NNLO corrections in massive QED

Tim Engel[1,2]**, Pulak Banerjee**[1]**, Adrian Signer**[1,2]**, Yannick Ulrich**[1,2]

[1] Laboratory for Particle Physics, Paul Scherrer Institut, CH-5232 Villigen PSI, Switzerland
[2] Physik-Institut, Universität Zürich, Winterthurerstrasse 190, CH-8057 Zürich, Switzerland

Why massive?

Low-energy precision experiments with leptons such as MEG, Mu3e, MUSE and MUonE require a precise knowledge of the Standard Model background. For this reason we are developing a Monte Carlo code for muons and other leptons (McMule). This includes the fully differential NNLO QED corrections to $\mu \to e\nu\nu$, $\mu e \to \mu e$ and $\ell p \to \ell p$. At the level of the required precision electron mass effects cannot be neglected. On the technical side this entails advantages as well as disadvantages:

- no collinear singularties
  → simple subtraction scheme for numerical phase space integration (FKS$^\ell$)
- multi-scale loop integrals
  → solution: 'massification' for analytical loop integration

The FKS$^\ell$ subtraction scheme

We use the simple exponentiating structure of soft singularities in QED

$$\sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)} = e^{-\alpha\hat{\mathcal{E}}} \sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)f}$$

where $\mathcal{M}_n^{(\ell)f}$ are free from infrared (IR) poles and all singularities are contained in the integrated eikonal $\hat{\mathcal{E}}$. The subtraction is performed in terms of the distribution

$$\int \mathrm{d}\xi \left(\frac{1}{\xi}\right)_c f(\xi) \equiv \int_0^1 \mathrm{d}\xi \, \frac{f(\xi) - f(0)\theta(\xi_c - \xi)}{\xi}$$

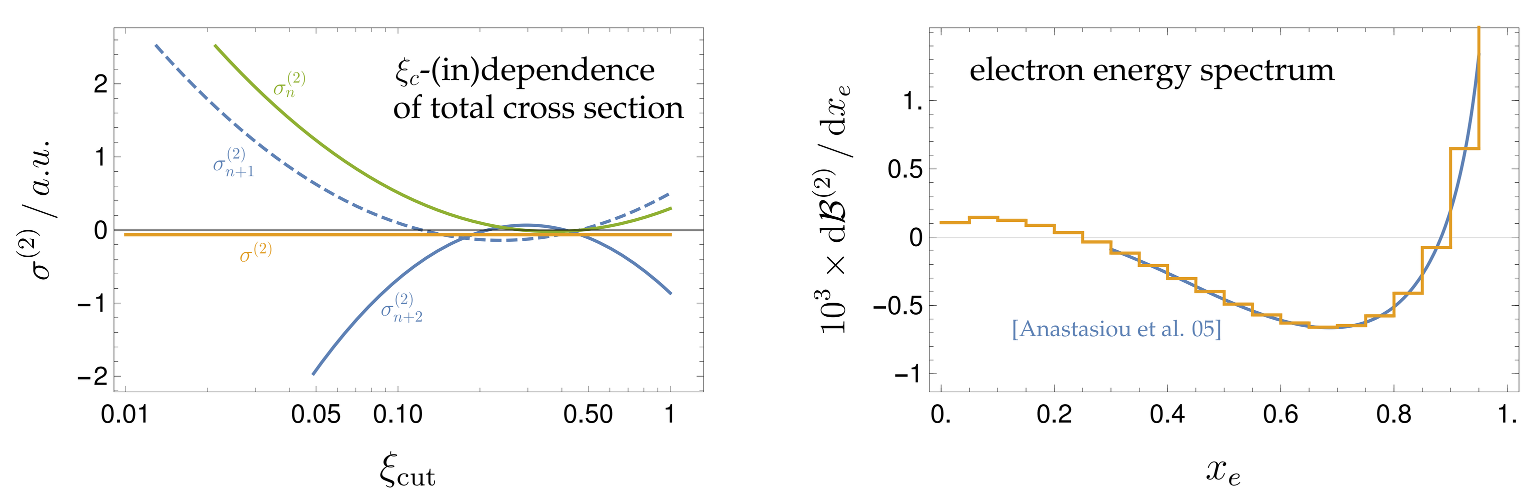
with the dimensionless photon energy ξ and the arbitrary cut parameter ξ_c.

