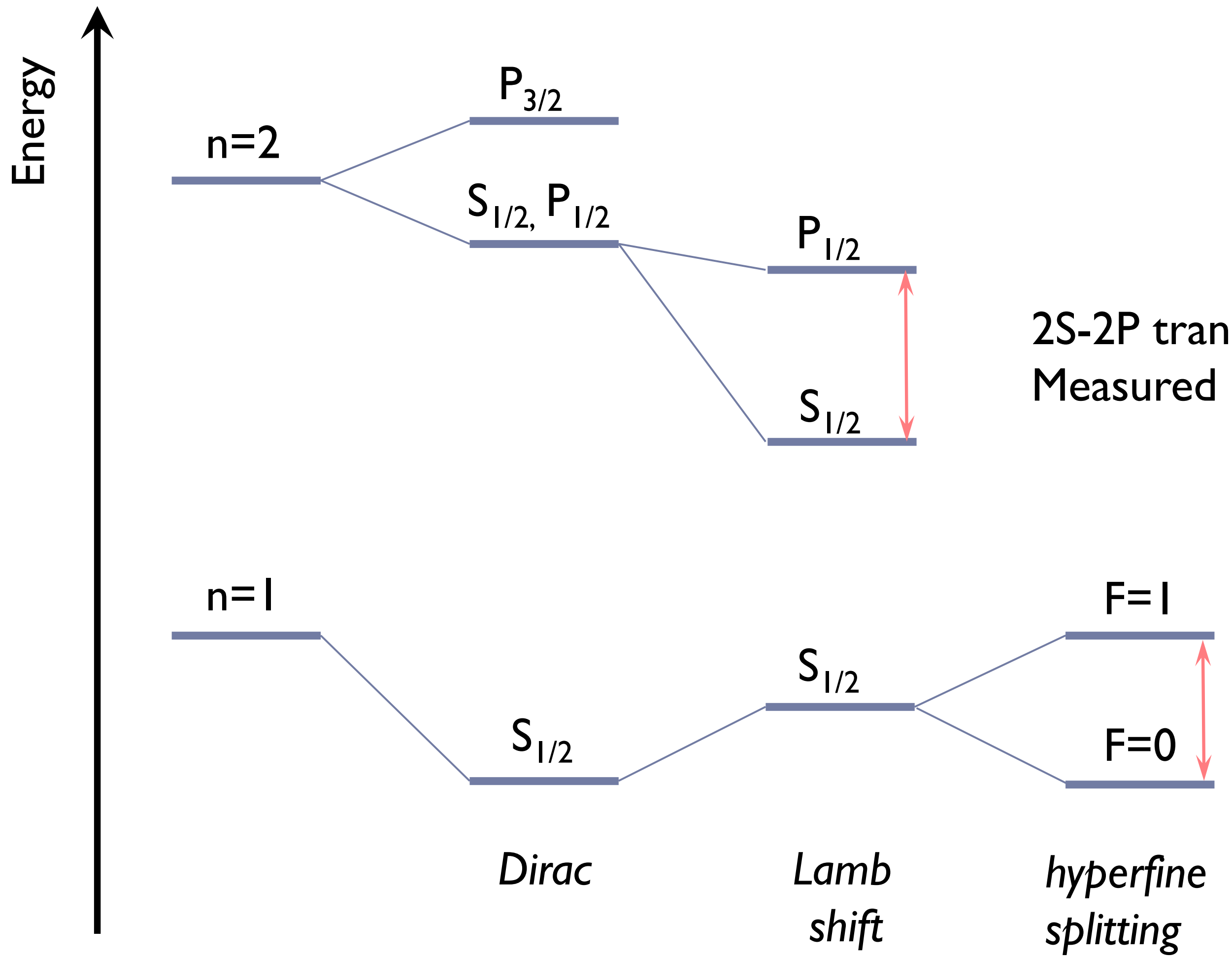
Optical cavity simulations for the HyperMu experiment

M. Marszalek

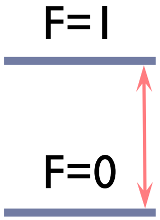
Hyperfine splitting in muonic hydrogen



2S-2P transition → charge radius
Measured in 2010.

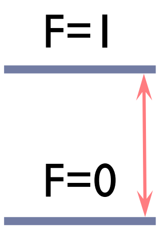
Ground state hyperfine splitting → Zemach radius
Work in progress!

Contributions to the hyperfine splitting



$$\Delta E_{\mathrm{hfs}} = E_F \left(1 + \Delta_{\mathrm{QED}} + \Delta_{\mathrm{TPE}} + \Delta_{\mathrm{weak+hVP}}\right)$$

