PSI 2019

High-precision QED predictions for low-energy lepton experiments

Yannick Ulrich

Paul Scherrer Institut / Universität Zürich

23RD OCTOBER 2019

QED theory support

what this talk is not about

- bound-state QED
- hadronic effects

what this talk is about

- tailored predictions
- in perturbative QED
- of background studies
- for lepton experiments (MEG, Mu3e, MUSE, MUonE etc)

take LHC techniques and apply to lepton experiments

QCD v QED

LHC techniques (N)NLO, (N)NLL, NLO+PS...

- higher-order technology is mature
- $\alpha_s(Q_{\mathsf{LHC}}) \gg \alpha_{\mathsf{em}} \sim 1\% \rightarrow$ much higher precision
- but large logarithms like $\alpha_{\mathsf{em}} \log \frac{m_e^2}{m_\mu^2} \sim 10\%$
- no PDF $\rightarrow$ less hadronic problems
- people involved $\mathcal{O}(10^3)$ v. $\mathcal{O}(10)$
- very few examples at NNLO done: Bhabha scattering and muon decay
- more required: $\mu e \rightarrow \mu e$, $\ell p \rightarrow \ell p$, $ee \rightarrow ee$, $ee \rightarrow \gamma\gamma$, ...

No. active scales →

loop order ↓

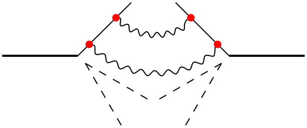
	2	3	4	5
LO	'trivial'			$\mu \to \nu\bar{\nu}e\gamma\gamma$
NLO			$\mu \to \nu\bar{\nu}e\gamma$ $\tau \to \nu\bar{\nu}e\gamma$	$\mu \to \nu\bar{\nu}eee$
NNLO	$(\ell p \to \ell p)_{\text{red}}$ $(\mu e \to \mu e)_{\text{red}}$	$\mu \to \nu\bar{\nu}e$	$(\mu e \to \mu e)_{\text{full}}$	
NNNLO	$(\mu e \to \mu e)_{\text{red}}$			terra incognita

double radiative muon decay

background to $\mu \to eX$, $X \to \gamma\gamma$ @ MEG

phase space | matrix element

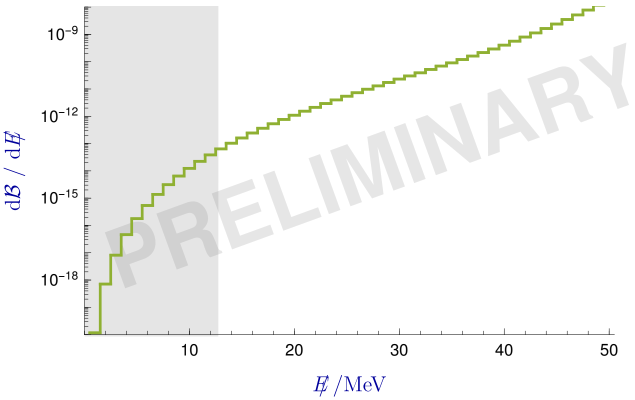
$$\underbrace{\int_{\mathrm{MEG}} \mathrm{d}\Phi_5}_{\text{numerical}}$$



numerical

trivial, analytic

MEG I cuts for photon $\sphericalangle(\gamma_1, \gamma_2) > 16.4^\circ$ at LO



less than $\mathcal{B} < 10^{-13}$ in the shaded region

No. active scales $\longrightarrow$

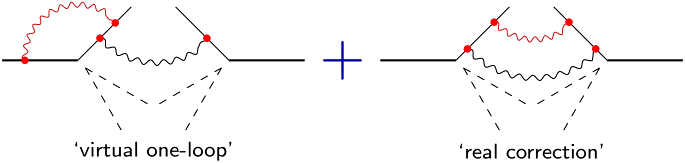
loop order $\downarrow$

	2	3	4	5
LO	'trivial'			$\mu \to \nu\bar{\nu}e\gamma\gamma$
NLO			$\mu \to \nu\bar{\nu}e\gamma$ $\tau \to \nu\bar{\nu}e\gamma$	$\mu \to \nu\bar{\nu}eee$
NNLO	$(\ell p \to \ell p)_{\text{red}}$ $(\mu e \to \mu e)_{\text{red}}$	$\mu \to \nu\bar{\nu}e$	$(\mu e \to \mu e)_{\text{full}}$	
NNNLO	$(\mu e \to \mu e)_{\text{red}}$			terra incognita

why higher-order?

