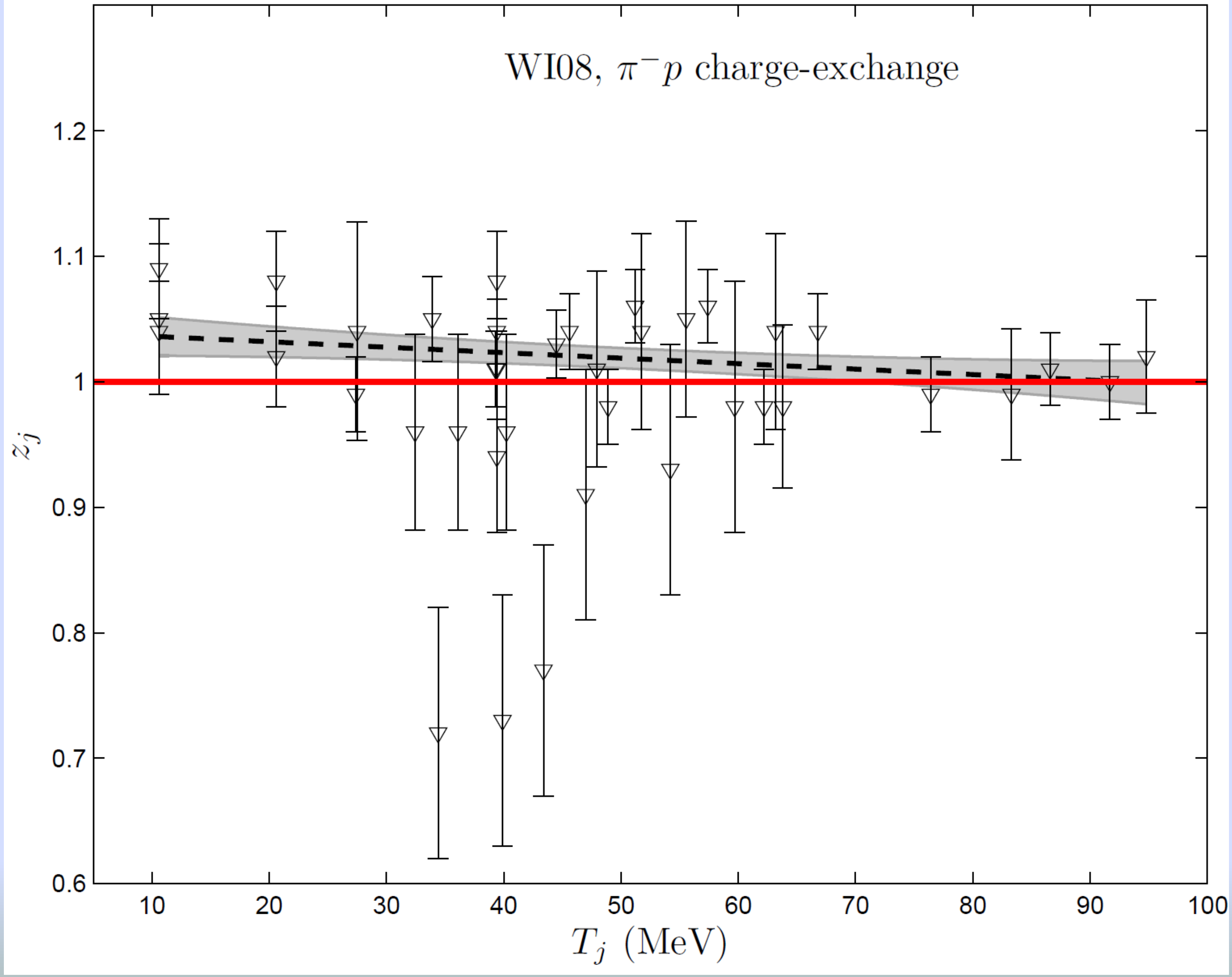
E. Matsinos, G. Rasche, *Systematic effects in the low-energy behavior of the current SAID solution for the pion-nucleon system*, Int. J. Mod. Phys. E 26 (2017) 1750002