Contributions to the hyperfine splitting

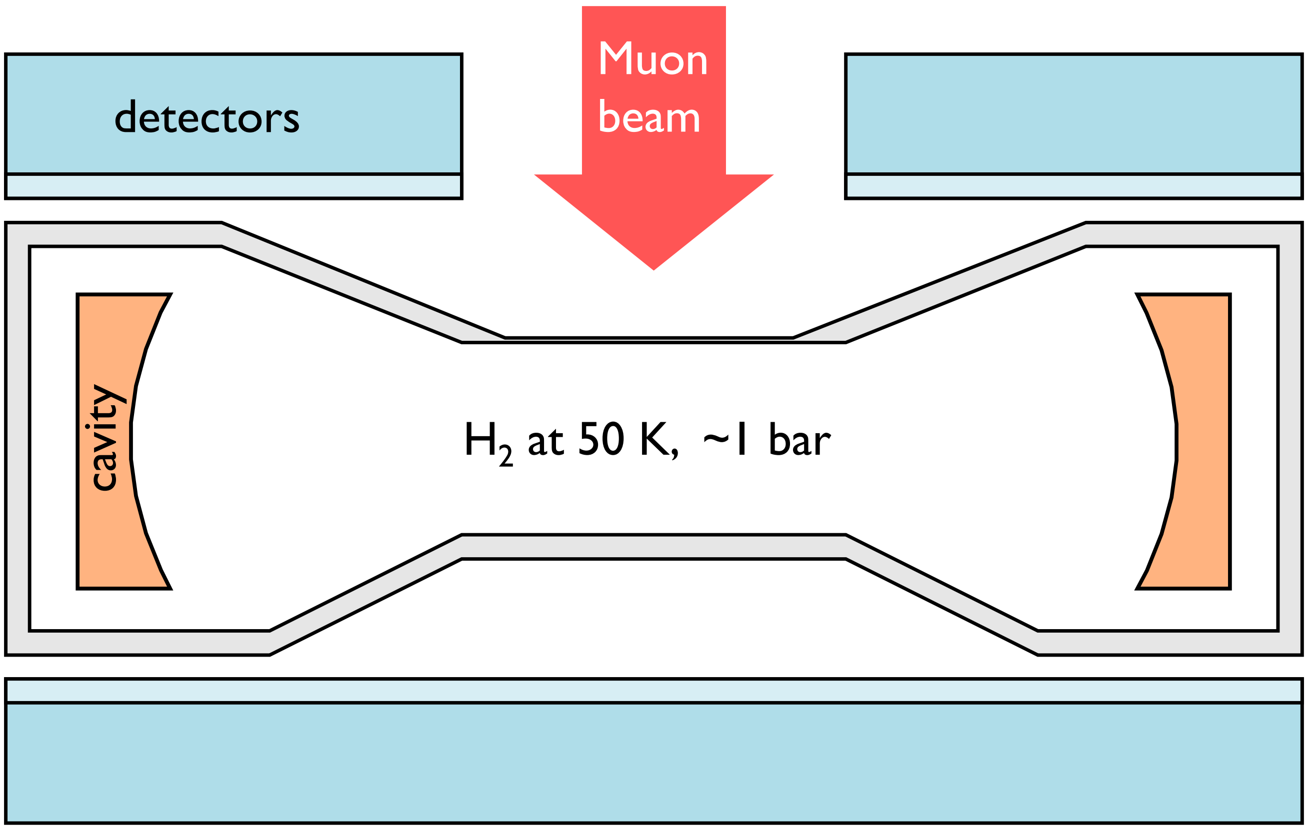


$$\Delta E_{\text{hfs}} = E_F \left(1 + \Delta_{\text{QED}} + \Delta_{\text{TPE}} + \Delta_{\text{weak+hVP}}\right)$$

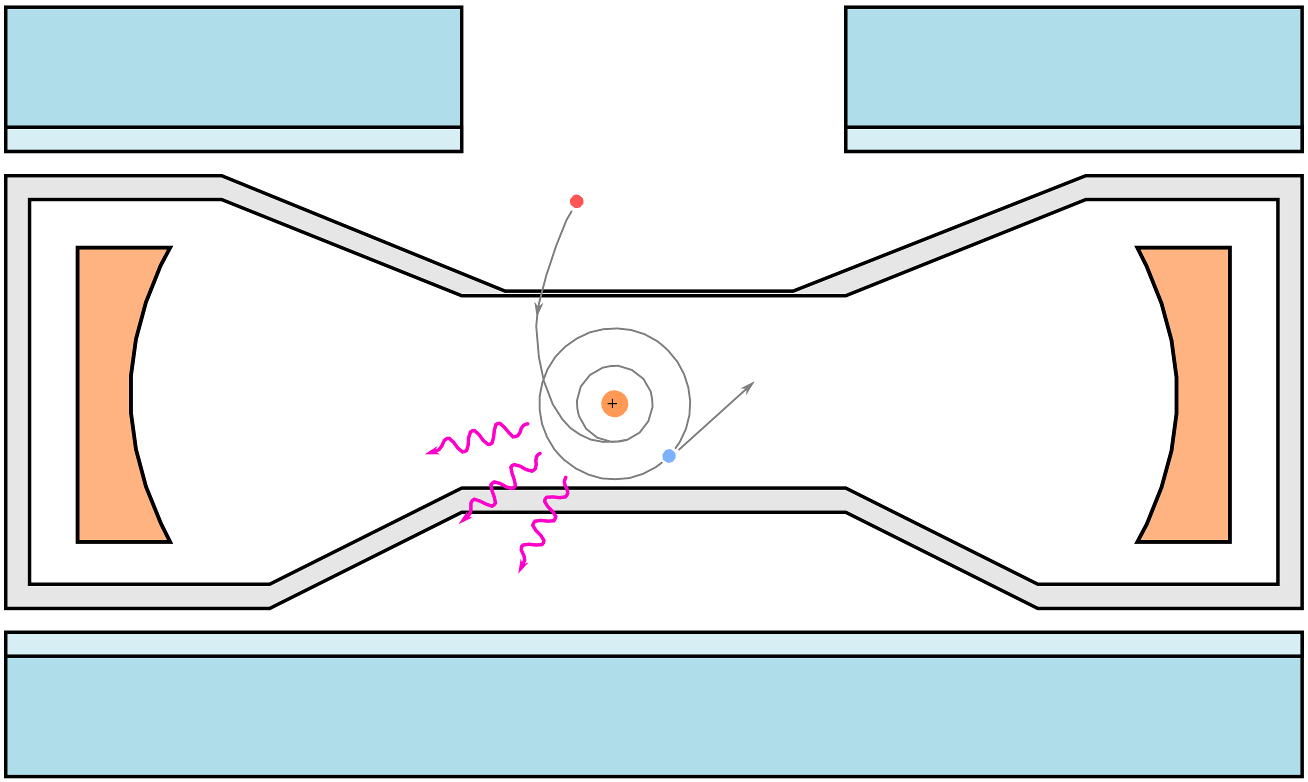
$$\Delta_{\text{TPE}} = \Delta_{\text{Z}} + \Delta_{\text{recoil}} + \Delta_{\text{pol}}$$

$$\Delta_{\text{Z}} = \frac{8Z\alpha m_r}{\pi} \int_0^\infty \frac{dQ}{Q} \left(G_E(Q^2) \frac{G_{M(Q^2)}}{1+\kappa_p} - 1 \right) = -2(Z\alpha) m_r R_Z$$