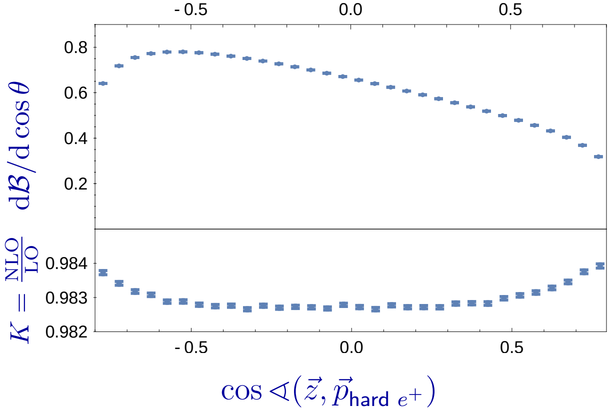
- reliable background studies (e.g. $\not{E} \to 0$, angular distr.)
- fiducial measurement $\to$ PDG value (e.g. τ decay)

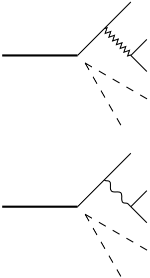
how higher-order?



- virtual and real correction separately divergent
- from LHC pheno: dimensional regularisation (no m_γ etc.)
- we use FKS subtraction [Frixione, Kunszt, Signer 1995]

light new physics would change angular distributions



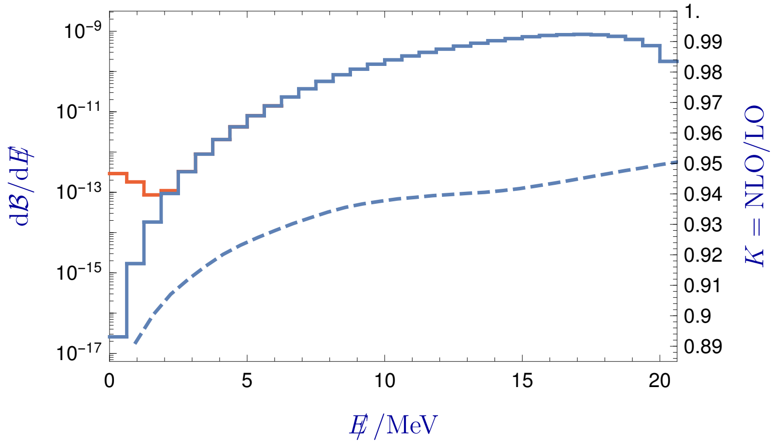


[cf. poster #73 by N. Berger]

[Pruna, Signer, YU 2016]

invisible energy spectrum MEG

exactly one photon $E_\gamma > 40\mathrm{MeV}$ in the detector. $\mathcal{B}_{\text{NP}} \simeq 4.2 \cdot 10^{-13}$



- large K (dashed) $\Rightarrow$ large logs $\sim \log \frac{\not{E}^2_{\text{cut}}}{M^2}$

[Pruna, Signer, YU 2017]

radiative τ decay

	$100 \times \mathcal{B}$		
BaBar	$1.847(54)$		[Oberhof et al 2015]
LO	1.834	0.2σ	
NLO	$1.645(1)$	3.7σ	[Fael, Mercolli, Passera 2015]
$\epsilon_{\mathsf{HO}} \times$ BaBar	$1.740(50)$	1.4σ	