Analysis of the scale factors of WI08



E. Matsinos, G. Rasche, *Systematic effects in the low-energy behavior of the current SAID solution for the pion-nucleon system*, Int. J. Mod. Phys. E 26 (2017) 1750002

Analysis of the scale factors of WI08

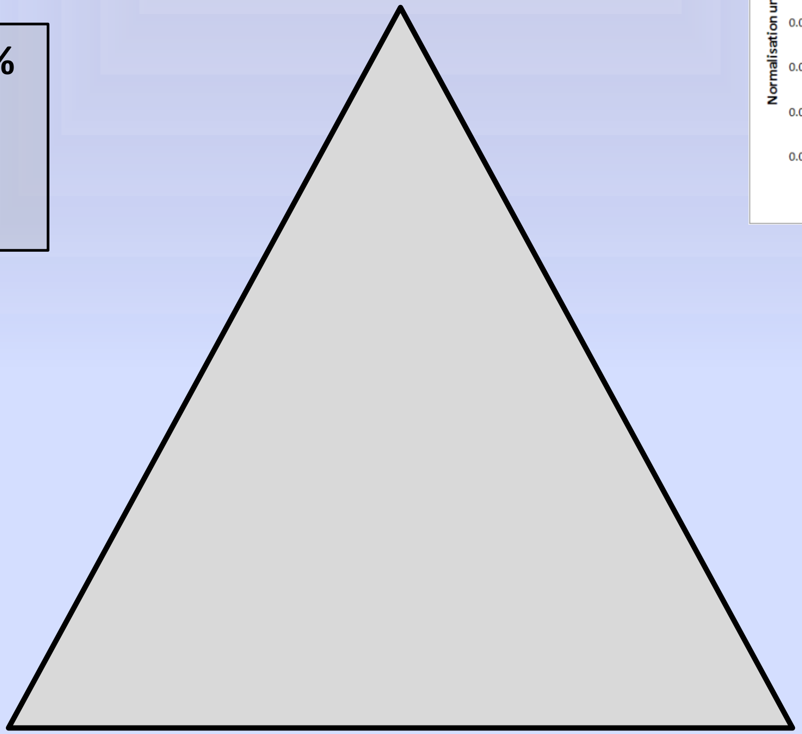


E. Matsinos, G. Rasche, *Systematic effects in the low-energy behavior of the current SAID solution for the pion-nucleon system*, Int. J. Mod. Phys. E 26 (2017) 1750002

How may the discrepancies be explained?

Do scattered pions evade detection?
Erroneous absolute normalisation?
Underestimated normalisation uncertainties?

DCS/AP data sets with Δz_j<3%
π^+ p: 15/45 (33.3%)
π^- p ES: 10/51 (19.6%)
π^- p CX: 5/38 (13.2%)



The absolute normalisation of the bulk of the low-energy data is reliable

Good

The residual EM effects are small

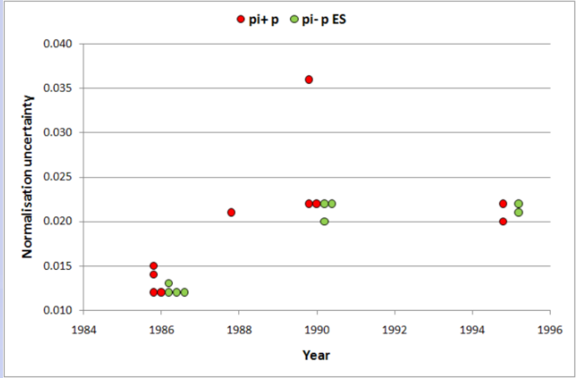
Cheap

Isospin invariance is fulfilled

Fast

New assessment of EM effects

Physics-wise, the most interesting possibility!



At the end of the day, what has been learnt?

