$\tau \to \mu\mu\mu$ at a rate of one out of 10^{14} tau decays?

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Lepton flavor changing in neutrinoless τ decays

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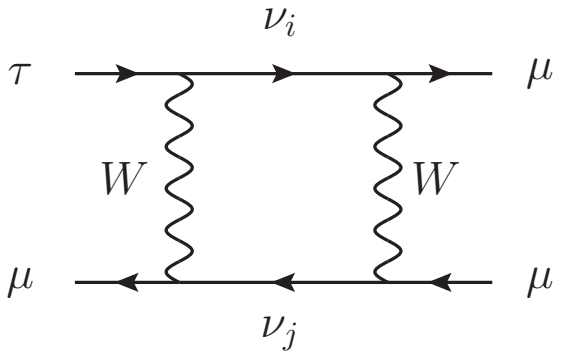
Abstract. Neutrino oscillations, as recently reported by the Super-Kamiokande collaboration, imply that lepton numbers could be violated, and $\tau^\pm \to \mu^\pm + \ell^+ + \ell^-, \tau^\pm \to \mu^\pm + \rho^0$ are some typical examples. We point out that in these neutrinoless modes, the GIM cancelation is much milder with only a logarithmic behavior $\log(m_j/m_k)$ where $m_{j,k}$ are the neutrino masses. This is in sharp contrast with the vanishingly small amplitude $\tau^\pm \to \mu^\pm + \gamma$ strongly suppressed by the quadratic power $(m_j^2 - m_k^2)/M_W^2$. In comparison with the hopelessly small branching ratio $B(\tau^\pm \to \mu^\pm + \gamma) \approx 10^{-40}$, the $B(\tau^\pm \to \mu^\pm + \ell^+ + \ell^-)$ could be larger than 10^{-14}. The latter mode, if measurable, could give one more constraint to the lepton mixing angle $\sin 2\theta_{jk}$ and the neutrino mass ratio m_j/m_k, and therefore is complementary to neutrino oscillation experiments.

- Starting point: SM + $m_\nu \neq 0$ (either Dirac or Majorana)
- Neutrino oscillations can induce τ and μ decays with CLFV



$$\mathcal{L}_{\text{weak}} = -\frac{g}{\sqrt{s}} \sum_{\ell=e,\mu,\tau} \bar{\ell}_L \gamma_\alpha \nu_{\ell L} W^{\alpha\dagger}$$

for massive neutrinos $\nu_{\ell L} = \sum_{i=1}^{3} U_{\ell i}\, \nu_{iL}$

$$\Gamma(L \to \ell\ell\ell) = \frac{\alpha^2 G_F^2 m_L^5}{(4\pi)^5} \left(\sum_{i=2}^{3} U_{\ell i} U_{Li}^* \frac{\Delta m_{i1}^2}{M_W^2} \log \frac{\Delta m_{i1}^2}{M_W^2} \right)^2$$

$\Delta m_{ij}^2 = m_{\nu i}^2 - m_{\nu j}^2$, $m_1 < m_2 < m_3$ and $m_1 \to 0$. Petkov, Sov. J. Nucl. Phys. 25 (1977) 340

with $\sum_i m_{\nu i} < 0.170$ eV (95% CL)

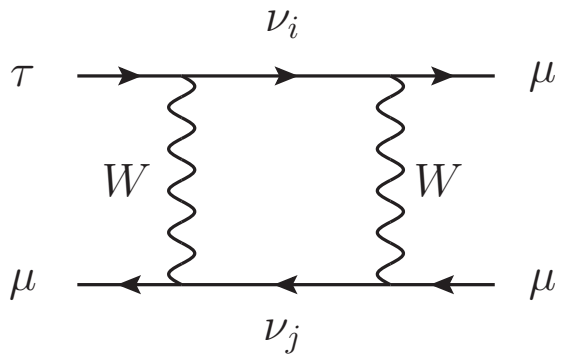
Plank + BAO, Astron. Astrophys. 594 (2016) A13

- $\Delta m_{ij}^2 / M_W^2 < 4 \times 10^{-25}$
- $\log \frac{M_W^2}{\Delta m_{ij}^2} \sim 60$

process	BR
$\tau \to \mu\mu\mu$	2.0×10^{-53}
$\tau \to eee$	1.1×10^{-54}
$\mu \to eee$	4.1×10^{-54}

$\tau \to \mu\mu\mu$ in the ZML

Petkov, Sov. J. Nucl. Phys. 25 (1977) 340



- In the 70s the calculation was too difficult with full momentum dependence.
- Solution: set momenta and masses of external particles to zero!

