Dispersive treatment of the radiative corrections to the pion vector form factor

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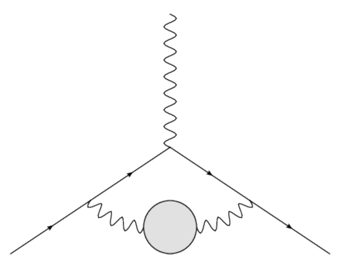
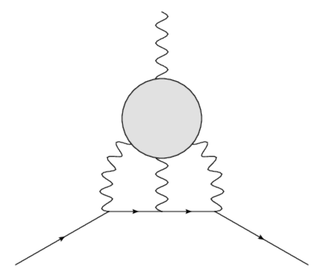
1. Introduction

Currently, there is a discrepancy of about 3.5σ between the Standard Model prediction of the anomalous magnetic moment of the muon $(g-2)_\mu$ and its experimental measurement. On the theoretical side, most of the uncertainty comes from the hadronic sector, namely from the hadronic vacuum polarization (HVP) and hadronic light-by-light (HLbL) contributions.



HVP contribution to $(g-2)_\mu$



HLbL contribution to $(g-2)_\mu$

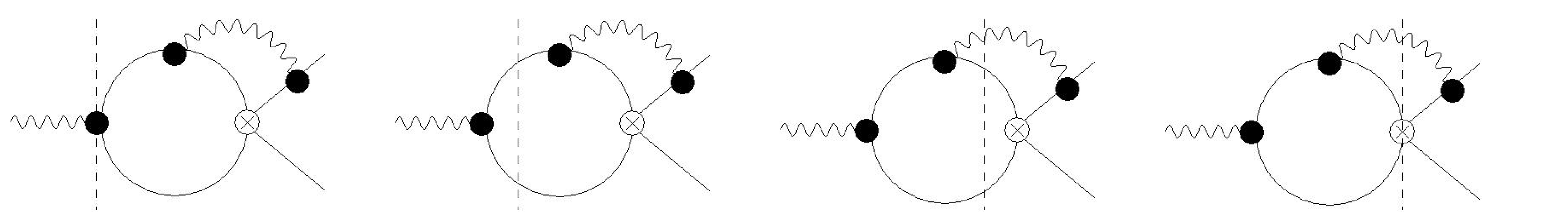
At the present level of uncertainty, the order α radiative corrections to the pion vector form factor become relevant in the HVP contribution to $(g-2)_\mu$. The aim of this project is to improve our understanding of these corrections in a model-independent way, based on unitarity and dispersion relations.

2. Dispersive method

The pion vector form factor can be calculated as a once-subtracted dispersion relation

$$F_v^\pi(s) \quad = \quad 1+\frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\mathrm{Im}F_v^\pi(s')}{s'(s'-s)}.$$

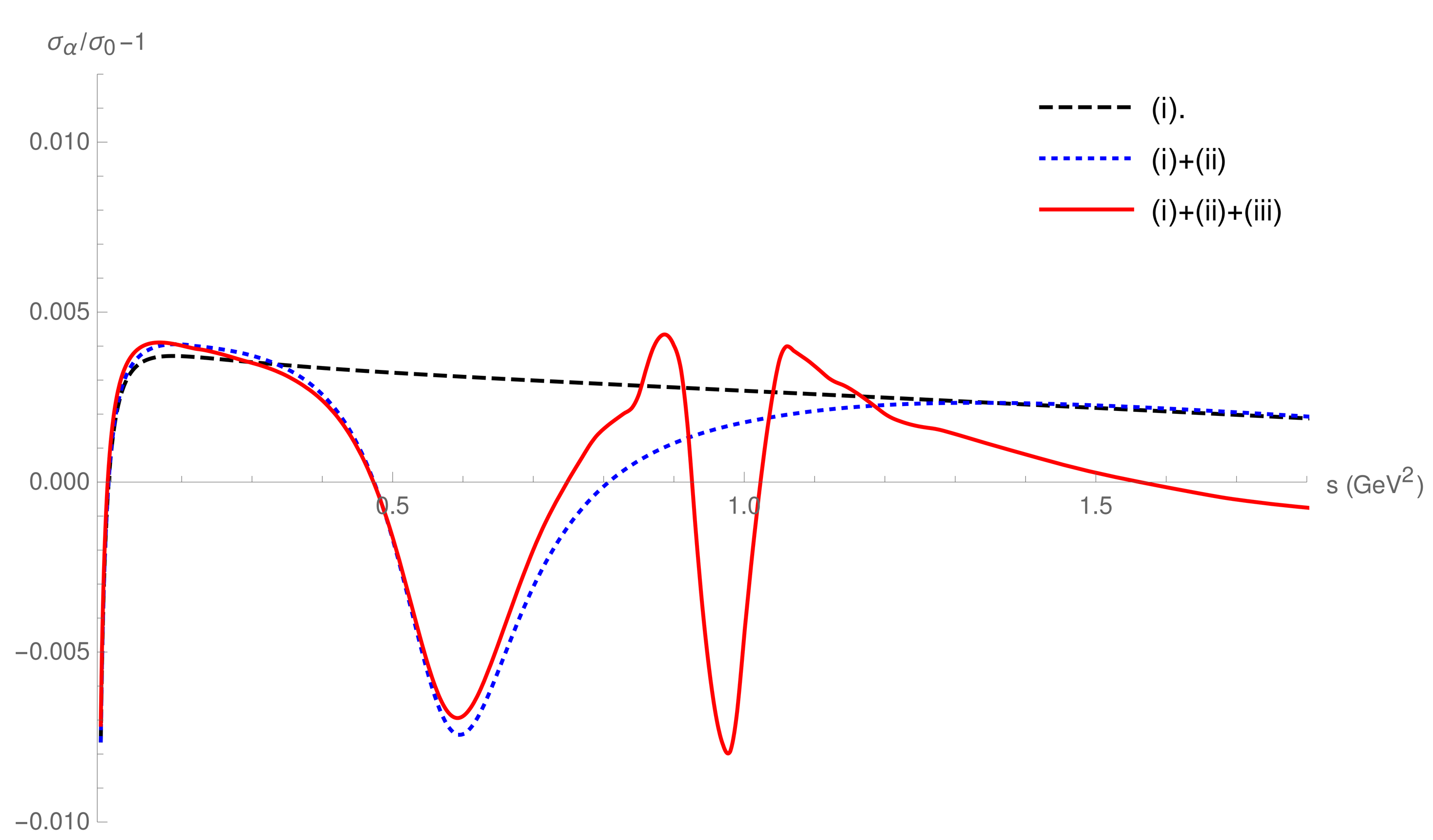
Starting from the unitarity relation, we calculate the contributions to the imaginary part by cutting the relevant diagrams displayed in 2.1. in all possible ways. We restrict ourselves to topologies with at most two pions (and possibly a photon) in the intermediate state. Taking the case of topology (iii) as an illustration, there are four cuts:



