UNIVERSITÄT BERN

UNIVERSITÄT BERN
AEC ALBERT EINSTEIN CENTER FOR FUNDAMENTAL PHYSICS

Pseudoscalar Contribution to the Muon g-2

Franziska Hagelstein (AEC Bern)

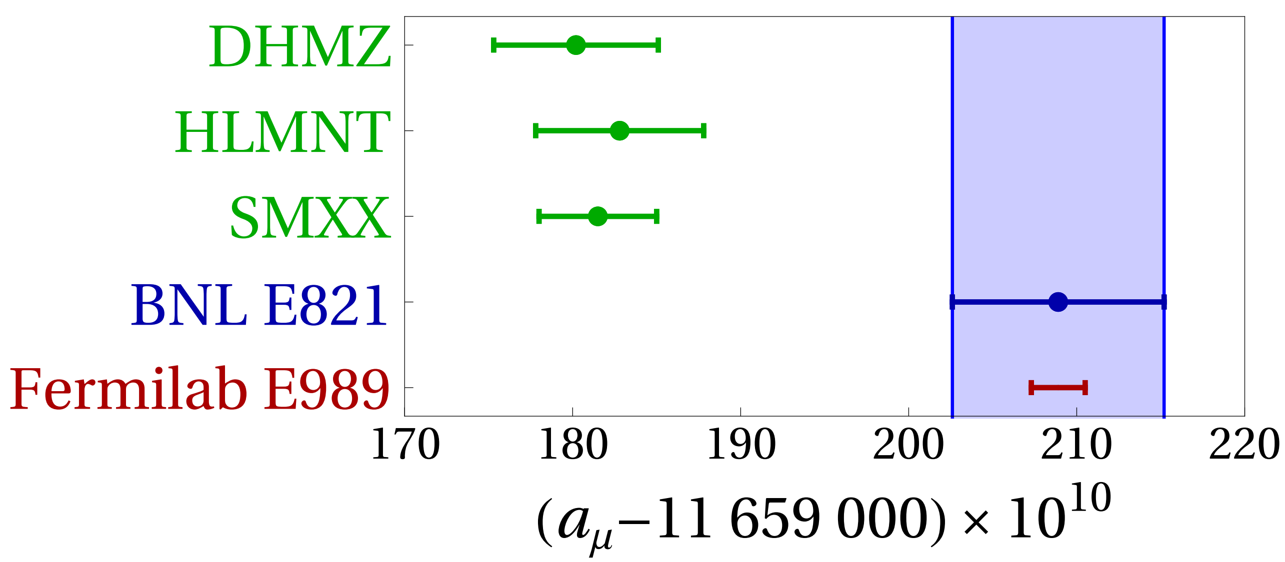
in collaboration with

G. Colangelo (AEC - Bern), M. Hoferichter (INT - Seattle),
L. Laub (AEC - Bern) and P. Stoffer (University of California - San Diego)

MUON ANOMALIE g-2

- Low-energy observables measured to high precision provide stringent tests of the Standard Model (SM) of particle physics.

- 3 to 4 σ discrepancy between the SM prediction and the experimental value of the muon anomalous magnetic moment $a_\mu = (g-2)_\mu/2$ from BNL E821



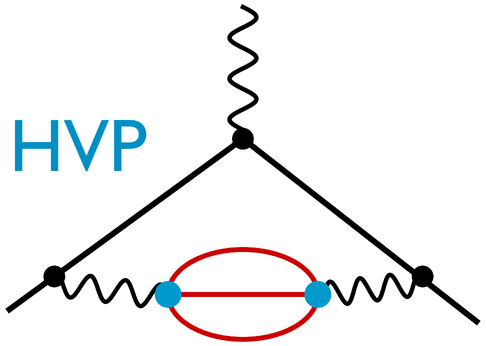
$$a_\mu^{\text{exp.}} = [11\,659\,209.1 \pm 6.3] \times 10^{-10}$$
$$a_\mu^{\text{th.}} = [11\,659\,178.3 \pm 4.3] \times 10^{-10}$$

- Fermilab E989 experiment is expected to reduce the experimental uncertainty by a factor of 4 (talk by P. Winter) & J-PARC experiment will provide independent cross check

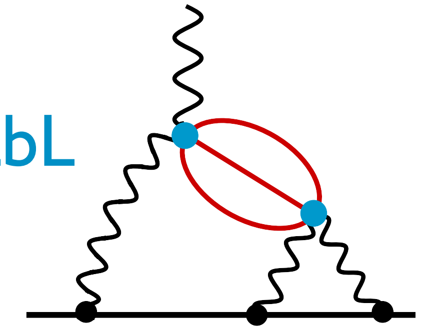
- Uncertainty of the SM prediction is dominated by hadronic corrections

HADRONIC CORRECTIONS

HVP



HLbL



$$a_\mu^{\text{LO HVP}} = [689.46 \pm 3.25] \times 10^{-10}$$

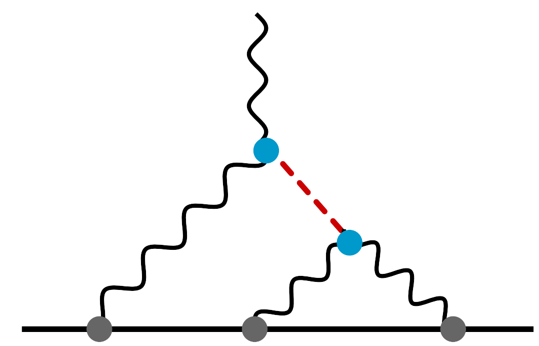
$$a_\mu^{\text{HLbL}} = [10.34 \pm 2.88] \times 10^{-10}$$

- Leading uncertainty presently comes from hadronic vacuum polarization (HVP)
- Soon hadronic light-by-light scattering (HLbL) will be leading uncertainty

pseudoscalar-pole contribution:

$$a_\mu^{\pi^0\text{–pole}} = 62.6^{+3.0}_{-2.5} \times 10^{-11}$$

M. Hoferichter et al., PRL 121, 112002 (2018)



- QCD is non-perturbative at low energies, therefore we use dispersion relations, lattice QCD and effective field theories
- Short-distance constraints (SDCs) are important for a model-independent approach towards hadronic corrections, because mixed- and high-energy regions cannot be constrained from data

DATA-DRIVEN DISPERSIVE APPROACH TO HVP

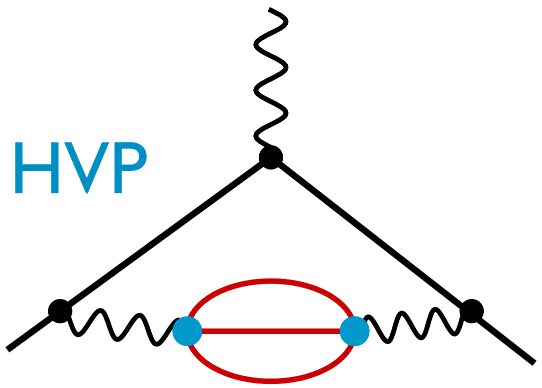
- HVP is calculated with a systematic data-driven dispersive approach:

$$a^{\mathrm{HVP}} = \frac{\alpha^2}{3\pi^2}\int_{m_\pi^2}^{\infty} \mathrm{d}s\, \frac{R_\gamma^{\mathrm{had}}(s)\, K(s/m^2)}{s}$$

$$K(s/m^2) = \int_0^1 \mathrm{d}x\, \frac{x^2(1-x)}{x^2 + (1-x)\, s/m^2}$$

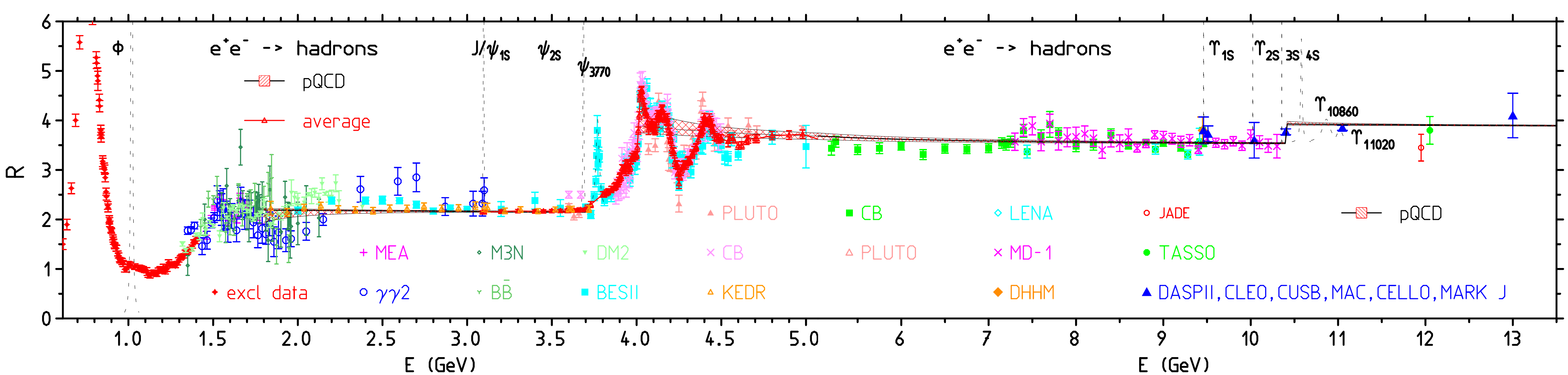
$$R_\gamma^{\mathrm{had}}(s) = \frac{\sigma(e^+e^- \to \gamma^* \to \text{hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)}$$

HVP



F. Jegerlehner, Springer Tracts Mod. Phys. 274 (2017)

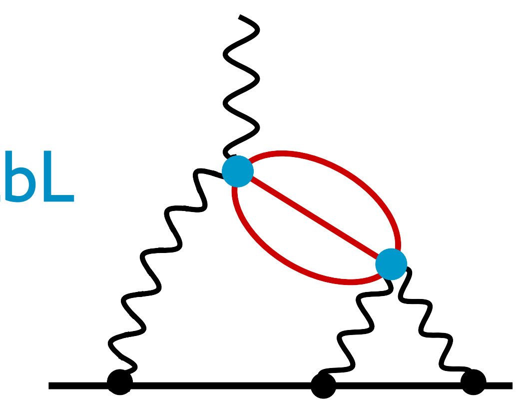
M. Davier, Nucl. Part. Phys. Proc. 287-288, 70 (2017)



Poster: "Dispersive treatment of the radiative corrections to the pion vector form factor" (J. Monnard, AEC Bern)

- We also want a model-independent dispersive approach to study HLbL!

