The fitting of the hyperfine splitting of the $5 \to 4$ transitions in muonic Re-185 and Re-187

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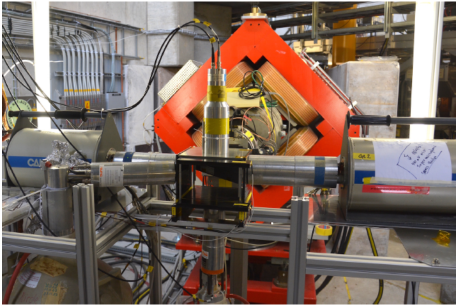
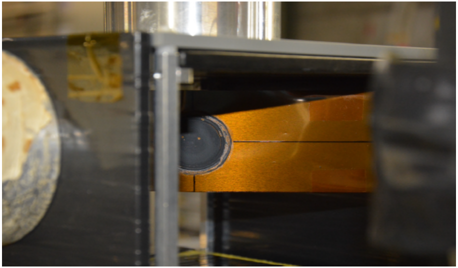
Apparatus during beam time 2016





- Four HPGe detectors, only two operating properly working:
  Relative efficiency:
  - 20 % (GeR)
  - 75 % (GeL)

  Energy resolution at 1332.5 keV:
  - 0.92 keV (GeR)
  - 1.23 keV (GeL)
- one brillance detector
- six plastic scintillators:
  - muon veto
  - muon entrance
  - four veto detectors
- target:
  500 mg Re-185 (97.6 %)
  500 mg Re-187 (99.6 %)
  1000 mg Pb-208

Energy levels in μRe-185 and μRe-187

Re-185, Re-187 are:

- ✓ stable elements
- ✓ nuclear charge radii have not been measured yet
- ✓ nuclear spin at ground level $I = 5/2 \rightarrow$ HF effect in the energy splitting levels ($\overrightarrow{F} = \overrightarrow{I} + \overrightarrow{J}$)

- Energy displacement due to the electric quadrupole interaction:

$$\Delta E_F\,(E2) = A_2\, 6\, \frac{K(K+1) - 4/3\, I(I+1)J(J+1)}{4I(2I-1)J(2J-1)}$$

where $K = F(F+1) - I(I+1) - J(J+1)$.

- Energy displacement due to the magnetic dipole interaction:

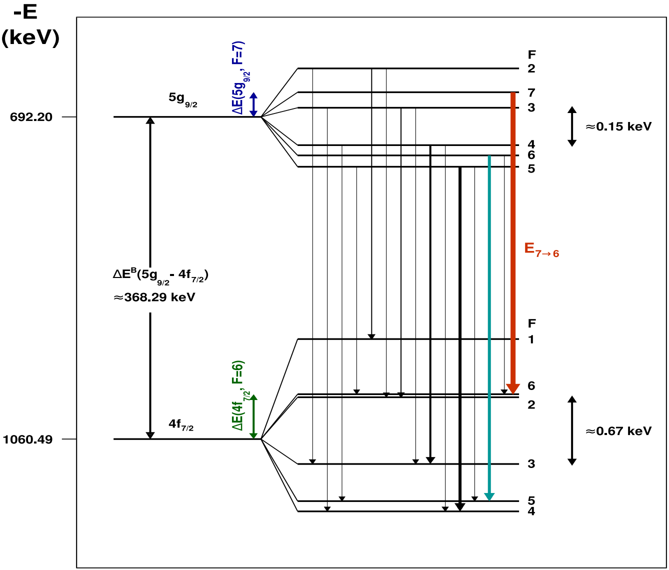
$$\Delta E_F(M1) = \frac{A_1}{2}\,\{F(F+1) - I(I+1) - J(J+1)\}$$

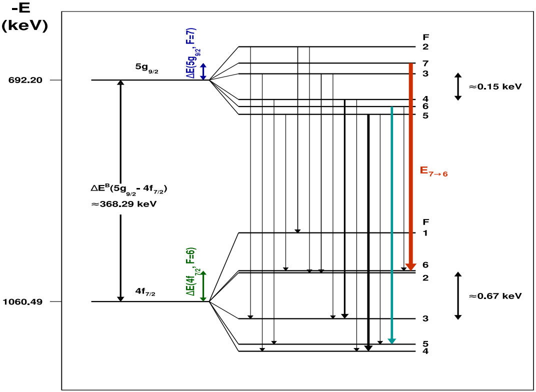
A_2 : E2 hf constant $\rightarrow Q$ dependence

A_1 : M1 hf constant $\rightarrow \mu_N$ dependence

far away from the nucleus total Q, μ_N

$5g_{9/2} \rightarrow 4f_{7/2}$ hyperfine transition in Re-185



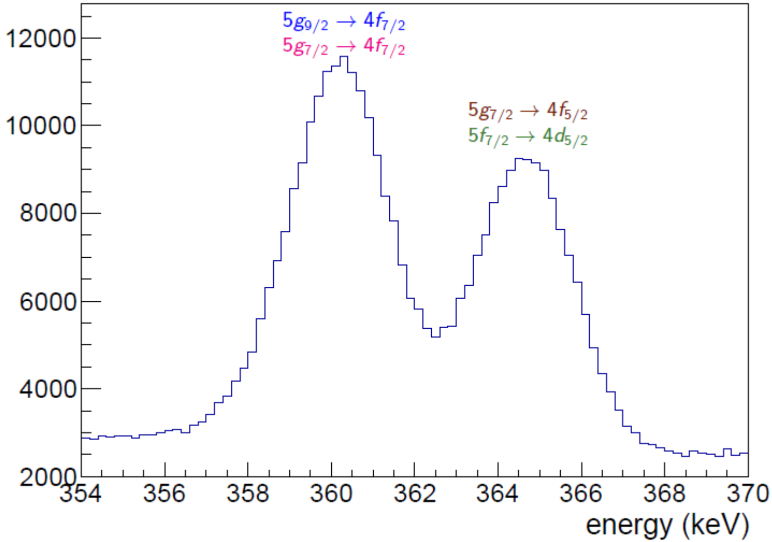


$$E_{7\to6} = E_7 - E_6' = \Delta E^B\left(5g_{9/2} - 4f_{7/2}\right) + \Delta E\left(5g_{9/2}, F=7\right) + \Delta E\left(4f_{7/2}, F=6\right)$$
$$E_{6\to5} = E_6 - E_5' = \Delta E^B\left(5g_{9/2} - 4f_{7/2}\right) + \Delta E\left(5g_{9/2}, F=6\right) + \Delta E\left(4f_{7/2}, F=5\right)$$
$$\xrightarrow{(-)}$$

$$\Rightarrow E_{6\to5} = E_{7\to6} + \Delta E(\mu_N, Q)$$

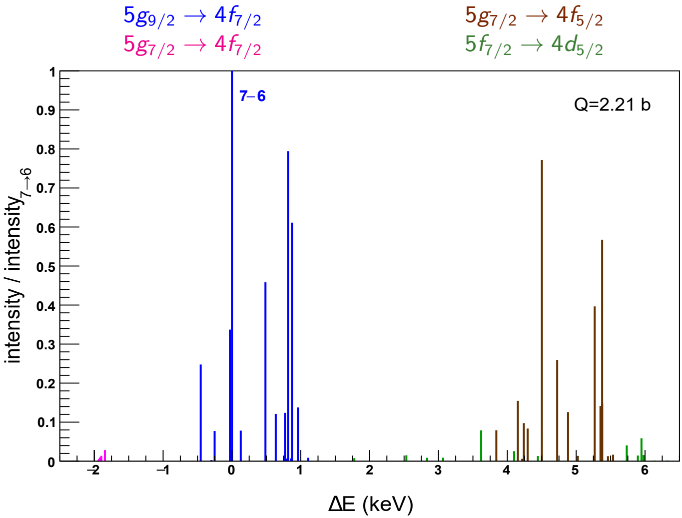
$$E_i = E_0 + \Delta E(\mu_N, Q)$$
$$\Delta E(\mu_N, Q) = Q^2 \cdot c_{quad_i} + Q \cdot c_{lin_i} + c_{const_i}$$

