Monte Carlo Studies of μp diffusion for the hyperfine-splitting experiment at PSI

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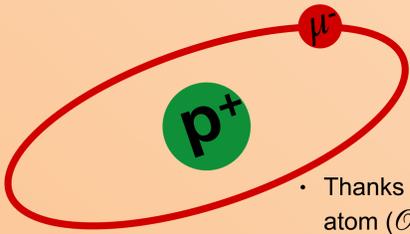
Motivation for the experiment

- Thanks to the high muon mass, μp is a strongly bound atom ($\mathcal{O}$(keV) binding energies)

- The muon is much closer to the proton than the electron in electronic hydrogen

$\Rightarrow$ **The proton structure has a strong impact on the energy levels of muonic hydrogen**

$1s_{1/2}$ F=1 F=0
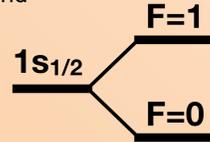
- Aim: Measurement of the ground-state hyperfine splitting in muonic hydrogen

- The experiment is sensitive to higher order corrections of the hyperfine splitting:

$$E_{HFS} = \left(1 + \Delta_{structure} + \Delta_{weak} + \Delta_{QED} \right) \cdot E_F$$

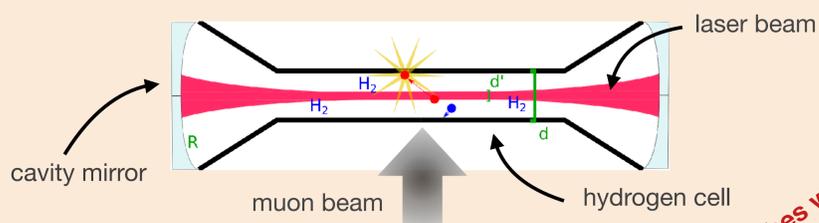- **Learn about electro-magnetic structure of the proton**
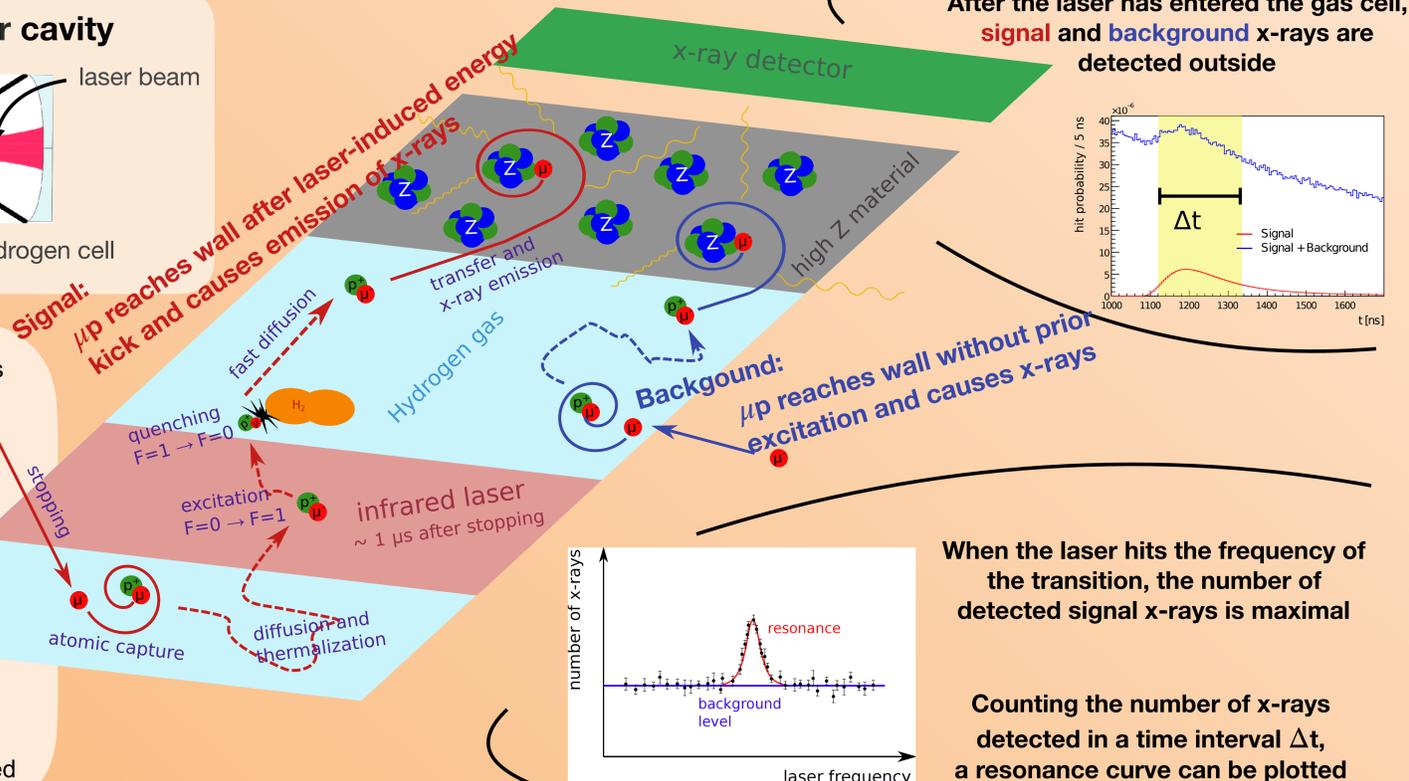- **Extract Zemach radius of the proton**