- The analyses
  - of the π^+p and π^-p ES measurements, and
  - of the π^+p and π^-p CX measurements

  yield **different results** for the model parameters and for the phase shifts
- Differences in the **isovector** part of the πN scattering amplitude
- The reproduction of the absolute normalisation of the π^-p CX data sets on the basis of the results obtained from the $\pi^\pm$p ES measurements (via the use of the triangle identity) is poor
- There is a general difficulty to accommodate the π^-p CX measurements into a joint analysis
- When forcing the π^-p CX measurements into a joint analysis, effects are observed in the overall quality of the fits, as well as in the resulting scale factors of the experiments; similar effects have been observed in the output of the current SAID solution (WI08)
- The departure from the triangle identity at low energy may be due to
  - experimental discrepancies (e.g., incorrectness of the absolute normalisation of the bulk of the meson-factory low-energy πN measurements),
  - sizeable residual EM effects (e.g., use of the hadronic masses), or
  - violation of isospin invariance

E. Matsinos, G. Rasche, *Update of the phase-shift analysis of the low-energy πN data*, arxiv:1706.05524

List of collaborators

- Boris L. Birbrair
- Nadia Fettes
- Agim Gashi
- Pieter F.A. Goudsmit
- Anatoli B. Gridnev
- Hans-Jörg Leisi
- Geoffrey C. Oades
- Günther Rasche
- William S. Woolcock

Left to right: H.J. Leisi, B.L. Birbrair, and A.B. Gridnev; Saint Petersburg, 1990

P.F.A. Goudsmit; Schinznach Dorf, 2010

G. Rasche; Schinznach Dorf, 2010


Thank you

On the πN σ/Σ terms

Kinematics

Mandelstam variables

$$s=(p+q)^2$$
$$u=(p-q')^2$$
$$t=(q-q')^2$$

$$s+u+t=2m_p^2+2m_c^2\equiv\Lambda$$

The amplitudes depend on two variables which may be chosen at will; one of the standard choices is to use ν and t

$$\nu=\frac{s-u}{4m_p}$$

First condition for scattering

$$\vec{q}^{\,2}=\frac{(s-(m_p+m_c)^2)(s-(m_p-m_c)^2)}{4s}\geq 0$$

Second condition for scattering

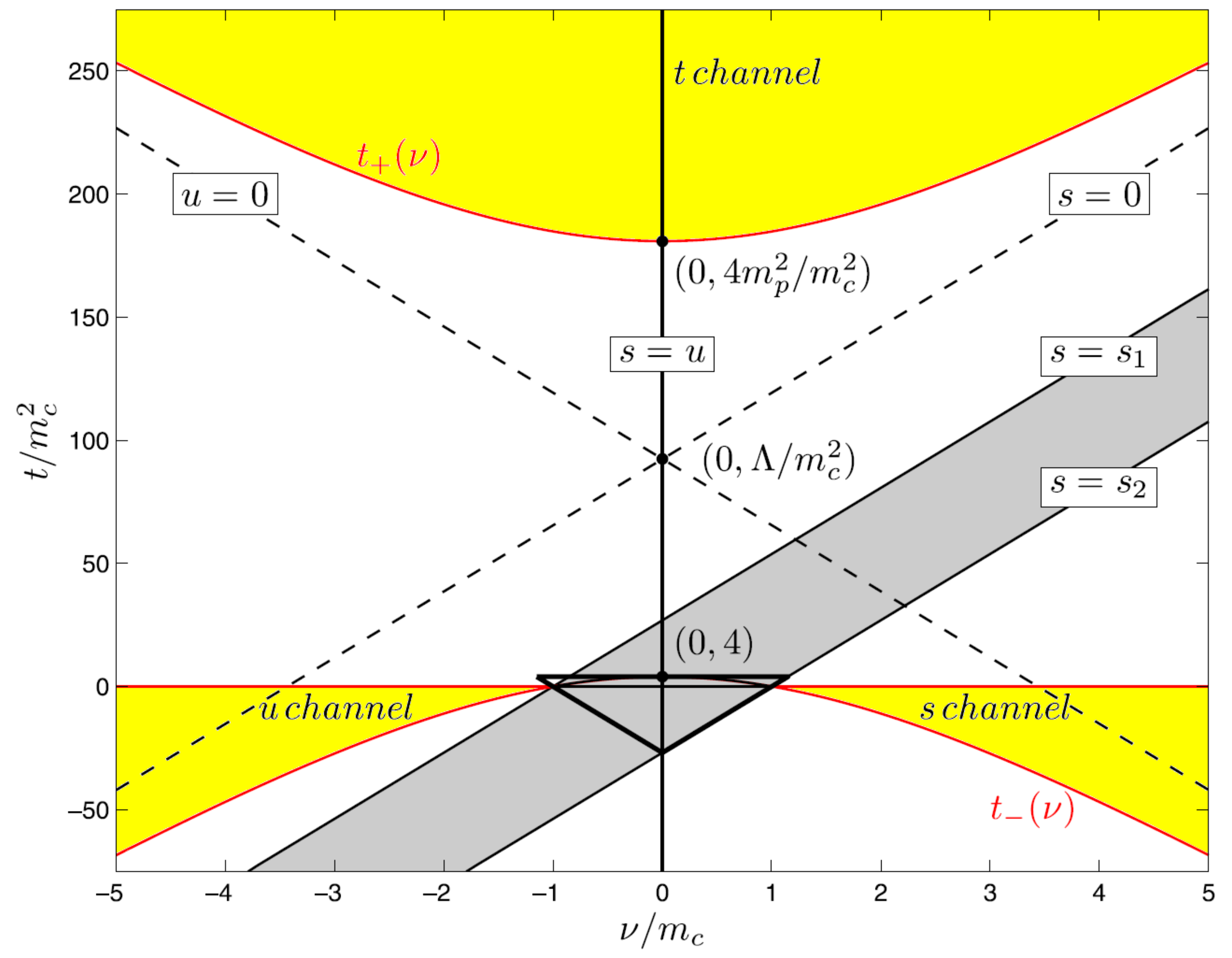
$$\begin{vmatrix} m_p^2 & \frac{s-m_p^2-m_c^2}{2} & \frac{t-2m_p^2}{2} \\ \frac{s-m_p^2-m_c^2}{2} & m_c^2 & \frac{u-m_p^2-m_c^2}{2} \\ \frac{t-2m_p^2}{2} & \frac{u-m_p^2-m_c^2}{2} & m_p^2 \end{vmatrix}\geq 0$$