Experimental spectrum of the 5g-4f, 5f-4d transitions in Re-185



5g-4f, 5f-4d HFS in Re-185

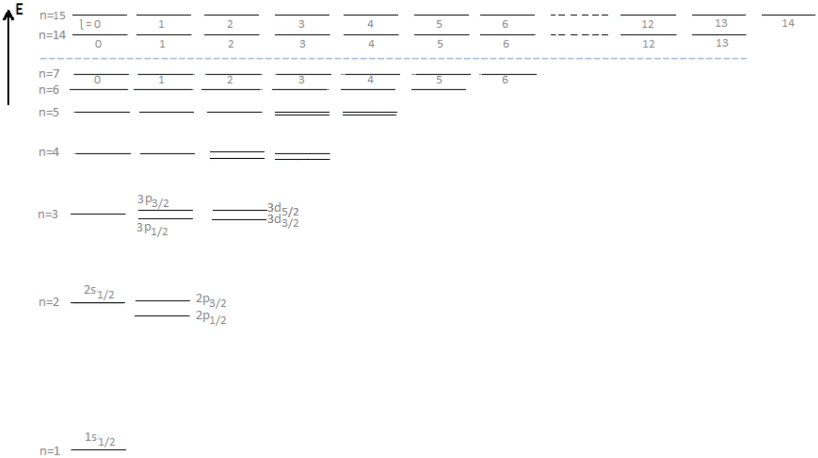
$$E_i = E_0 + \Delta E(\mu_N, Q)$$
$$\Delta E(\mu_N, Q) = Q^2 \cdot c_{quad_i} + Q \cdot c_{lin_i} + c_{const_i}$$



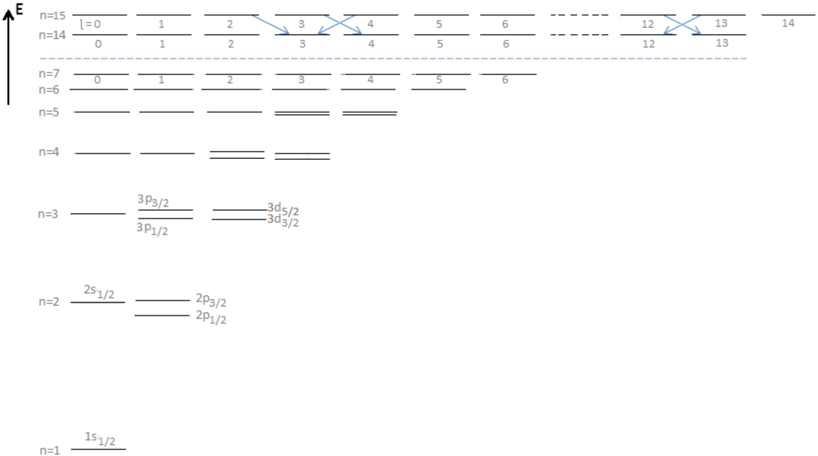
61 transitions

***Theoretical energies and intensities calculated by N. Michel, Max-Planck-Institut für Kernphysik in Heidelberg**

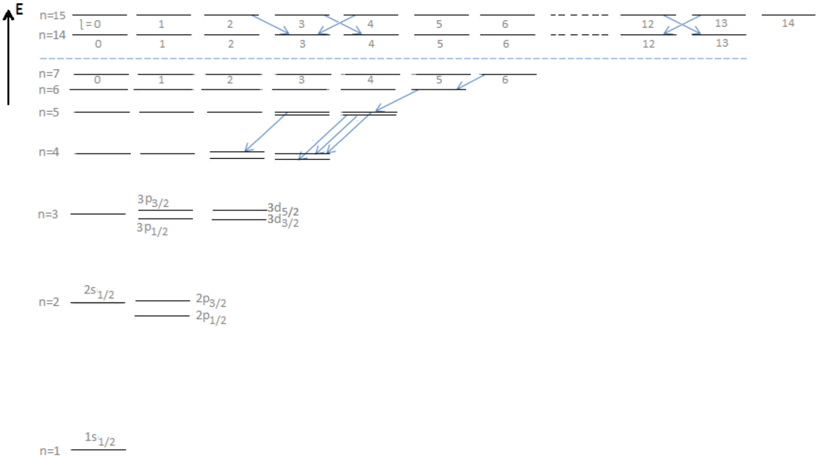
Muon cascade calculations



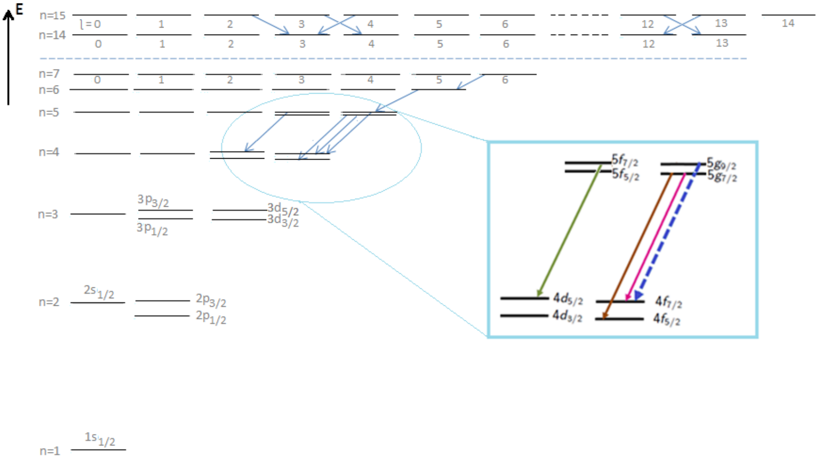
Muon cascade calculations



Muon cascade calculations



Muon cascade calculations



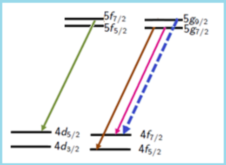
5g-4f, 5f-4d HFS in Re-185

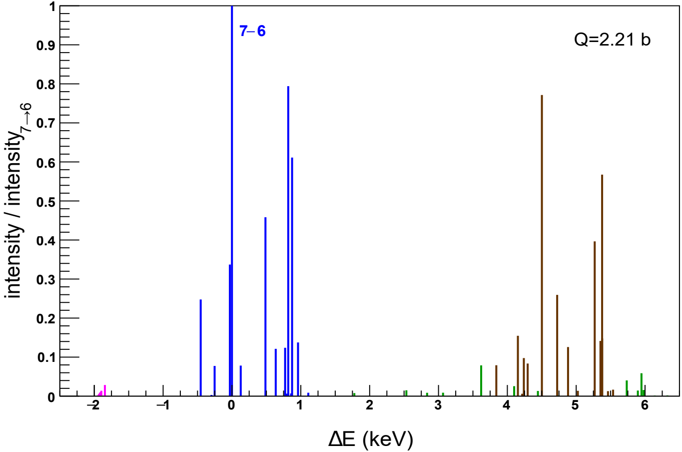
$5g_{9/2} \rightarrow 4f_{7/2}$

$5g_{7/2} \rightarrow 4f_{7/2}$

$5g_{7/2} \rightarrow 4f_{5/2}$

$5f_{7/2} \rightarrow 4d_{5/2}$





61 transitions

Absolute intensities scale:

$$\frac{I_{(5g_{7/2}\rightarrow 4f_{5/2})}}{I_{(5g_{9/2}\rightarrow 4f_{7/2})}} = 0.7714$$

$$\frac{I_{(5f_{7/2}\rightarrow 4d_{5/2})}}{I_{(5g_{9/2}\rightarrow 4f_{7/2})}} = 0.0789\ \textbf{??}$$