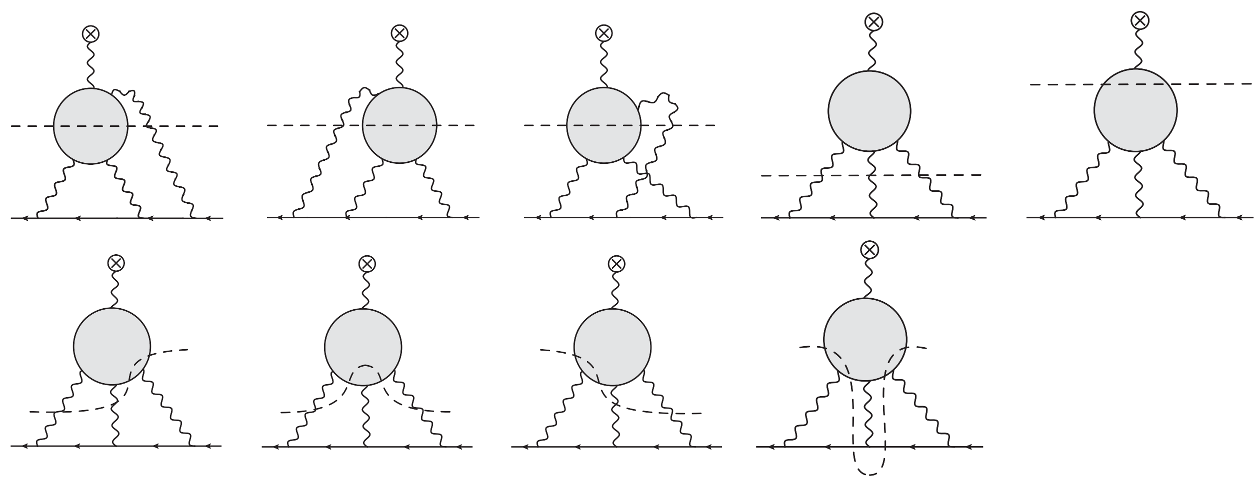
HLbL



DISPERSIVE APPROACH TO HLBL

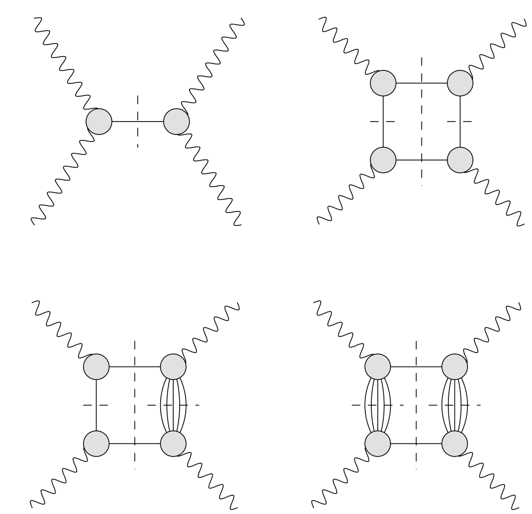
- HLbL: no analogue of the simple dispersive formula

dispersive formula for the e.m. vertex function:



V. Pauk and M. Vanderhaeghen, Phys. Rev. D 90, 113012 (2014)

dispersive formula for the light-by-light scattering amplitude:

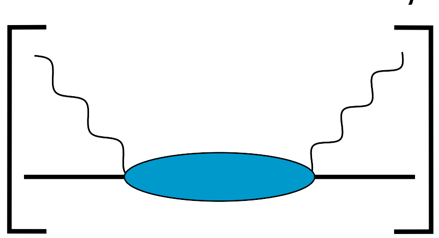


G. Colangelo, et al., JHEP 1509 (2015) 074

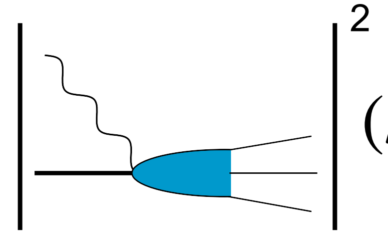
Schwinger sum rule (a dispersive formula for Compton scattering):

$$a_\mu = \frac{m^2}{\pi^2 \alpha} \int_{\nu_0}^{\infty} \mathrm{d}\nu \left[\frac{\sigma_{LT}(\nu, Q^2)}{Q} \right]_{Q^2=0}$$

CS amplitude, a_μ



$$(\nu, Q^2) = \frac{2}{\pi} \int_{\nu_0}^{\infty} \mathrm{d}\nu' \frac{\nu'}{\nu'^2 - \nu^2}$$



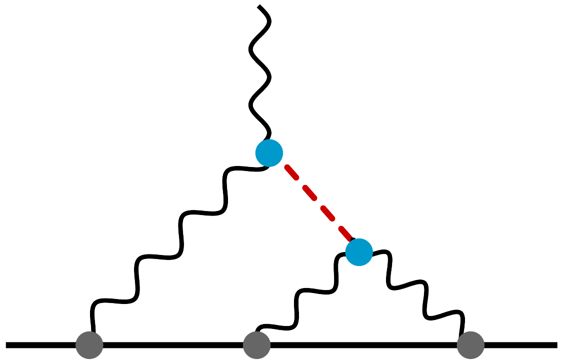
$$(\nu', Q^2)$$

Cross sections, structure functions

FH and V. Pascalutsa, PRL 120 (2018) 072002 and 1907.06927 (2019)

PSEUDOSCALAR-POLE CONTRIBUTION

Reference	π^0-pole	η-pole	η'-pole	PS-pole
Knecht & Nyffeler	5.8(1.0)	1.3(0.1)	1.2(0.1)	8.3(1.2)
Melnikov & Vainshtein	7.65	1.8	1.8	11.4(1.0)
Masjuan & Sanchez-Puertas	$6.30 \div 6.41$	$1.62 \div 1.63$	$1.43 \div 1.47$	9.43(0.53)



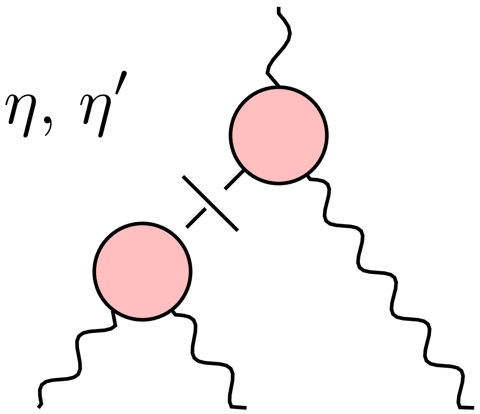
- Pseudoscalar-pole (in particular Pion-pole) contributions are the leading HLbL contributions
- Mixed- and high-energy regions need to be estimated for a full evaluation
- Issue: pseudoscalar-pole contribution does not have the asymptotic behaviour dictated by QCD
- Effective solution proposed by Melnikov & Vainshtein is incompatible with low-energy properties of the HLbL tensor
  K. Melnikov and A. Vainshtein, Phys. Rev. D 70, 113006 (2004)
- SDCs can be satisfied with a summation over an infinite tower of pseudoscalar poles

SDC FOR MIXED- AND HIGH ENERGIES

- Relevant part of the HLbL tensor:

$$\Pi_1^{\mathrm{P-pole}} = -\frac{F_{P\gamma^*\gamma^*}(-Q_1^2, -Q_2^2)\, F_{P\gamma\gamma^*}(-Q_3^2)}{Q_3^2 + M_P^2}$$

G. Colangelo, et al., JHEP 1704 (2017) 161

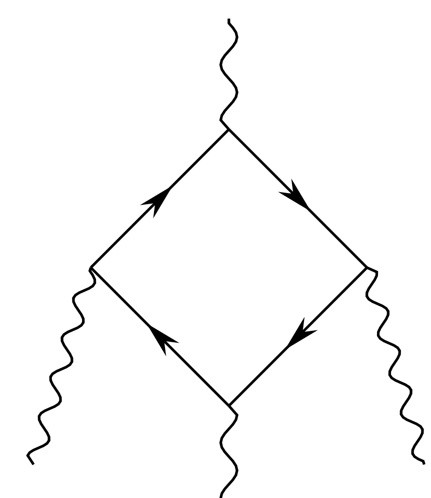


- SDCs for asymptotic $(Q^2 \equiv Q_1^2 \approx Q_2^2 \approx Q_3^2 \gg \Lambda_{\mathrm{QCD}}^2)$ and mixed energy region $(Q^2 \equiv Q_1^2 \approx Q_2^2 \gg Q_3^2)$ (Melnikov & Vainshtein '04) follow from the operator product expansion (OPE):

$$\lim_{Q\to\infty} \sum_{n=0}^{\infty} \hat{\Pi}_1^{\pi(n)-\mathrm{pole}}(Q^2, Q^2, Q^2) = -\frac{1}{9\pi^2}\frac{1}{Q^4}$$

$$\lim_{Q_3\to\infty} \lim_{Q\to\infty} \sum_{n=0}^{\infty} \hat{\Pi}_1^{\pi(n)-\mathrm{pole}}(Q^2, Q^2, Q_3^2) = -\frac{1}{6\pi^2}\frac{1}{Q^2 Q_3^2}$$