HyperMu in a nutshell



HyperMu in a nutshell



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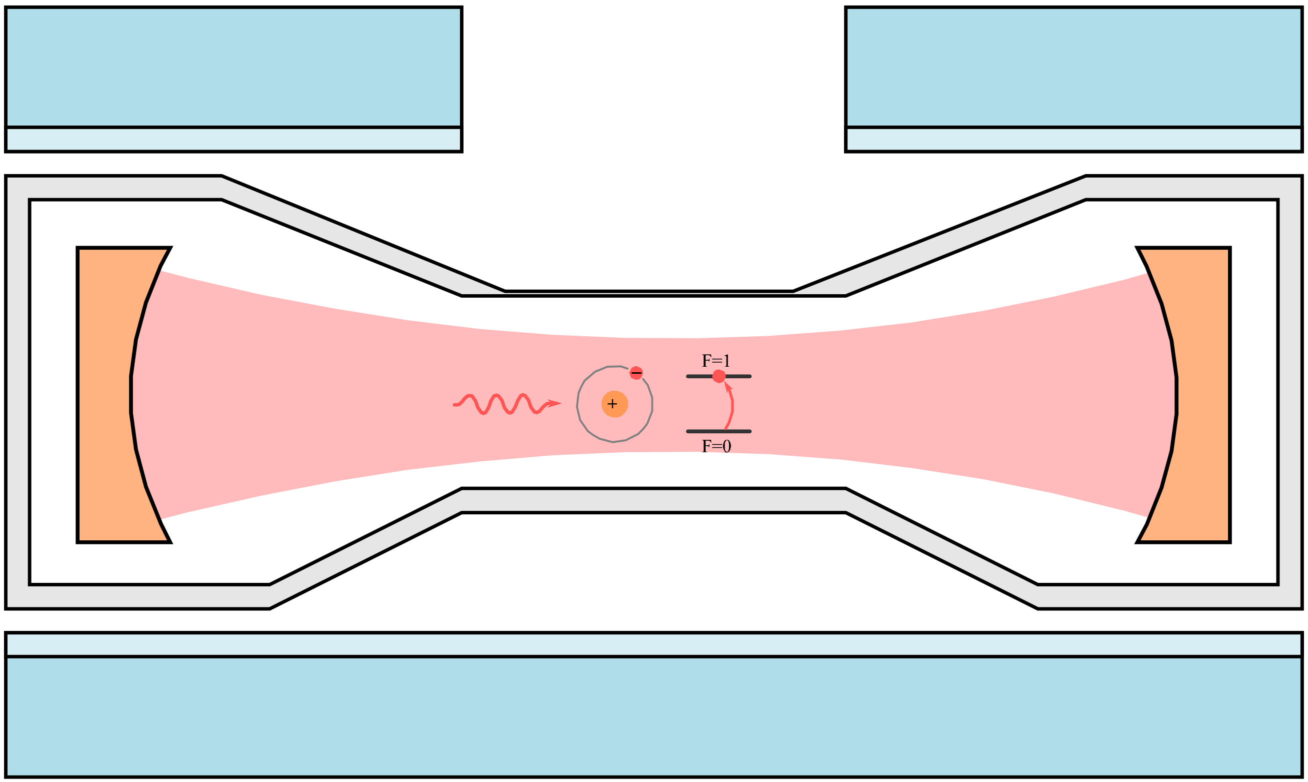

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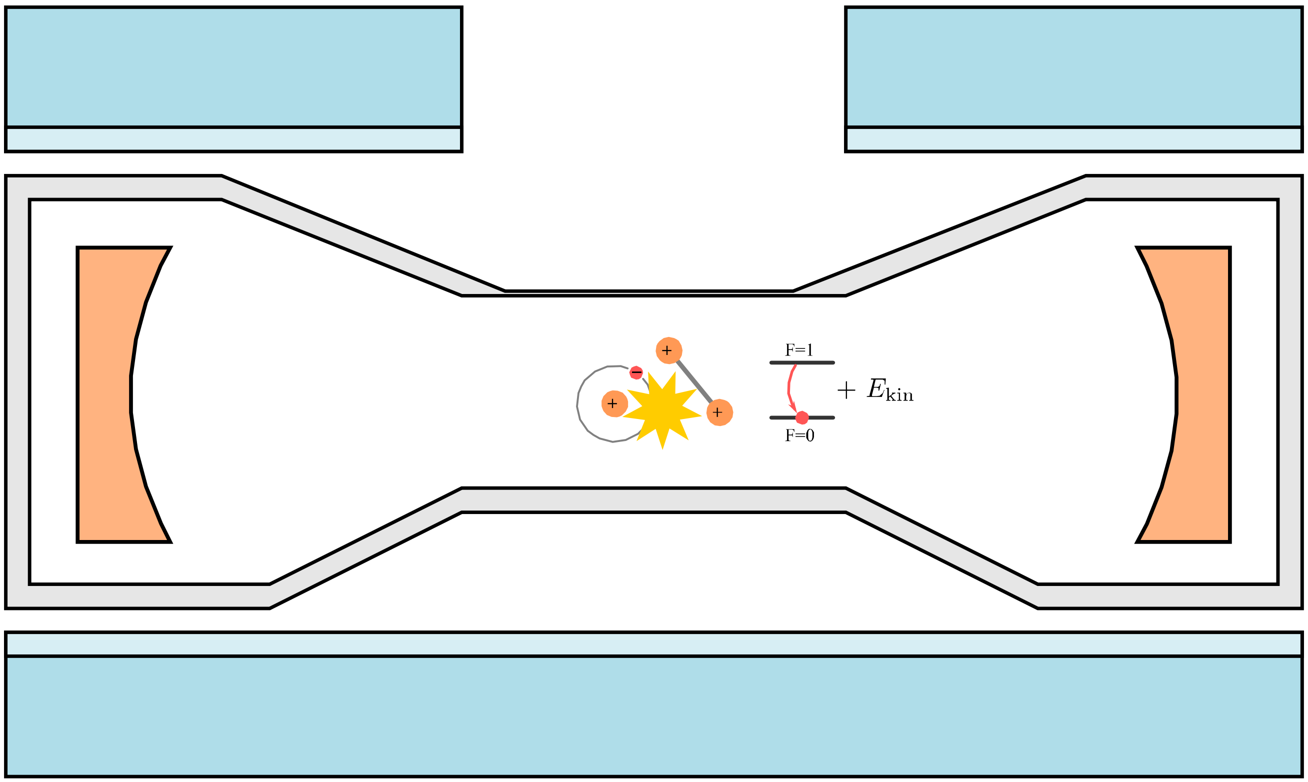