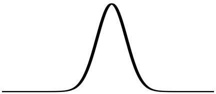
$$\frac{I_{(5g_{7/2}\rightarrow 4f_{7/2})}}{I_{(5g_{9/2}\rightarrow 4f_{7/2})}} = 0.02857$$

***Theoretical energies and intensities calculated by N. Michel, Max-Planck-Institut für Kernphysik in Heidelberg**

Instrumental line-shape

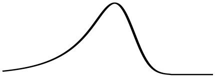
1. Gaussian shape → broadening of a line in an ideal detector (statistical fluctuations and electronic noise):

$$G(E) = \frac{1}{\sqrt{2\pi}\sigma} exp\left(-\frac{(E-m)^2}{2\sigma^2}\right)$$



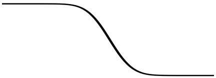
2. Hypermet shape → low-energy exponential tail (incomplete charge collection):

$$T(E) = \frac{1}{2\ b} \cdot exp\left(\frac{E-m}{b} + \frac{\sigma^2}{2b^2}\right) \cdot erfc\left(\frac{E-m}{\sqrt{2}\sigma} + \frac{\sigma}{\sqrt{2}b}\right)$$



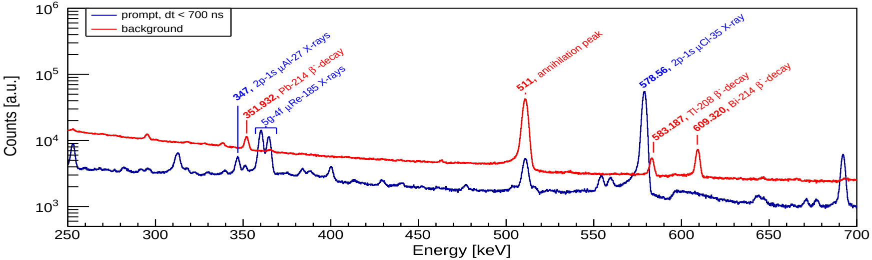
3. Step-like shelf shape → discontinuous background under each peak (accumulation of Compton scattering and pair production effects):

$$S(E) = \frac{A}{2} \cdot erfc\left(\frac{E-m}{\sqrt{2}\sigma}\right)$$

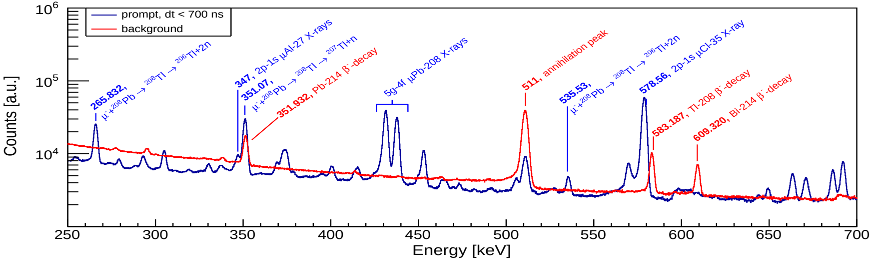


Energy spectra in Re-185 and Pb-208

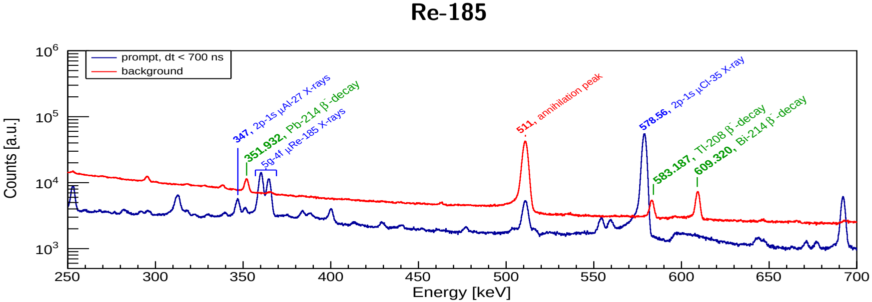
Re-185

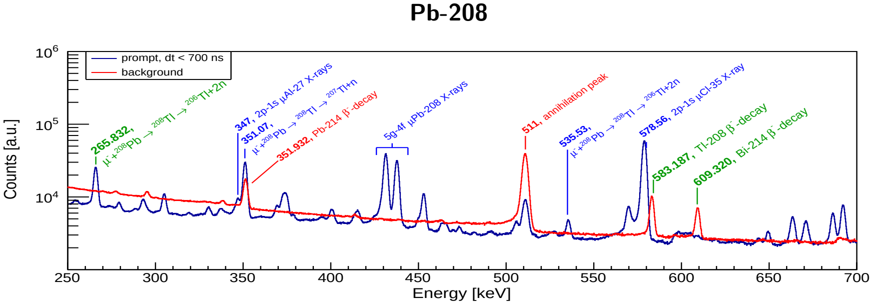


Pb-208



Energy spectra in Re-185 and Pb-208





Line-shape determination

Total PDF in RooFit: $P(E) = N_{signal} \cdot (f_G \, G(E) + f_T \, T(E) + S(E)) + B$

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1. $\sigma = \alpha_\sigma \cdot E + \beta_\sigma$ (detector resolution)

Line-shape determination

Total PDF in RooFit: $P(E) = N_{signal} \cdot (f_G\, G(E) + f_T\, T(E) + S(E)) + B$

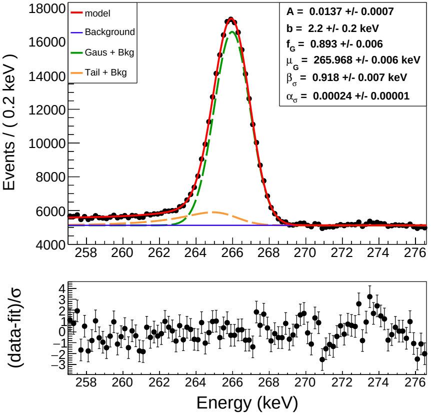
1. $\sigma = \alpha_\sigma \cdot E + \beta_\sigma$ (detector resolution)
2. Simultaneous fit of the 265.83, 351.93, 583.19, 609.32 keV peaks
   Common parameters: f_G, b, A, α_σ, β_σ

