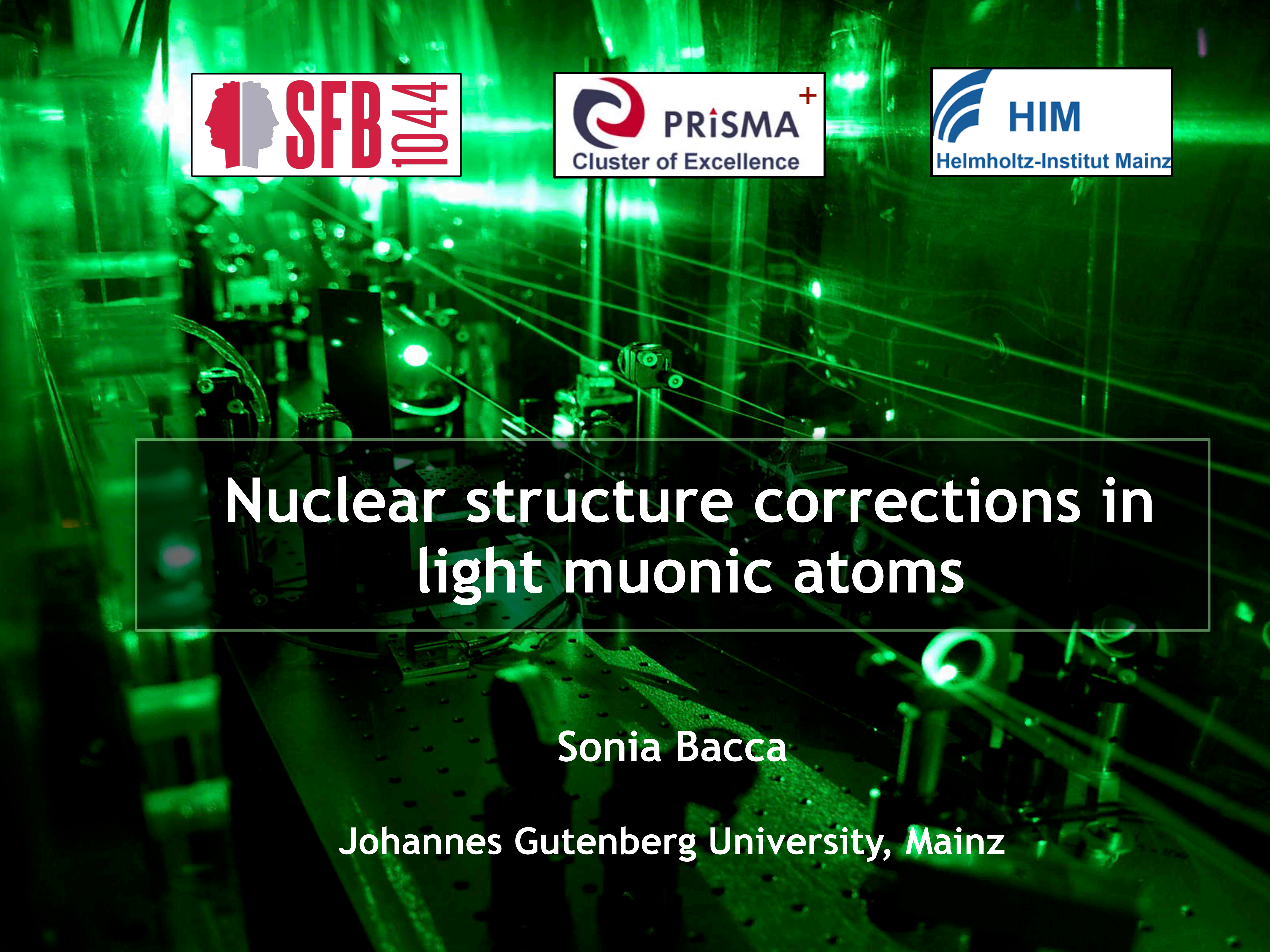


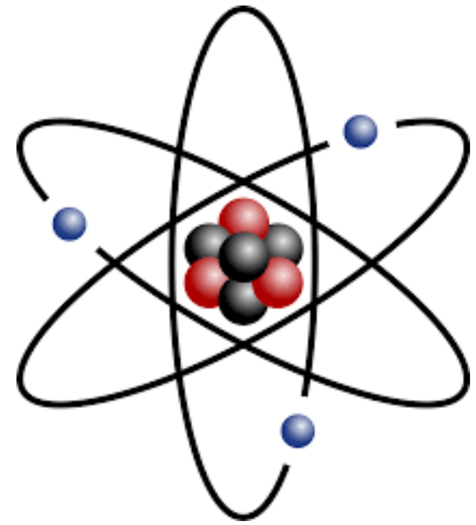
Nuclear structure corrections in light muonic atoms

Sonia Bacca

Johannes Gutenberg University, Mainz

What are muonic atoms?

Exotic atoms



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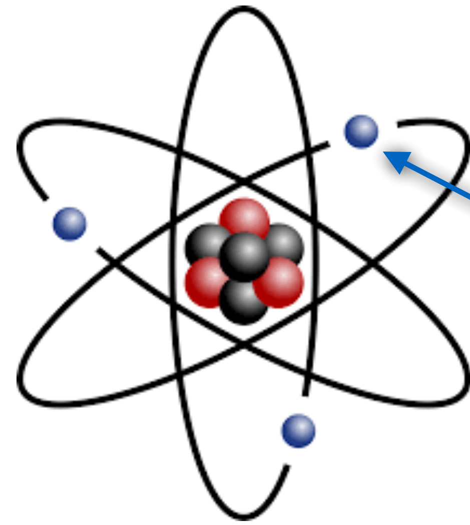

Electron replaced with a muon

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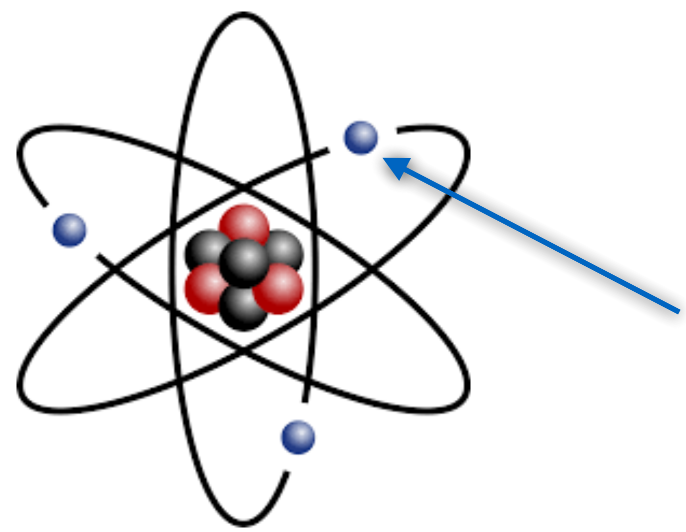
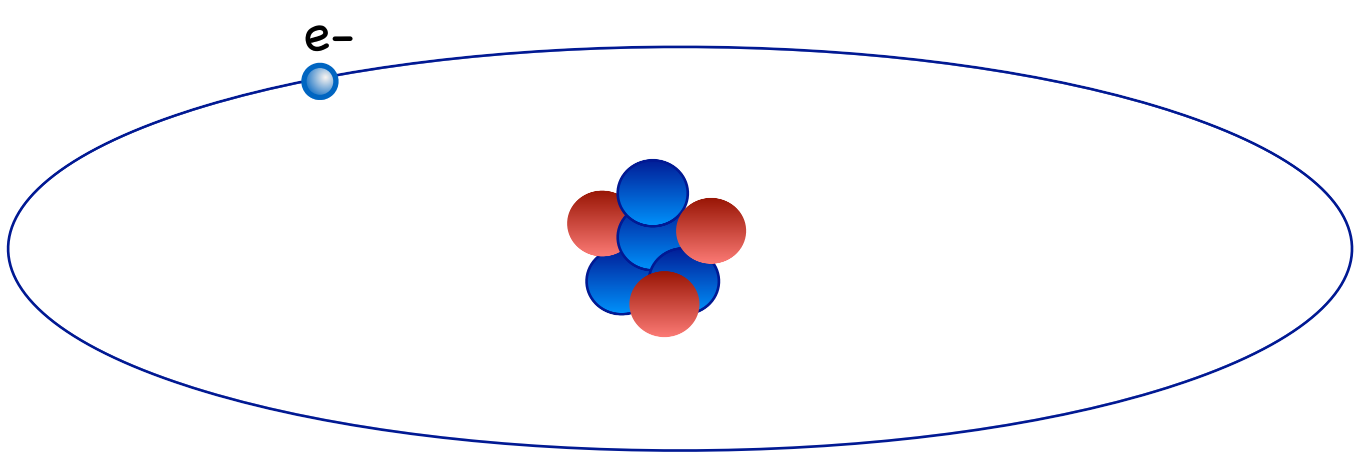


Hydrogen-like systems

Ordinary atoms



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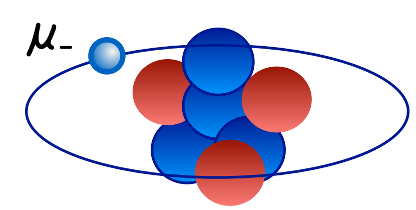

muon more sensitive to the nucleus

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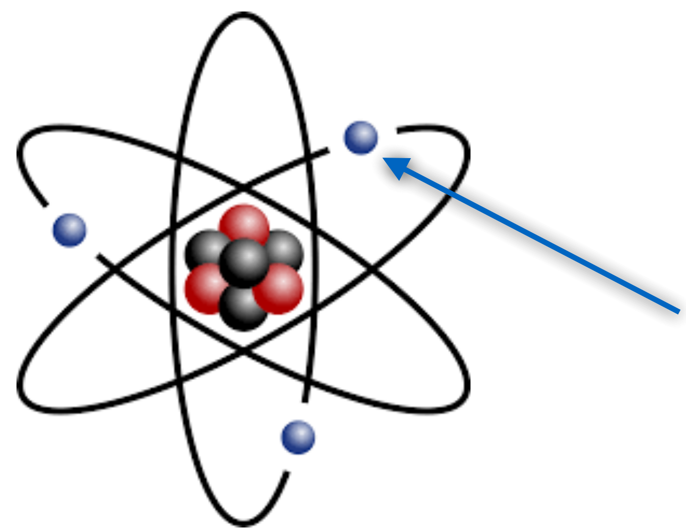

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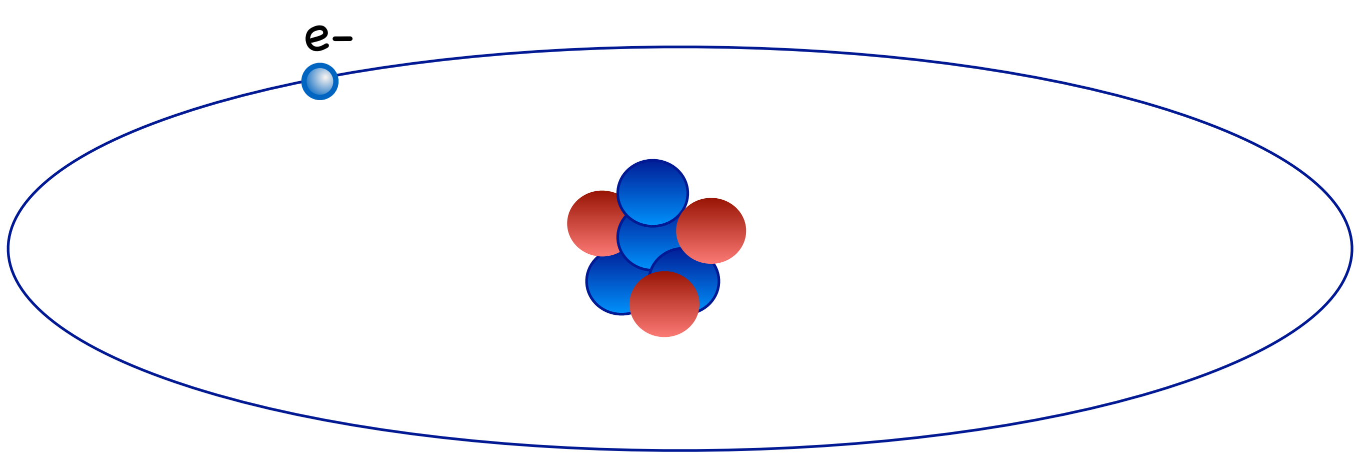
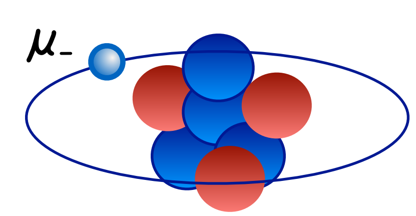




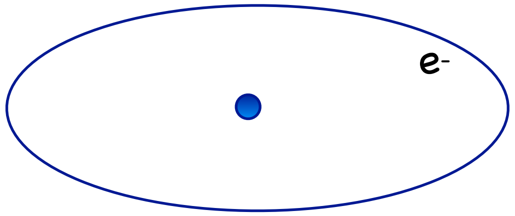
muon more sensitive to the nucleus

Can be used as a precision probe for the nucleus

Proton Radius Puzzle

The proton charge radius is measured from:

- electron-proton interactions: 0.8770 ± 0.0045 fm
  - eH spectroscopy
  - e-p scattering



- muonic -proton interactions: 0.8409 ± 0.0004 fm
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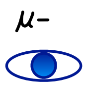

Pohl *et al.*, Nature (2010)
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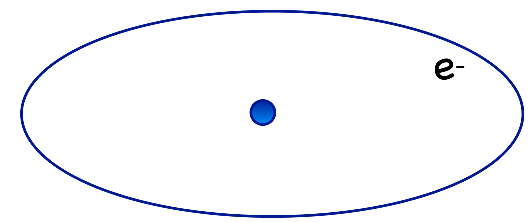

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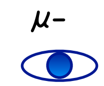

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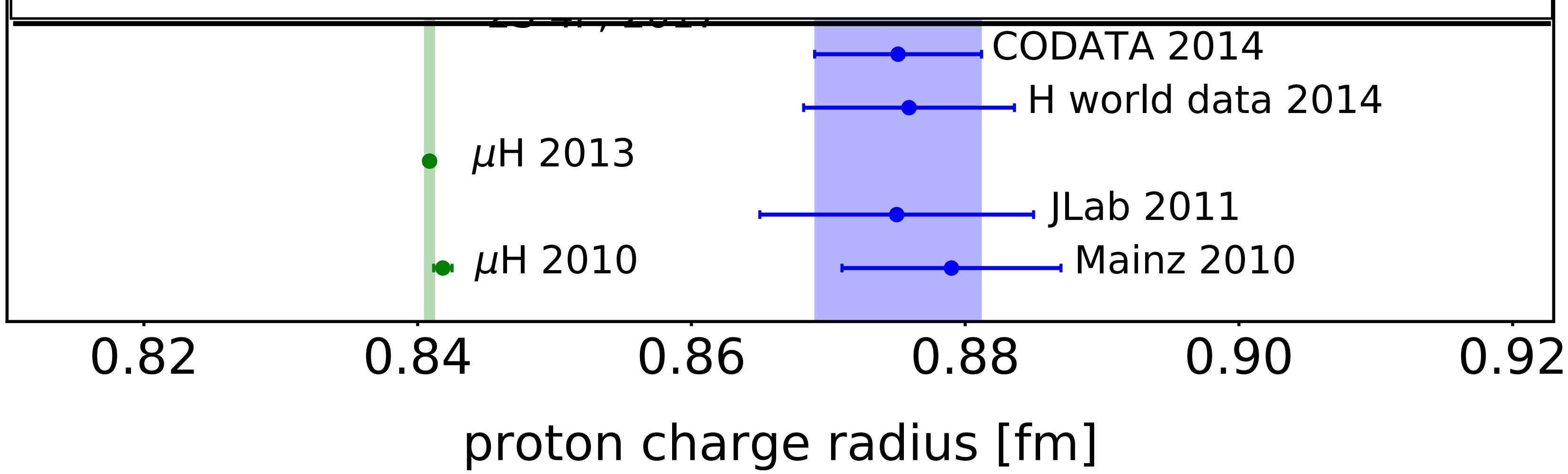
Is lepton universality violated?