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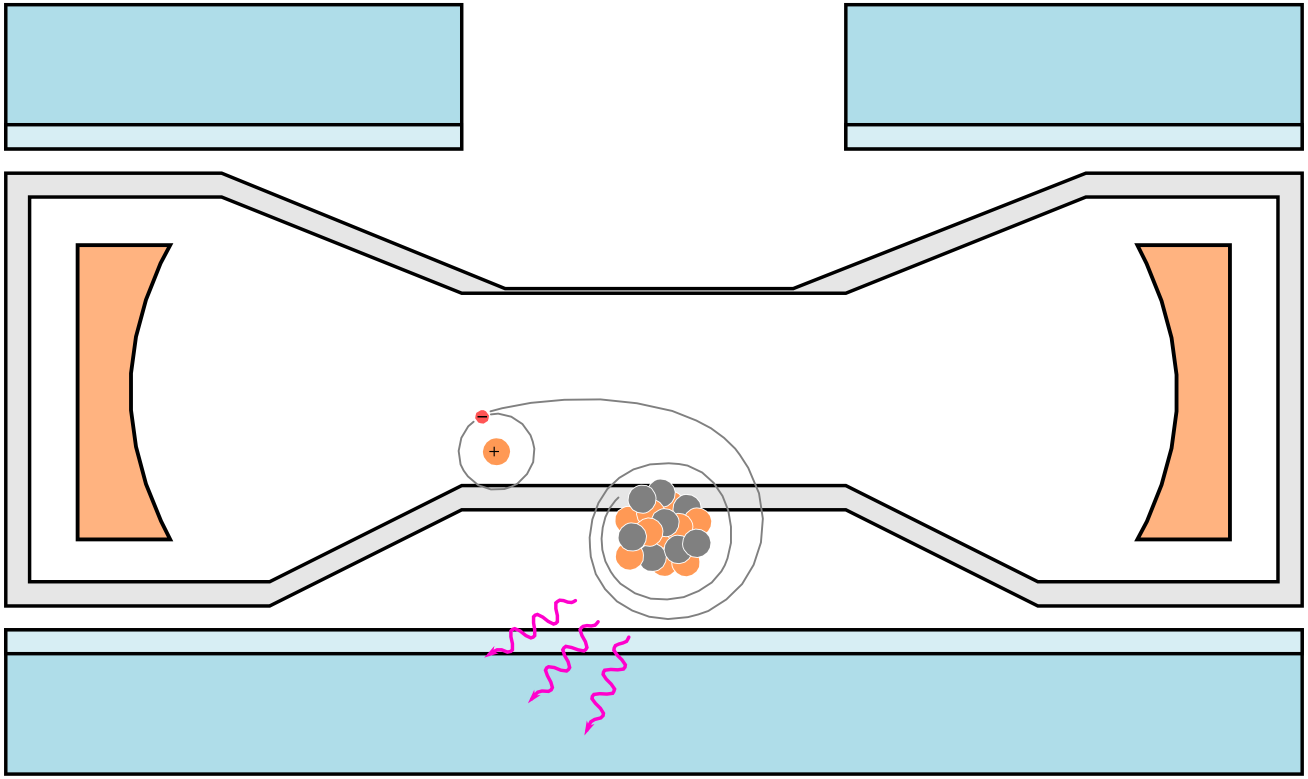
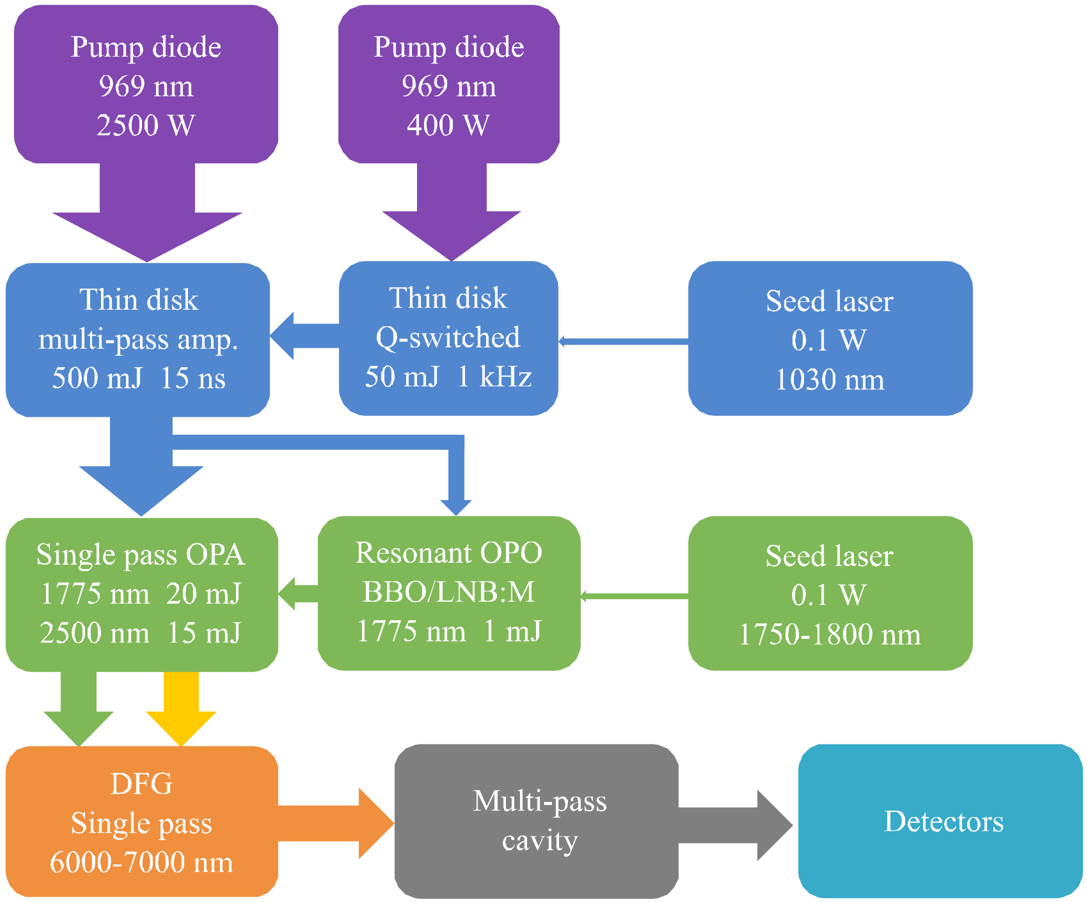
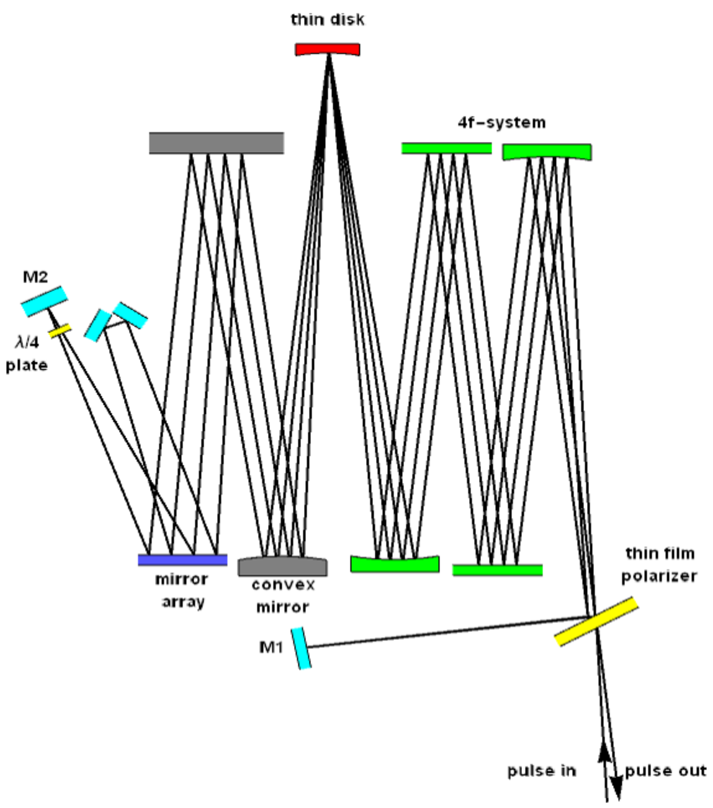