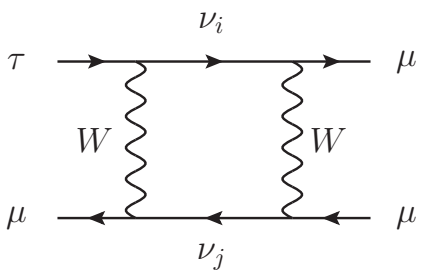
$\Gamma(\mu \to eee)$ was computed in the Zero Momentum Limit (ZML).

$$M_W \gg m_\nu \gg \mathcal{P}$$

$\mathcal{P}$ is any scale associated to the external particles: $m_\tau, m_\mu, p_\tau, p_{\mu 1-3}$

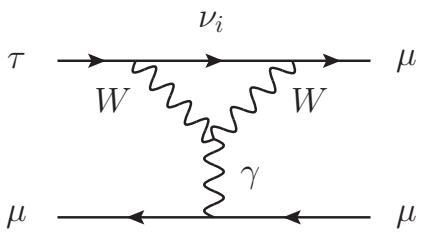
$x_i = m_{\nu i}^2 / M_W^2$ dependence

Boxes



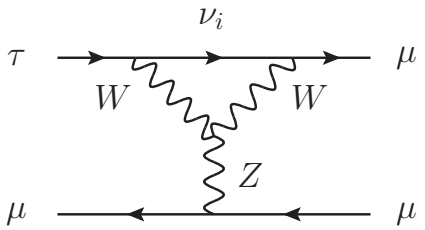
$$= x\left[1 + \log x\right]$$

γ penguins



$$= -4x$$

Z penguins



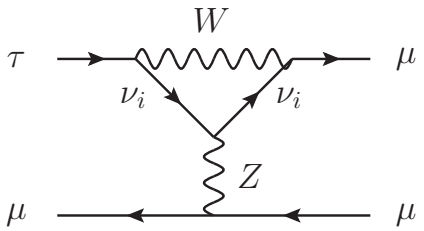
$$= x\left[3 + \log x\right]$$

They are the leading term in the $x \to 0$ limit of the Inami-Lim functions

Inami,Lim, Prog.Theor.Phys. 65 (1981) 297

$\tau \to \mu\mu\mu$ at a rate of one out of 10^{14}!

Pham, EPJC 8 (1999) 513



- Without neglecting $\mathcal{P}$, this diagram behaves like $\log x$ because there is a pseudo singularity at $q^2 \to 0$.

- There must be an infrared divergence associated to the limit $m_\nu \to 0$ and $q^2 \to 0$.
- $\mathrm{BR} \propto \left[\sum_{i=2}^{3} U_{\mu i} U^*_{\tau i} \log(m_i^2/m_1^2)\right]^2$
- $\mathrm{BR}(\tau \to \mu\ell^+\ell^-) \geq 10^{-14}$,

With Pham expression and current PDG values:

process	BR $m_1 = 10^{-2}$ eV	BR $m_1 = 10^{-8}$ eV
$\tau \to \mu\mu\mu$	10^{-16}	10^{-13}
$\tau \to eee$	10^{-16}	10^{-14}
$\mu \to eee$	10^{-22}	10^{-17}

Current limits:

- $\mathrm{BR}^{\mathrm{exp}}(\tau \to \ell\ell'\ell') \lesssim 10^{-8}$

  HFLAV Collaboration, hep-ex/1909.12524

- $\mathrm{BR}^{\mathrm{exp}}(\mu \to eee) < 1.0 \times 10^{-12}$ 90% CL

  SINDRUM Coll, NPB 299 (1988) 1

Expected limits:

- MU3E
  $\mu \to eee \sim 10^{-15} - 10^{-16}$
- HL-LHC (3000 fb^{-1})
  $\tau \to \mu\mu\mu \sim 10^{-9}$
- Belle II (50 ab^{-1})
  $\tau \to \mu\mu\mu \sim 10^{-10}$

Cerri et al, hep-ph/1812.07736. Belle II physics book, 1808.10567

A first numerical check

Hernandez-Tome, Lopez Castro, Roig, EPJC 79 (2019) 1

- They calculate $\tau \to \mu\mu\mu$ in the physical limit with numerical evaluation of the amplitudes.
- However they neglect γ penguins, so the result is gauge dependent!
- Well known that there is a delicate cancellation of gauge dependence between boxes, Z and γ penguins.