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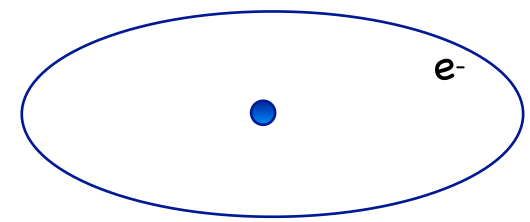

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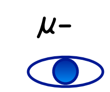

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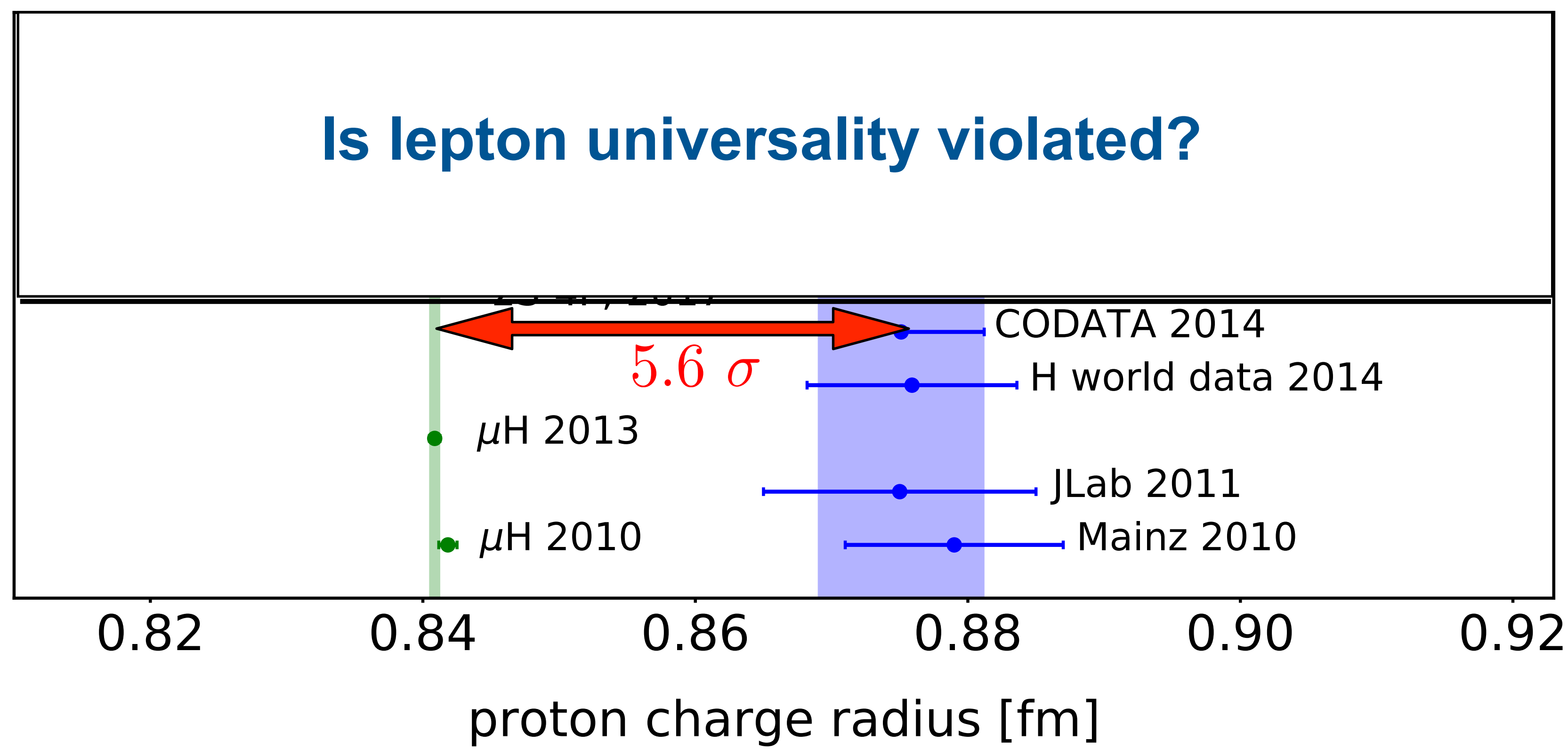

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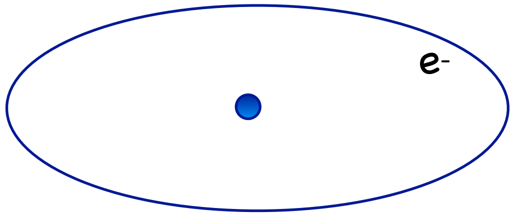

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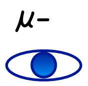

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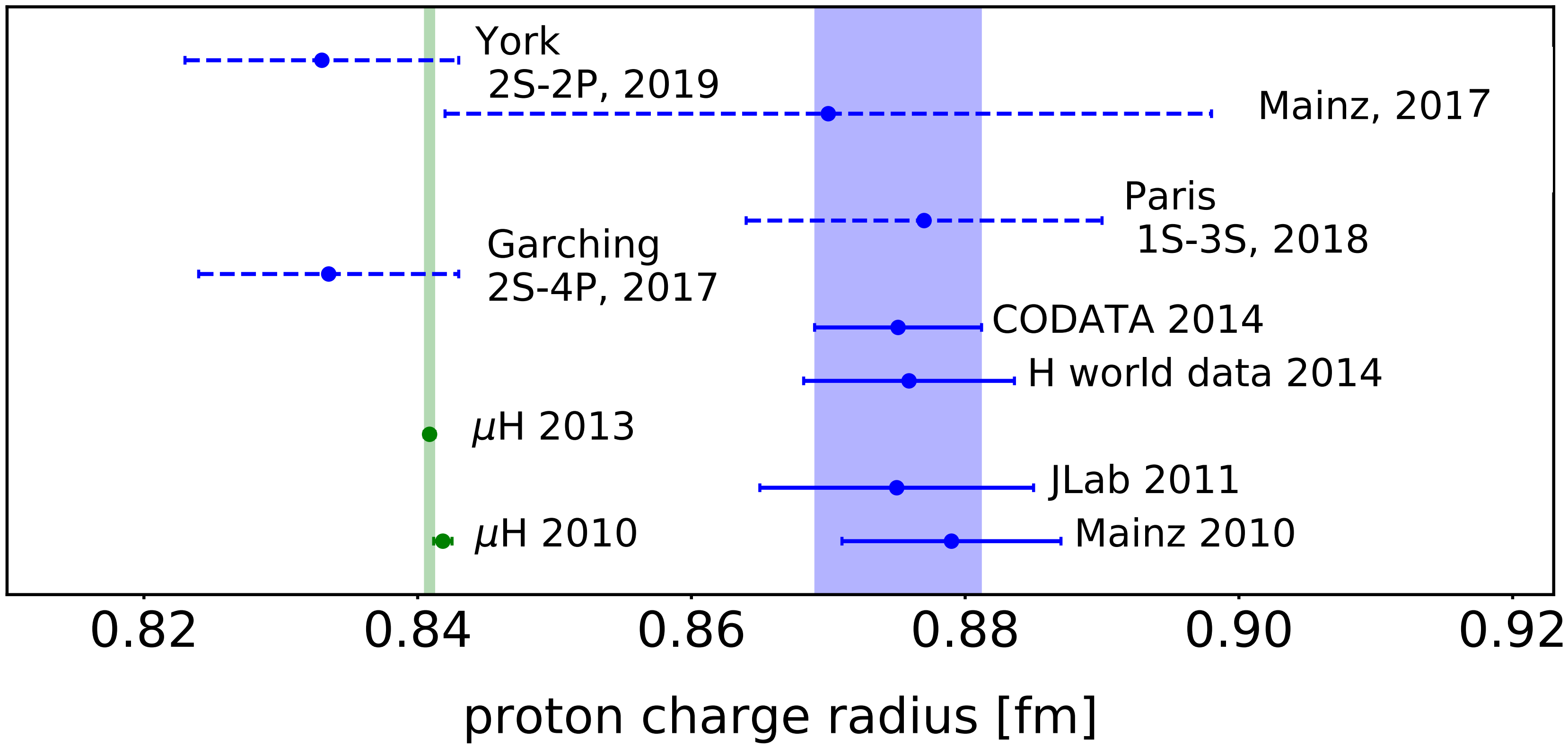

Experimental efforts

- **Higher precision electron scattering experiments**

  Q^2 from 10^{-4} GeV^2 to 10^{-2} GeV^2 Jefferson Lab

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  PAUL SCHERRER INSTITUT PSI

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Other light muonic atoms

Strong experimental program at PSI from the CREMA collaboration to unravel the mystery by studying other muonic atoms:

- μD (results released)
- μ^4He$^+$ (analyzing data)
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- μ^3H (impossible)
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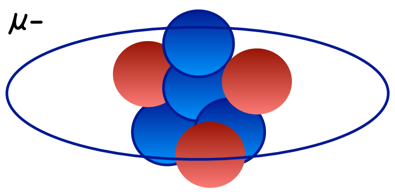

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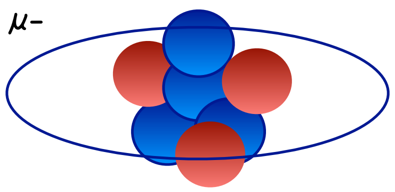

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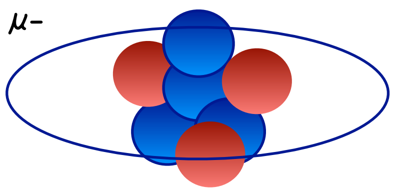

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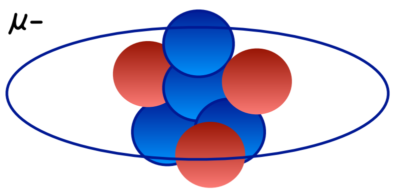

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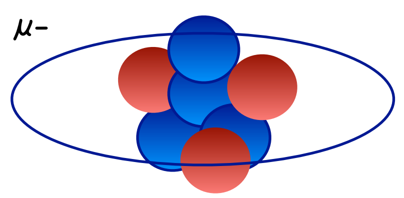

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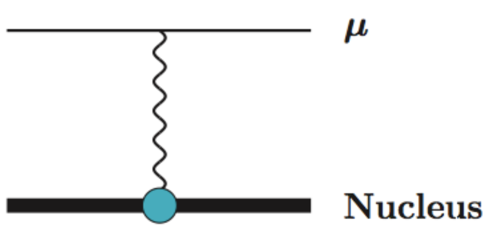

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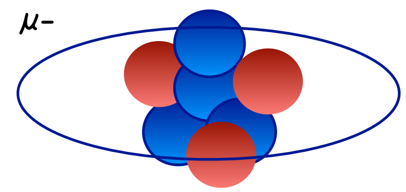

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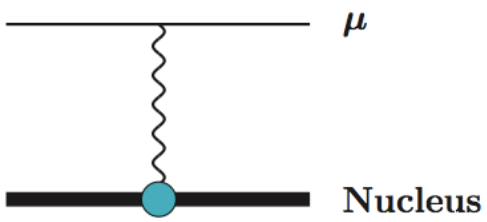
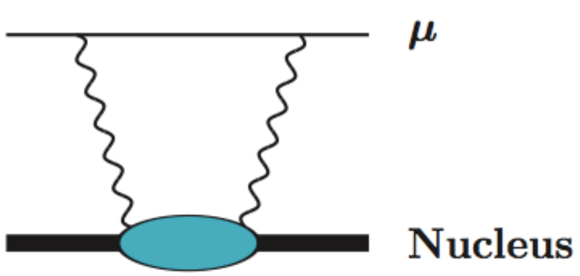


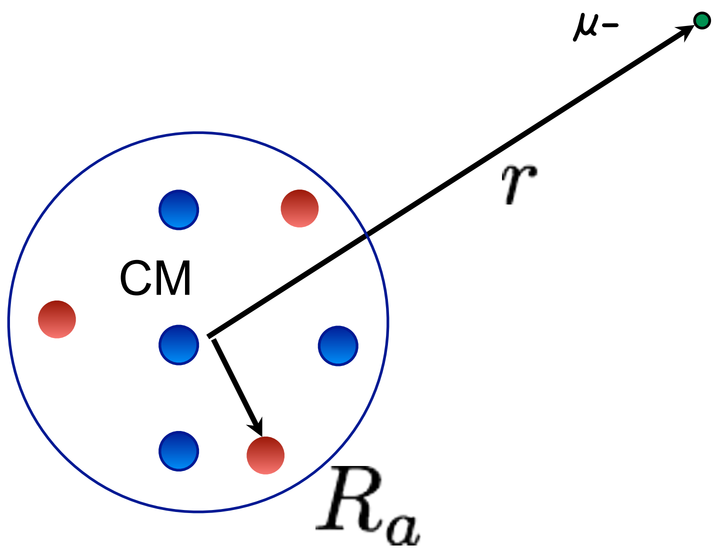
unknown
NS corrections



Theoretical derivation of TPE

$$H = H_N + H_\mu + \Delta V$$

$$H_\mu = \frac{p^2}{2m_r} - \frac{Z\alpha}{r}$$



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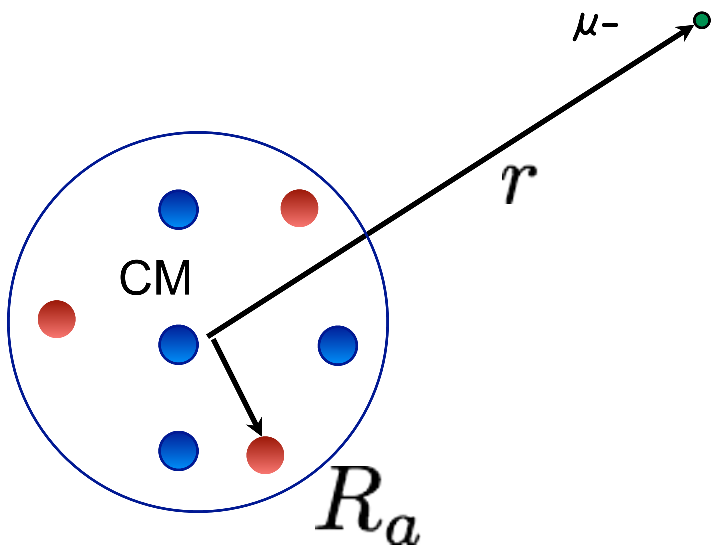

Perturbative potential: correction to the bulk Coulomb

$$\Delta V = \sum_a^Z \alpha \left(\frac{1}{r} - \frac{1}{|\vec{r} - \vec{R}_a|} \right)$$

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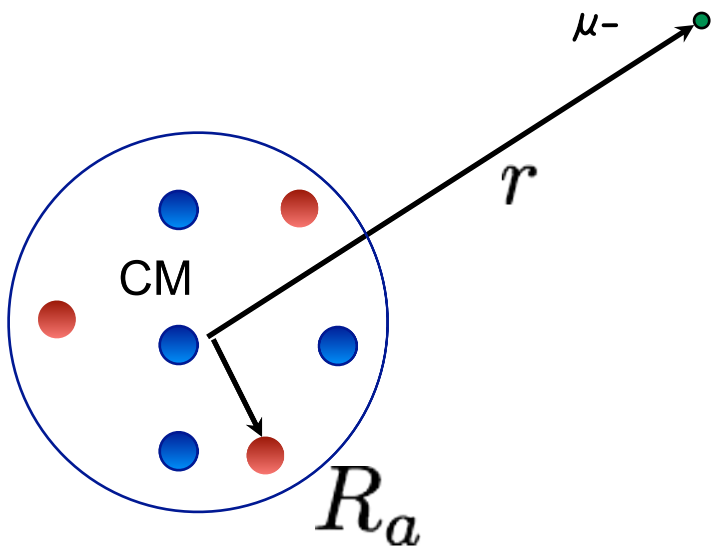

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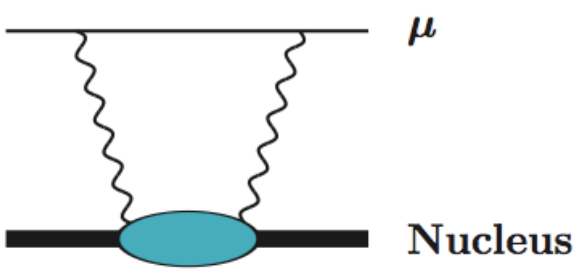
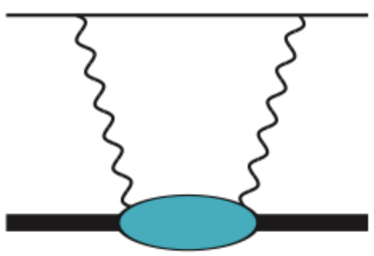
Using perturbation theory at second order one obtains the expression for TPE up to order $(Z\alpha)^5$