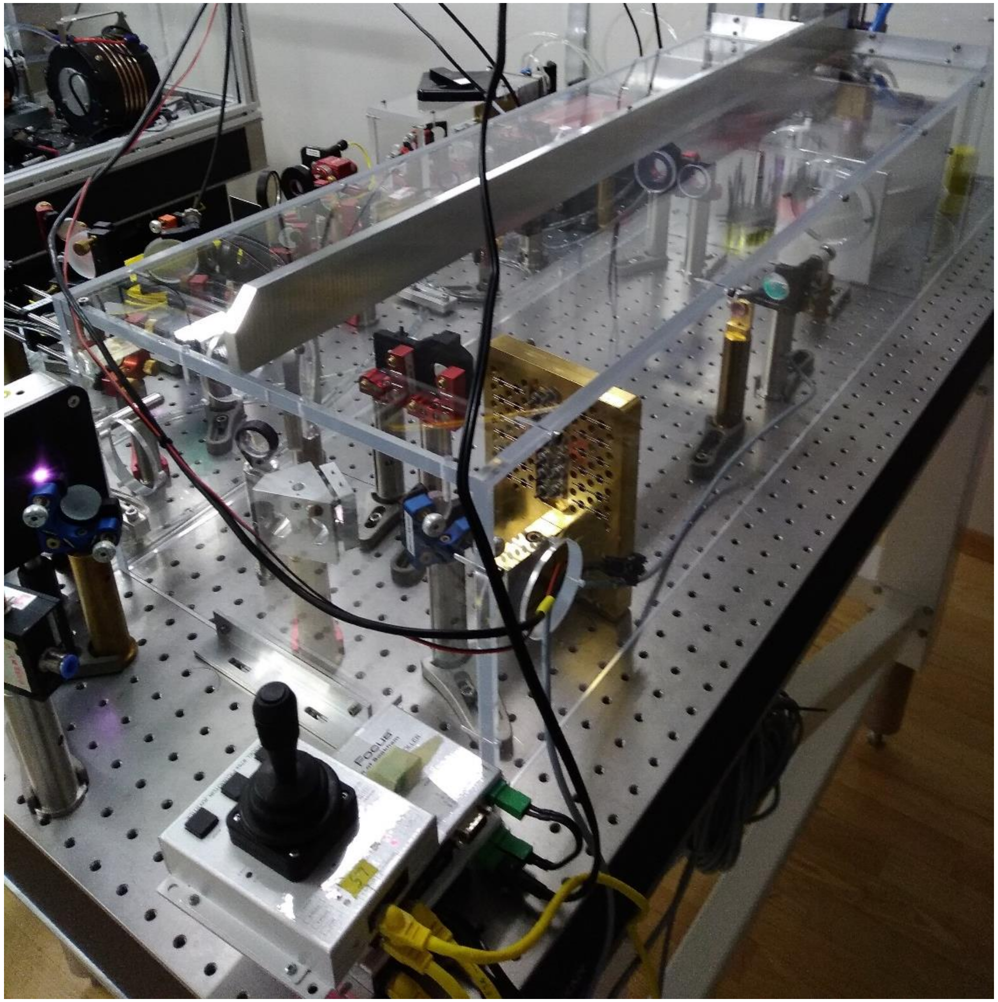
HyperMu in a nutshell



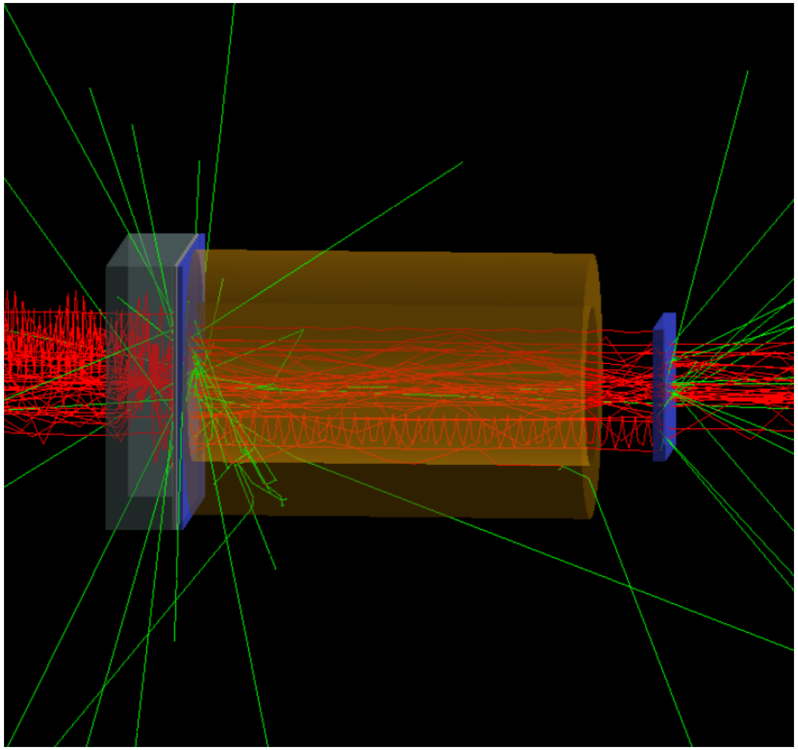
What’s up at PSI?

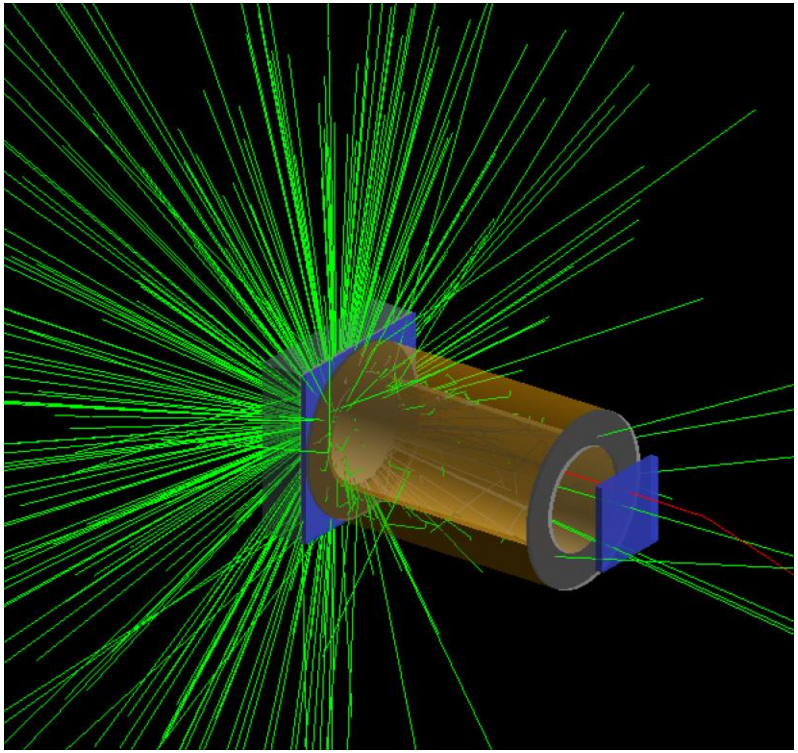
The laser system at ETH – Manu & Karsten