Line-shape determination - Simultaneous fit

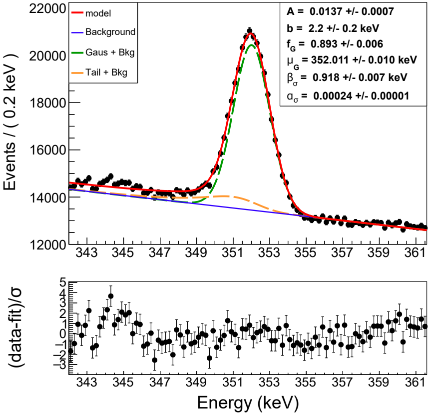
Total PDF in RooFit: $P(E) = N_{signal} \cdot (f_G\, G(E) + f_T\, T(E) + S(E)) + B$

Common parameters: f_G, b, A, α_σ, β_σ from $\sigma = \alpha_\sigma \cdot E + \beta_\sigma$

265.832 keV



351.932 keV

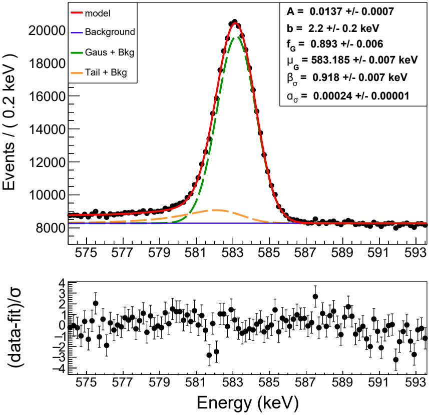


Line-shape determination - Simultaneous fit

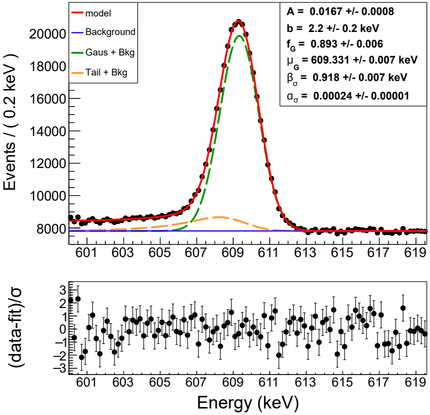
Total PDF in RooFit: $P(E) = N_{signal} \cdot (f_G\, G(E) + f_T\, T(E) + S(E)) + B$

Common parameters: f_G, b, A, α_σ, β_σ from $\sigma = \alpha_\sigma \cdot E + \beta_\sigma$

583.187 keV



609.320 keV

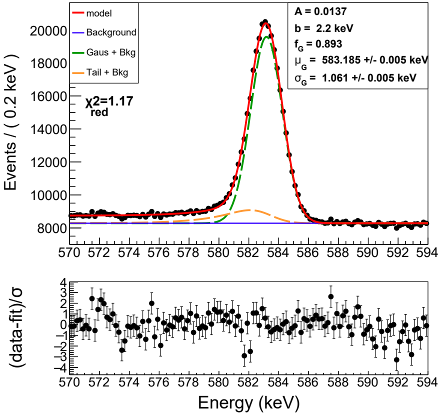


Line-shape determination - χ^2 comparison

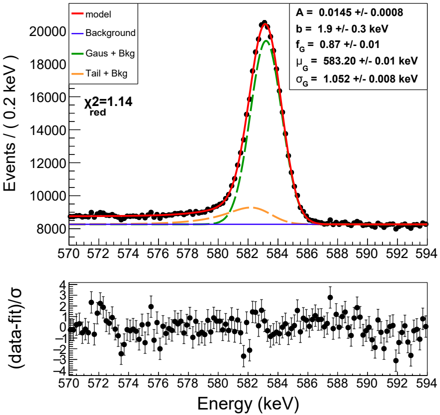
Total PDF in RooFit: $P(E) = N_{signal} \cdot (f_G\, G(E) + f_T\, T(E) + S(E)) + B$

583.187 keV

fixed f_G, b, A from simultaneous fit

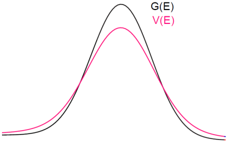


all parameters free



Fit of the 5g-4f, 5f-4d HF transitions

Signal PDF in RooFit: $P_{signal}(E) = N_{signal} \cdot (f_G\, \boldsymbol{V(E)} + f_T\, T(E) + S(E))$



Voigt profile due to natural linewidth of:

$5g_{9/2} \rightarrow 4f_{7/2}$
$5g_{7/2} \rightarrow 4f_{7/2}$
$5g_{7/2} \rightarrow 4f_{5/2}$
$5f_{7/2} \rightarrow 4d_{5/2}$

Fixed parameters:

- line-shape parameters (f_G, b, $\sigma = \alpha_\sigma \cdot E + \beta_\sigma$)
- $\frac{I_{(5g_{7/2} \rightarrow 4f_{5/2})}}{I_{(5g_{9/2} \rightarrow 4f_{7/2})}}$
- $\frac{I_{(5g_{7/2} \rightarrow 4f_{7/2})}}{I_{(5g_{9/2} \rightarrow 4f_{7/2})}}$

Fitted parameters:

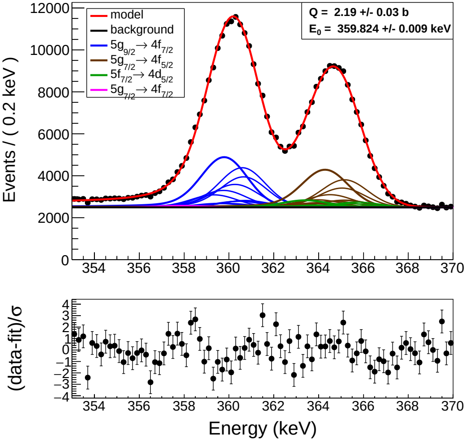
- Q
- $E_{(5g_{9/2},\, F=7 \rightarrow 4f_{7/2},\, F=6)}$
- $N_{signal_{(5g_{9/2},\, F=7 \rightarrow 4f_{7/2},\, F=6)}}$
- A (step height)
- B (background constant)
- $\frac{I_{(5f_{7/2} \rightarrow 4d_{5/2})}}{I_{(5g_{9/2} \rightarrow 4f_{7/2})}}$

Fit of the 5g-4f, 5f-4d HF transitions in Re-185

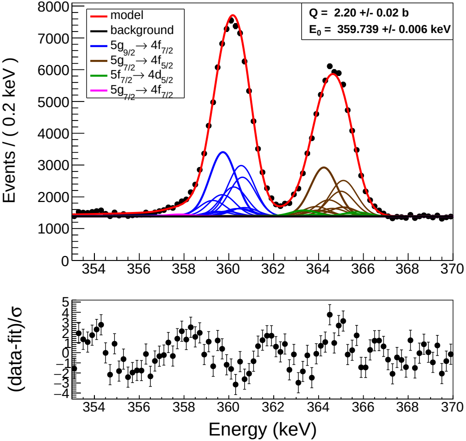
Lineshape parameters obtained from the simultaneous fits:

detector	$f_G = 1 - f_T$	b (keV)	α_σ	β_σ (keV)
GeL	0.893 ± 0.006	2.2 ± 0.2	0.00024 ± 0.00001	0.918 ± 0.007
GeR	0.93 ± 0.01	5.0 ± 0.8	0.00034 ± 0.00001	0.466 ± 0.004