or

'Kibble function' →

$$\boxed{\left(su-\left(m_p^2-m_c^2\right)^2\right)t}\geq 0$$

Kinematics



E. Matsinos, G. Rasche, *Aspects of the ETH model of the pion-nucleon interaction*, Nucl. Phys. A 927 (2014) 147-194

The πN σ/Σ terms

σ (MeV)	Σ (MeV)	Reference
59 ± 7		1
59.1 ± 1.9(stat.) ± 3.0(syst.)		2
58 ± 5		3
	79 ± 7	4
	70.3 ± 3.0	This programme (hadronic model/$\pi^{\pm}$p ES)
	68.0 ± 2.4(stat.) ± 1.7(syst.)	This programme (Olsson's method/$\pi^{\pm}$p ES)

1. J.M. Alarcón, J. Martin Camalich, J.A. Oller, *Chiral representation of the πN scattering amplitude and the pion-nucleon sigma term*, Phys. Rev. D 85 (2012) 051503
2. M. Hoferichter, J. Ruiz de Elvira, B. Kubis, Ulf-G. Meißner, *Precision determination of the pion-nucleon σ-term from Roy-Steiner equations*, Phys. Rev. Lett. 115 (2015) 092301
3. J. Ruiz de Elvira, M. Hoferichter, B. Kubis, Ulf-G. Meißner, *Extracting the sigma-term from low-energy pion-nucleon scattering*, arXiv:1706.01465
4. M.M. Pavan, R.A. Arndt, I.I. Strakovsky, R.L. Workman, *The pion-nucleon Σ term is definitely large: results from a G.W.U. analysis of πN scattering data*, πN Newslett. 16 (2002) 110-115

The πN Σ term

$$\widetilde{D}^{+}(\nu,t)=\widetilde{A}^{+}(\nu,t)+\nu\widetilde{B}^{+}(\nu,t)$$

$$\Sigma=F_{\pi}^{2}\,\Re[\widetilde{D}^{+}(\nu=0,t=2m_{c}^{2})]$$

The ($\nu=0,t=m_c^2$) point is known as **Cheng-Dashen point** and lies in the **unphysical** region. To obtain the value of the D amplitude at the Cheng-Dashen point, one first needs to extrapolate the D amplitude from the physical region (where it is determined) into the unphysical one (analyticity constraint).