Theoretical derivation of TPE

- Non relativistic term

Take non-relativistic kinetic energy in muon propagator
Neglect Coulomb force in the intermediate state
Expand the muon matrix elements in powers of $\eta = \sqrt{\frac{m_r}{m_N}} \sim 0.17$



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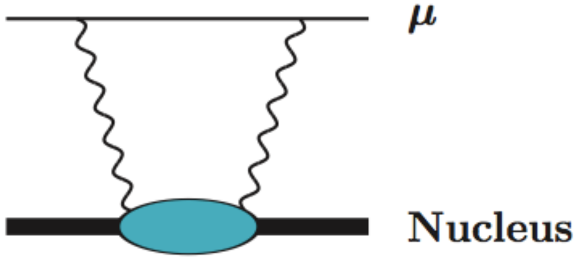

★ **Leading-order term,** related to the energy-weighted integral of the dipole response function

$$\delta_{D1}^{(0)} = -\frac{2\pi m_r^3}{9}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega \sqrt{\frac{2m_r}{\omega}} S_{D_1}(\omega)$$

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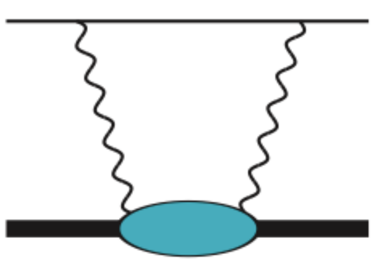

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★ **Next to leading-order term,** related to Zemach moment elastic contribution

$$\delta_{Z3}^{(1)} = \frac{\pi}{3} m_r (Z\alpha)^2 \phi^2(0) \iint d^3R d^3R' |\boldsymbol{R} - \boldsymbol{R}'|^3 \rho_0^p(\boldsymbol{R}) \rho_0^p(\boldsymbol{R}')$$

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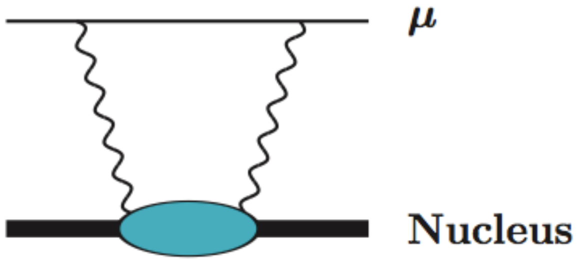

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★ **Next-to-next to leading-order term,** related to monopole and quadrupole response functions…

$$S_{R^2}(\omega), S_Q(\omega), S_{D1D3}(\omega)$$

Theoretical derivation of TPE

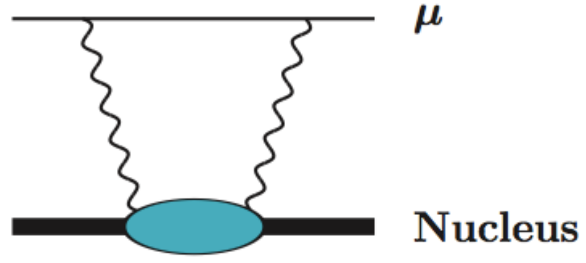
- Coulomb term

Consider the Coulomb force in the intermediate states

Naively $\delta_C^{(0)} \sim (Z\alpha)^6$, actually logarithmically enhanced

$\delta_C^{(0)} \sim (Z\alpha)^5 \log(Z\alpha)$ Friar (1977), Pachucki (2011)

Related to the dipole response function



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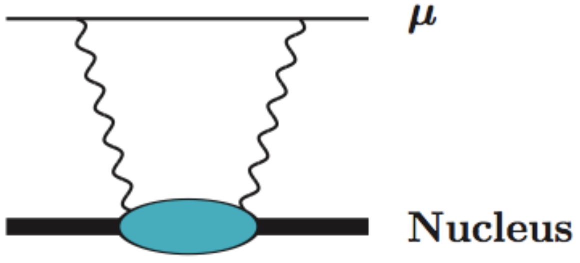

Relativistic terms

Take the relativistic kinetic energy in muon propagator
Related to the dipole response function

$$\delta_{L(T)}^{(0)} = \frac{2m_r^3}{9}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega\, K_{L(T)}\left(\frac{\omega}{m_r}\right) S_{D_1}(\omega)$$

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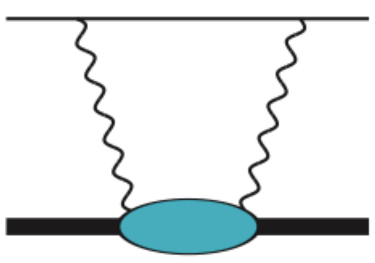

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Finite nucleon-size corrections

Consider finite nucleon-size by including their charge distributions and obtain terms, e.g.,

$$\delta_{R1}^{(1)} = -8\pi m_r (Z\alpha)^2 \phi^2(0) \int\int d^3R d^3R' |\boldsymbol{R}-\boldsymbol{R}'| \left[\frac{2}{\beta^2}\rho_0^{pp}(\boldsymbol{R},\boldsymbol{R}') - \lambda\rho_0^{np}(\boldsymbol{R},\boldsymbol{R}')\right]$$

Theoretical derivation of TPE

$$\delta_{\mathrm{TPE}} = \boxed{\delta^{A}_{\mathrm{Zem}}} + \delta^{N}_{\mathrm{Zem}} + \boxed{\delta^{A}_{\mathrm{pol}}} + \delta^{N}_{\mathrm{pol}}$$

Theoretical derivation of TPE

$$\delta_{\mathrm{TPE}} = \boxed{\delta^A_{\mathrm{Zem}}} + \delta^N_{\mathrm{Zem}} + \boxed{\delta^A_{\mathrm{pol}}} + \delta^N_{\mathrm{pol}}$$

$$\delta^A_{\mathrm{pol}} = \delta^{(0)}_{D1} + \delta^{(1)}_{R3} + \cancel{\delta^{(1)}_{Z3}} + \delta^{(2)}_{R^2} + \delta^{(2)}_{Q} + \delta^{(2)}_{D1D3} + \delta^{(0)}_{C}$$
$$+\delta^{(0)}_{L} + \delta^{(0)}_{T} + \delta^{(0)}_{M} + \delta^{(1)}_{R1} + \cancel{\delta^{(1)}_{Z1}} + \delta^{(2)}_{NS}$$

$$\delta^A_{\mathrm{Zem}} = -\cancel{\delta^{(1)}_{Z3}} - \cancel{\delta^{(1)}_{Z1}}$$

Friar an Payne ('97)

Need to calculate δ_{TPE} and related uncertainties.

A matter of precision

The uncertainty of the extracted radius depends on the precision of the TPE

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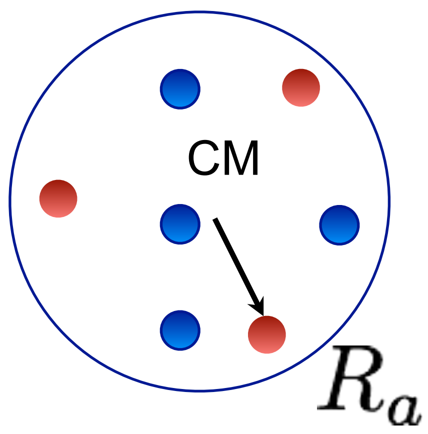
*Leidemann, Rosenfelder ’95 using few-body methods

Ab Initio Nuclear Theory

- Solve the Schrödinger equation for few-nucleons

$$H_N|\psi_i\rangle = E_i|\psi_i\rangle$$

$$H_N = T + V_{NN}(\Lambda) + V_{3N}(\Lambda) + \ldots$$



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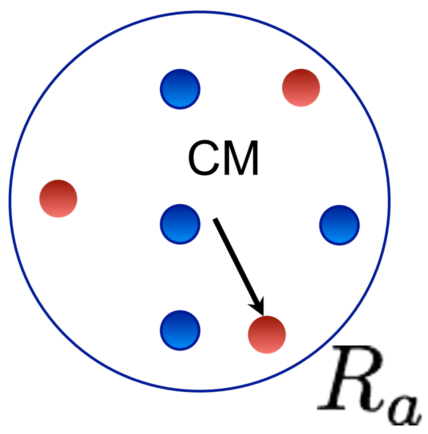

Hyper-spherical harmonics expansions and Lorenz integral transform method for A=3,4,6,7

Barnea, Leidemann, Orlandini PRC **61** (2000) 054001

Efros, *et al.*, JPG.: Nucl.Part.Phys. **34** (2007) R459

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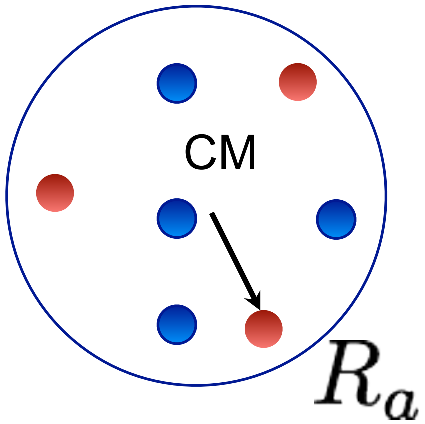

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For A=2 we use an harmonic oscillator expansion

- We will use nuclear interactions derived from traditional potentials (AV18+UIX) and from chiral effective filed theory (at various orders)

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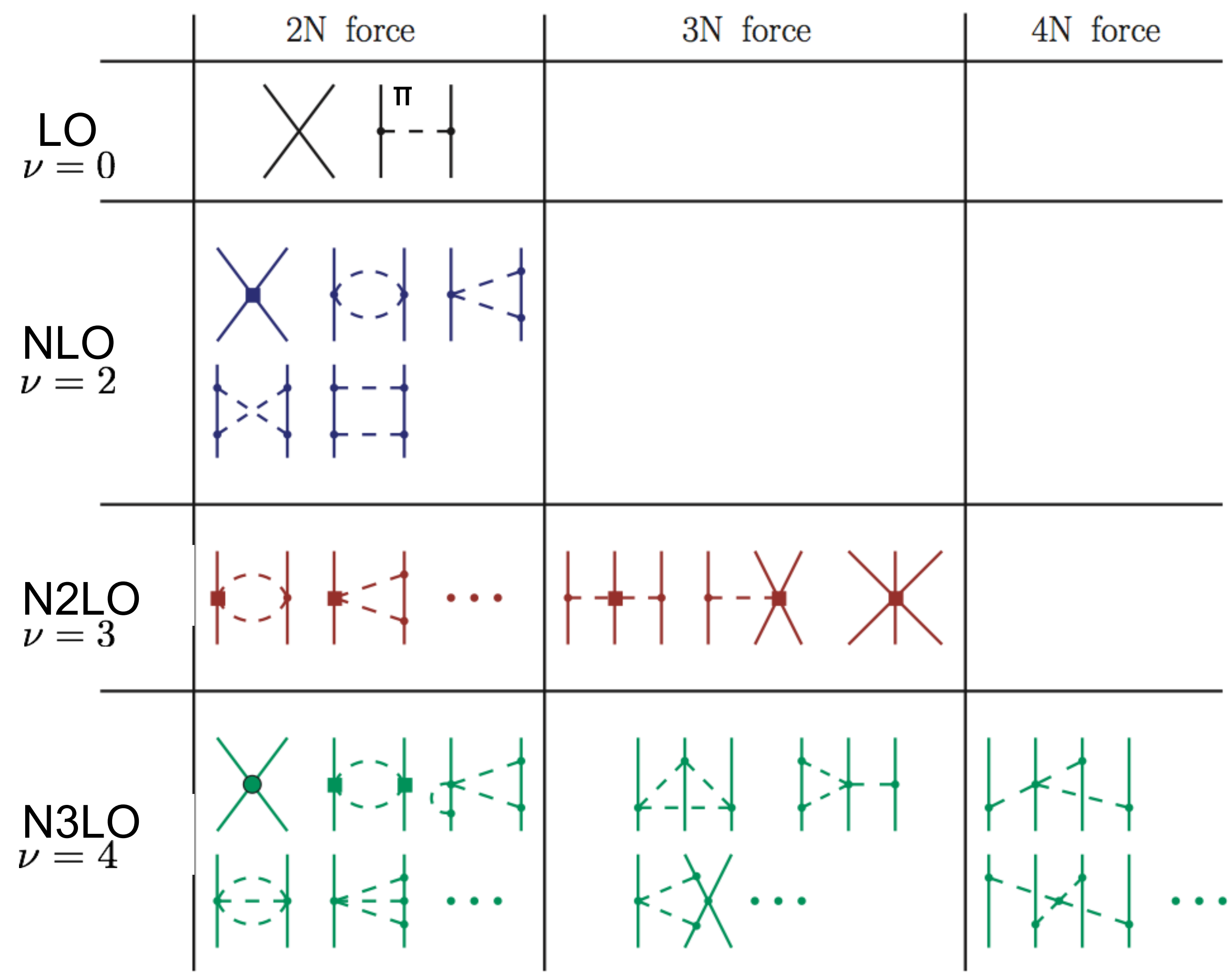
Chiral effective filed theory

Systematic expansion $$\mathcal{L} = \sum_{\nu} c_{\nu} \left(\frac{Q}{\Lambda_b} \right)^{\nu}$$



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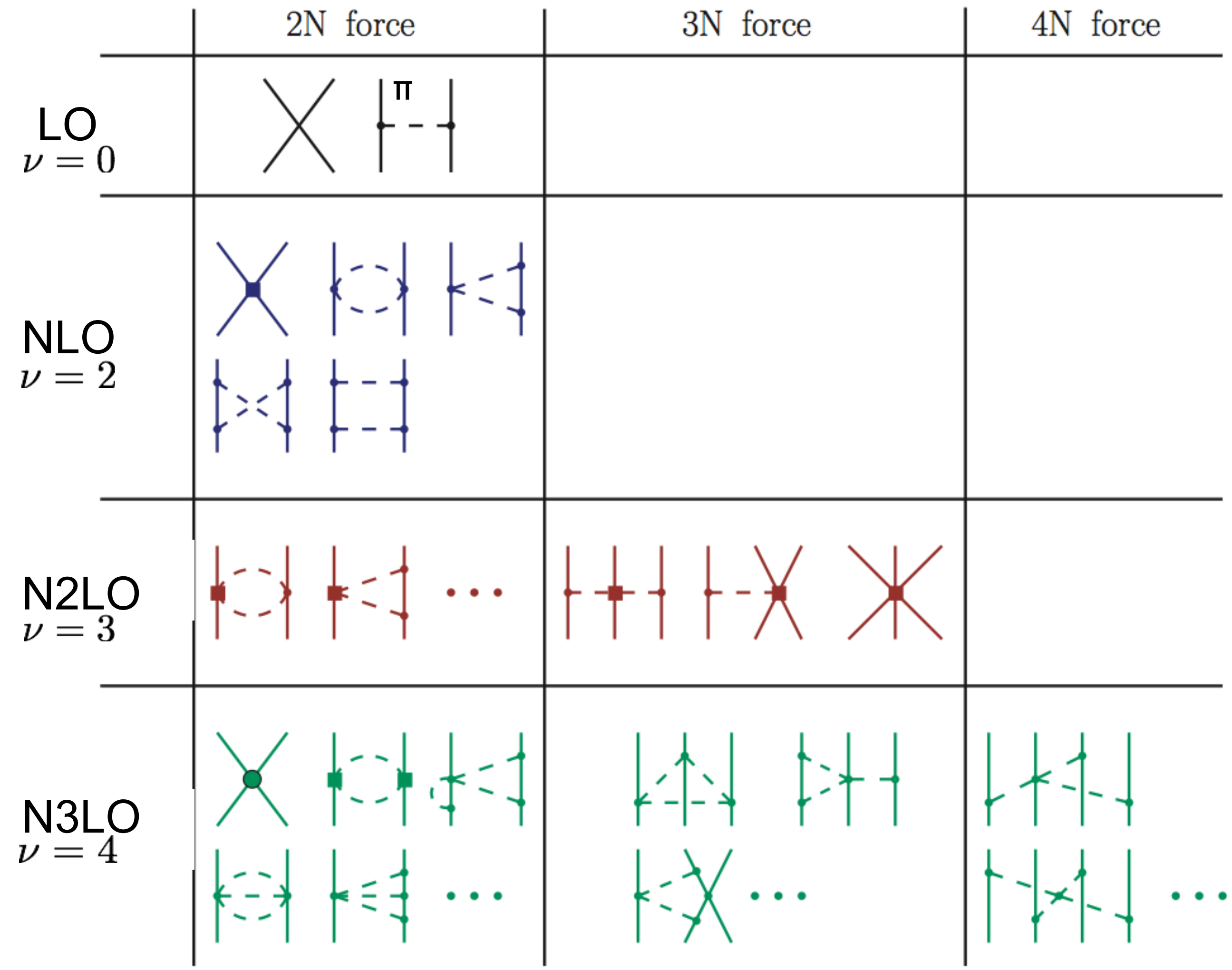
Details of short distance physics not resolved, but captured in low energy constants (LEC)