- BaBar uses very restrictive cuts, unfolds using LO MC
- implement cuts at NLO and LO and compare

$$\epsilon_{\mathsf{HO}} = \frac{\epsilon_{\mathsf{NLO}}}{\epsilon_{\mathsf{LO}}} = \frac{\mathcal{B}_{\mathsf{NLO}}(10\,\mathrm{MeV})}{\mathcal{B}_{\mathsf{NLO}}(\textsc{BaBar})} \frac{\mathcal{B}_{\mathsf{LO}}(\textsc{BaBar})}{\mathcal{B}_{\mathsf{LO}}(10\,\mathrm{MeV})} = 0.92$$

- large logs $\sim \log \frac{E_{\mathsf{cut}}^2}{M^2}$

No. active scales →

loop order ↓

	2	3	4	5
LO	'trivial'			$\mu\to\nu\bar\nu e\gamma\gamma$
NLO			$\mu\to\nu\bar\nu e\gamma$ $\tau\to\nu\bar\nu e\gamma$	$\mu\to\nu\bar\nu eee$
NNLO	$(\ell p\to\ell p)_{\text{red}}$ $(\mu e\to\mu e)_{\text{red}}$	$\mu\to\nu\bar\nu e$	$(\mu e\to\mu e)_{\text{full}}$	
NNNLO	$(\mu e\to\mu e)_{\text{red}}$			terra incognita

(see also poster #72 by Tim Engel)

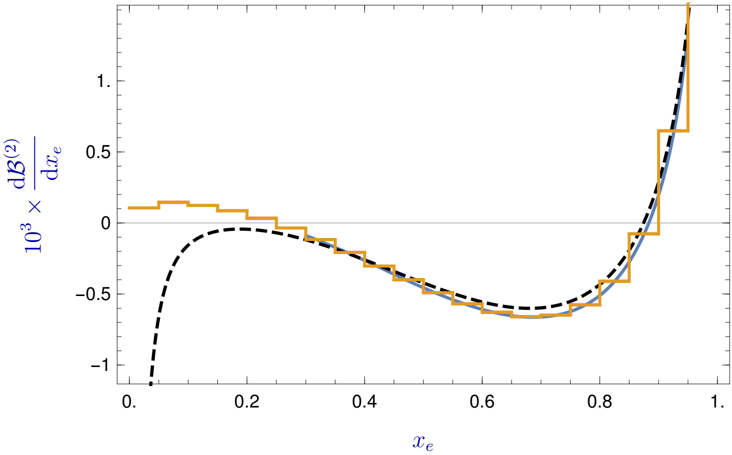
muon decay at NNLO: history calculations

- inclusive NNLO for G_F [Stuart, van Ritbergen 1999]
- logarithms $\log^{\{1,2\}} \frac{m_e^2}{m_\mu^2}$ of $\mathrm{d}\Gamma/\mathrm{d}E_e$ [Arbuzov, Czarnecki, Gaponenko 2002, Arbuzov, Melinkov 2002]
- fully inclusive, numeric energy spectrum [Anastasiou, Melnikov, Petriello 2005]

how?

- analytic two-loop integrals [Chen 2018] and form factors [Engel, Gnendiger, Signer, YU 2018]
- fully differential Monte Carlo [Engel, Signer YU, 2019] using FKS^2

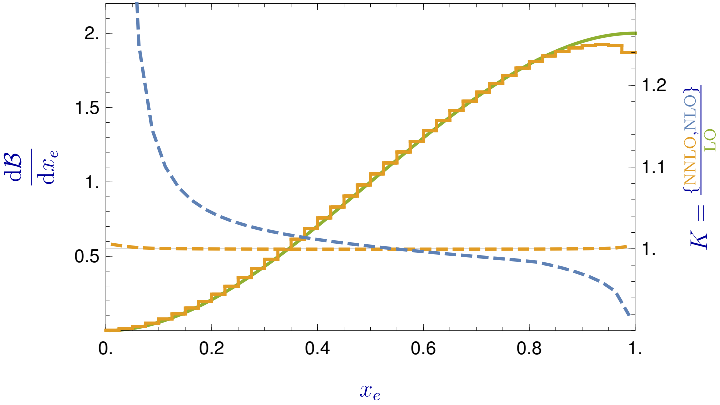
electron energy spectrum



our result, Anastasiou et al, logarithms

more exclusive spectrum

total photon energy within $\cos\sphericalangle(\vec{p}_e, p_\gamma) > 0.8$ is $\sum E_\gamma < 10\mathrm{MeV}$