The wave over the amplitudes indicates that the pseudovector pole term (contribution of the N graphs) is removed prior to the extrapolation.

On the πN σ/Σ terms

Scalar form factor σ(t): the matrix element of the u- and d-quark QCD Hamiltonian mass term between two proton states with 4-momenta p and p'

$$\bar{u}(p')\sigma(t)u(p) = \frac{m_a}{2m_p}\langle p' | \bar{u}u + \bar{d}d | p \rangle$$

$$m_a = \frac{m_u + m_d}{2}$$

The πN σ term: $\sigma \equiv \sigma(0)$

The strange-quark content of the proton: $y = \frac{2\langle p | \bar{s}s | p \rangle}{\langle p | \bar{u}u + \bar{d}d | p \rangle}$

$$\sigma = \frac{m_a}{2m_p}\,\frac{\langle p | \bar{u}u + \bar{d}d - 2\bar{s}s | p \rangle}{1-y} \equiv \frac{\hat{\sigma}}{1-y}$$

An estimate of y may be obtained from σ and $\hat{\sigma}$

Via the πN data and the Σ term

Mass breaking in the QCD Hamiltonian

$$\boxed{\sigma = \Sigma - \Delta_\sigma - \Delta_R}$$

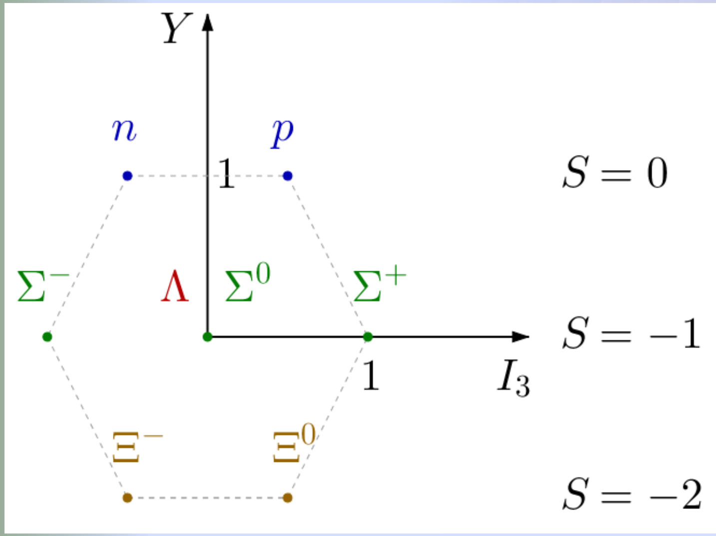
J. Gasser, H. Leutwyler, M.E. Sainio, *Form factor of the σ-term*, Phys. Lett. B253 (1991) 260-264 $\Delta_\sigma = 15.2 \pm 0.4\,\mathrm{MeV}$

V. Bernard, N. Kaiser, Ulf-G. Meißner, *Critical analysis of baryon masses and sigma-terms in heavy baryon chiral perturbation theory*, Z. Phys. C 60 (1993) 111-119 $\Delta_\sigma \approx 15\,\mathrm{MeV}$

This programme (hadronic model/$\pi^\pm$p ES) $\Delta_\mathrm{R} = 0.627 \pm 0.011\,\mathrm{MeV}$

$$\boxed{\sigma = \Sigma - (15.8 \pm 0.4)\,\mathrm{MeV}}$$

Strange-quark content of the nucleon



M. Gell-Mann, R. Oakes, B. Renner, *Behavior of current divergences under $SU_3 \times SU_3$*, Phys. Rev. 175 (1968) 2195-2199

$$\hat{\sigma} \approx \frac{(m_\Xi + m_\Sigma - 2m_N)m_c^2}{2(m_K^2 - m_c^2)} \approx 27.7\,\mathrm{MeV}$$

J. Gasser, *Hadron masses and the sigma commutator in light of chiral perturbation theory*, Ann. Phys. 136 (1981) 62-112
J. Gasser, H. Leutwyler, *Quark masses*, Phys. Rep. 87 (1982) 77-169
B. Borasoy, *Sigma-terms in heavy baryon chiral perturbation theory revisited*, Eur. Phys. J. C8 (1999) 121-130