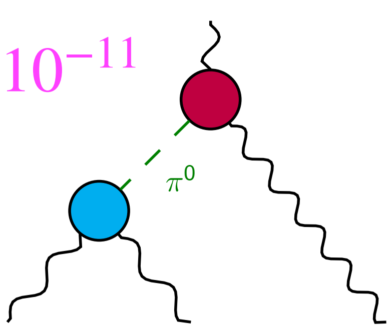
- Leading term in the OPE for HLbL corresponds to the perturbative quark loop Bijnens et al., 1908.03331 (2019)



SDC FOR TRANSITION FORM FACTOR

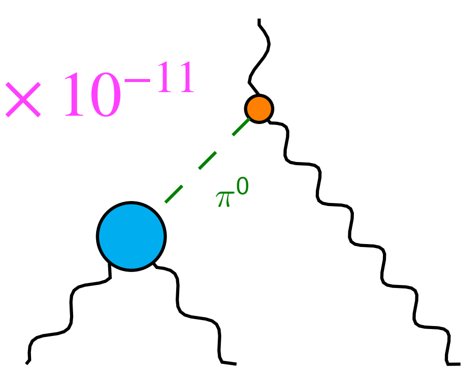
- SDCs for pseudoscalar transition form factor
  - Chiral Anomaly: $F_{\pi^0\gamma\gamma}(0,0) = -\frac{1}{4\pi^2 f_\pi}$
  - Brodsky-Lepage limit: $\lim_{Q^2\to\infty} F_{\pi^0\gamma\gamma^*}(Q^2) = -\frac{2f_\pi}{Q^2}$
  - Symmetric pQCD limit: $\lim_{Q^2\to\infty} F_{\pi^0\gamma^*\gamma^*}(Q^2,Q^2) = -\frac{2f_\pi}{3Q^2}$
- Melnikov & Vainshtein replaced the external photon vertex with the transition form factor at real-photon point (dropped Q^2 dependence)
  - Prescription is incompatible with low-energy properties of the HLbL tensor

$a_\mu^{\pi^0-\text{pole}} = 62.6\times 10^{-11}$



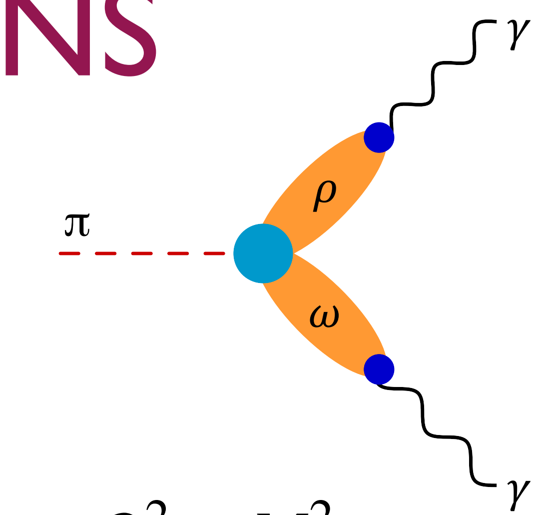
$$-\frac{\mathcal{F}_{\pi^0\gamma^*\gamma^*}(Q_1^2,Q_2^2)\mathcal{F}_{\pi^0\gamma\gamma^*}(Q_3^2)}{Q_3^2+m_{\pi^0}^2} \longrightarrow -\frac{2f_\pi}{3Q^2}\,\frac{1}{Q_3^2}\,\frac{2f_\pi}{Q_3^2}$$

$a_\mu^{\pi^0-\text{pole}} = 76.5\times 10^{-11}$



$$\overline{\Pi}_1^{\text{OPE},(3)} \longrightarrow -\frac{1}{6\pi^2 Q^2 Q_3^2} = -\frac{2f_\pi}{3Q^2}\,\frac{1}{Q_3^2}\,\frac{1}{4\pi^2 f_\pi}$$

INFINITE TOWERS OF MESONS



- Start from a large-N$_c$ Regge model:

Broniowski and Ruiz Arriola, Phys. Rev. D74, 034008 (2006)

$$F_{\pi^0\gamma^*\gamma^*}(-Q_1^2,-Q_2^2) \propto \sum_{V_\rho,V_\omega}\left[\frac{1}{D^1_{V_\rho}D^2_{V_\omega}}+\frac{1}{D^1_{V_\omega}D^2_{V_\rho}}\right] \qquad \text{with } D^i_X := Q_i^2+M_X^2$$

  - Symmetric Momenta: $F_{\pi^0\gamma^*\gamma^*}(-Q^2,-Q^2) \propto \sum_{n=0}^{\infty}\frac{1}{[Q^2+M^2_{V(n)}]^2}$

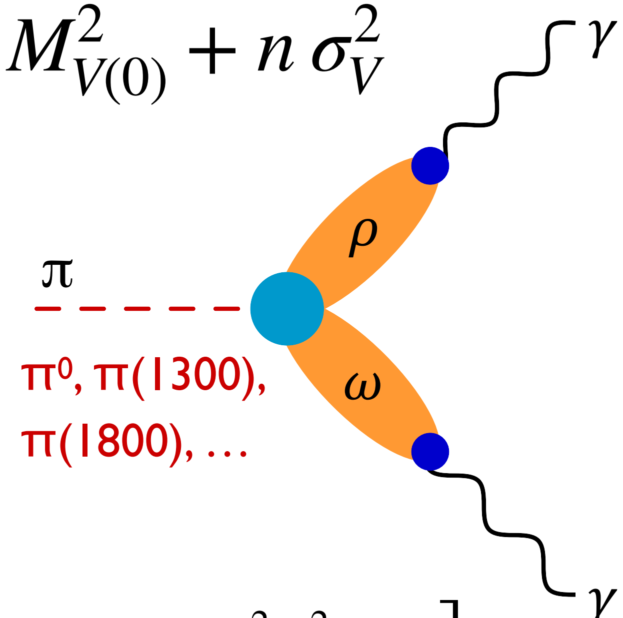
$$= \frac{1}{\sigma_V^4}\,\psi^{(1)}\left(\frac{M_V^2+Q^2}{\sigma_V^2}\right)$$

  - Each term in the sum is of $\mathcal{O}(1/Q^4)$, but the infinite sum satisfies the symmetric pQCD limit $\lim_{Q^2\to\infty} F_{\pi^0\gamma^*\gamma^*}(Q^2,Q^2) = -\frac{2f_\pi}{3Q^2}$

- In the same way, the SDCs on the HLbL tensor will be satisfied

LARGE-Nc REGGE MODEL

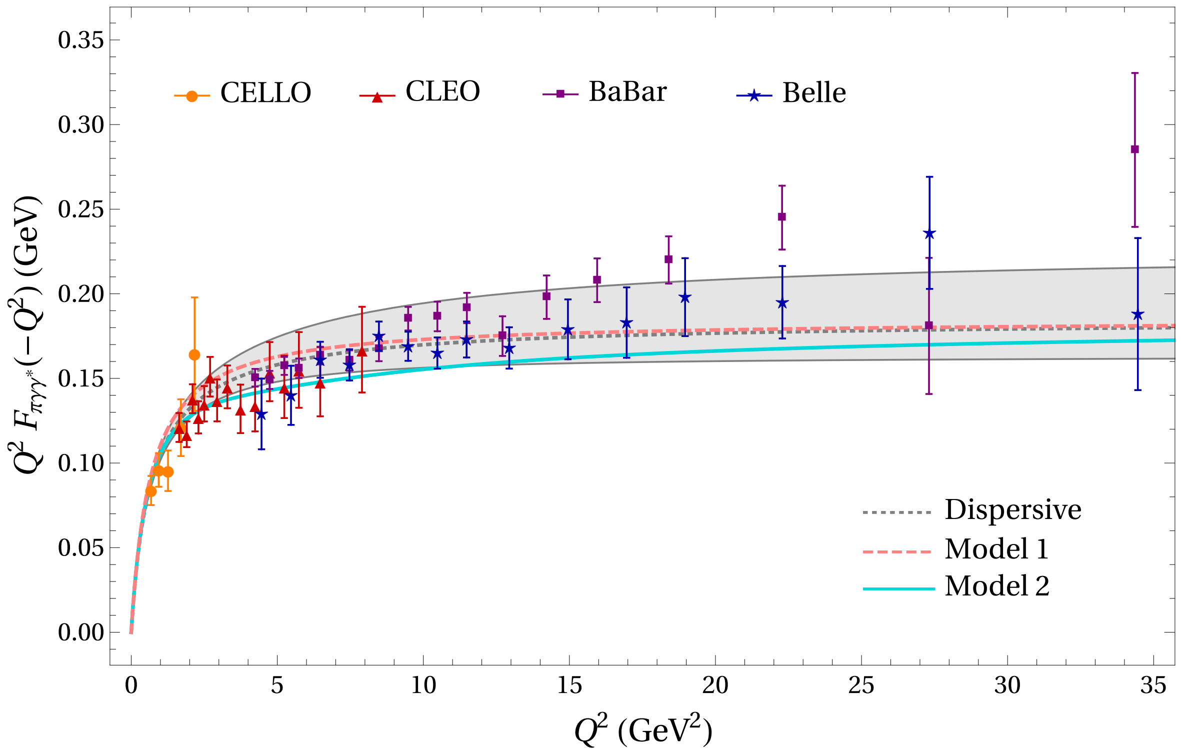
- Vector-meson-dominance model for transition form factors of radially-excited pseudoscalar mesons
  - Large-N_c limit — spectrum of the theory in any sector (set of quantum numbers) reduces to an infinite tower of narrow resonances
  - Regge ansatz for masses of radially-excited mesons $M^2_{V(n)} = M^2_{V(0)} + n\,\sigma^2_V$
  - Minimal model that satisfies all constraints on the transition form factors and HLbL tensor
  - Reproduce phenomenological constraints