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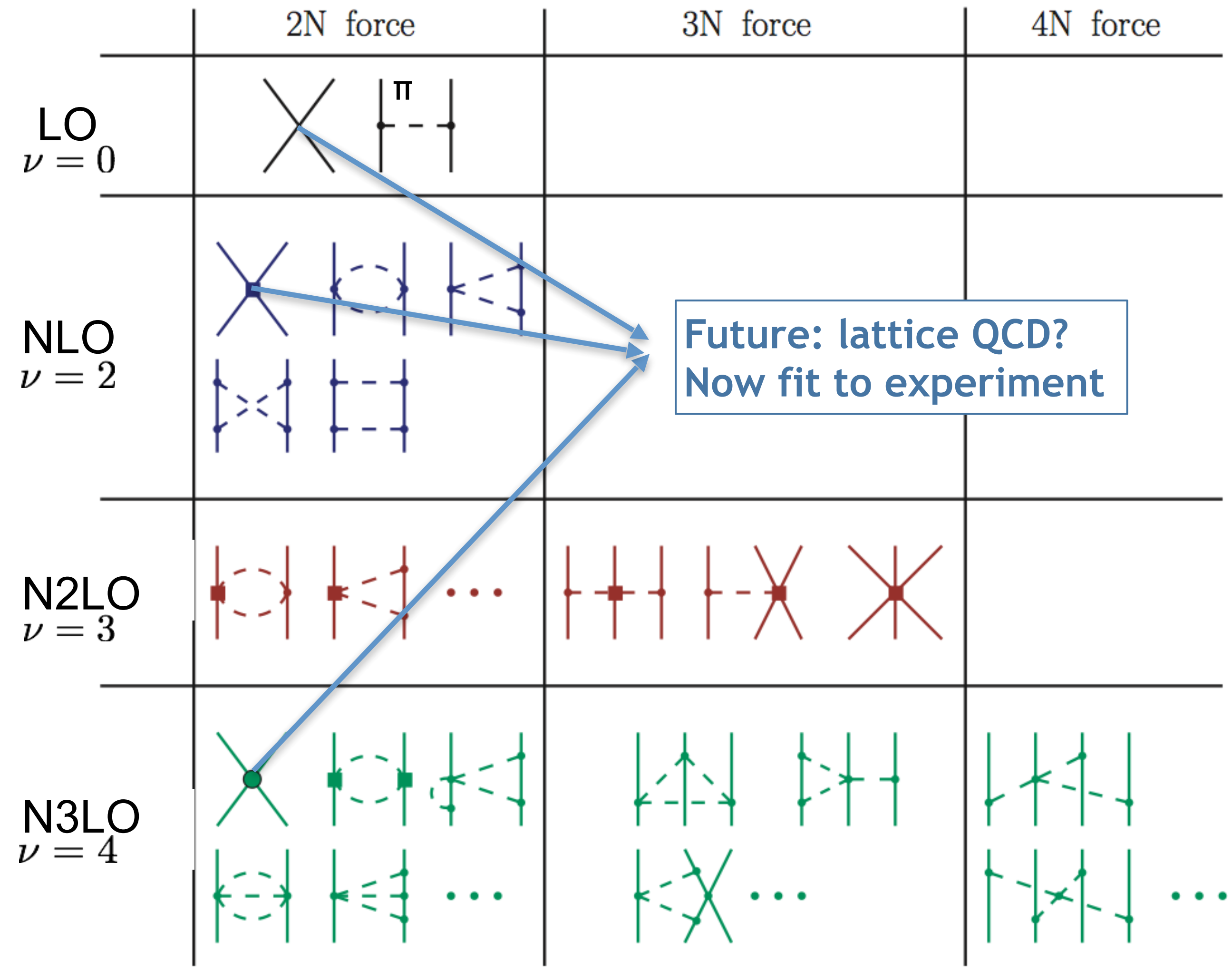

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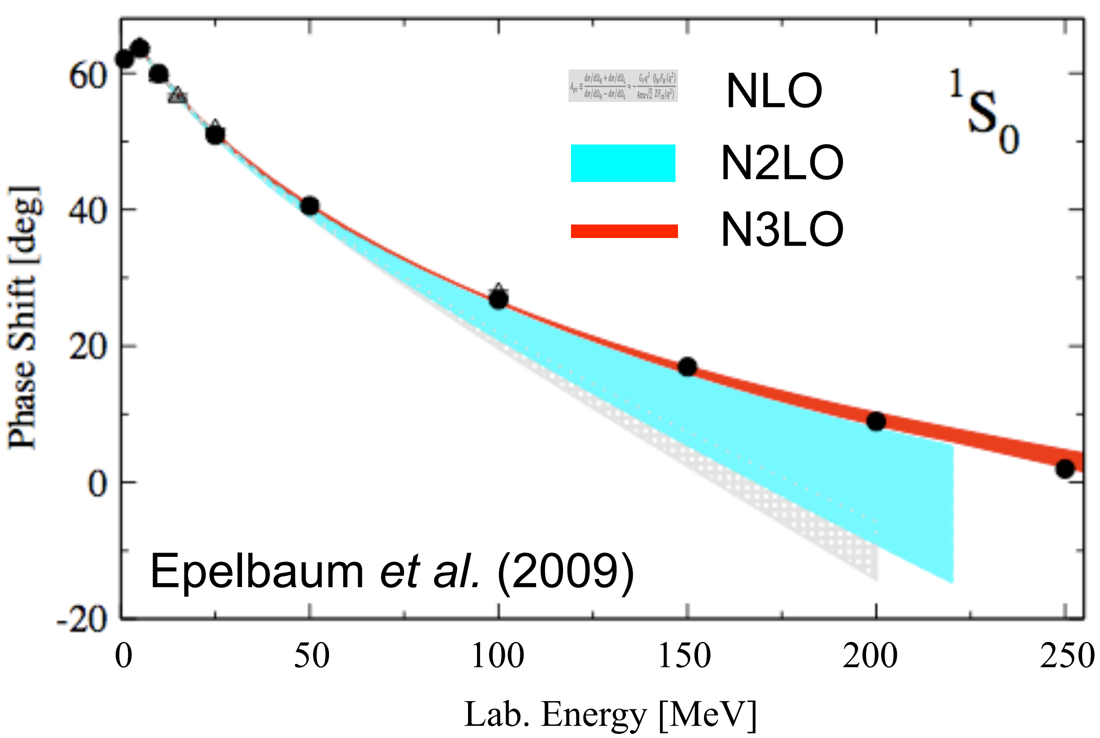
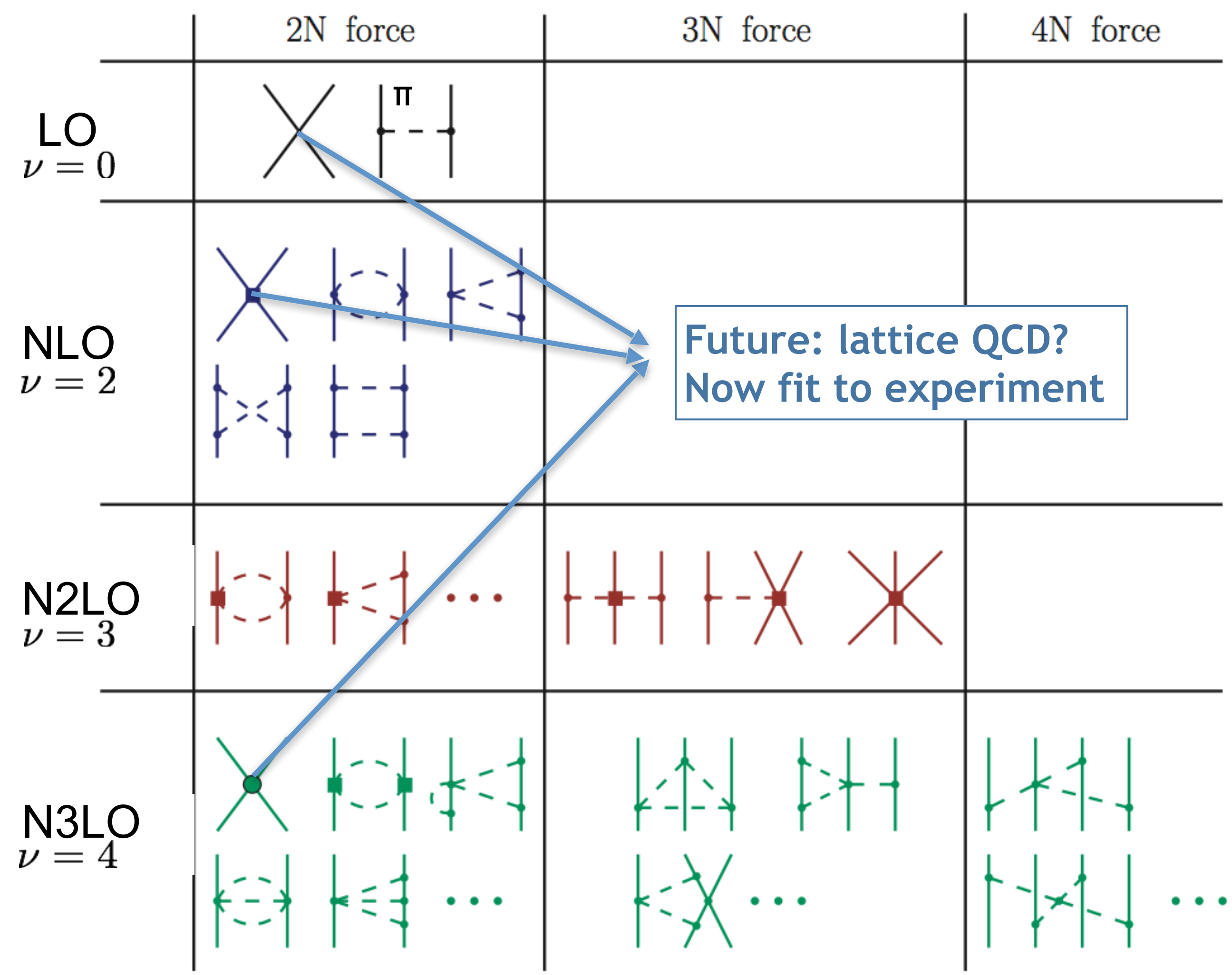
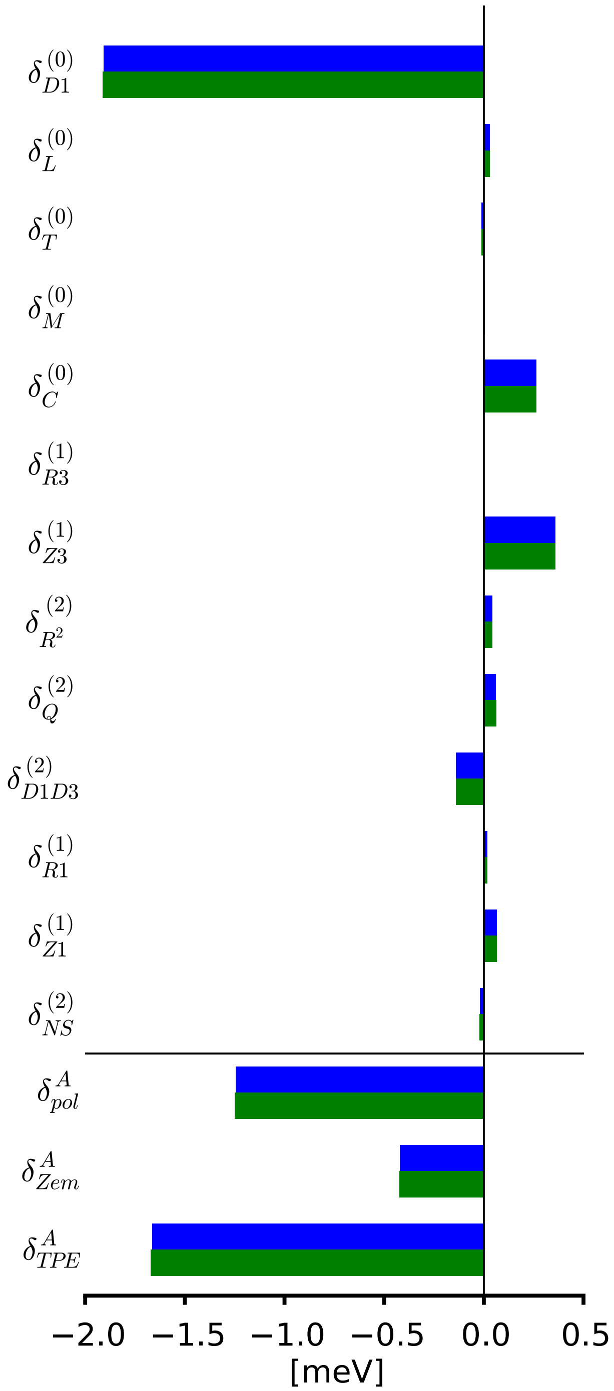
LEC fit to experiment - NN sector -





Muonic Deuterium



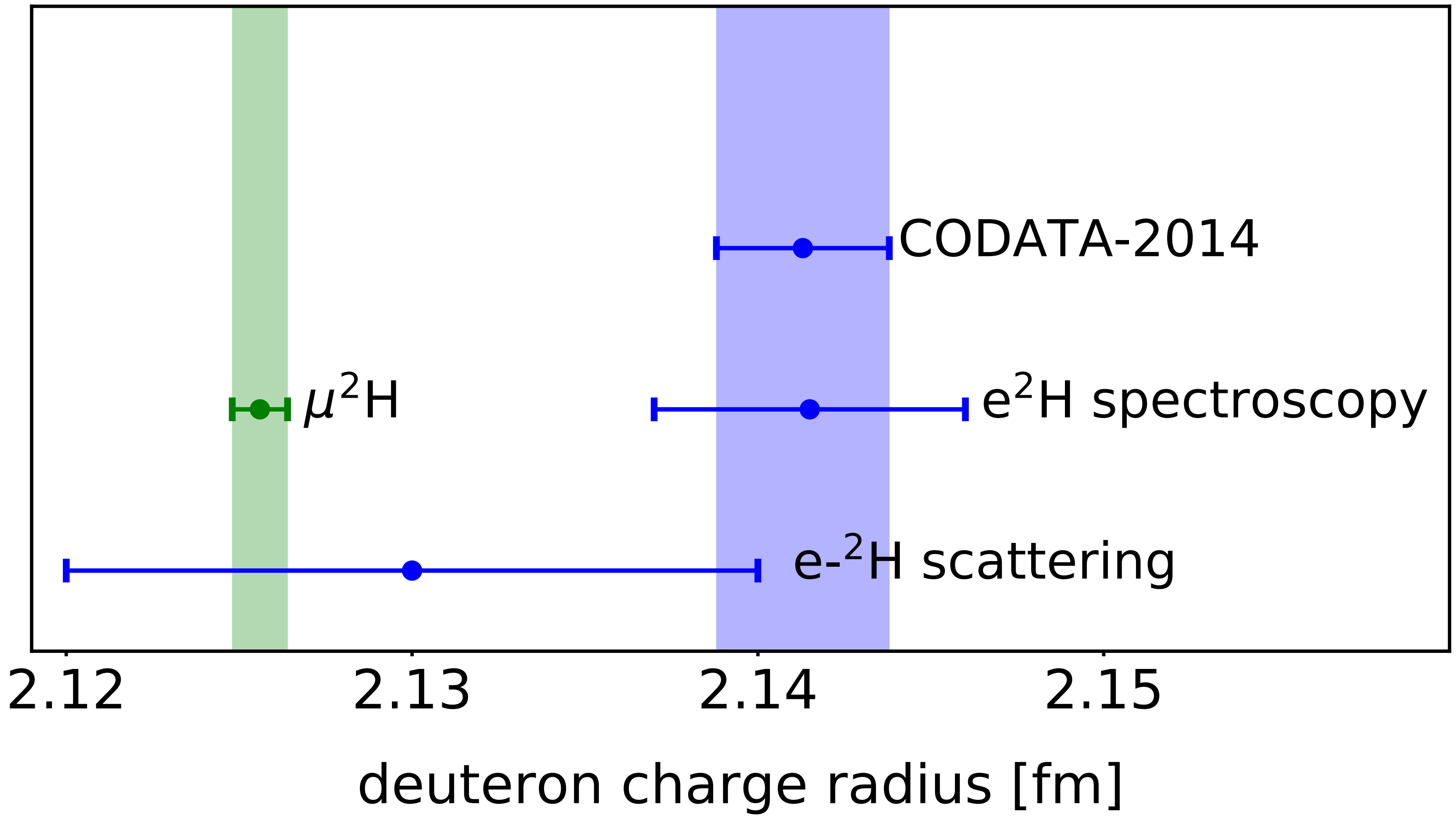
	AV18	χEFT
$\delta_{D1}^{(0)}$	-1.907	-1.912
$\delta_{L}^{(0)}$	0.029	0.029
$\delta_{T}^{(0)}$	-0.012	-0.012
$\delta_{M}^{(0)}$	0.003	0.003
$\delta_{C}^{(0)}$	0.262	0.262
$\delta_{R3}^{(1)}$	-	-
$\delta_{Z3}^{(1)}$	0.357	0.359
$\delta_{R^2}^{(2)}$	0.042	0.041
$\delta_{Q}^{(2)}$	0.061	0.061
$\delta_{D1D3}^{(2)}$	-0.139	-0.139
$\delta_{R1}^{(1)}$	0.017	0.017
$\delta_{Z1}^{(1)}$	0.064	0.064
$\delta_{NS}^{(2)}$	-0.020	-0.021
δ_{pol}^{A}	-1.243	-1.248
δ_{Zem}^{A}	-0.421	-0.423
δ_{TPE}^{A}	-1.664	-1.671