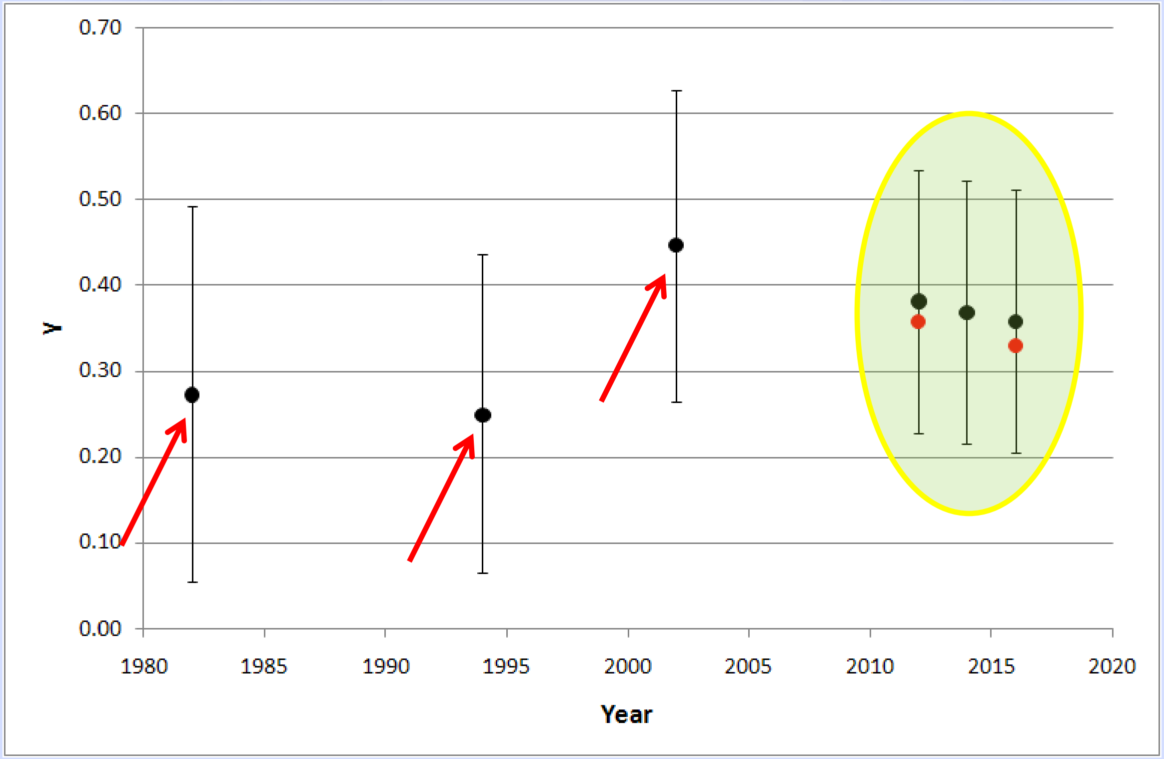
$$\boxed{\hat{\sigma} = 35 \pm 5\,\mathrm{MeV}}$$

R. Koch, *A new determination of the πN sigma term using hyperbolic dispersion relations in the (ν^2,t) plane*, Z. Phys. C 15 (1982) 161-168

$$\boxed{\Sigma = 64 \pm 8\,\mathrm{MeV} \Rightarrow \sigma = 48 \pm 8\,\mathrm{MeV}}$$

The meson-factory measurements require an enhanced isoscalar component in the πN scattering amplitude, thus suggesting that **a sizeable fraction of the nucleon mass might be due to the strange quark**!