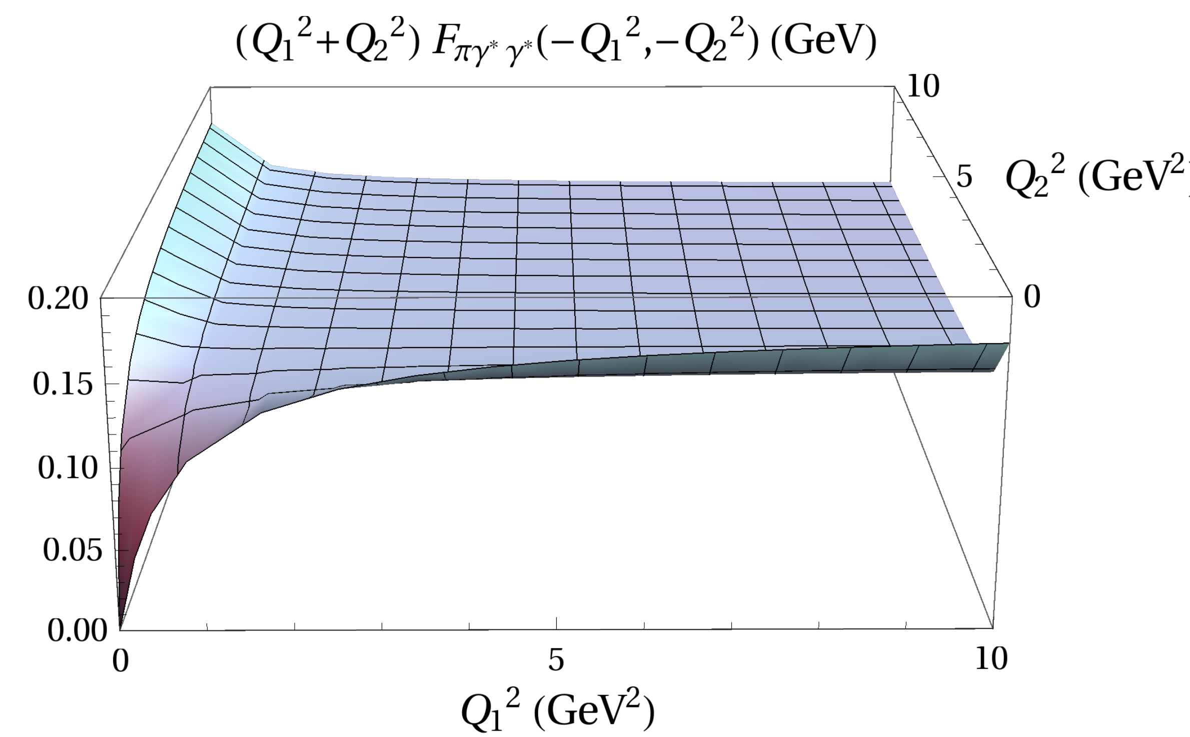


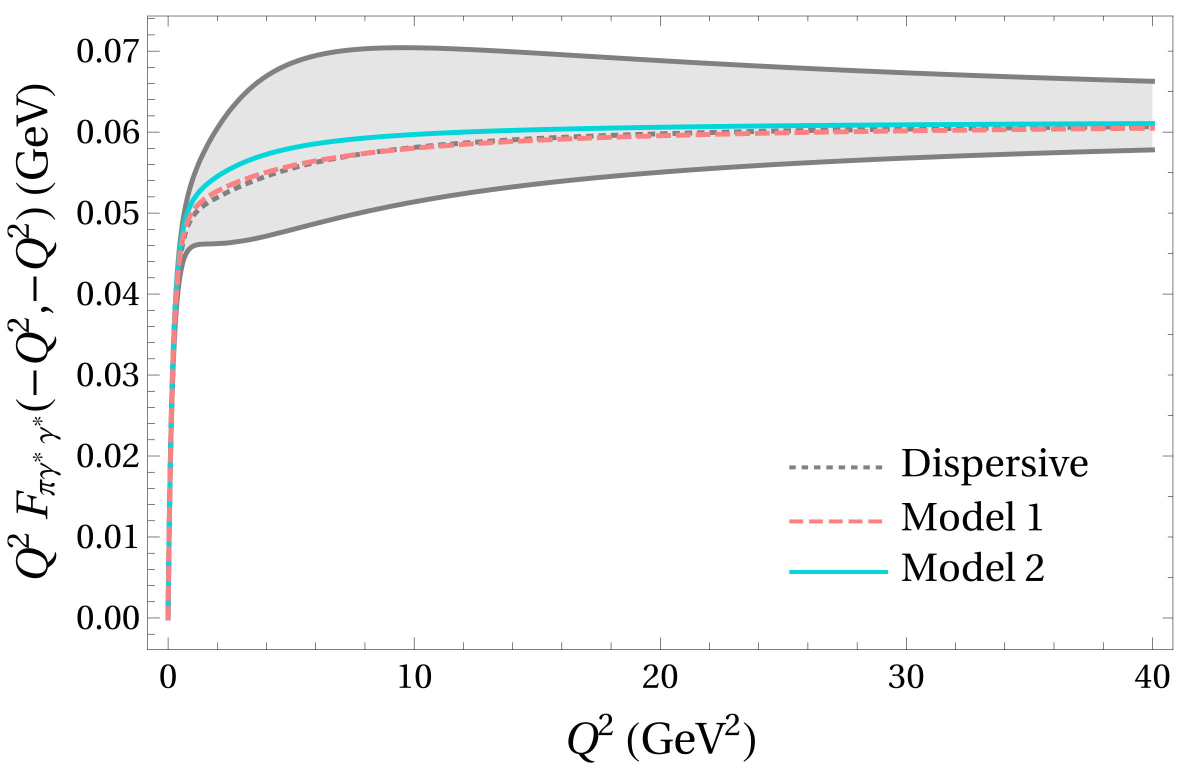
$$F_{\pi(n)\gamma^*\gamma^*}(-Q_1^2,-Q_2^2) = \frac{1}{8\pi^2 F_\pi}\left\{\left(\frac{M_\rho^2 M_\omega^2}{D^1_{\rho(n)}D^2_{\omega(n)}} + \frac{M_\rho^2 M_\omega^2}{D^2_{\rho(n)}D^1_{\omega(n)}}\right)\left[c_{\text{anom}} + \frac{1}{\Lambda^2}\left(c_A M^2_{+,n} + c_B M^2_{-,n}\right) + c_{\text{diag}}\frac{Q_1^2 Q_2^2}{\Lambda^2(Q_+^2 + M^2_{\text{diag}})}\right]\right.$$
$$\left. + \frac{Q_-^2}{Q_+^2}\left[c_{\text{BL}} + \frac{1}{\Lambda^2}\left(c_A M^2_{-,n} + c_B M^2_{+,n}\right)\right]\left(\frac{M_\rho^2 M_\omega^2}{D^1_{\rho(n)}D^2_{\omega(n)}} - \frac{M_\rho^2 M_\omega^2}{D^2_{\rho(n)}D^1_{\omega(n)}}\right)\right\}$$

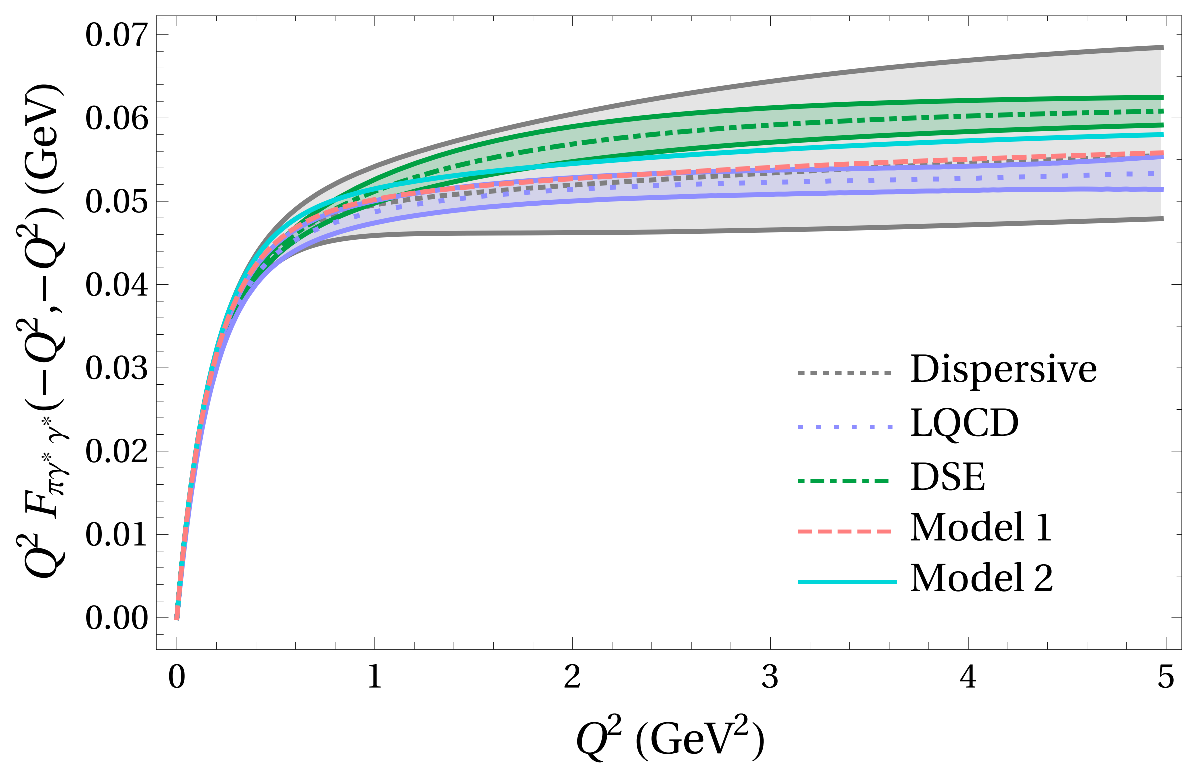
with $M^2_{\pm,n} = \frac{1}{2}\left(M^2_{\omega(n)} \pm M^2_{\rho(n)}\right)$, $Q^2_\pm = Q_1^2 \pm Q_2^2$, $D^j_V = Q_j^2 + M_V^2$