Idea
$$\int \mathrm{d}\xi \, \mathcal{M}(\xi) = \underbrace{\int \mathrm{d}\xi \, \big(\mathcal{M}(\xi) - \mathcal{M}_{\rm CT}\big)}_{\text{complicated \& finite} \to \text{numerical}} + \underbrace{\int \mathrm{d}\xi \, \mathcal{M}_{\rm CT}}_{\text{divergent \& easy} \to \text{analytical}}$$

FKS (NLO)
$$\sigma^{(1)} = \sigma_n^{(1)}(\xi_c) + \sigma_{n+1}^{(1)}(\xi_c)$$
$$\sigma_n^{(1)}(\xi_c) = \int \mathrm{d}\Phi_n^{d=4} \left(\mathcal{M}_n^{(1)} + \hat{\mathcal{E}}(\xi_c)\,\mathcal{M}_n^{(0)}\right)$$
$$\sigma_{n+1}^{(1)}(\xi_c) = \int \mathrm{d}\Phi_{n+1}^{d=4} \left(\frac{1}{\xi_1}\right)_c \left(\xi_1 \, \mathcal{M}_{n+1}^{(0)f}\right)$$

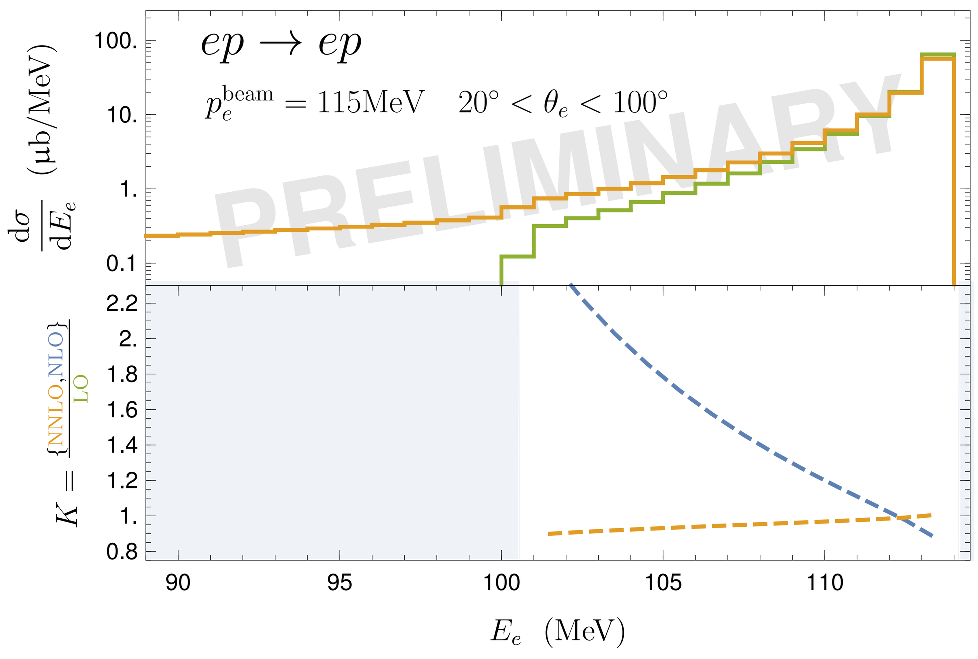
FKS^2 (NNLO)
$$\sigma^{(2)} = \sigma_n^{(2)}(\xi_c) + \sigma_{n+1}^{(2)}(\xi_c) + \sigma_{n+2}^{(2)}(\xi_c)$$
$$\sigma_n^{(2)}(\xi_c) = \int \mathrm{d}\Phi_n^{d=4} \left(\mathcal{M}_n^{(2)} + \hat{\mathcal{E}}(\xi_c)\,\mathcal{M}_n^{(1)} + \frac{1}{2!}\mathcal{M}_n^{(0)}\hat{\mathcal{E}}(\xi_c)^2\right)$$
$$\sigma_{n+1}^{(2)}(\xi_c) = \int \mathrm{d}\Phi_{n+1}^{d=4} \left(\frac{1}{\xi}\right)_c \left(\xi \, \mathcal{M}_{n+1}^{(1)f}(\xi_c)\right)$$
$$\sigma_{n+2}^{(2)}(\xi_c) = \int \mathrm{d}\Phi_{n+2}^{d=4} \left(\frac{1}{\xi_1}\right)_c \left(\frac{1}{\xi_2}\right)_c \left(\xi_1 \xi_2 \, \mathcal{M}_{n+2}^{(0)f}\right)$$