GeL



GeR

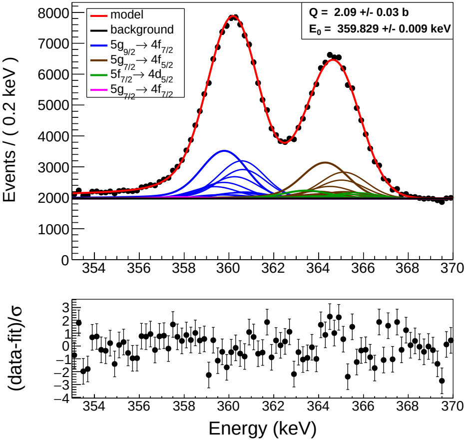


Fit of the 5g-4f, 5f-4d HF transitions in Re-187

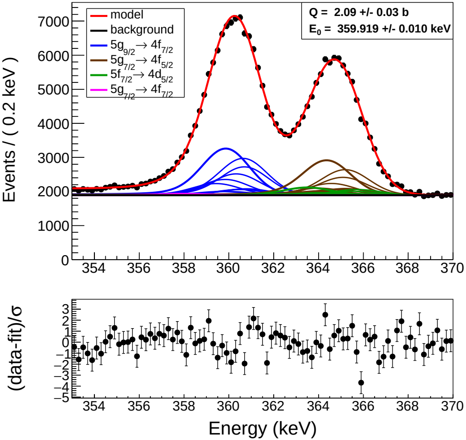
Lineshape parameters obtained from the simultaneous fits:

detector	$f_G = 1 - f_T$	b (keV)	α_σ	β_σ (keV)
GeL, $group_1$	0.893 ± 0.006	2.2 ± 0.2	0.00024 ± 0.00001	0.918 ± 0.007
GeL, $group_2$	0.87 ± 0.02	1.2 ± 0.2	0.00017 ± 0.00005	0.98 ± 0.02

GeL, $group_1$



GeL, $group_2$



Fit of the 5g-4f, 5f-4d HF transitions in Re-187

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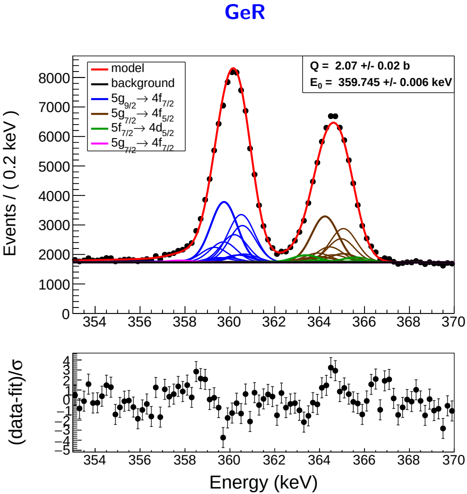
detector	$f_G = 1 - f_T$	b (keV)	α_σ	β_σ (keV)
GeR	0.93 ± 0.01	5.0 ± 0.8	0.00034 ± 0.00001	0.466 ± 0.004



Fit of the 5g-4f, 5f-4d HF transitions in Re-185 and Re-187

Fitting results:

Re-185

Dataset	Q (b)	E_0 (keV)	$RI(5f_{7/2} \rightarrow 4d_{5/2})$	χ^2_{red} (keV)
GeL	2.19 ± 0.03	359.824 ± 0.009	0.143 ± 0.008	1.532
GeR	2.20 ± 0.02	359.739 ± 0.006	0.094 ± 0.007	2.537

Re-187

Dataset	Q (b)	E_0 (keV)	$RI(5f_{7/2} \rightarrow 4d_{5/2})$	χ^2_{red} (keV)
GeL, $group_1$ GeL, $group_2$	2.09 ± 0.03	359.829±0.009 359.919±0.010	0.169 ± 0.007	1.268 1.164
GeR	2.07 ± 0.02	359.745 ± 0.006	0.119 ± 0.008	1.826

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GeR	2.20 ± 0.02	359.739 ± 0.006	0.094 ± 0.007	2.537
GeL	2.27 ± 0.03	359.810 ± 0.007	0.094 (fixed)	2.043
GeR	2.14 ± 0.02	359.754 ± 0.006	0.143 (fixed)	2.980

Re-187

Dataset	Q (b)	E_0 (keV)	RI($5f_{7/2} \rightarrow 4d_{5/2}$)	χ^2_{red} (keV)
GeL, $group_1$ GeL, $group_2$	2.09 ± 0.03	359.829±0.009 359.919±0.010	0.169 ± 0.007	1.268 1.164
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GeL	2.27 ± 0.03	359.810 ± 0.007	0.094 (fixed)	2.043
GeR	2.14 ± 0.02	359.754 ± 0.006	0.143 (fixed)	2.980

Re-187

Dataset	Q (b)	E_0 (keV)	RI($5f_{7/2} \rightarrow 4d_{5/2}$)	χ^2_{red} (keV)
GeL, $group_1$ GeL, $group_2$	2.09 ± 0.03	359.829±0.009 359.919±0.010	0.169 ± 0.007	1.268 1.164
GeR	2.07 ± 0.02	359.745 ± 0.006	0.119 ± 0.008	1.826
GeL, $group_1$ GeL, $group_2$	2.15 ± 0.03	359.817±0.009 359.907±0.010	0.119 (fixed)	1.678 1.379
GeR	2.01 ± 0.02	359.761 ± 0.006	0.169 (fixed)	2.238

Preliminary result

Preliminary result of the current analysis:

$$\mathbf{Q_{\mathit{Re}-185} = (2.20 \ \pm \ 0.02 \ (\text{stat}) \ \pm \ 0.08 \ (\text{RI})) \ b}$$

$$\mathbf{Q_{\mathit{Re}-187} = (2.08 \ \pm \ 0.02 \ (\text{stat}) \ \pm \ 0.06 \ (\text{RI})) \ b}$$

[1] J. Konijn et al., Nucl. Phys. A **360**, 187 (1981)

Preliminary result

Preliminary result of the current analysis:

$$\mathbf{Q}_{\boldsymbol{Re}-185} = (\mathbf{2.20} \pm \mathbf{0.02}\ (\text{stat}) \pm \mathbf{0.08}\ (\text{RI}))\ \mathbf{b}$$

$$\mathbf{Q}_{\boldsymbol{Re}-187} = (\mathbf{2.08} \pm \mathbf{0.02}\ (\text{stat}) \pm \mathbf{0.06}\ (\text{RI}))\ \mathbf{b}$$

Result obtained by similar analysis by means of muonic X-ray spectroscopy in the past[1]:

$$\mathbf{Q}_{\boldsymbol{Re}-185} = (\mathbf{2.21} \pm \mathbf{0.04})\ \mathbf{b}$$

$$\mathbf{Q}_{\boldsymbol{Re}-187} = (\mathbf{2.09} \pm \mathbf{0.04})\ \mathbf{b}$$

[1] J. Konijn et al., Nucl. Phys. A **360**, 187 (1981)

Preliminary result

Preliminary result of the current analysis:

$$\mathbf{Q_{Re-185} = (2.20 \pm 0.02\ (\text{stat}) \pm 0.08\ (\text{RI}))\ b}$$

$$\mathbf{Q_{Re-187} = (2.08 \pm 0.02\ (\text{stat}) \pm 0.06\ (\text{RI}))\ b}$$