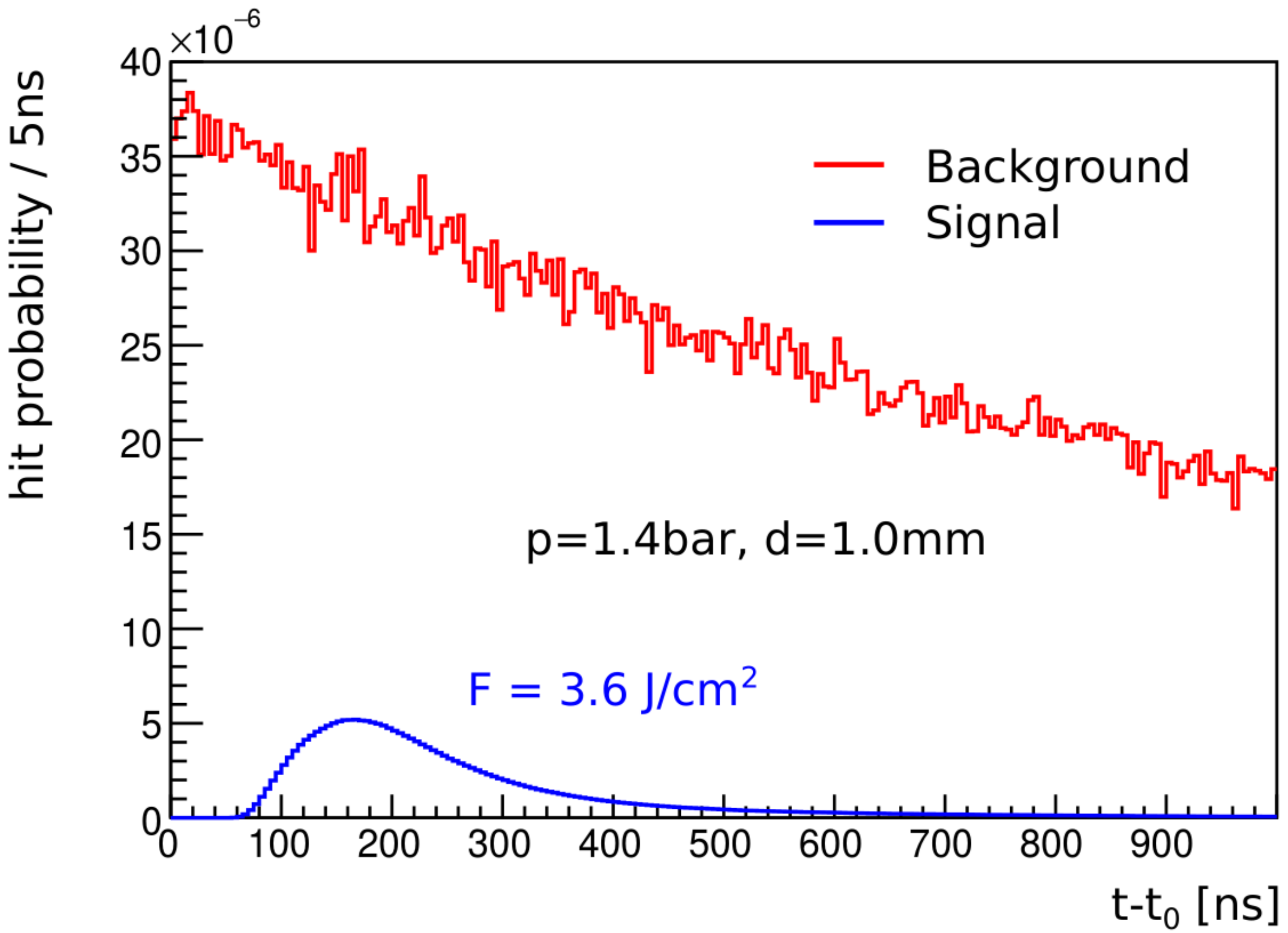


Detectors - Laura





μp atom diffusion - Jonas

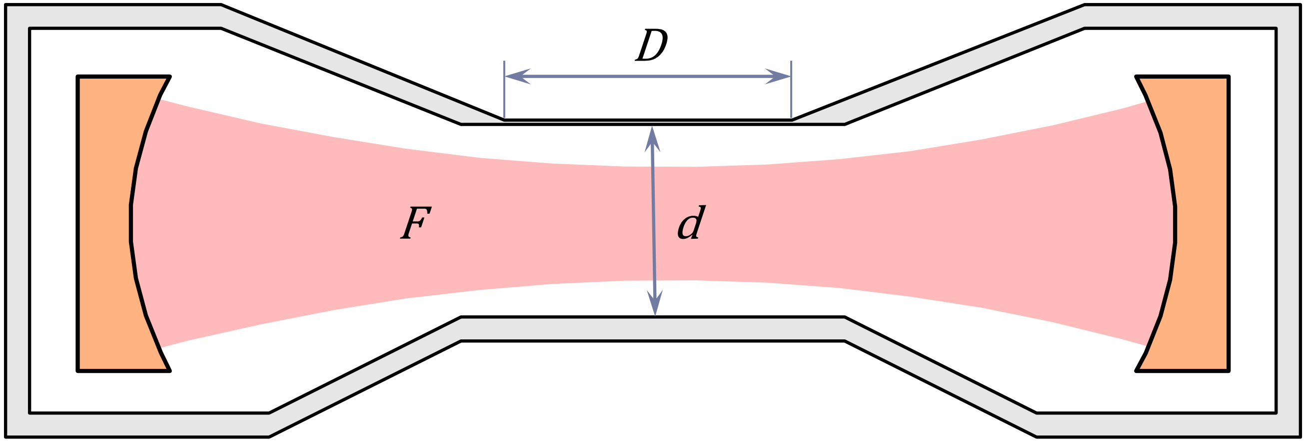


The cavity - Mirek

- Find the best geometry
- Optimize its parameters
- Estimate the performance

$F > 5\ \mathrm{J \cdot cm^{-2}}$

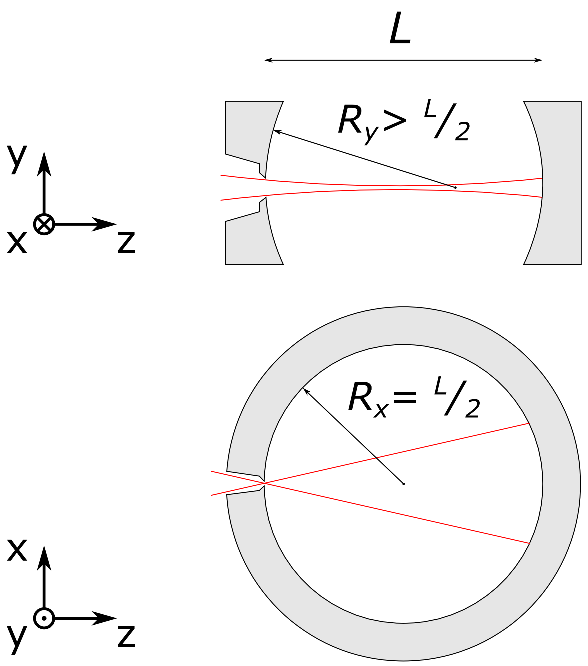
$d \leq 1\ \mathrm{mm}$



Simulations of the cavity

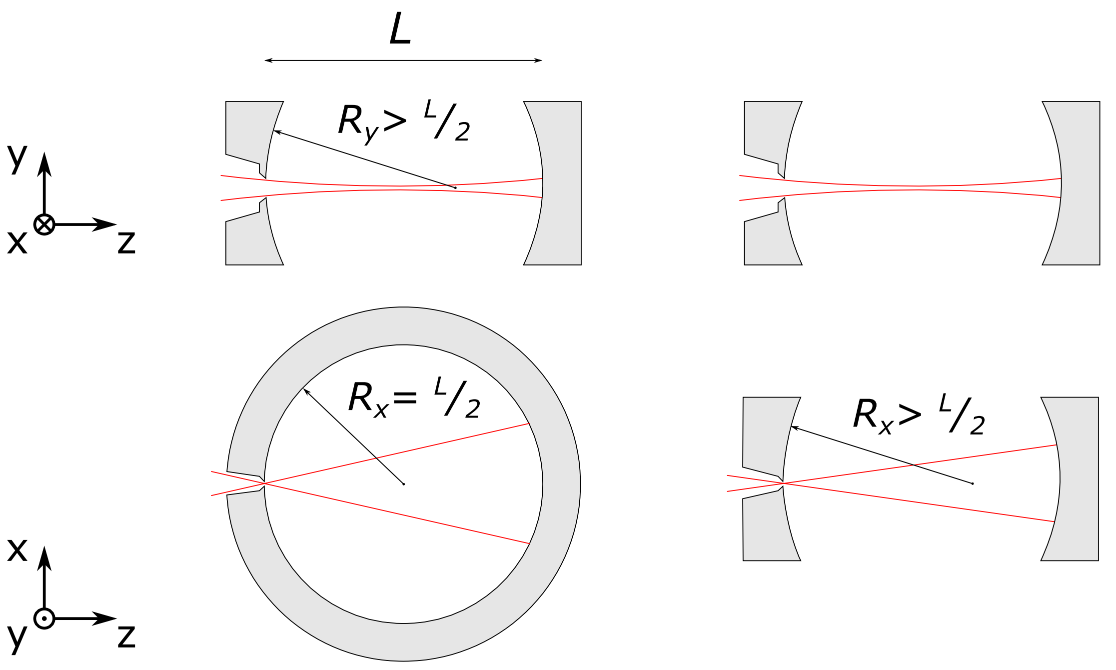
- Cavity geometries
- Methods of simulation
- Figures of merit
- Comparison of designs
- Summary