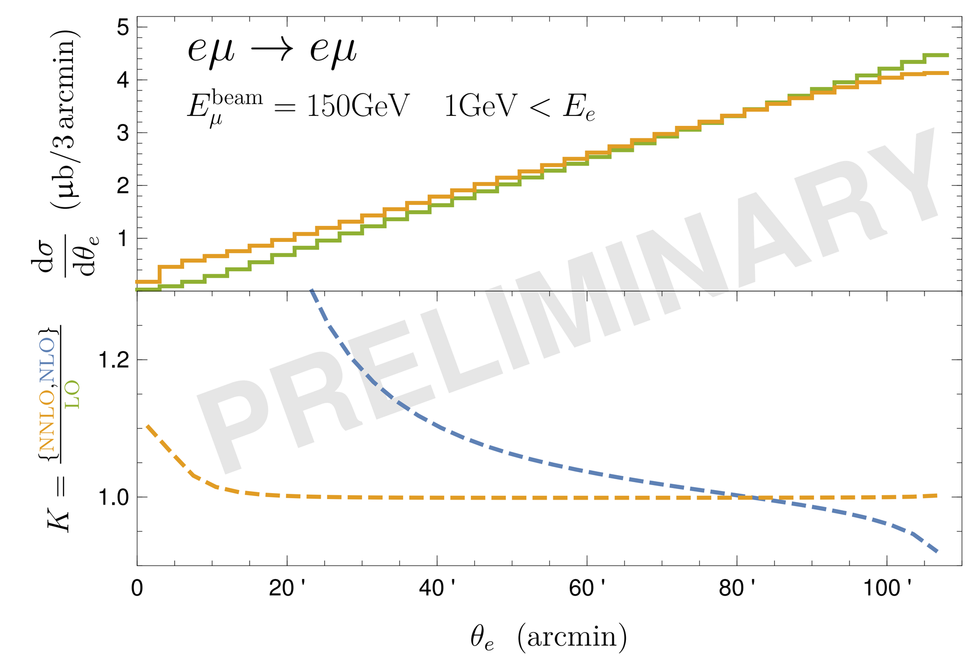
Proof of concept: muon decay @ NNLO



Results

After the successful application of FKS^2 to the muon decay, we are using the developed methodology for the other processes of interest. We show preliminary results for the dominant emission-from-e-line-only for $ep \to ep$ (MUSE) and $e\mu \to e\mu$ (MUonE).



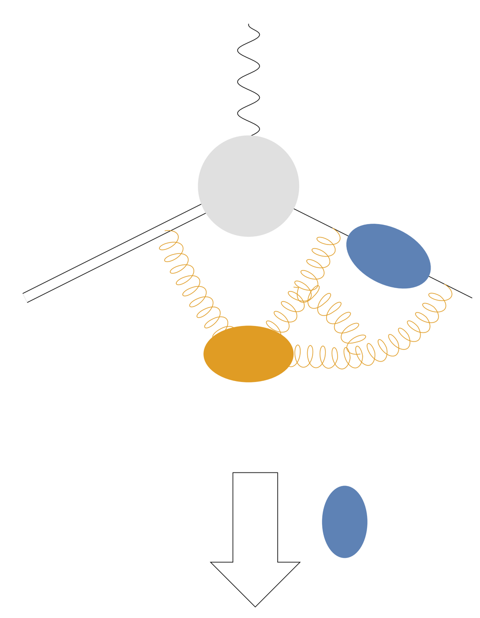


Outlook

As a next step we will take mixed emissions from the p/μ and e line into account. Contrary to the emission-from-e-line-only (above results), the loop integration with the full electron-mass dependence is not feasible. However, since the electron mass m is small, it is sufficient to determine the logarithmically enhanced terms $\sim \log m$. These correspond to regularized collinear divergences. We can therefore exploit the universal structure of IR singularities with the 'massification' of the massless amplitudes:

Simple process ($\mu \to e\nu\nu$):

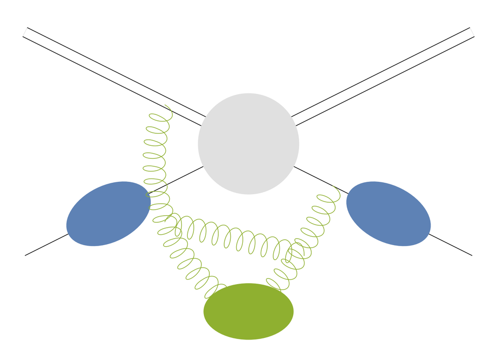
$\mathcal{A}_\mu(m) = \mathcal{S} \times Z \times \mathcal{A}_\mu(0) + \mathcal{O}(m \log m)$
$Z \supset \log m$: process indep. jet fct.
$\mathcal{S} \supset \log m$: process dep. soft fct. (easy)



Z

Complex process ($\mu e \to \mu e$):

$\mathcal{A}_{\mu e}(m) = \mathcal{S}' \times Z \times Z \times \mathcal{A}_{\mu e}(0) + \mathcal{O}(m \log m)$



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Contact: tim.engel@psi.ch