Result obtained by similar analysis by means of muonic X-ray spectroscopy in the past[1]:

$$\mathbf{Q_{Re-185} = (2.21 \pm 0.04)\ b}$$

$$\mathbf{Q_{Re-187} = (2.09 \pm 0.04)\ b}$$

$$\frac{Q_{Re-185}}{Q_{Re-187}} = 1.056709(17)$$

[1] J. Konijn et al., Nucl. Phys. A **360**, 187 (1981)

Preliminary result

Preliminary result of the current analysis:

$$\mathbf{Q_{\mathit{Re}-185} = (2.20 \pm 0.02\ (\text{stat}) \pm 0.08\ (\text{RI}))\ b}$$

$$\mathbf{Q_{\mathit{Re}-187} = (2.08 \pm 0.02\ (\text{stat}) \pm 0.06\ (\text{RI}))\ b}$$

$$\frac{Q_{Re-185}}{Q_{Re-187}} = (1.06 \pm 0.01\ (\text{stat}) \pm 0.05\ (\text{RI}))$$

Result obtained by similar analysis by means of muonic X-ray spectroscopy in the past[1]:

$$\mathbf{Q_{\mathit{Re}-185} = (2.21 \pm 0.04)\ b}$$

$$\mathbf{Q_{\mathit{Re}-187} = (2.09 \pm 0.04)\ b}$$

$$\frac{Q_{Re-185}}{Q_{Re-187}} = 1.056709(17)$$

[1] J. Konijn et al., Nucl. Phys. A **360**, 187 (1981)

Conclusions

1. Development of the analysis tool for the fitting of the experimentally measured energies of the $5g - 4f$, $5f - 4d$ hyperfine transitions in Re-185 and Re-187.

2. Extraction of the quadrupole moment in Re-185 and Re-187 (preliminary).

Outlook

1. Finalize the analysis for the extraction of the quadrupole moment in Re-185 and Re-187.
2. Extraction of the nuclear charge radius of Re-185 and Re-187.

Thank you!

Backup Slides

Previous measurements of Q in Re-185 and Re-187

Spectroscopic quadrupole moments Q (in b) in ^{185}Re and ^{187}Re [a])

^{185}Re	^{187}Re	Method	Ref.
2.8 ±0.7	2.6 ±0.6	h.f.s. optical	[31]
2.9 ±0.7	2.9 ±0.7	h.f.s. optical	[32]
2.3 ±0.9	2.2 ±0.9	h.f.s. optical	[33]
1.7 ±0.4	1.5 ±0.4	CE γ (NaI) $(0\rightarrow 1)$	[34]
2.7 ±0.7	2.1 ±0.5	CE e^- $(0\rightarrow 1)$	[35]
1.5 ±0.4		CE e^- $(0\rightarrow 2)$	[35]
1.5 ±0.4	1.8 ±0.5	CE γ (NaI) $(0\rightarrow 1)$	[36]
1.8 ±0.5	2.0 ±0.5	CE γ (NaI) $(0\rightarrow 2)$	[36]
2.1 ±0.5	1.9 ±0.5	CE e^- $(0\rightarrow 1)$	[37]
2.0 ±0.2	1.9 ±0.2	CE γ (NaI) $(0\rightarrow 1)$	[38]
2.1 ±0.2	2.0 ±0.2	CE γ (NaI) $(0\rightarrow 1)$	[38]
2.24±0.11	2.0 ±0.1	CE γ (NaI) $(0\rightarrow 1)$	[25]
2.26±0.11	2.2 ±0.1	CE γ (NaI) $(0\rightarrow 2)$	[25]
2.5 ±0.3	2.1 ±0.3	CE γ (NaI) $(0\rightarrow 1)$	[39]
2.4 ±0.4	2.2 ±0.4	CE γ (NaI) $(0\rightarrow 2)$	[39]
1.92±0.10	1.99±0.10	CE inel. $(0\rightarrow 1)$	[40]
1.99±0.10	2.01±0.10	CE inel. $(0\rightarrow 2)$	[40]
	1.95±0.17	$T_{1/2}$ $(1\rightarrow 0)$	[23]
2.18±0.02	2.07±0.02	π^- atom	present results
2.21±0.04	2.09±0.04	μ^- atom	present results

Discrepancy of fitted RI with cascade calculations

In $\mu^{176}\mathrm{Lu}$[2]:

"The resulting intensity of the 5f-4d transition corresponds to $(8.1 \pm 2.1)\,\%$ of the 5g-4f transition. The cascade calculation returns a value of $4.5\,\%$."

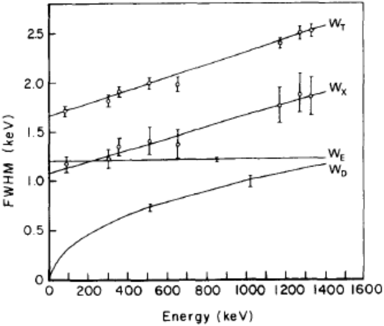
[2] W. Dey Thesis, Swiss Federal Institute of Technology, Zurich, Dissertation No. 5473, 1975

Energy resolution in Germanium detector

Overall energy resolution in Ge detector is determined by a combination of three factors as:

$$W_T^2 = W_D^2 + W_X^2 + W_E^2 \qquad (1)$$

- W_D: statistical fluctuation in the number of charge carriers created
- W_X: due to incomplete charge collection (more significant at detectors of large volume and low average electric field)
- W_E: accounts for the broadening effects due to electronic noise



FWHM variation with gamma-ray energy of an 86 cm^3 HPGe detector

Natural line-width

- Natural width of the line $\sim$ quantum mechanical uncertainty in the energy E of levels with finite lifetimes

  If an energy level above ground state has energy E and lifetime Δt, it has energy uncertainty: $\Delta E \Delta t \sim \hbar$ (faster the transition, larger the width). Then, the photon emitted in a transition from this level to the ground level has a range of possible frequencies:

$$\Delta\nu \sim \frac{\Delta E}{h} \sim \frac{1}{2\pi\Delta t}$$

- The FWHM of measured X-ray peak Γ_{m} is related to the natural (lorentzian) width Γ_{n} and the instrumental resolution Γ_{i} (assumed to be Gaussian) by,

$$\Gamma_{\mathrm{n}} = \Gamma_{\mathrm{m}} - \Gamma_{\mathrm{i}}^2/\Gamma_{\mathrm{m}}$$

Natural line-widths for transitions in Re-185

$$\Gamma_n(5g_{9/2} \rightarrow 4f_{7/2}) = 18.559\,\text{eV}$$

$$\Gamma_n(5g_{7/2} \rightarrow 4f_{5/2}) = 18.086\,\text{eV}$$

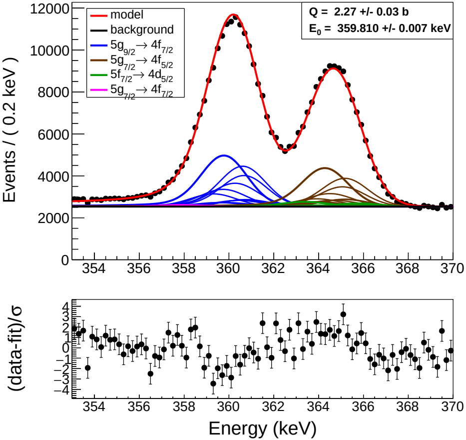
$$\Gamma_n(5f_{7/2} \rightarrow 4d_{5/2}) = 11.212\,\text{eV}$$

$$\Gamma_n(5g_{7/2} \rightarrow 4f_{7/2}) = 0.636\,\text{eV}$$

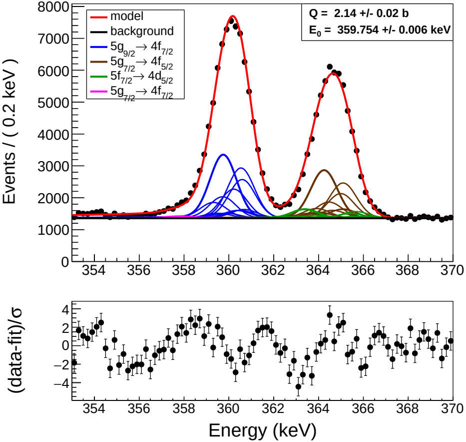
Fit of the 5g-4f, 5f-4d HF transitions in Re-185