J. Hernandez et al, Phys. Lett. B **736**, 344 (2014)

AV18 in agreement with Pachucki (2011)+ Pachucki, Wienczek (2015)

Deuteron radius puzzle

Pohl et al., Science **353**, 669 (2016)



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Hernandez et al., PLB **736**, 334 (2014)

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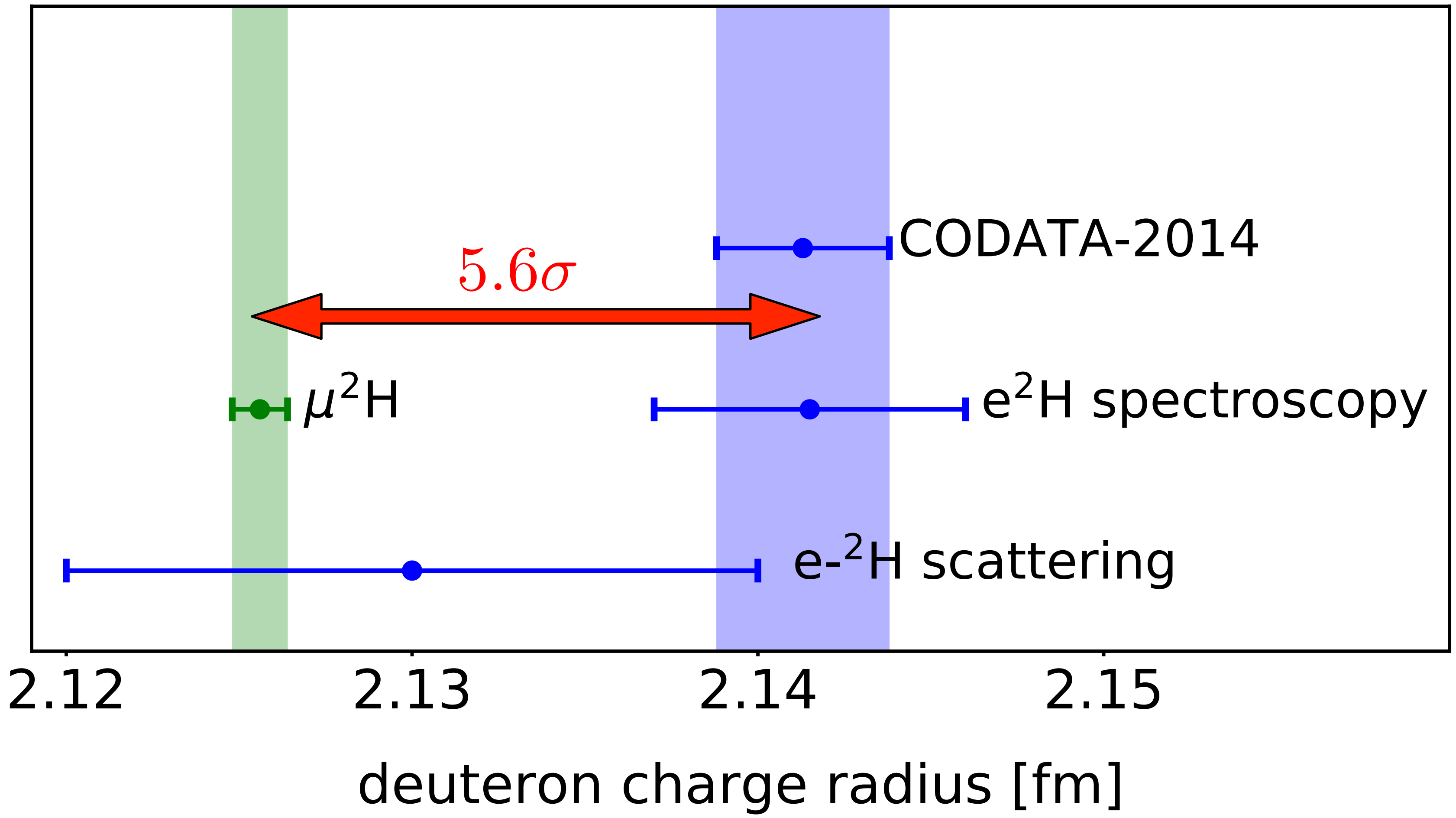

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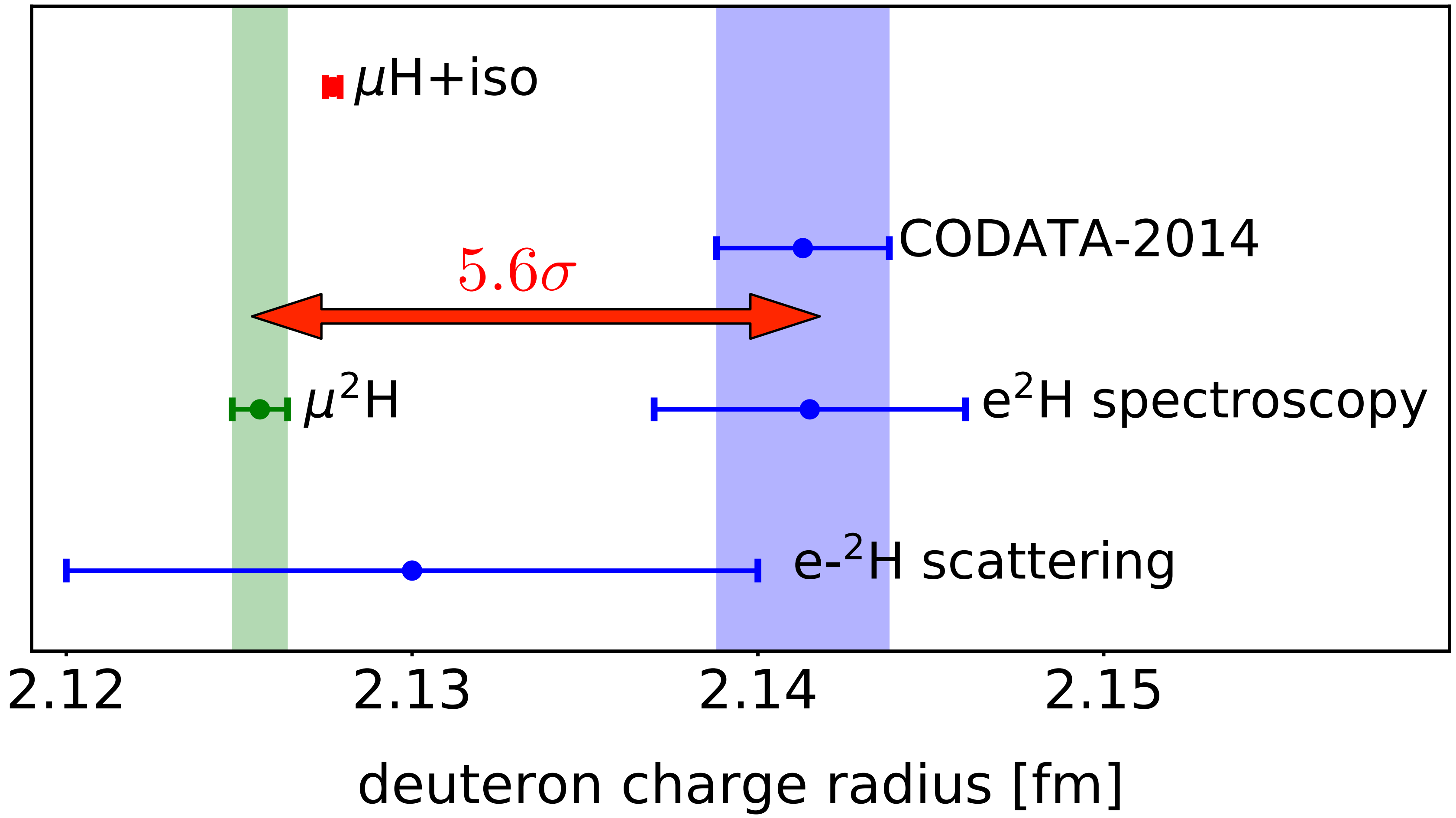

$$\Delta E_{2S-2P} = \delta_{\mathrm{QED}} + \mathcal{A}_{\mathrm{OPE}} \langle r_c^2 \rangle + \delta_{\mathrm{TPE}}$$

μH+iso: r_p from μH and deuterium isotopic shift $r^2_d - r^2_p$: Parthey et al., PRL **104** 233001 (2010)

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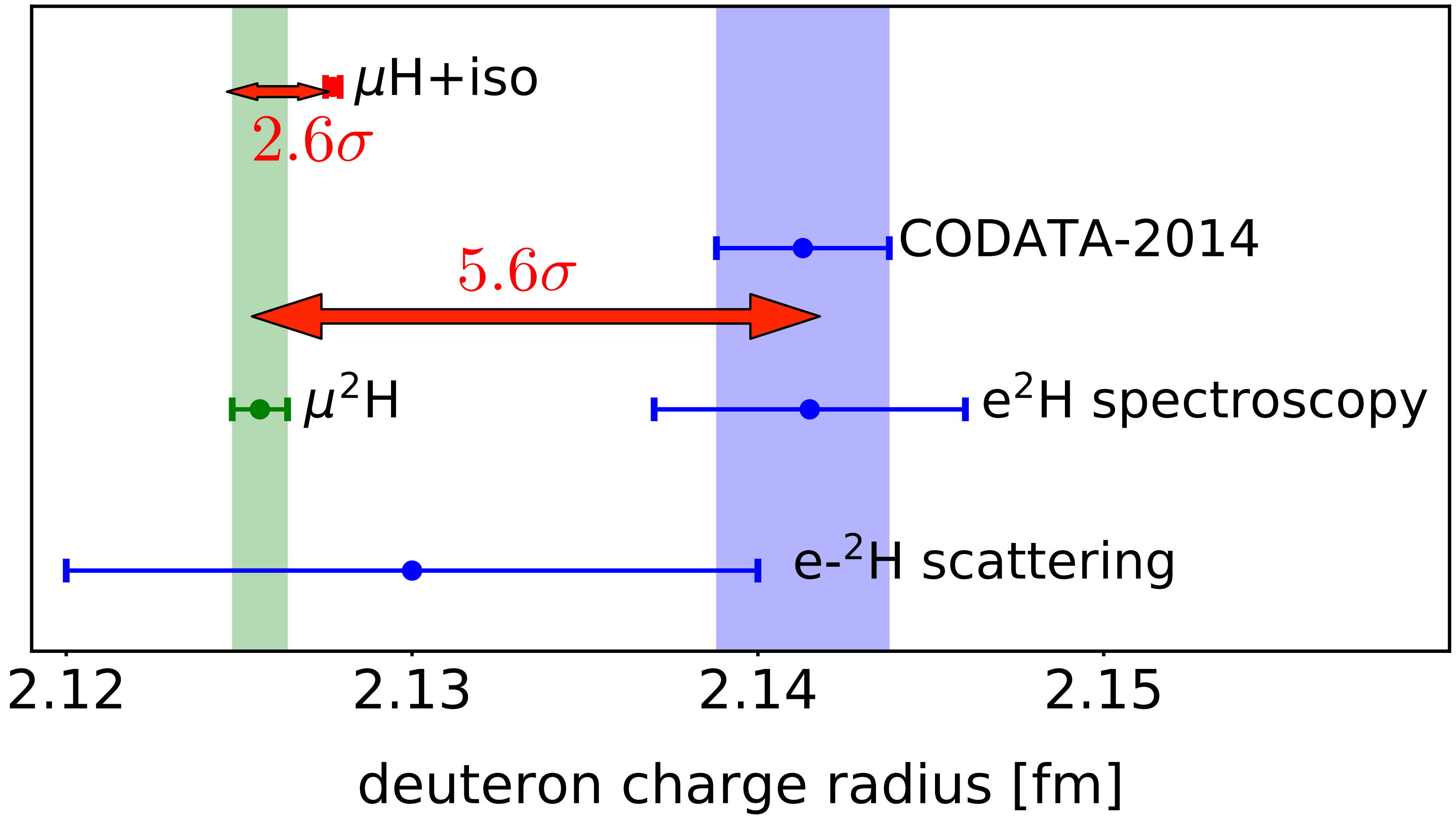

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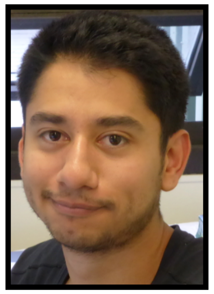
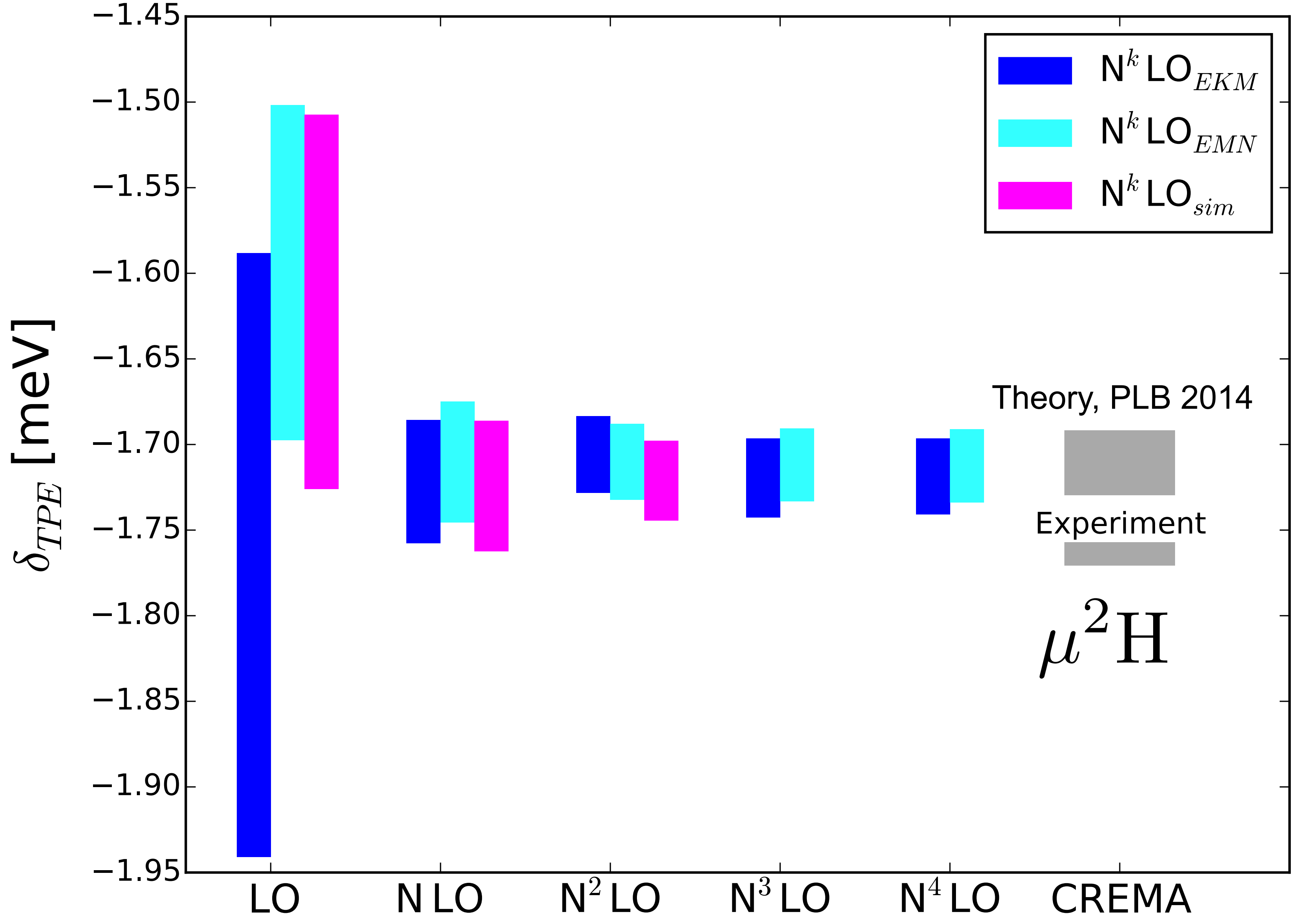
Order-by-order chiral expansion