Cavity geometries



toroidal

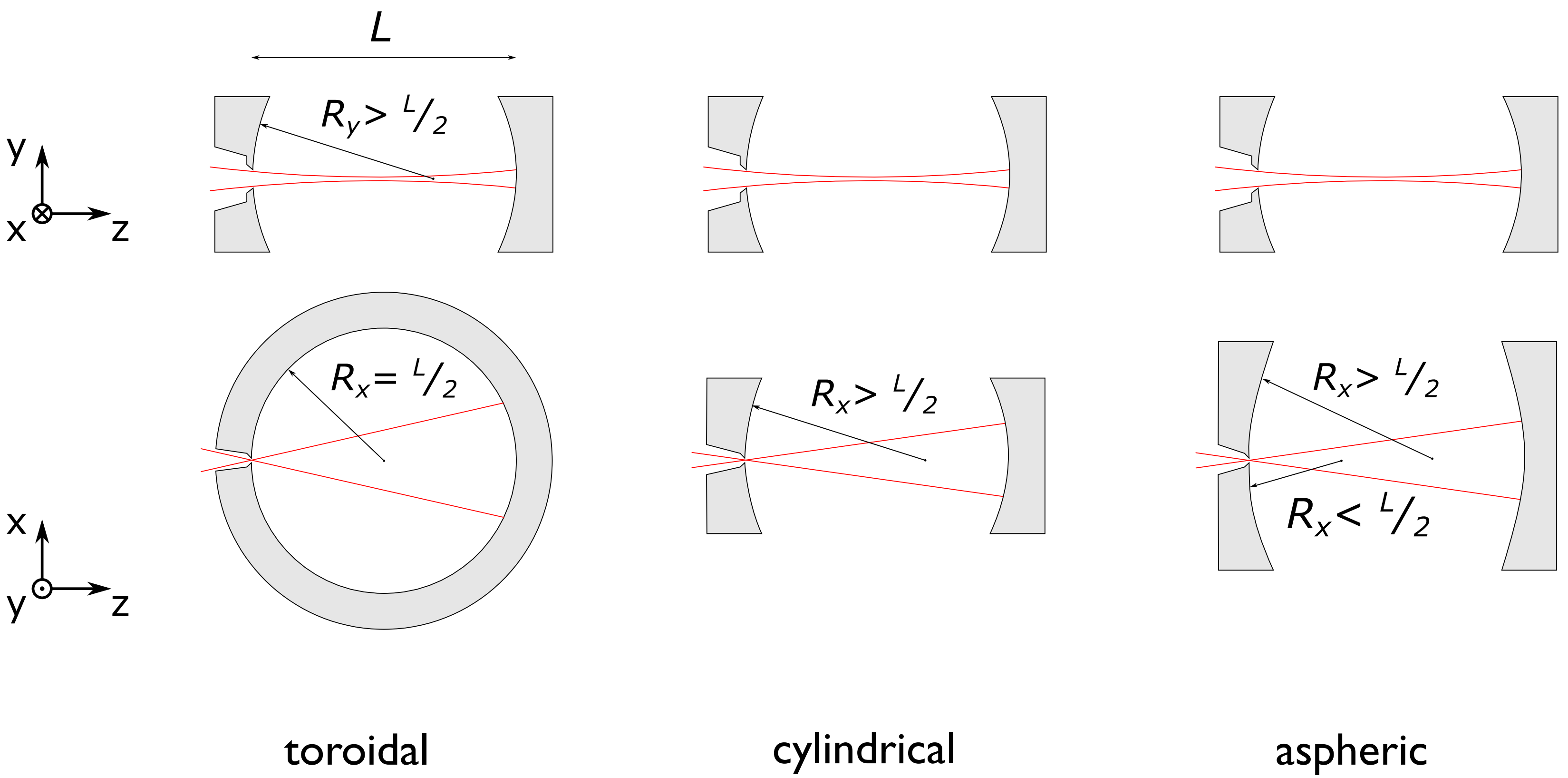
Cavity geometries



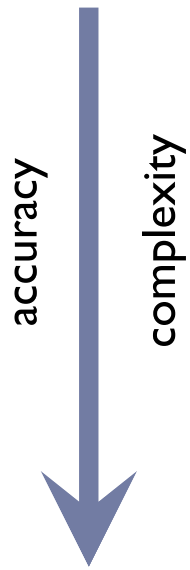
toroidal

cylindrical

Cavity geometries

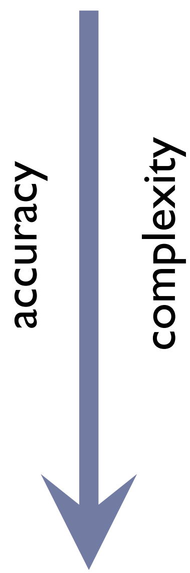


Methods of simulation



- Gaussian beam propagation
- Ray tracing
- Diffraction integrals
- Maxwell equations

Methods of simulation



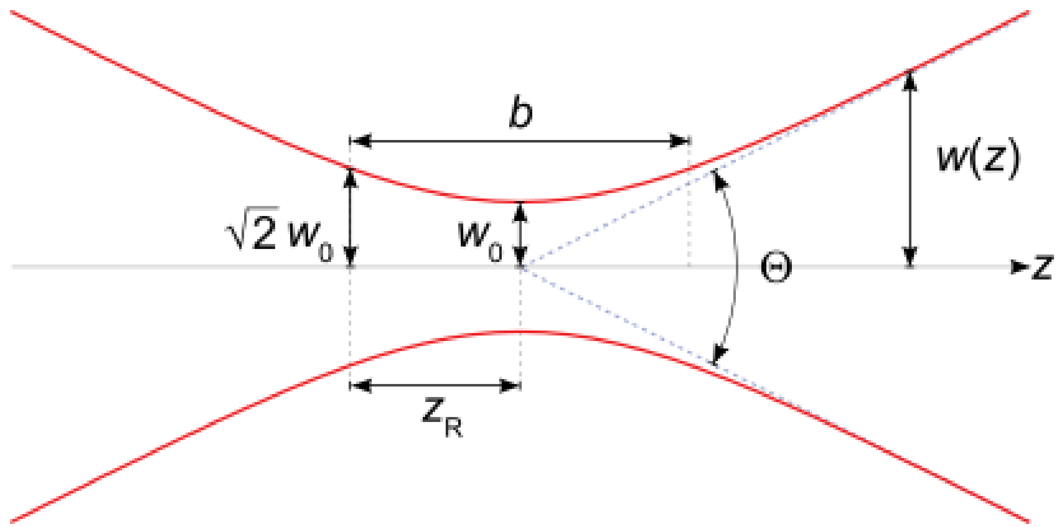