PION TRANSITION FORM FACTOR

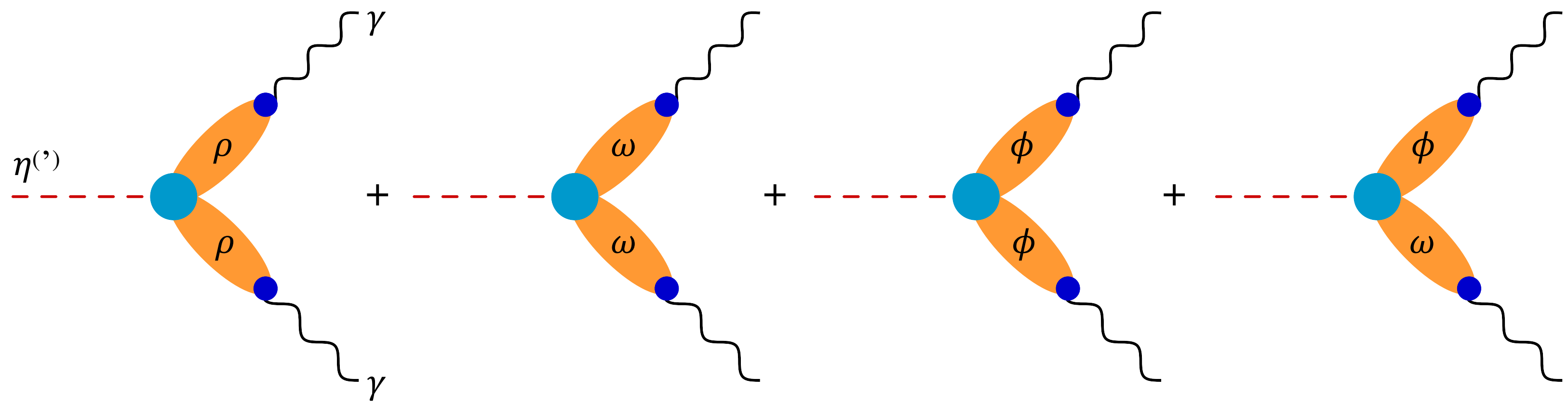




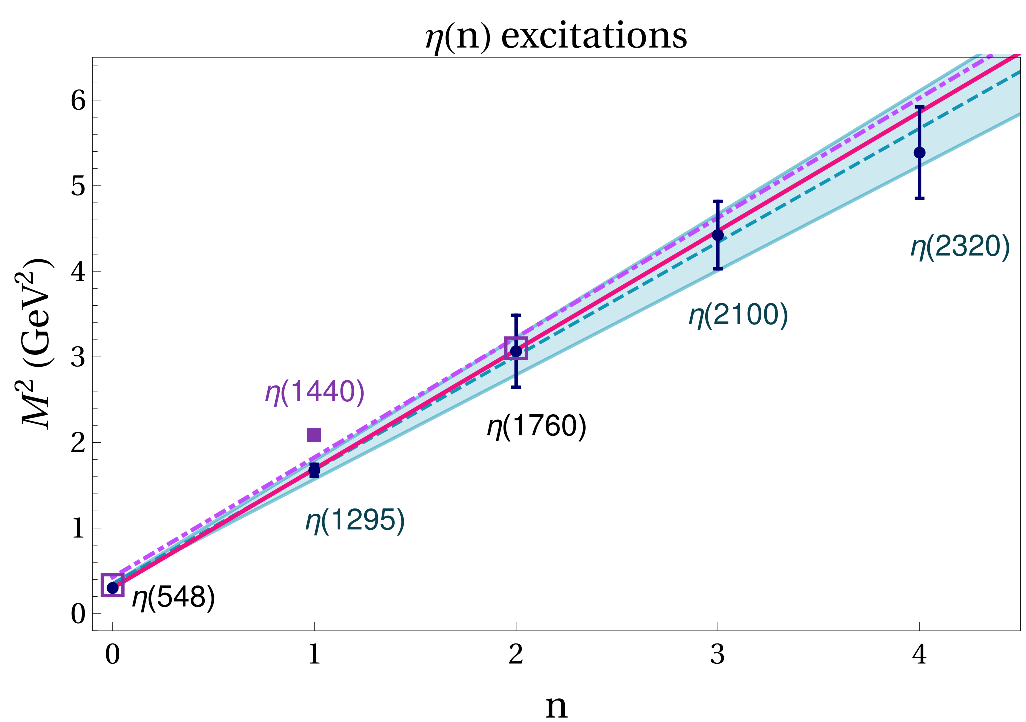


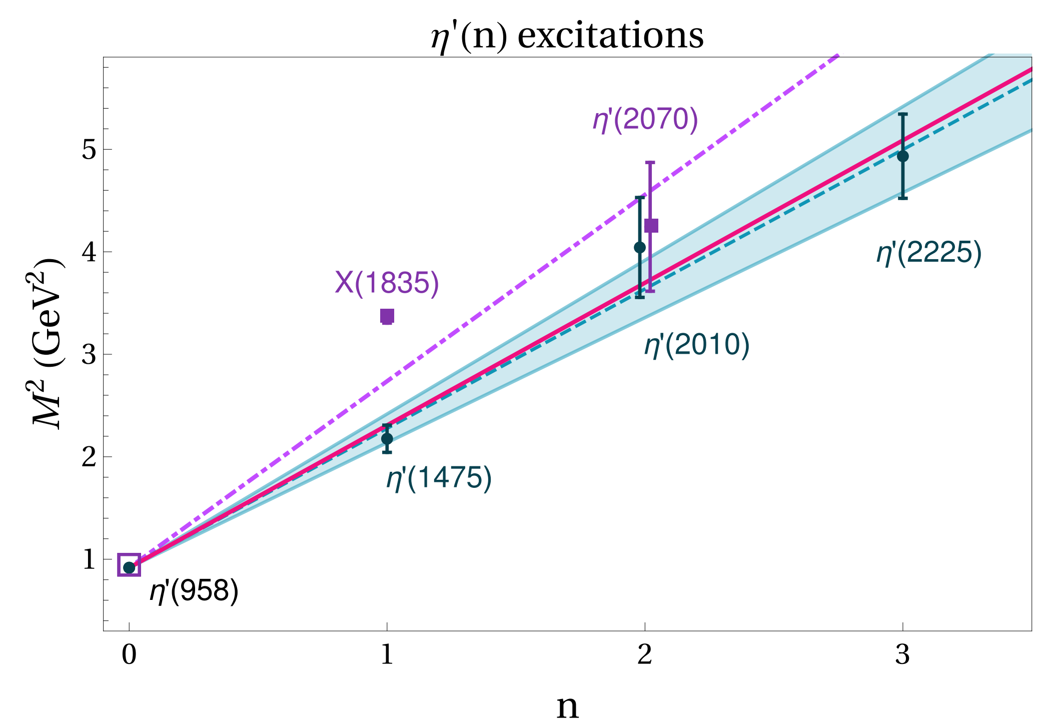


ETA TRANSITION FORM FACTORS

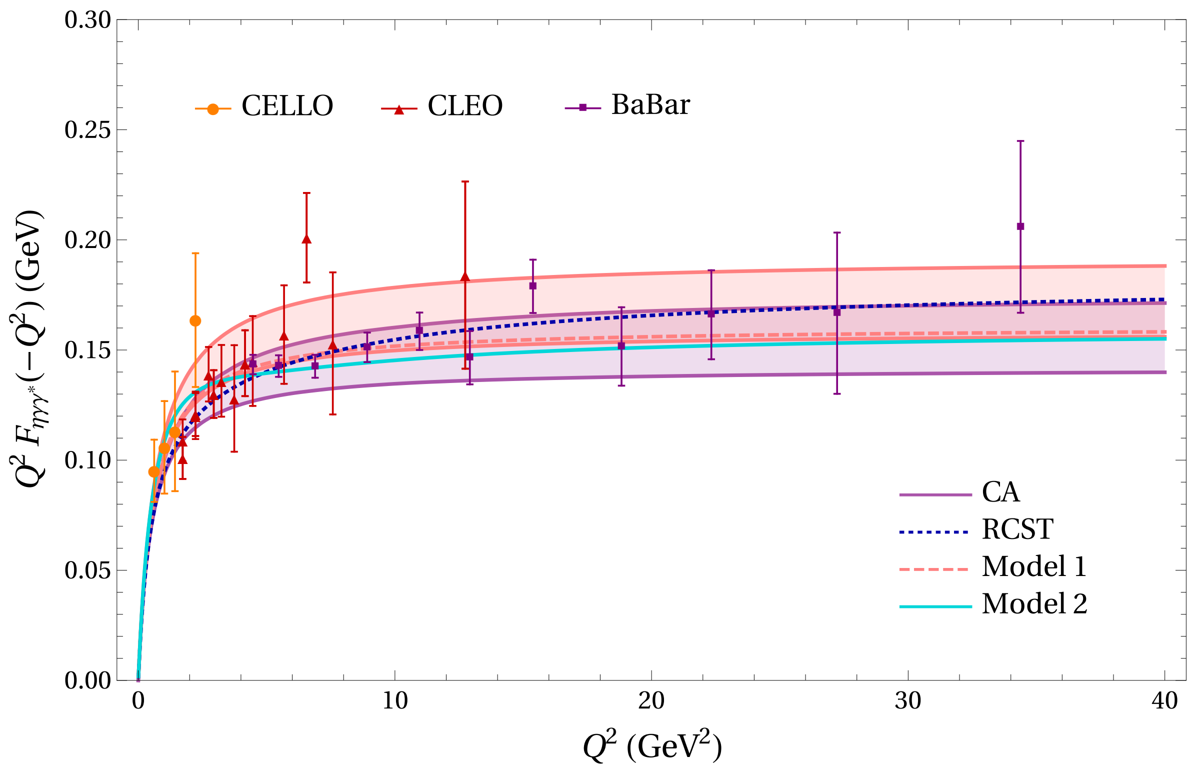


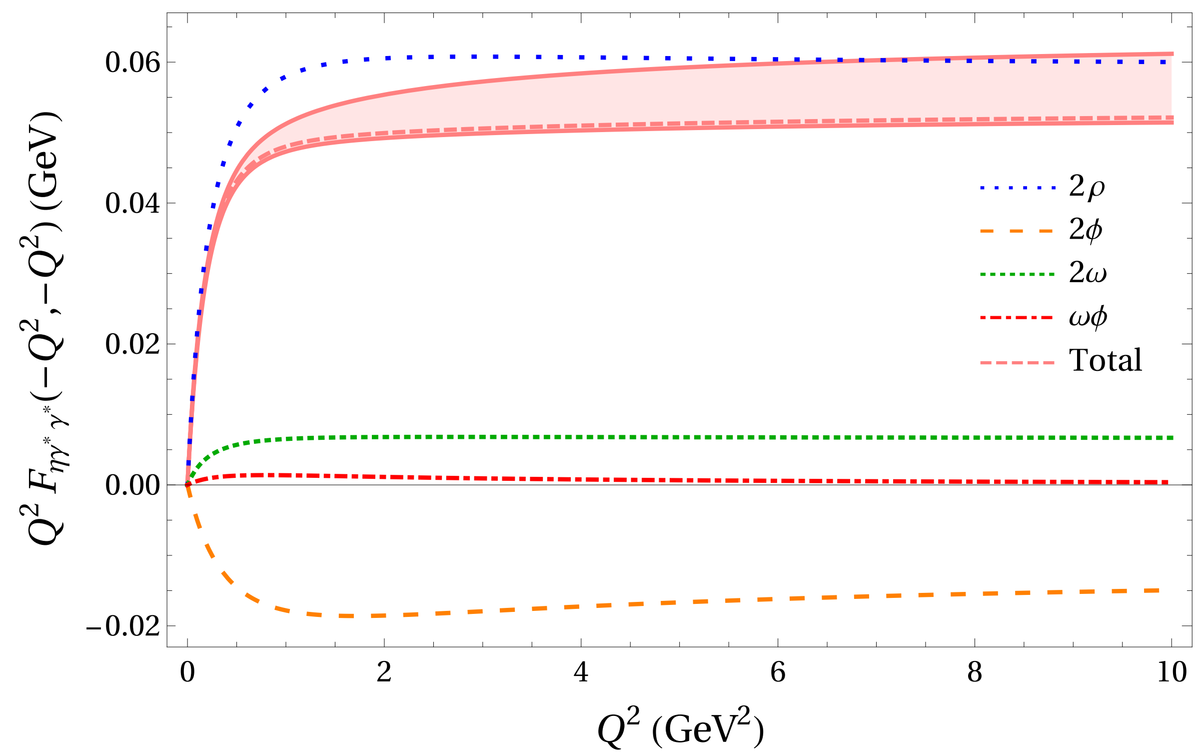
- Vector-meson-dominance model with of isoscalar-isoscalar and isovector-isovector pairs
- Relative coupling strengths follow from effective Lagrangian
- $\eta - \eta'$ and $\phi - \omega$ mixings must be considered