Statistical and systematic uncertainty analysis

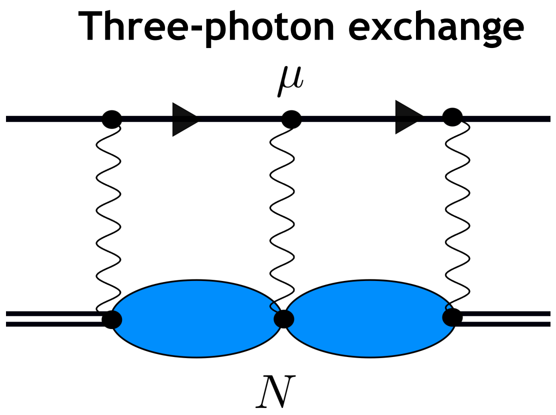
J. Hernandez et al., Phys. Lett. B **778**, 377 (2018)





Only sightly mitigate the “small” proton radius puzzle (2.6 to 2 σ)

Higher order corrections in α



Pachucki et al., Phys. Rev. A **97** 062511 (2018)

$(Z\alpha)^6$ correction, negligible

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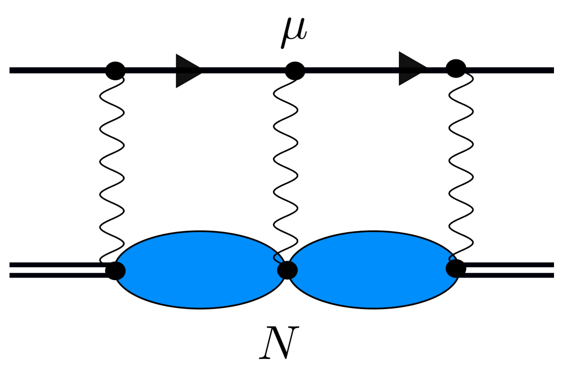

Pachucki et al., Phys. Rev. A **97** 062511 (2018)

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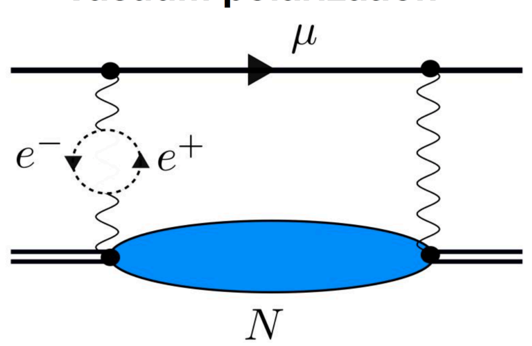

One the many α^6 corrections, supposedly the largest

Kalinowski, Phys. Rev. A **99** 030501 (2019)

$$\delta_{\mathrm{TPE}} = -1.750^{+14}_{-16}\ \mathrm{meV}$$ Theory

$$\delta_{\mathrm{TPE}} = -1.7638(68)\ \mathrm{meV}$$ Exp

Consistent within 1σ

solves the small deuteron-radius puzzle

Large deuteron-radius puzzle still unsolved

Reduction of Uncertainties

Atom	Exp uncertainty on ΔE_{2S-2P}	Uncertainty on TPE prior to the discovery of the puzzle	Uncertainty on TPE: ab initio
μ^2H	0.003 meV	0.03 meV	0.02 meV
μ^3He^+	0.08 meV	1 meV	0.3 meV
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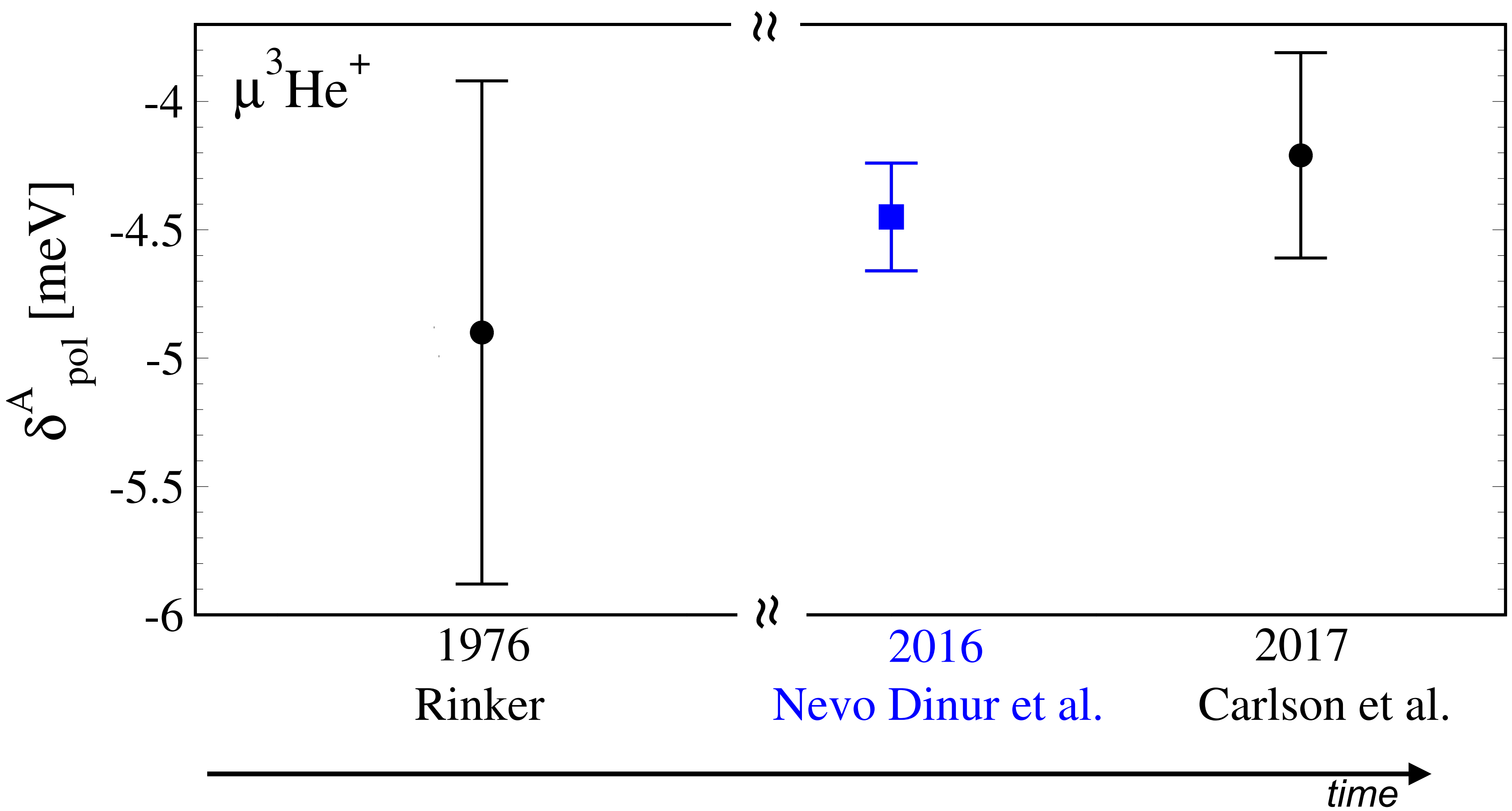
C.Ji et al., JPG: Part. Nucl. **45**, 093002 (2018)



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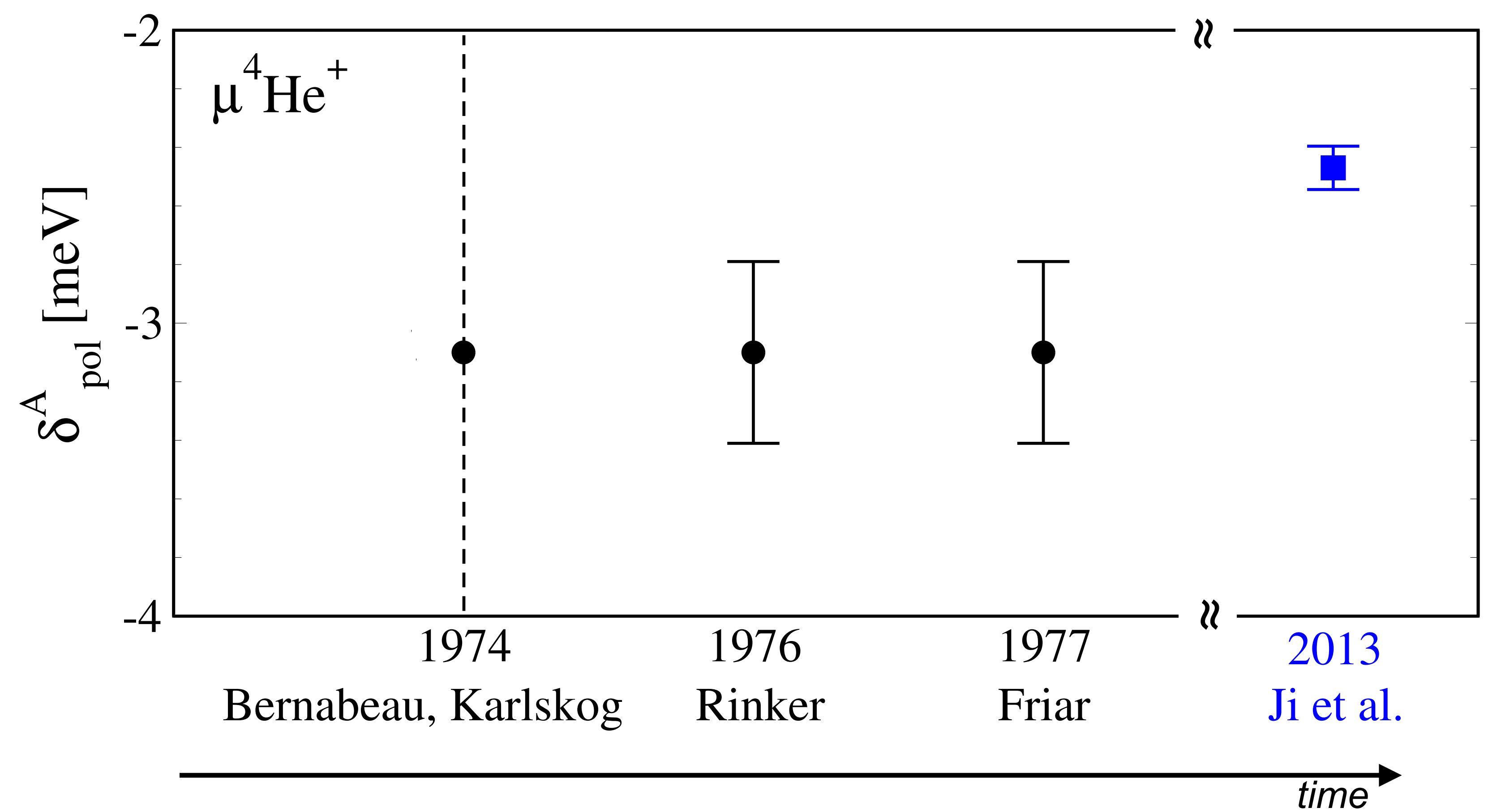
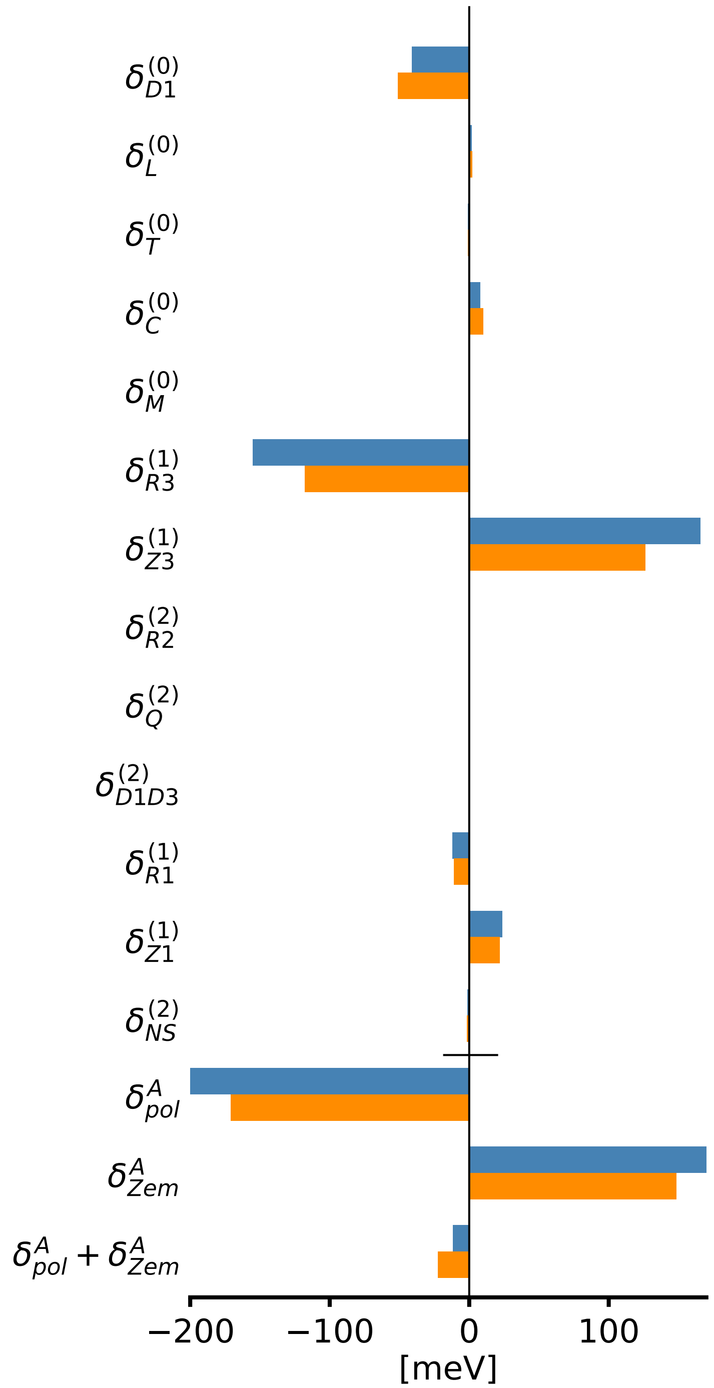