  Buchalla, Buras, Harlander NPB 349 (1991) 1
- BR are found to be a few order of magnitude smaller than in the ZML!

GOAL: we want to determine the analytic form of

$$\Gamma(L \to \ell\ell\ell) = \frac{\alpha^2 G_F^2 m_L^5}{(4\pi)^5} \left(\sum_{i=2}^{3} U_{\ell i} U_{Li}^* \quad ??? \quad \right)^2$$

in the physical limit $M_W \gg \mathcal{P} \gg m_\nu$.

Details of the calculation

- We consider the full set of 60 one-loop diagrams:
  Boxes + Z penguins + γ penguins.
- Amplitudes are generated automatically in `FeynArts`.
- They are computed by `FormCalc` and reduced to master integrals with full dependence on $M_W, \mathcal{P}, m_\nu$.

T. Hahn, Comput. Phys. Commun. 140 (2001) 418;
T. Hahn, S. Passehr and C. Schappacher, J. Phys. Conf. Ser. 762 (2016) 012065

CHALLENGE: expand one-loop integrals in the limit

$$M_W \gg \mathcal{P} \gg m_\nu$$

Method of regions

Beneke, Smirnov, NPB 522 (1998) 321; Smirnov, Springer Tracts Mod. Phys. 177

We want to calculate in the limit $0 < m \ll q$

$$\int_0^\infty \frac{dk}{(k+m)(k+q)}$$

<u>Large Region</u> $k \sim q$

$$\int_0^\infty \frac{k^{-\varepsilon} dk}{k+q} \left[\frac{1}{k} - \frac{m}{k^2} + \dots \right]$$

<u>Small Region</u> $k \sim m$

$$\int_0^\infty \frac{k^{-\varepsilon} dk}{k+m} \left[\frac{1}{q} - \frac{k}{q^2} + \dots \right]$$

Method of regions

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We want to calculate in the limit $0 < m \ll q$

$$\int_0^\infty \frac{dk}{(k+m)(k+q)} = \frac{\log\left(\frac{q}{m}\right)}{q-m} = \frac{\log\left(\frac{q}{m}\right)}{q} \sum_{n=0}^{\infty} \left(\frac{m}{q}\right)^n$$

<u>Large Region</u> $k \sim q$

$$\int_0^\infty \frac{k^{-\varepsilon} dk}{k+q} \left[\frac{1}{k} - \frac{m}{k^2} + \dots\right] = \left[-\frac{1}{\varepsilon q} + \frac{\log q}{q}\right] \sum_{n=0}^{\infty} \left(\frac{m}{q}\right)^n$$

<u>Small Region</u> $k \sim m$

$$\int_0^\infty \frac{k^{-\varepsilon} dk}{k+m} \left[\frac{1}{q} - \frac{k}{q^2} + \dots\right] = \left[\frac{1}{\varepsilon q} - \frac{\log m}{q}\right] \sum_{n=0}^{\infty} \left(\frac{m}{q}\right)^n$$

- The master integrals are recomputed from scratch using the method of regions + `Package-X` Patel, Comput. Phys. Commun. 197 (2015) 276
- First step: $M_W \gg \mathcal{P} \sim m_\nu$

$$\mathcal{A} = \sum_n f_n(\mathcal{P}^2, m_\nu^2) \left(\frac{1}{M_W^2}\right)^n$$