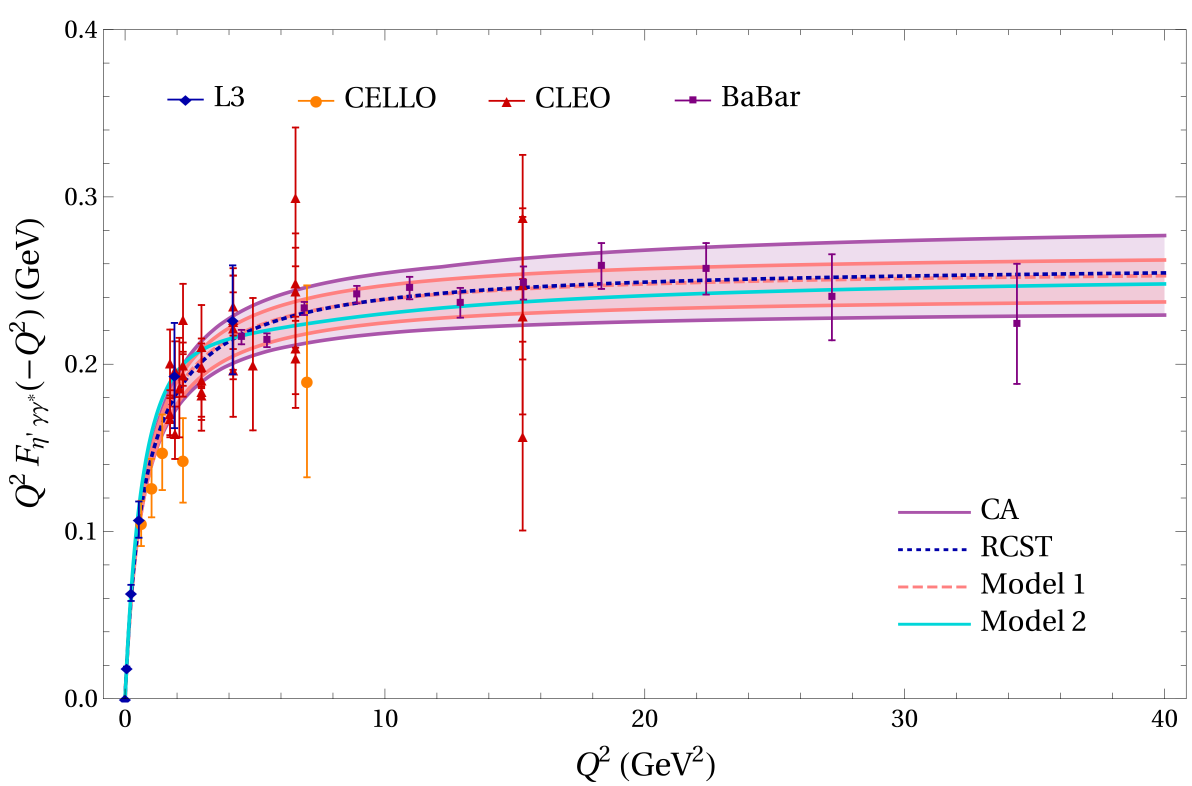


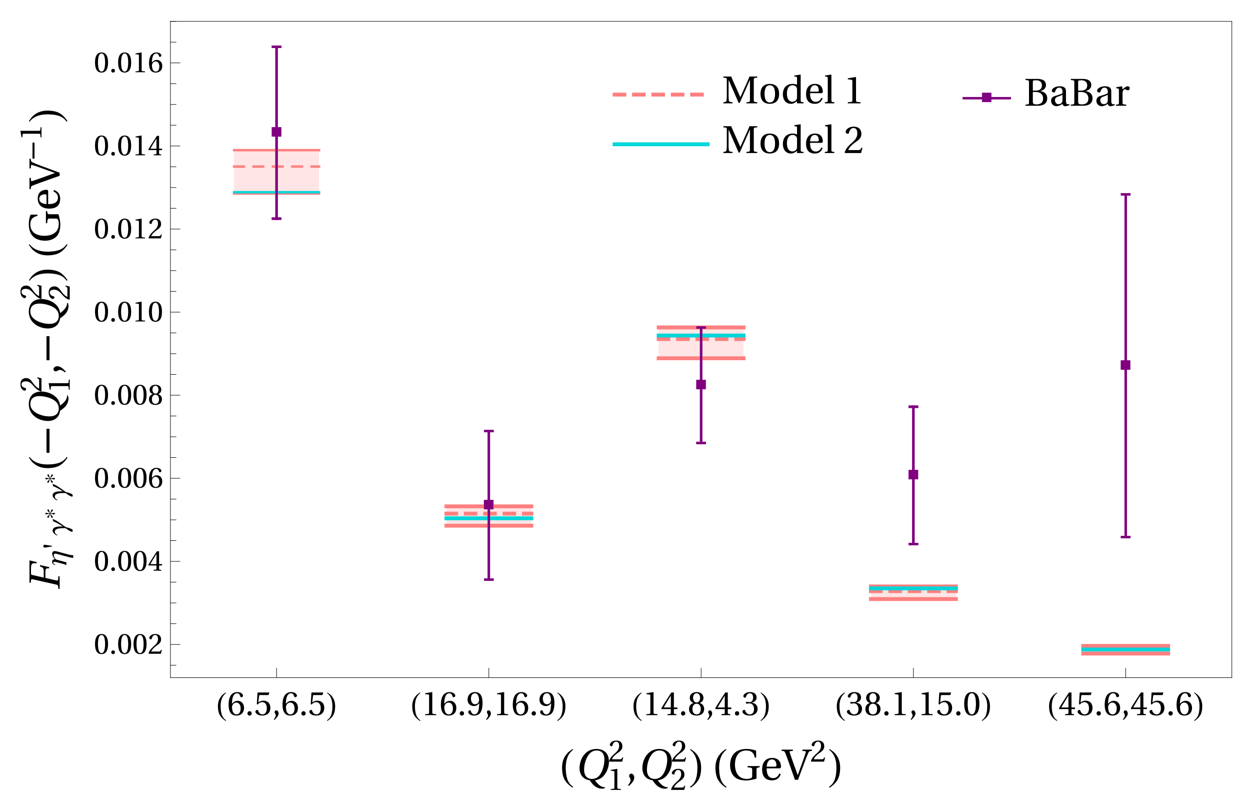


ETA TRANSITION FORM FACTORS



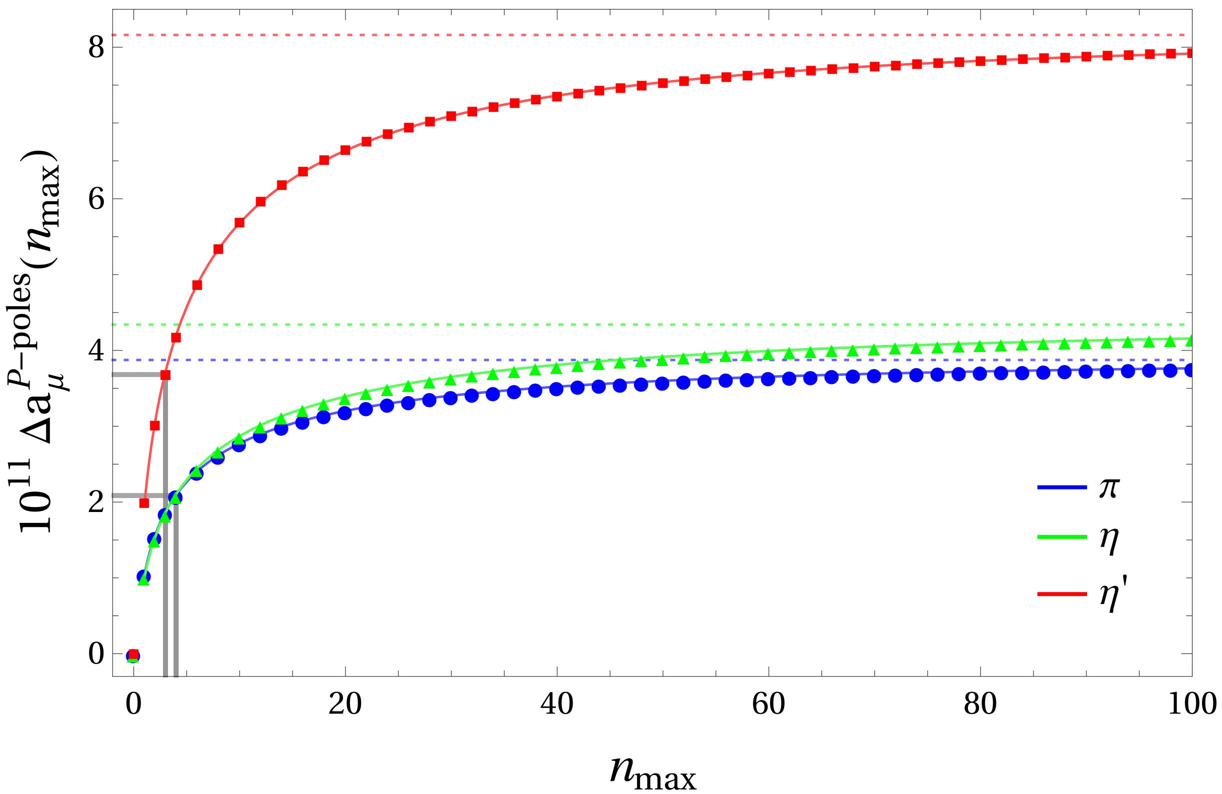


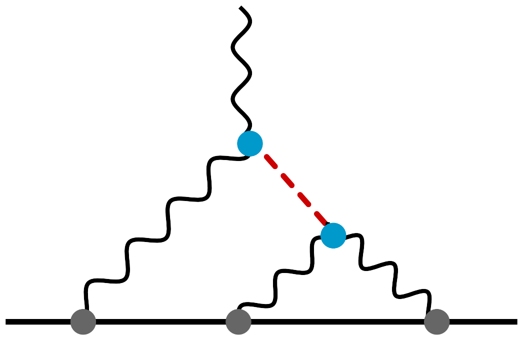




SUM OF PSEUDOSCALAR-POLE CONTRIBUTIONS

$$\Delta a_\mu^{P-\text{poles}}(n_\text{max}) = \sum_{n=1}^{n_\text{max}} a_\mu^{P(n)-\text{pole}}$$





$$\Delta a_\mu^{\pi-\text{poles}} = 2.7\,(0.4)_\text{Model}\,(1.2)_\text{syst} \times 10^{-11} = 2.7\,(1.3) \times 10^{-11}$$

$$\Delta a_\mu^{\eta-\text{poles}} = 3.4\,{}^{+0.9}_{-0.7}\Big|_\text{Model}\,(0.9)_\text{syst} \times 10^{-11} = 3.4\,{}^{+1.3}_{-1.1} \times 10^{-11}$$

$$\Delta a_\mu^{\eta'-\text{poles}} = 6.5\,(1.1)_\text{Model}\,(1.7)_\text{syst} \times 10^{-11} = 6.5\,(2.0) \times 10^{-11}$$