Muonic Lithiums, unpublished

Preliminary



	$\mu^6\mathrm{Li}^{2+}$	$\mu^7\mathrm{Li}^{2+}$
$\delta_{D1}^{(0)}$	-41.0	-51.0
$\delta_{L}^{(0)}$	1.66	2.04
$\delta_{T}^{(0)}$	-0.75	-0.92
$\delta_{C}^{(0)}$	7.85	9.89
$\delta_{M}^{(0)}$	-	-
$\delta_{R3}^{(1)}$	-155.0	-118.0
$\delta_{Z3}^{(1)}$	165.5	126.4
$\delta_{R2}^{(2)}$	-	-
$\delta_{Q}^{(2)}$	-	-
$\delta_{D1D3}^{(2)}$	-	-
$\delta_{R1}^{(1)}$	-12.0	-11.0
$\delta_{Z1}^{(1)}$	23.48	22.03
$\delta_{NS}^{(2)}$	-1.41	-1.75
δ_{pol}^{A}	-200.65	-170.74
δ_{Zem}^{A}	188.98	148.43
$\delta_{pol}^{A} + \delta_{Zem}^{A}$	-11.67	-22.31

S. Li Muli et al.

Summary and Outlook

- Ab initio calculations have allowed to microscopically compute TPE and to substantially reduce uncertainties

- Independently on the nature of the puzzle, these calculations are needed to support any spectroscopic measurement with muonic atoms

- In the future we will investigate the hyperfine splitting of muonic deuterium and the Lamb shift in muonic Lithium and Beryllium atoms

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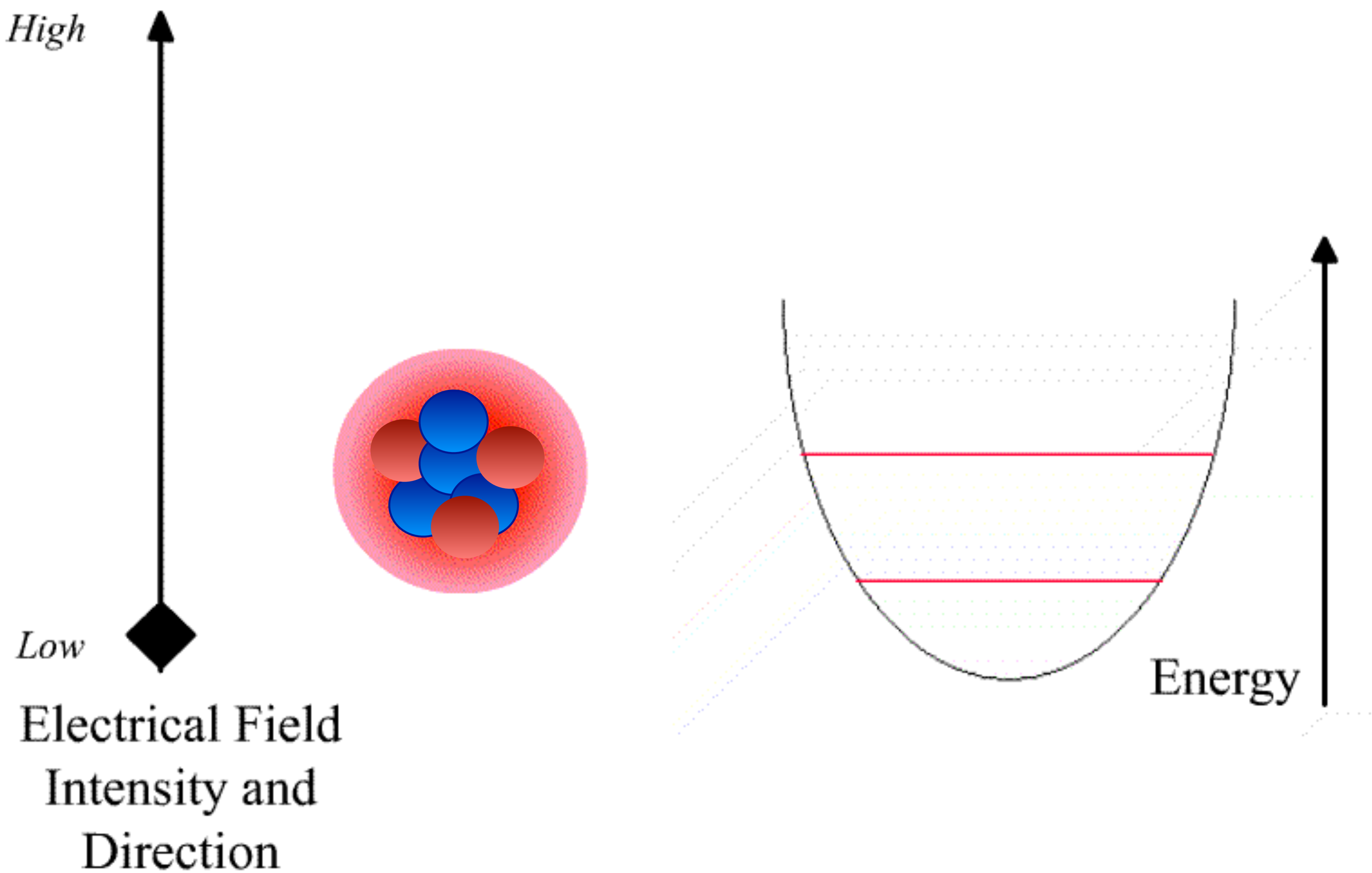
Thank you for your attention!

Backup Slides

Concept of polarizability



$$\mathbf{P} = \alpha_{\mathbf{E1}} \mathbf{E}$$

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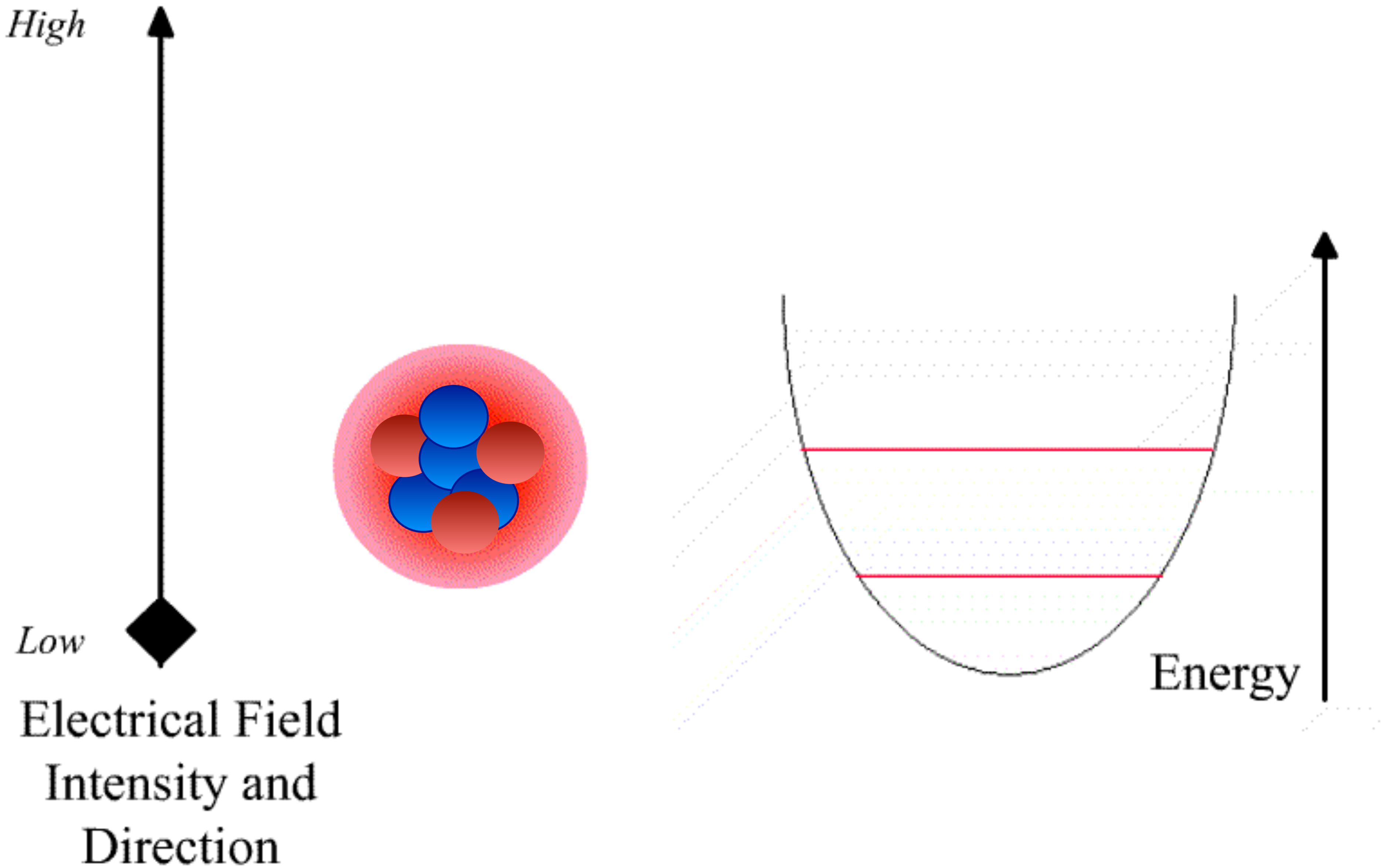

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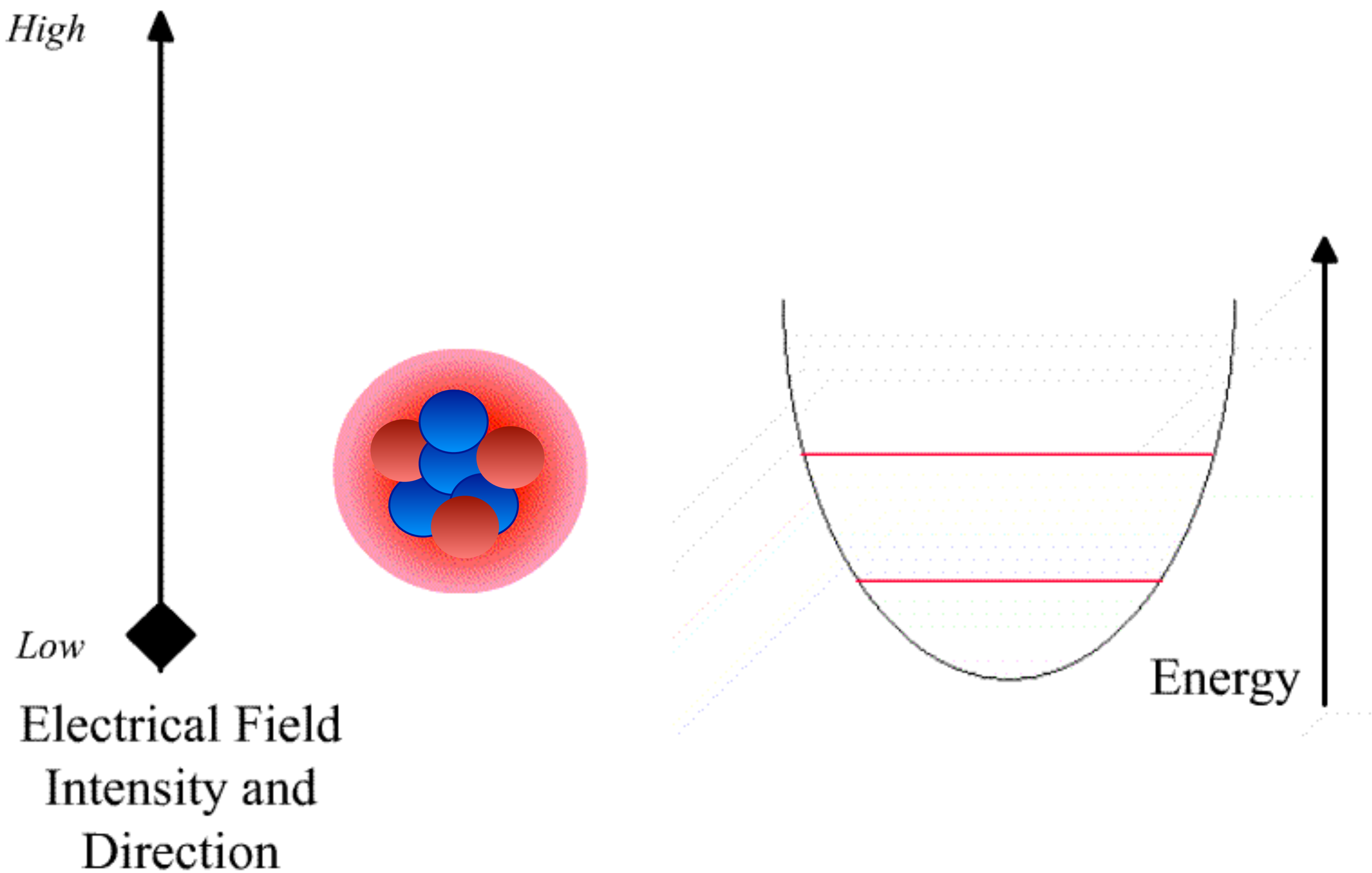

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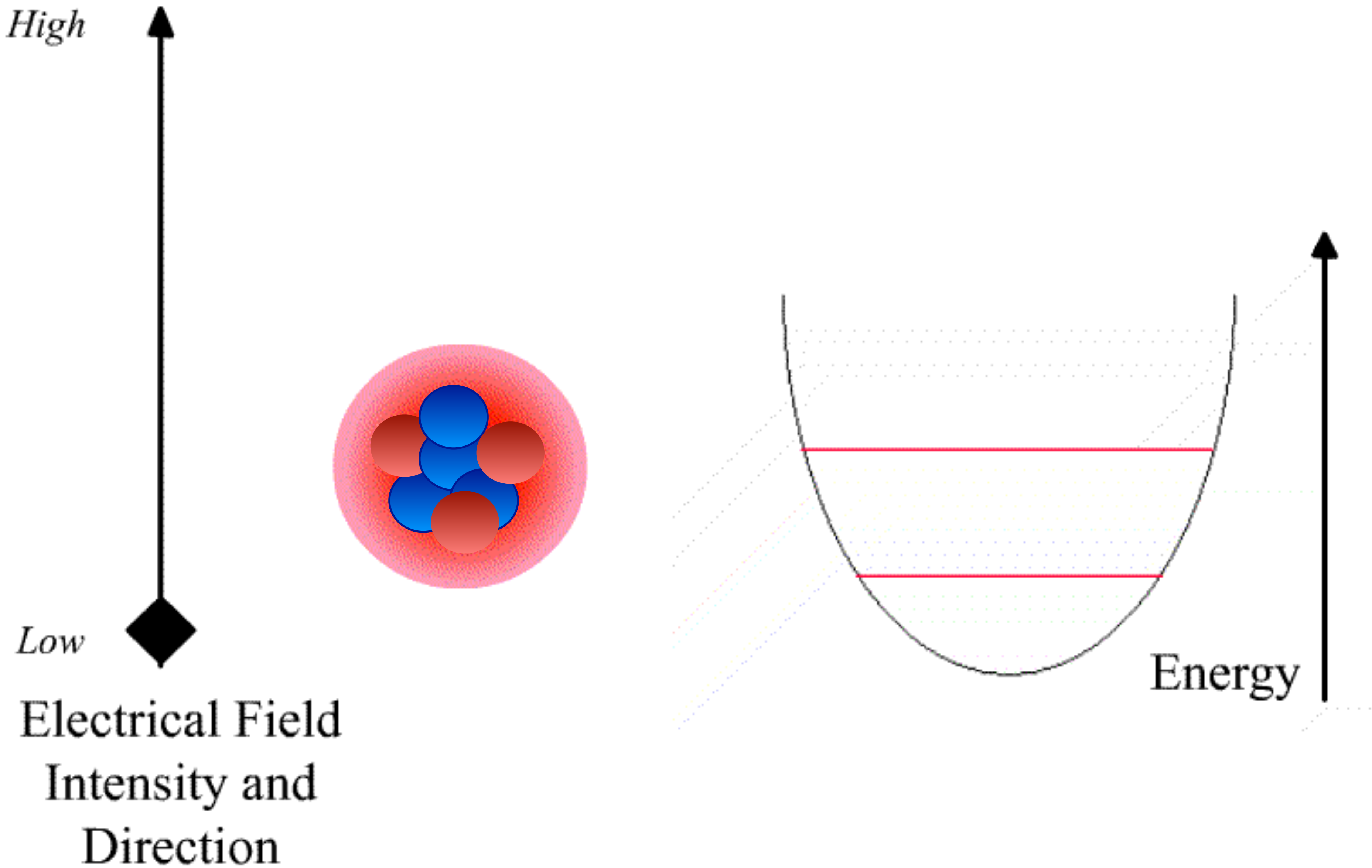