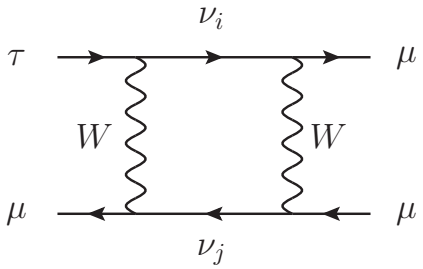
- Second step: $M_W \gg \mathcal{P} \gg m_\nu$

$$\mathcal{A} = \sum_{nmij} c_{nm,ij} \left(\frac{\mathcal{P}^2}{M_W^2}\right)^n \left(\frac{m_\nu^2}{\mathcal{P}^2}\right)^m \log^i\left(\frac{\mathcal{P}^2}{M_W^2}\right) \log^j\left(\frac{m_\nu^2}{\mathcal{P}^2}\right)$$

- Use GIM mechanism and keep only the leading term in m_ν

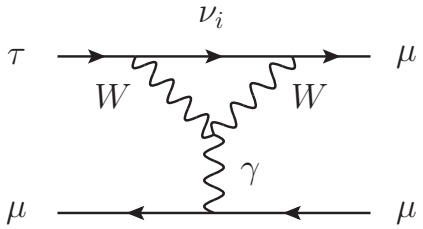
$$\mathcal{A} \to \frac{\mathcal{P}^4}{M_W^4} \frac{m_\nu^2}{\mathcal{P}^2} \log^i\left(\frac{\mathcal{P}^2}{M_W^2}\right) \log^j\left(\frac{m_\nu^2}{\mathcal{P}^2}\right)$$

Boxes



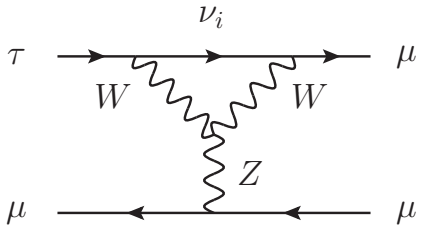
$$\propto \frac{\mathcal{P}^4}{M_W^4}\frac{m_\nu^2}{\mathcal{P}^2}\log\left(\frac{m_\tau^2}{M_W^2}\right)$$

γ penguins



$$\propto \frac{\mathcal{P}^4}{M_W^4}\frac{m_\nu^2}{\mathcal{P}^2}$$

Z penguins



$$\propto \frac{\mathcal{P}^4}{M_W^4}\frac{m_\nu^2}{\mathcal{P}^2}\log\left(\frac{m_\tau^2}{M_W^2}\right)$$

The logarithimc enhancement $\log(m_\nu/M_W)$ of the ZML disappears!
Only a mild $\log(M_W/m_\tau) = 3.8$

Decay rate in the physical limit

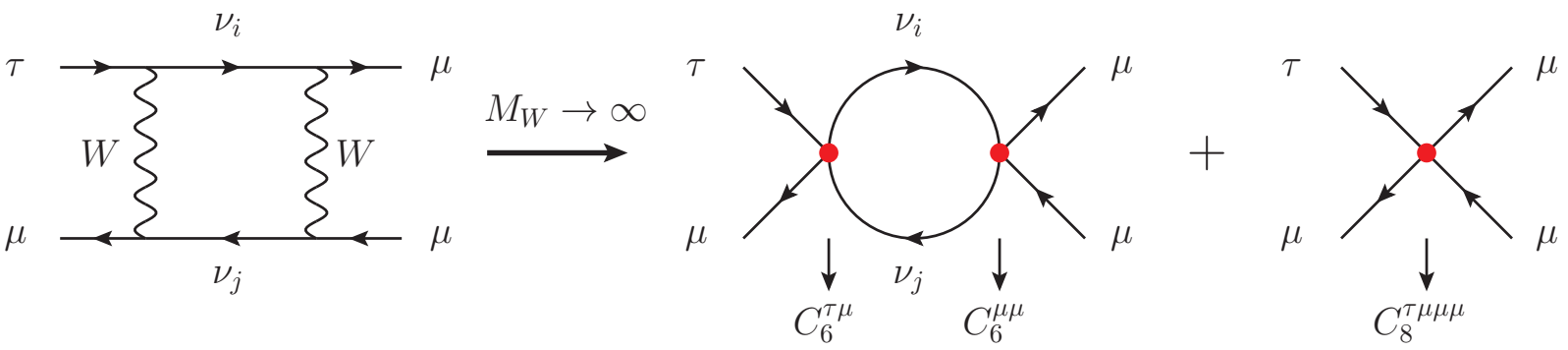
Neglecting terms of order m_ℓ/m_L in the phase space integration, we obtain:

$$\Gamma(L \to \ell\ell\ell) = \frac{G_F^2 \alpha^2 m_L^5}{(4\pi)^5} \left(\sum_{i=2}^{3} U_{\ell i} U_{Li}^* \frac{\Delta m_{i1}^2}{M_W^2} \right)^2 \left[\log^2 x_L + 2 \log x_L \right.$$
$$\left. - \frac{1}{6} \log x_\ell + \frac{19}{18} + \frac{17}{18}\pi^2 + \frac{1}{\sin^2\theta_W} \left(\log x_L - \frac{11}{12} \right) + \frac{3}{8 \sin^4 \theta_W} \right]$$

with $m_1 < m_2 < m_3$, $x_L = m_L^2/M_W^2$ and $x_\ell = m_\ell^2/M_W^2$.

	ZML	PL	ZML/PL
$\tau \to \mu\mu\mu$	2.0×10^{-53}	5.8×10^{-55}	34
$\tau \to eee$	1.1×10^{-54}	3.3×10^{-56}	34
$\mu \to eee$	4.1×10^{-54}	2.9×10^{-55}	14

with $m_3 < m_1 < m_2$ we obtain similar results.



$$C_8^{\tau\mu\mu\mu}(\mu) = C_8^{\tau\mu\mu\mu}(M_W) + \gamma \log\left(\frac{\mu^2}{M_W^2}\right) C_6^{\tau\mu}(M_W) C_6^{\mu\mu}(M_W)$$

- **ZML** the operator can evolve between M_W and m_ν: $\log\left(\frac{m_\nu^2}{M_W^2}\right)$.
- **PL** the operator evolves only between M_W and m_τ: $\log\left(\frac{m_\tau^2}{M_W^2}\right)$.

Conclusions

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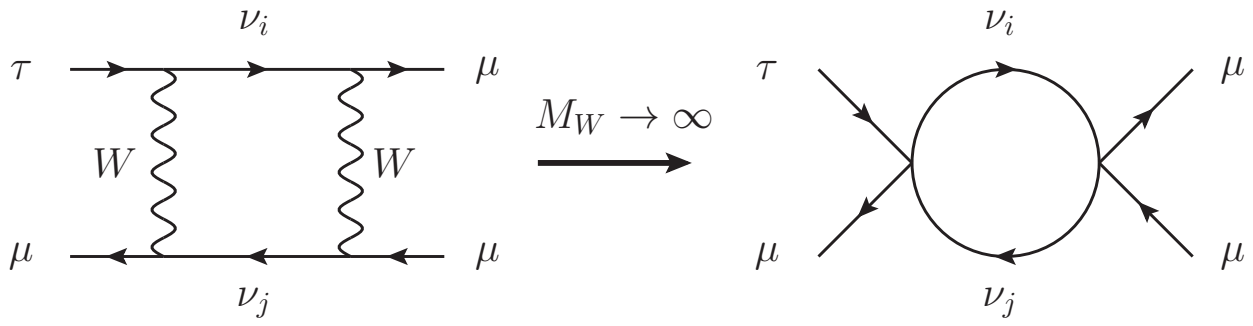
- We win the price for the smallest (but non zero) branching ratio ever calculated in particle physics.
- We re-confirm that any signal of CLFV detected by Mu3e, Belle II, HL-LHC and co. will be New Physics, beyond SM + $m_\nu \neq 0$.

Backup



$$(\sqrt{\mathcal{P}})^4 \times \left(\frac{1}{M_W^2}\right)^2 \times \mathcal{P}^2 \left(\frac{m_\nu^2}{\mathcal{P}^2}\right)$$

- Mass dimension of the amplitude: $[\mathcal{A}] = 0$
- External spinors: $\sqrt{P}$
- W-boson propagators: $1/M_W^2$
- Vacuum-polarization-like loop: $\mathcal{P}^2 \left(1 + \frac{m_\nu^2}{\mathcal{P}^2} + \dots\right)$