NNLO K-factor dashed, large logs in tail $\Rightarrow$ resummation

No. active scales →

loop order ↓

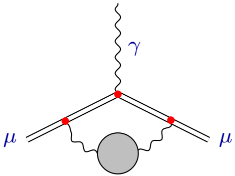
	2	3	4	5
LO	‘trivial’			$\mu \to \nu\bar{\nu}e\gamma\gamma$
NLO			$\mu \to \nu\bar{\nu}e\gamma$ $\tau \to \nu\bar{\nu}e\gamma$	$\mu \to \nu\bar{\nu}eee$
NNLO	$(\ell p \to \ell p)_{\text{red}}$ $(\mu e \to \mu e)_{\text{red}}$	$\mu \to \nu\bar{\nu}e$	$(\mu e \to \mu e)_{\text{full}}$	
NNNLO	$(\mu e \to \mu e)_{\text{red}}$			terra incognita

(see also poster #72 by Tim Engel)

- measure HVP using t-channel μ-e scattering at 10ppm

$$a_\mu^{\text{HLO}} = \frac{\alpha}{\pi}\int_0^1 \mathrm{d}x(1-x)\,\underbrace{\Delta\alpha_{\text{had}}\Big(\frac{x^2}{x-1}m_\mu^2\Big)}_{\propto \mathrm{d}\sigma/\mathrm{d}t}$$

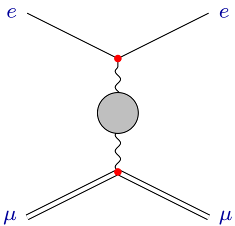
[Carloni Calame et al. 2015, Abbiendi et al 2016]



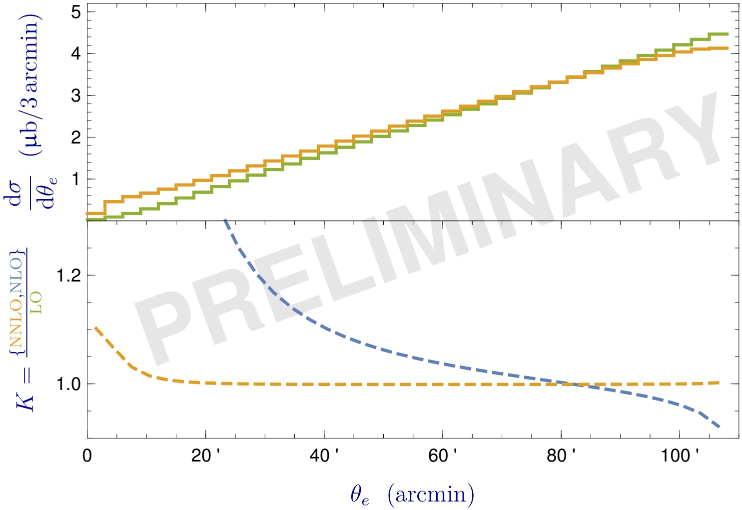
- proposed experiment @ CERN's M2

[Matteuzzi, YU et al 2019 (LOI)]

- huge theory effort [Alacevich et al 2018, Mastrolia et al 2017 and 2018, Fael, Passera 2019]



⇒ NNLO emission-from-e-line-only (largest part) [Banerjee, Engel, Signer, YU soon] using [Bernreuther et al. 2004] with FKS^2

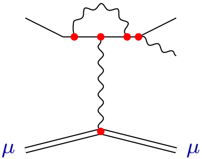

PRELIMINARY
$\frac{d\sigma}{d\theta_e}$ (μb/3 arcmin)
5
4
3
2
1
$K = \frac{\{NNLO, NLO\}}{LO}$
1.2
1.0
0
20 '
40 '
60 '
80 '
100 '
θ_e (arcmin)

- extract form factors at high precision at low Q^2 (MESA, MUSE, Prad, QWeak, ...)