The ‘strange’ strange-quark content of the nucleon



R. Koch, *A new determination of the πN sigma term using hyperbolic dispersion relations in the (ν^2,t) plane*, Z. Phys. C 15 (1982) 161-168

P.F.A. Goudsmit, H.J. Leisi, E. Matsinos, B.L. Birbrair, A.B. Gridnev, *The extended tree-level model of the pion-nucleon interaction*, Nucl. Phys. A 575 (1994) 673-706

M.M. Pavan, R.A. Arndt, I.I. Strakovsky, R.L. Workman, *The pion-nucleon Σ term is definitely large: results from a G.W.U. analysis of πN scattering data*, πN Newslett. 16 (2002) 110-115

E. Matsinos, G. Rasche, *Aspects of the ETH model of the pion-nucleon interaction*, Nucl. Phys. A 927 (2014) 147-194

The voice of one crying in the wilderness

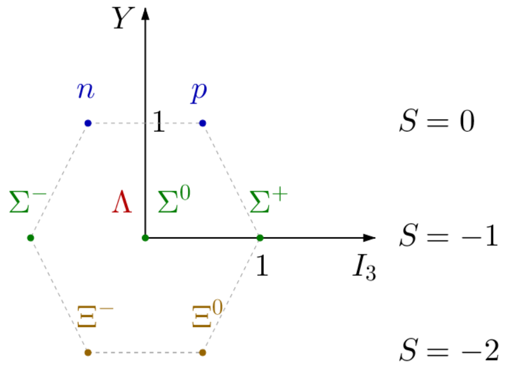
- M. Anselmino, M.D. Scadron, *Does the proton contain large strange quark components?*, Phys. Lett. B 229 (1989) 117-121
- G. Clément, M.D. Scadron, J. Stern, *Why a large sigma term does not require a large strange quark content in the nucleon*, J. Phys. G 17 (1991) 199-204
- M.D. Scadron, *On the strange quark content of nucleons*, Z. Phys. C 54 (1992) 595-597
- S.A. Coon, M.D. Scadron, *Is there a scalar form-factor suppression of the nucleon σ-term?*, J. Phys. G 18 (1992) 1923-1931

$$\widehat{\sigma}^{IMF} \approx \frac{\left(m_{\Xi}^{2} + m_{\Sigma}^{2} - 2m_{N}^{2}\right)m_{c}^{2}}{2m_{N}(m_{K}^{2} - m_{c}^{2})} \approx 65.1\,\mathrm{MeV}$$

Problems with the GL value?

Redetermination of $\hat{\sigma}$

J.M. Alarcón, L.S. Geng, J. Martin Camalich, J.A. Oller, *The strangeness content of the nucleon from effective field theory and phenomenology*, Phys. Lett. B 730 (2014) 342-346



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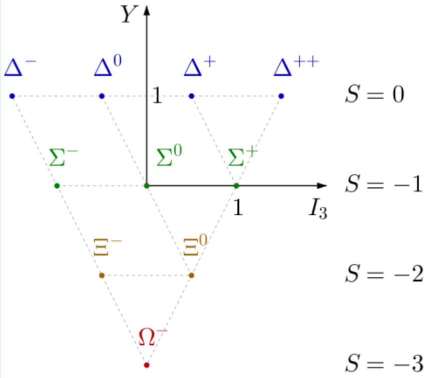