- Total effect of excited pseudoscalar mesons: $\Delta a_\mu^{\text{PS}-\text{poles}} = \Delta a_\mu^{\pi-\text{poles}} + \Delta a_\mu^{\eta-\text{poles}} + \Delta a_\mu^{\eta'-\text{poles}}$

$$= 12.6\,{}^{+1.6}_{-1.5}\Big|_\text{Model}\,(3.8)_\text{syst} \times 10^{-11}$$

$$= 12.6(4.1) \times 10^{-11}$$

- Original and updated MV result:

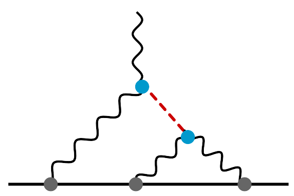
$$\Delta a_\mu^{\pi-\text{poles}}\Big|_\text{MV} = 13.5 \times 10^{-11} \quad [16.2 \times 10^{-11}]$$

$$\Delta a_\mu^{\eta-\text{poles}}\Big|_\text{MV} = 5.0 \times 10^{-11} \quad [10.0 \times 10^{-11}]$$

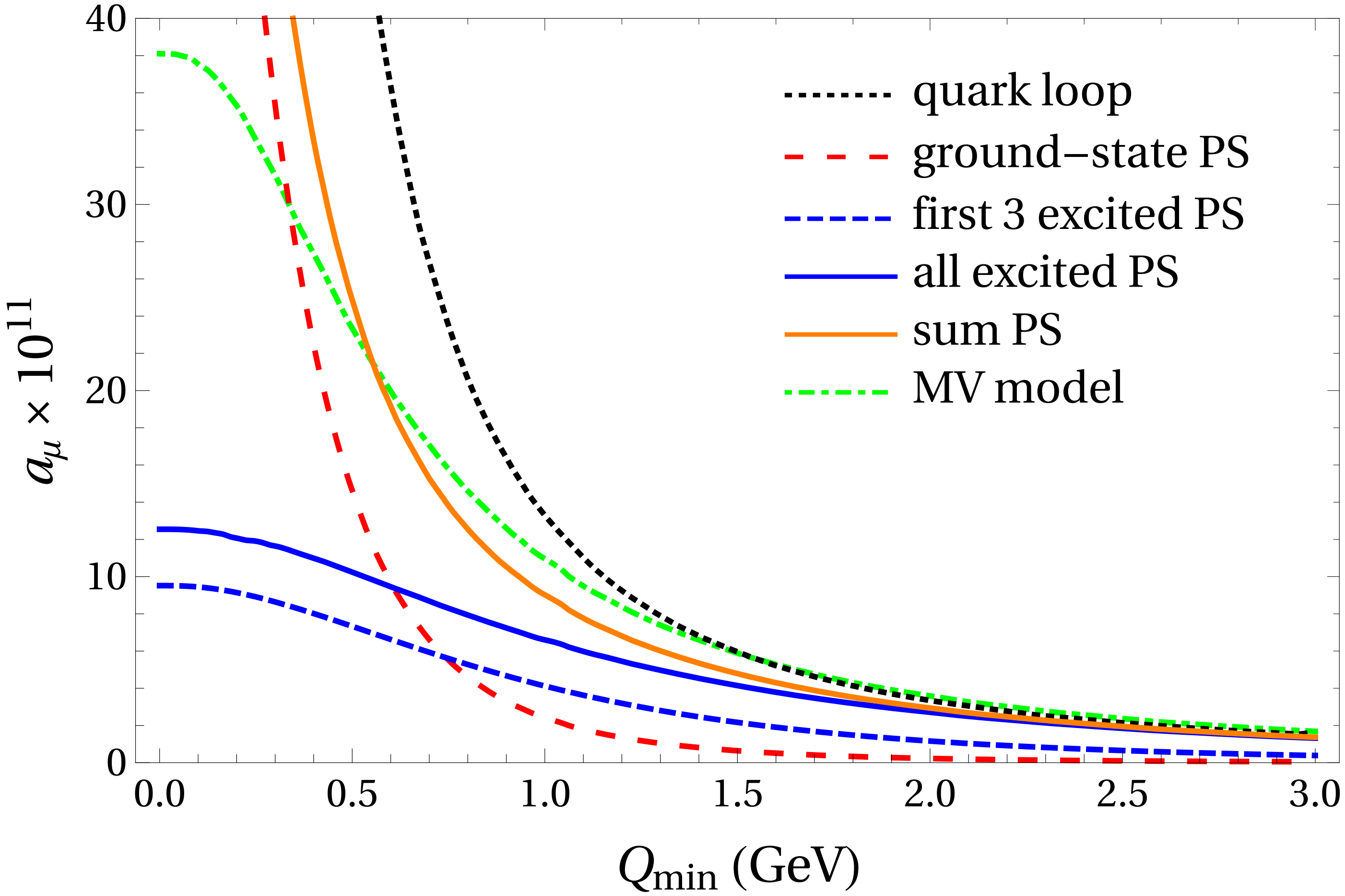
$$\Delta a_\mu^{\eta'-\text{poles}}\Big|_\text{MV} = 5.0 \times 10^{-11} \quad [12.1 \times 10^{-11}]$$