In practice, those cuts are related to phase-space integrals that lead to IR divergences. In fact, only topology (i) carries an IR-divergence that is cancelled by the soft-photon emission in the total inclusive cross-section. The others are IR-safe quantities, even though the contribution from the different cuts are separately divergent and need to be regularized.

3. Preliminary results

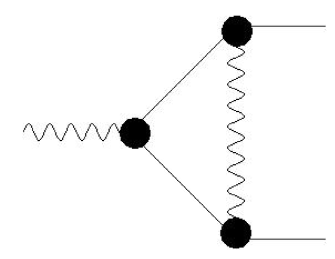
So far, we have calculated the contribution from topologies (i) and (ii), as well as the S- and P-wave contributions to (iii).
This plot describes the relative correction to the total inclusive cross-section $\sigma\left(e^-e^+\to\pi^+\pi^-(\gamma)\right)$. The correction is of the order of half a percent, which is small. One can clearly see the influence of the $\rho(770)$ and $f_0(980)$ resonances due to $\pi\pi$ rescattering effect. These structures may have a sizeable influence on the data.



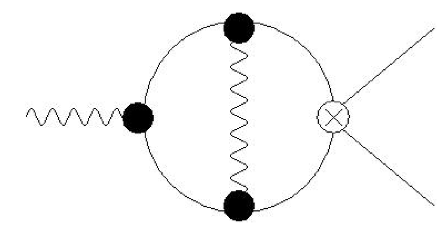
2.1. Topologies

The different $\mathcal{O}(\alpha)$ corrections to the process $\gamma^*\to\pi^-\pi^+$ are the following:

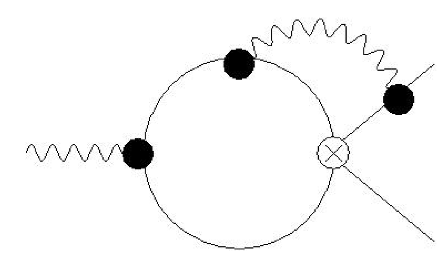
(i)



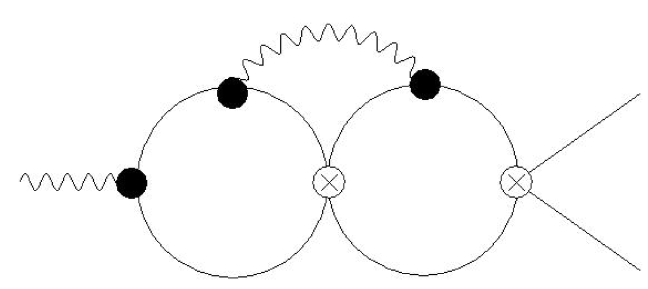
(ii)



(iii)



(iv)



Any further topology is already contained in that set, since the blobs account for all hadronic processes.

2.2. Subamplitudes

It can happen that a subamplitude present in the unitarity relation is an a priori unknown loop-diagram. In this case, we must apply the method on the subamplitude itself.

This process goes on until all subamplitudes consist exclusively of tree-diagrams. Those are either purely hadronic quantities, or pion-pole contributions to the corresponding process.

2.3. Hadronic Input

The vertices in the diagrams above are non-local and thus dressed with hadronic blobs. The black ones represent the purely hadronic pion vector form factor and the crossed ones the $\pi\pi$ scattering amplitudes in the isospin limit.

These hadronic quantities are taken as input and the latter is expanded in partial waves,

$$A^I(s,t)=32\pi\sum_l(2l+1)P_l\left(1+\tfrac{2t}{s-4m_\pi^2}\right)t_l^I(s),$$

where the series is dominated by the lowest ones at low energy.

For energies below $\sqrt{s}\;=\;1.42$ GeV, parametrizations of the partial waves coming from Roy equation analysis can be used. For higher energies, we rely on a Regge description.