$\boxed{\hat{\sigma}}$ $\longrightarrow$

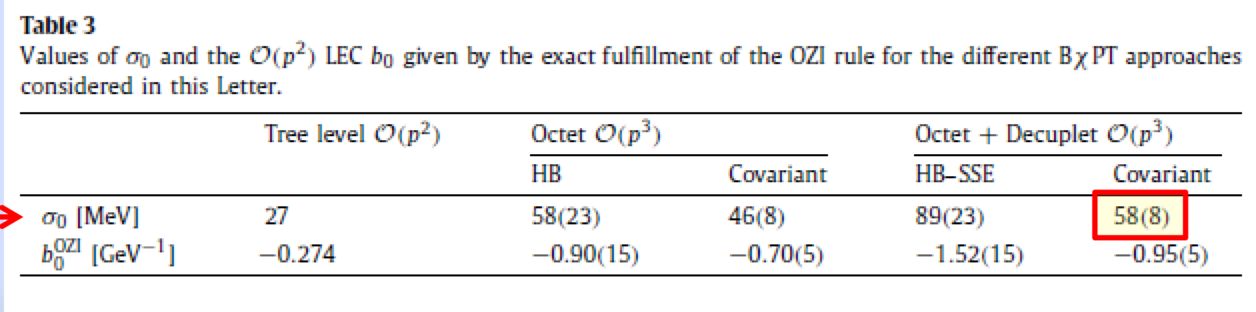
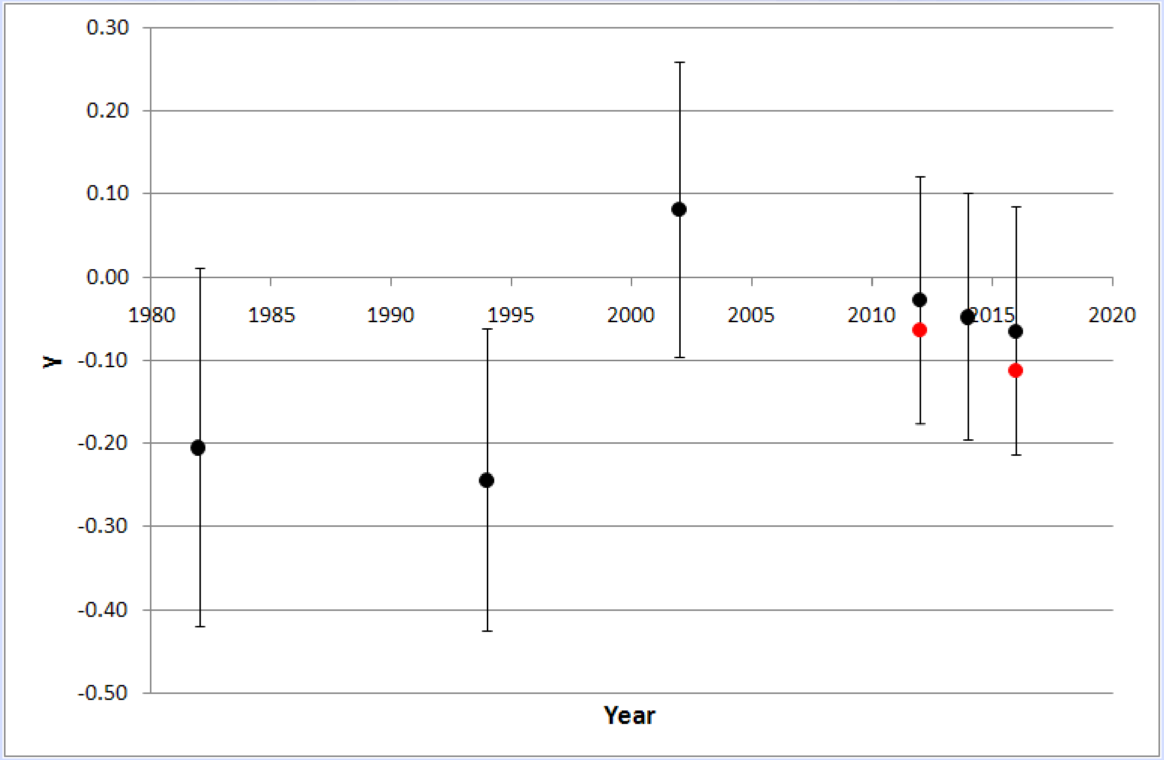


Table 3
Values of σ_0 and the $\mathcal{O}(p^2)$ LEC b_0 given by the exact fulfillment of the OZI rule for the different BχPT approaches considered in this Letter.

	Tree level $\mathcal{O}(p^2)$	Octet $\mathcal{O}(p^3)$		Octet + Decuplet $\mathcal{O}(p^3)$	
		HB	Covariant	HB–SSE	Covariant
σ_0 [MeV]	27	58(23)	46(8)	89(23)	58(8)
b_0^{OZI} [GeV^{-1}]	−0.274	−0.90(15)	−0.70(5)	−1.52(15)	−0.95(5)

The ‘not-so-strange’ strange-quark content of the nucleon



As a result, the larger values of the Σ term are not in conflict with our picture of the nucleon and our expectations for its small strange-quark content!

Personal opinion: the careful re-evaluation of both Δ_σ and $\widehat{\sigma}$ is called for!