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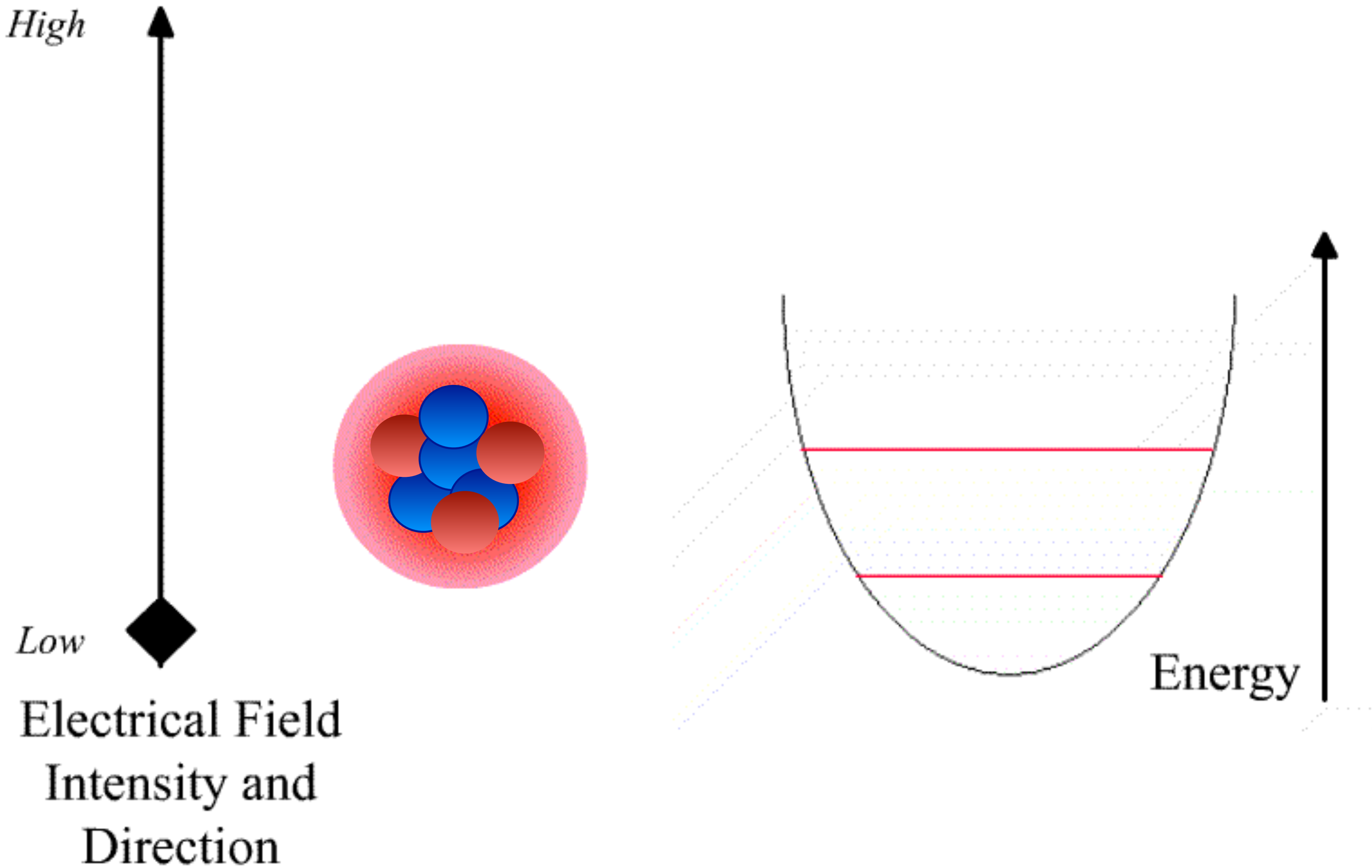

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- Nuclear model
- Isospin symmetry breaking
- Nucleon-size
- Truncation of multiples
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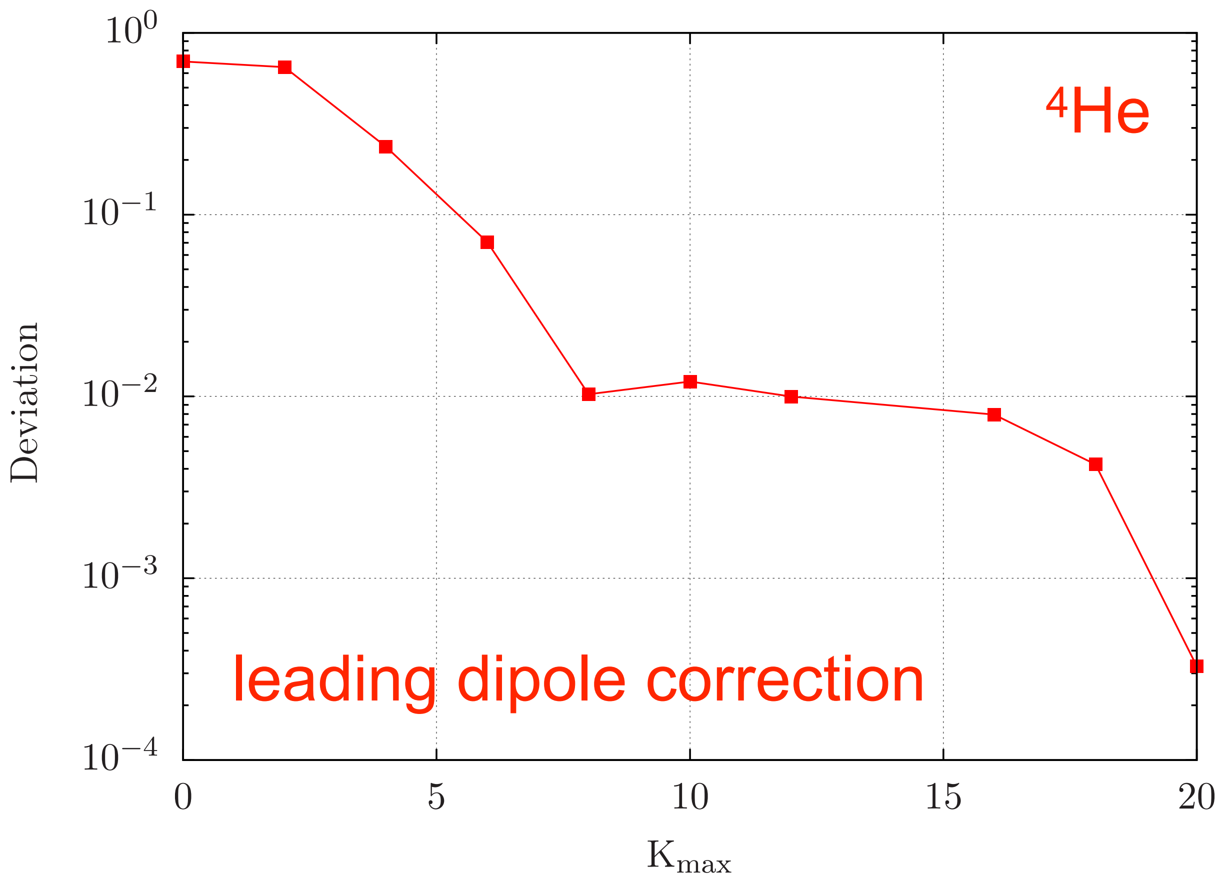
C.Ji et al., JPG: Part. Nucl. **45,** 093002 (2018)



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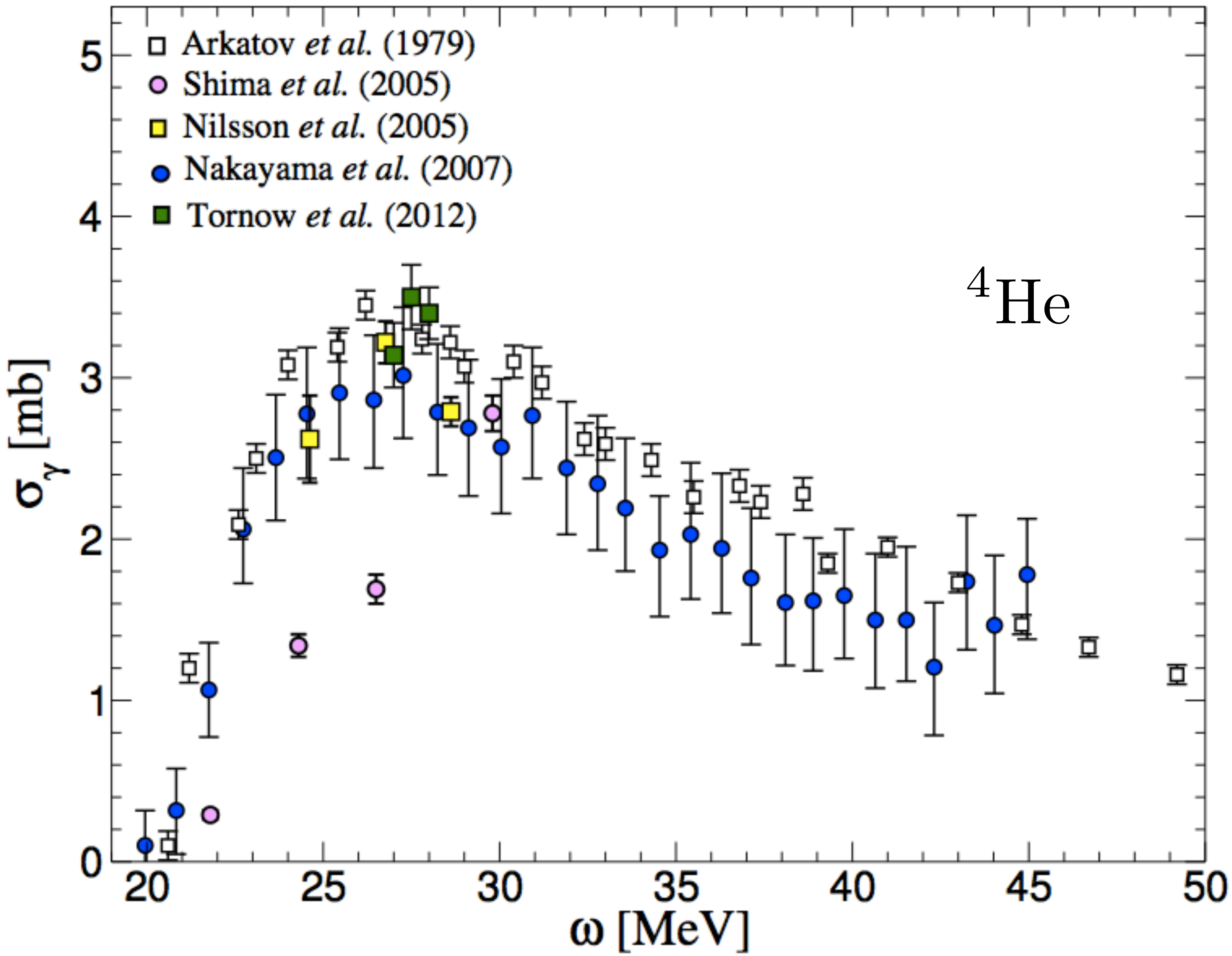
C.Ji et al., JPG: Part. Nucl. **45**, 093002 (2018)

	μ^2H			μ^3H			μ^3He$^+$			μ^4He$^+$		
	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$
Numerical	0.0	0.0	0.0	0.1	0.0	0.1	0.4	0.1	0.1	0.4	0.3	0.4
Nuclear-model	0.3	0.5	0.4	1.3	2.4	1.7	0.7	1.8	1.5	3.9	4.6	4.4
ISB	0.2	0.2	0.2	0.7	0.2	0.5	1.8	0.2	0.5	2.2	0.5	0.5
Nucleon-size	0.3	0.8	0.0	0.6	0.9	0.2	1.2	1.3	0.9	2.7	2.0	1.2
Relativistic	0.0	—	0.0	0.1	—	0.1	0.4	—	0.1	0.1	—	0.0
Coulomb	0.4	—	0.3	0.5	—	0.3	3.0	—	0.9	0.4	—	0.1
η-expansion	0.4	—	0.3	1.3	—	0.9	1.1	—	0.3	0.8	—	0.2
Higher $Z\alpha$	0.7	—	0.5	0.7	—	0.5	1.5	—	0.4	1.5	—	0.4
Total	1.0	0.9	0.8	2.3	2.2	2.0	4.2	2.2	2.1	5.5	5.1	4.6

An example



S.B. and Saori Pastore, Journal of Physics G.: Nucl. Part. Phys. **41**, 123002 (2014)

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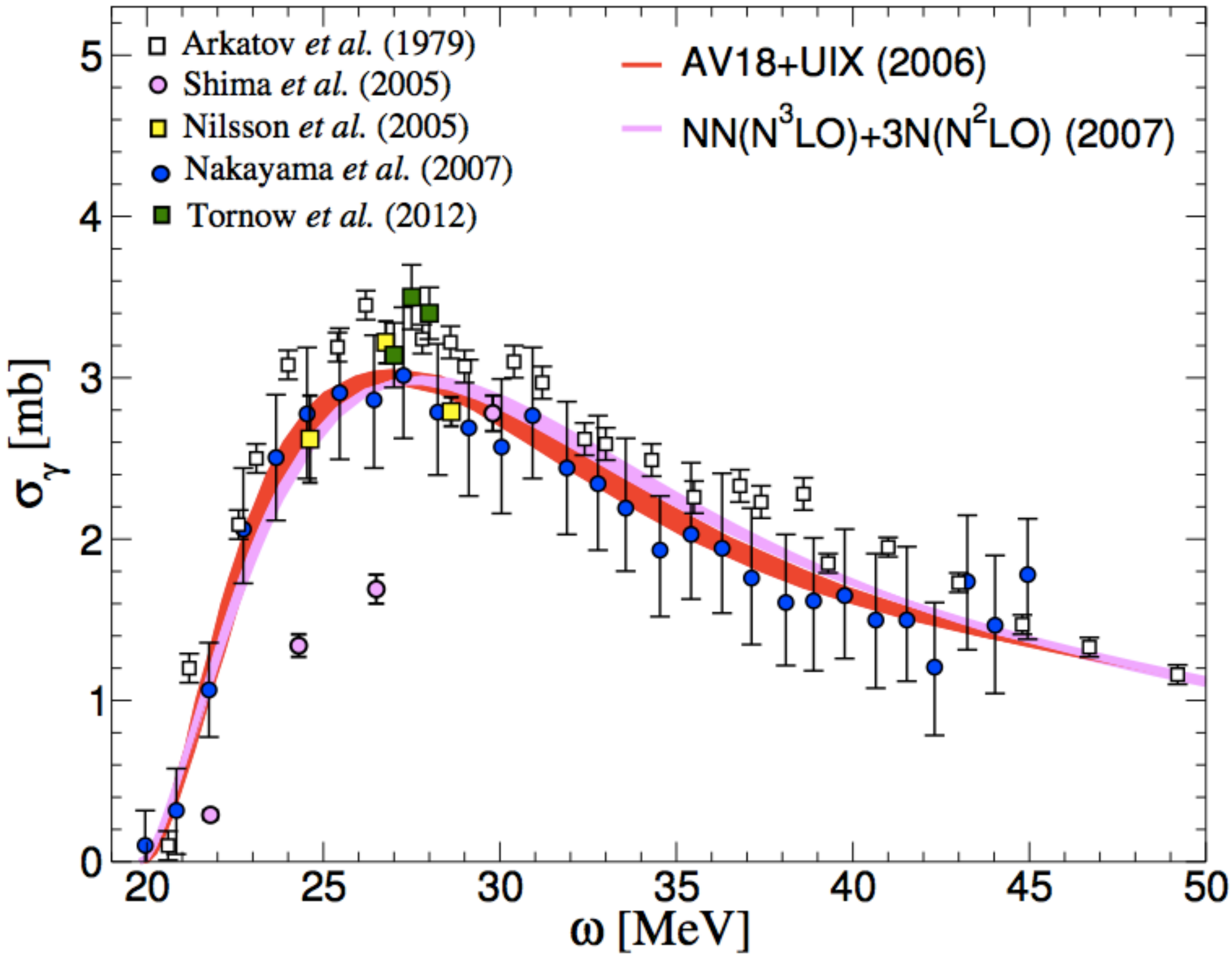

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