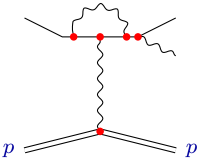
[cf. talks by Golossanov (#141) and Antognini (#63)]

$\Rightarrow$ QED corrections (cf. at NLO [Gramolin 2014, Akushevich et al. 2015] and NNLO [Bucoveanu, Spiesberger 2018])

- emission-from-ℓ-line-only relatively straight-forward

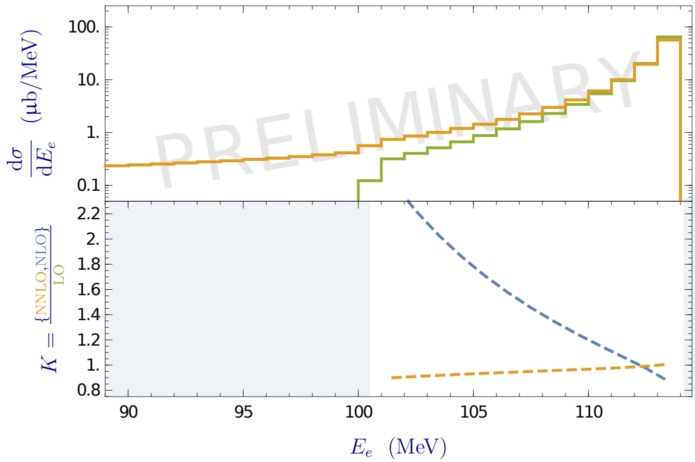


$\propto \bar{u}(m_\mu)\gamma_\mu u(m_\mu)$



$\propto \bar{u}(m_p)\Big(F_1\gamma^\mu + F_2 \frac{i\sigma^{\mu\nu}q_\nu}{2m}\Big)u(m_p)$

MUSE cuts $20° < \theta < 100°$, $p_{\text{in}} = 115$ MeV



- shaded region kinematically forbidden at tree-level

No. active scales →

loop order ↓	2	3	4	5
LO	'trivial'			$\mu \to \nu\bar{\nu}e\gamma\gamma$
NLO			$\mu \to \nu\bar{\nu}e\gamma$ $\tau \to \nu\bar{\nu}e\gamma$	$\mu \to \nu\bar{\nu}eee$
NNLO	$(\ell p \to \ell p)_{\text{red}}$ $(\mu e \to \mu e)_{\text{red}}$	$\mu \to \nu\bar{\nu}e$	$(\mu e \to \mu e)_{\text{full}}$	
NNNLO	$(\mu e \to \mu e)_{\text{red}}$			terra incognita

beyond NNLO

for emission-from-e-line-only in μ-e scattering

- extension FKS^ℓ is known
- three-loop $\gamma^* \to Q\bar{Q}$ being calculated [Henn, Smirnov, Smirnov, Steinhauser 2016, Lee, Smirnov, Smirnov, Steinhauser 2018, Ablinger, Blümlein, Marquard, Rana, Schneider 2018...]
- adapt two-loop $\gamma^* \to qqg$ [Garland, Gehrmann, Glover, Koukoutsakis, Remiddi 2002]

$\Rightarrow$ no conceptual problems for $\mathrm{N}^3\mathrm{LO}$

- but many practical problems

conclusion

- NNLO (and beyond) achievable for many processes
- numerical instabilities along the way

outlook

- resummation for tails $\Rightarrow$ integrate YFS PS !
- add $\ell p \to \ell p$, $e\mu \to e\mu$, $\mu \to eJ$, $ee \to ee$, $ee \to \gamma\gamma$ ...
- bundle all of this into one tool

...introducing McMule

(**M**onte **C**arlo for **Mu**ons and other **le**ptons)

P. Banerjee, T. Engel, A. Signer, YU, ...