Fixing RI at 'extreme' values

GeL



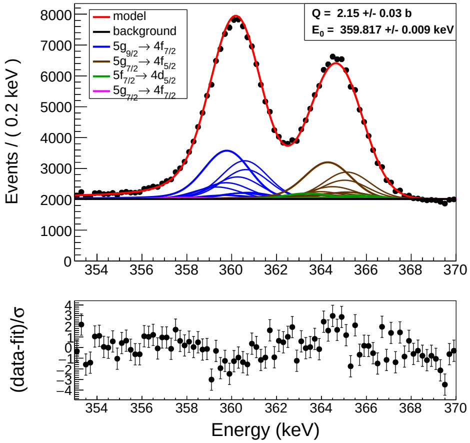
GeR



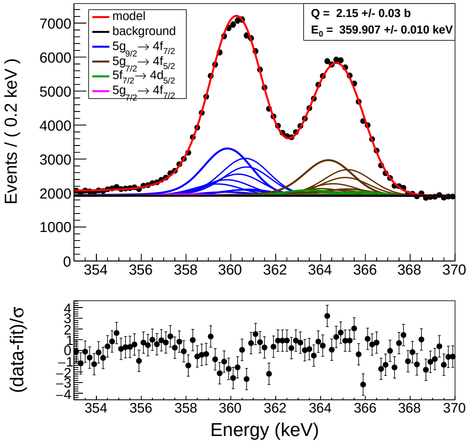
Fit of the 5g-4f, 5f-4d HF transitions in Re-187

Fixing RI at 'extreme' values

GeL, group$_1$



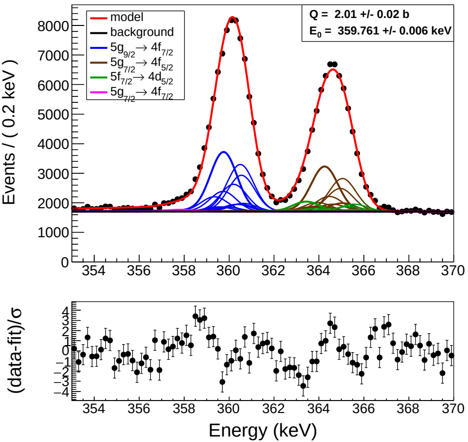
GeL, group$_2$



Fit of the 5g-4f, 5f-4d HF transitions in Re-187

Fixing RI at 'extreme' values

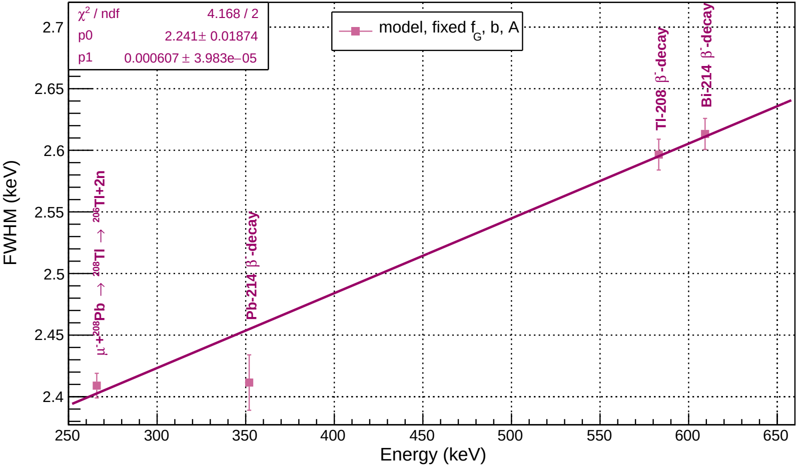
GeR



Line-shape determination

Lineshape parameters obtained from the simultaneous fit:

$f_G = 1 - f_T$	b (keV)	A	α_σ	β_σ (keV)
0.893 ± 0.006	2.2 ± 0.2	0.0137 ± 0.0007	0.00024 ± 0.00001	0.918 ± 0.007



Lineshape fit

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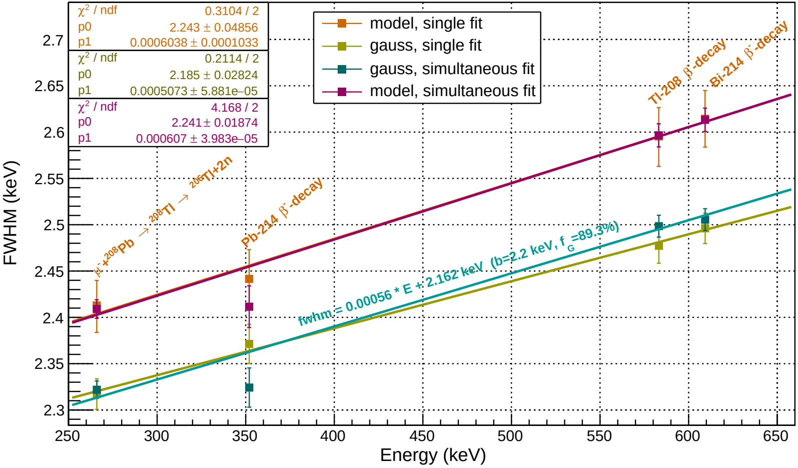
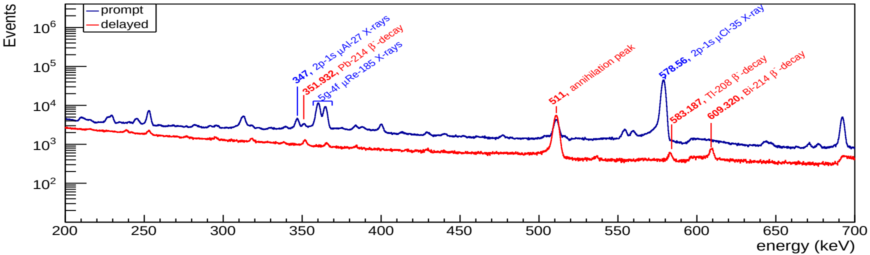
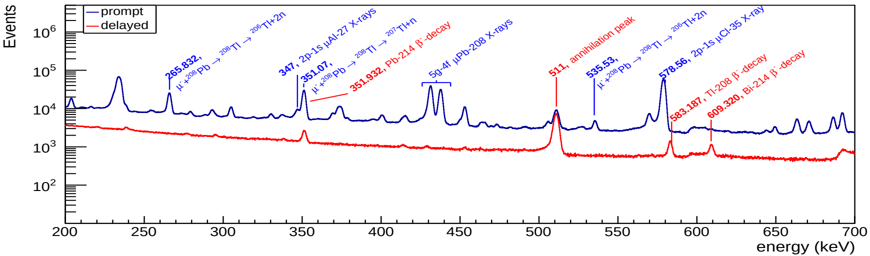