- **test bound-state QED**

$$R_Z = \frac{4}{\pi} \int_0^\infty \frac{dQ}{Q^2} \left(1 - \frac{G_E(Q^2)G_m(Q^2)}{1+\kappa} \right)$$

Principle of the measurement

Cross section of the target with laser cavity

laser beam, cavity mirror, muon beam, hydrogen cell

- After stopping in the H_2 gas, the muons form μp atoms

- In the first 1μs after formation, the μp atom diffuses and thermalises in the H_2 gas (T ~ 30-50 K)

- **After ~1 μs: Laser spreads over diffusion volume**

  - Laser excitation of μp to upper hyperfine level (F=1)

  - Direct de-excitation to F=0 in collision μp with a H_2 molecule

    $\Rightarrow$ **energy kick of ~0.1 eV**

- If a μp approaches a high-Z nucleus in the wall of the gas cell, the muon is transferred and x-rays are emitted

Signal: μp reaches wall after laser-induced energy kick and causes emission of x-rays

Background: μp reaches wall without prior excitation and causes x-rays

x-ray detector, high Z material, Hydrogen gas, infrared laser ~ 1 μs after stopping, quenching F=1 → F=0, excitation F=0 → F=1, atomic capture, diffusion and thermalization, fast diffusion, transfer and x-ray emission, stopping

After the laser has entered the gas cell, **signal** and **background** x-rays are detected outside

When the laser hits the frequency of the transition, the number of detected signal x-rays is maximal

Counting the number of x-rays detected in a time interval Δt, a resonance curve can be plotted
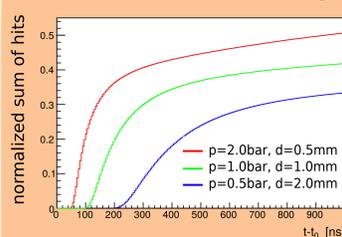

MC diffusion simulations

Description of the project:

- Implementation of molecular scattering processes[1] in Geant4

  $$\mu p (F) + H_2 \longrightarrow \mu p (F') + H_2$$

- Simulations are used to design the target and to predict the significance level reached by the measurement

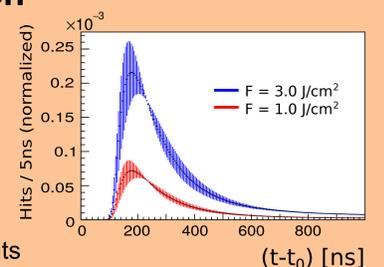### Pre-laser diffusion and thermalization

- After formation, μp atoms thermalise and are quenched to the F=0 state in molecular collisions

- Energies below ~0.2eV are too low for collisional excitation of upper hyperfine state (first kink: all μp in F=0)

spatial distribution of μp after 1 μs for target of length d = 1.0 mm
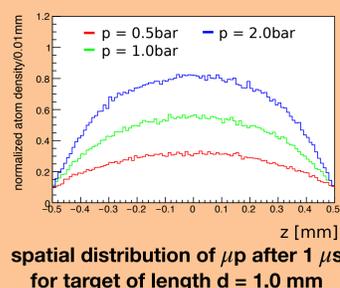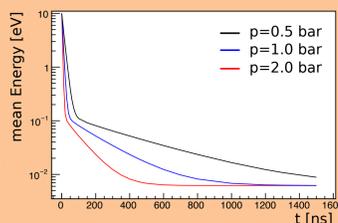
- Some μp atoms hit the wall during thermalisation and are lost for the further measurement

- For higher gas pressure, the μp distribution in the target after 1μs is strongly peaked in the central region (which is the main laser region)

Simulation of Signal hits after laser excitation

Cumulated probability for laser-excited μp atoms to hit wall
- steep slope means better Signal-to-background ratio -

- Simulations with laser-excited μp atoms help to understand the influence of various parameters on the signal-to-background ratio

- E.g., maximising the laser fluence F is crucial to obtain a sufficient number of laser-induced **signal** events

$$F = \int_{t_0}^\infty I(t) \, dt \sim \frac{E_{laser}}{1-R}$$

Probability to have a signal μp hit the wall in time intervals of 5ns

Combined simulations of Signal and Background

- A combination of **signal** and **background** simulations is used to optimise the target conditions

- As figure of merit, we use the statistical significance η per muon entering the target (assuming the laser is on resonance)

$$\eta = \frac{S}{\sqrt{S+B}}, \quad S, B \equiv \text{rates of laser-induced / background events within integration time window of length } \Delta t$$

- Some implications:
  - Short targets (small d) with high H_2 pressure are favourable
  - Optimal time window of integration: $\Delta t \, \epsilon \, [200,300]$ ns
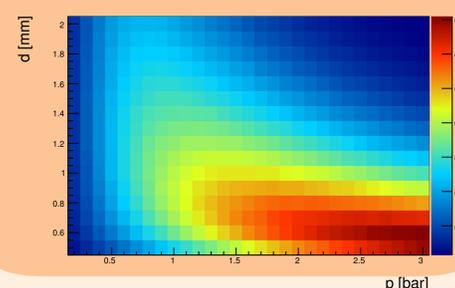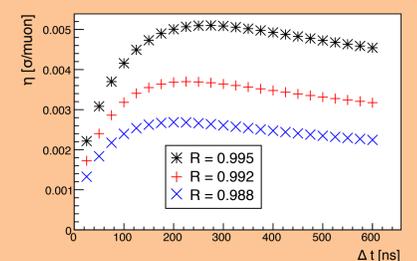
[1] Differential cross sections calculated by A. Adamczak, cf. A. Adamczak, Phys. Rev. A 74, 042718 (2006)