MATCHING TO PERTURBATIVE QUARK LOOP

$$a_\mu^{\mathrm{HLbL}} = \frac{2\alpha^3}{3\pi^2}\int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^{1} d\tau\, \sqrt{1-\tau^2}\, Q_1^3 Q_2^3 \sum_{i=1}^{12} T_i(Q_1,Q_2,\tau)\bar{\Pi}_i(Q_1,Q_2,\tau)$$



with $Q_3^2 = Q_1^2 + 2Q_1Q_2\tau + Q_2^2$



high-energy region

$$\Delta a_\mu^{\mathrm{PS-poles}}\Big|_{\mathrm{MV}} = 23.5 \times 10^{-11} \quad [38 \times 10^{-11}]$$

$$\Delta a_\mu^{\mathrm{LSDC}} = \left[8.7(5.5)_{\mathrm{PS-poles}} + 4.6(9)_{q-\mathrm{loop}}\right] \times 10^{-11} \sim 13(6) \times 10^{-11}$$

SUMMARY AND CONCLUSIONS

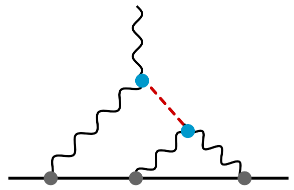
- Uncertainty of the SM prediction is dominated by hadronic corrections
- HVP is calculated with a systematic data-driven dispersive approach — we want a similar model-independent approach for HLbL
- Pseudoscalar-pole contributions are the leading HLbL contributions
- Mixed- and high-energy regions are not constraint by data and need to be estimated — SDCs from operator product expansion
- Infinite tower of radially-excited pseudoscalar-pole diagrams can satisfy the Melnikov-Vainshtein SDC in the mixed region and the asymptotic SDC
- Large-N_c Regge model for the pseudoscalar transition form factors
- Effect of SDCs is smaller than previously estimated:

$$\Delta a_\mu^{\mathrm{LSDC}} = \left[8.7(5.5)_{\mathrm{PS-poles}} + 4.6(9)_{q-\mathrm{loop}}\right] \times 10^{-11} \sim 13(6) \times 10^{-11}$$

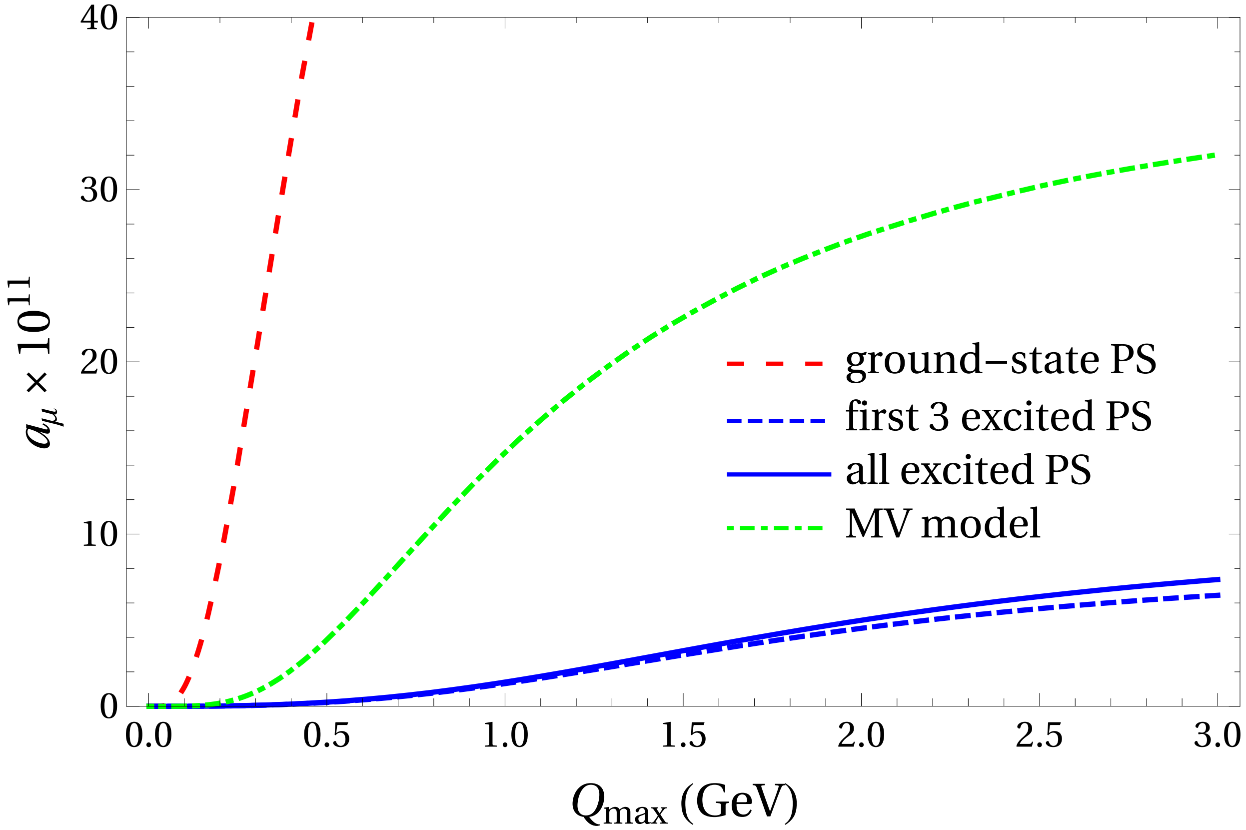
Back-up Slides

LOW- AND MIXED ENERGY REGION

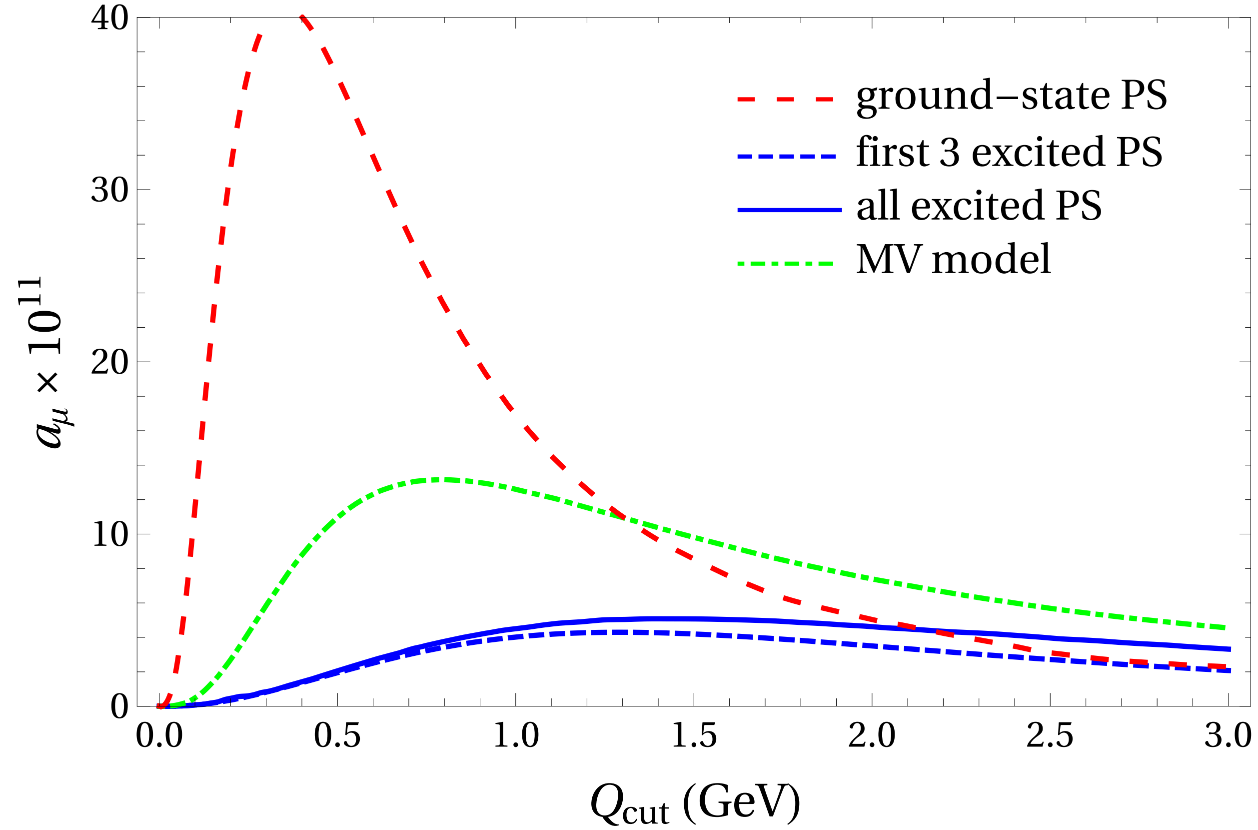
$$a_\mu^{\mathrm{HLbL}} = \frac{2\alpha^3}{3\pi^2}\int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^{1} d\tau \sqrt{1-\tau^2}\, Q_1^3 Q_2^3 \sum_{i=1}^{12} T_i(Q_1,Q_2,\tau)\bar{\Pi}_i(Q_1,Q_2,\tau)$$



with $Q_3^2 = Q_1^2 + 2Q_1Q_2\tau + Q_2^2$



low-energy region



mixed-energy region