Energy spectra in Re-185 and Pb-208

Re-185



Pb-208



Fine and Hyperfine transitions

Fine structure from the muon muon spin-orbit coupling:

$$\overrightarrow{J} = \overrightarrow{L} + \overrightarrow{S} \tag{2}$$

Selection rules: $\Delta L = \pm 1$, $\Delta J = 0, \pm 1 \longrightarrow$ $\begin{matrix} 5g_{9/2} \rightarrow 4f_{7/2} \\ 5g_{7/2} \rightarrow 4f_{5/2} \\ 5f_{7/2} \rightarrow 4d_{5/2} \\ 5f_{5/2} \rightarrow 4d_{3/2} \end{matrix}$

Hyperfine structure from the nuclear spin-muon total angular momentum coupling:

$$\overrightarrow{F} = \overrightarrow{I} + \overrightarrow{J} \tag{3}$$

$I(^{185}Re) = 5/2$ and $\Delta F = 0, \pm 1$

Energy levels in μRe-185

The total Hamiltonian for a muon bound to a nucleus:

$$H = H_N + H_\mu + H_{\mu-N} \tag{4}$$

Charge distribution in a multipole expansion:

$$\rho(\overrightarrow{r}) = \rho_0(r) + \sum_{lm} \rho_{lm} Y_{lm}(\theta, \phi) \tag{5}$$

spherical part: gross and fine structure
non-spherical part: hyperfine structure

Intrinsic quadrupole moment of the nucleus:

$$Q_0 = 2\sqrt{\frac{4\pi}{5}} \int \rho(\overrightarrow{r}) Y_{20}(\theta, \phi)\, r^2 d\tau \tag{6}$$

The electric quadrupole interaction (E2) is given by:

$$H(E2) = -\frac{e^2}{2}\sqrt{\frac{4\pi}{5}} Q_0 f(r) Y_{20}(\theta, \phi) \tag{7}$$

$f(r) \rightarrow r^{-3}$ when far away from nucleus

$f(r) \rightarrow$ dependent on the exact form of $\rho(\overrightarrow{r})$ when close to the nucleus

Theory

- Energy displacement due to the electric quadrupole interaction:

$$\Delta E_F\,(E2) = 6\,\frac{K(K+1) - 4/3\,I(I+1)J(J+1)}{4I(2I-1)J(2J-1)}A_2 \tag{8}$$

where $K = F(F+1) - I(I+1) - J(J+1)$.

$$A_2^0 = \frac{2J-1}{2J+2}\,e^2\,Q_0\,\left\langle \frac{1}{r^3} \right\rangle_{n,J} \tag{9}$$

- Energy displacement due to the magnetic dipole interaction:

$$\Delta E_F(M1) = \frac{1}{2IJ}\,\{F(F+1) - I(I+1) - J(J+1)\}\,A_1 \tag{10}$$

where $A_1 = IJ\alpha$, α.
Binding energy $E_B \propto m_\mu$. Thus, M1 is two orders of magnitude smaller than that of ordinary atoms

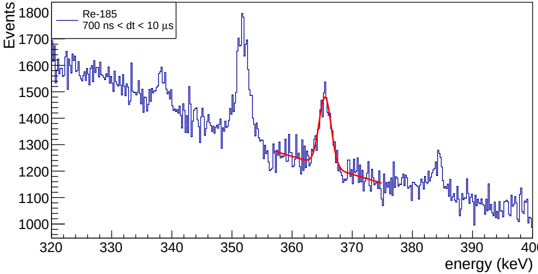
$$A_1^0 = \mu_\mu\,g_I\,\mu_N\,\frac{I(I+1)}{J(J+1)}\,\left\langle \frac{1}{r^3} \right\rangle_{n,J} \tag{11}$$

Peak under 5g-4f hf region in GeL

365.5 keV (14.59 ¯s): γ-line in $^{181}\mathrm{W}$

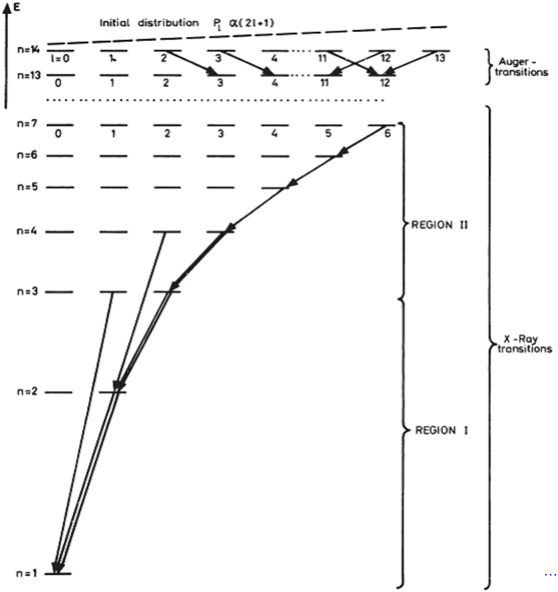
$$\mu^- + {}^{185}\mathrm{Re} \rightarrow {}^{185}\mathrm{W}^* + \nu_\mu \rightarrow {}^{181}\mathrm{W} + 4n$$

??



NO.	NAME	VALUE	ERROR
1	p0 **counts**	6.58095e+02	4.46067e+01
2	p1 **mean**	3.65489e+02	6.75885e-02
3	p2 **sigma**	1.00341e+00	6.99741e-02
4	p3	3.69186e+03	2.61751e+02
5	p4	-6.76684e+00	7.14220e-01

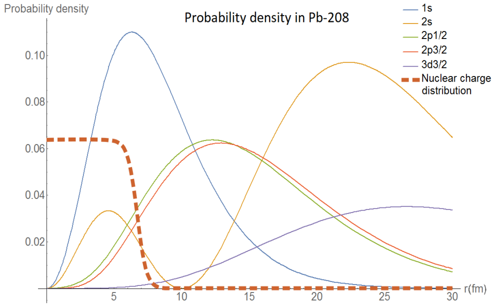
Muonic atom spectroscopy - Muonic cascade



- Cascade starts at high n
  - ✓ For $n \gtrsim 6 \rightarrow$ Auger emissions
  - ✓ For $n \lesssim 6 \rightarrow$ X-rays
- $n_{\mu^-} = 14$ is below the $n_{e^-} = 1$ (K-shell)
- statistical distribution $P_l \sim (2l+1)$: $(n, l = n-1) \rightarrow (n-1, l = n-2)$ most intense transitions.

...at 1s state muon is captured by nucleus or decays

Muonic atom spectroscopy



- Large overlap of the **low-lying states** with the nuclear charge distribution
- After $n \cong 6$, finite size effect is negligible

Two-parameter Fermi nuclear charge distribution:

$$\rho(r) = \frac{1}{1 + e^{\frac{r-c}{\alpha}}}$$

state	$(E_B^0 - E_B^{FS})/E_B^0$
$1s_{1/2}$	45.50 %
$2p_{1/2}$	8.16 %
$2p_{3/2}$	3.52 %
$4d_{3/2}$	0.11 %
$4d_{5/2}$	0.044 %
$5f_{5/2}$	0.0005 %
$6h_{9/2}$	$<$ 0.0005 %