- **Gaussian beam propagation**
- **Ray tracing**
- **Diffraction integrals**
- Maxwell equations

Gaussian beams

$$w^2(z) = w_0^2\left(1 + \left(\frac{z}{z_R}\right)^2\right)$$

$$z_R = \frac{\pi w_0^2}{\lambda}$$

$$\tan\theta = \frac{\lambda}{\pi w_0}$$



Gaussian beams

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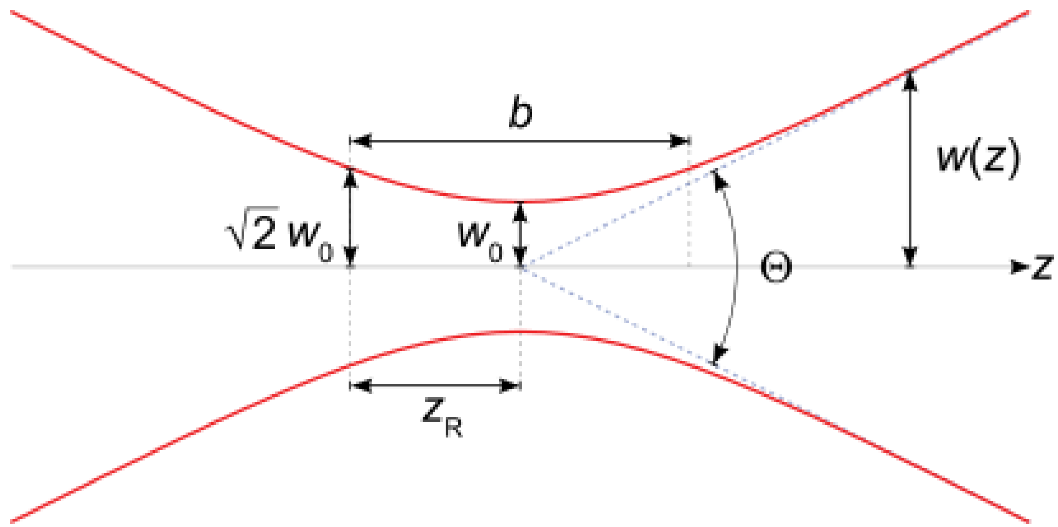
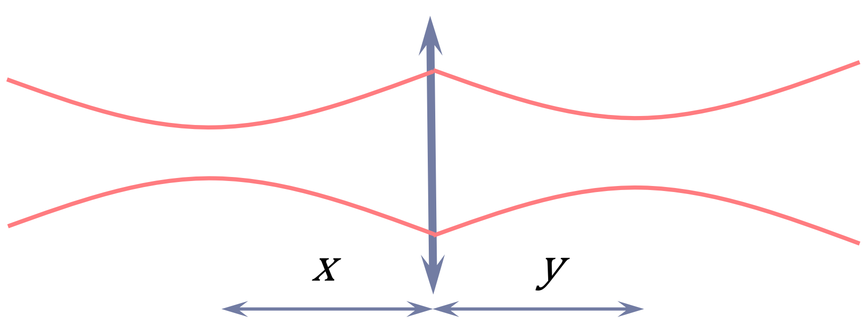


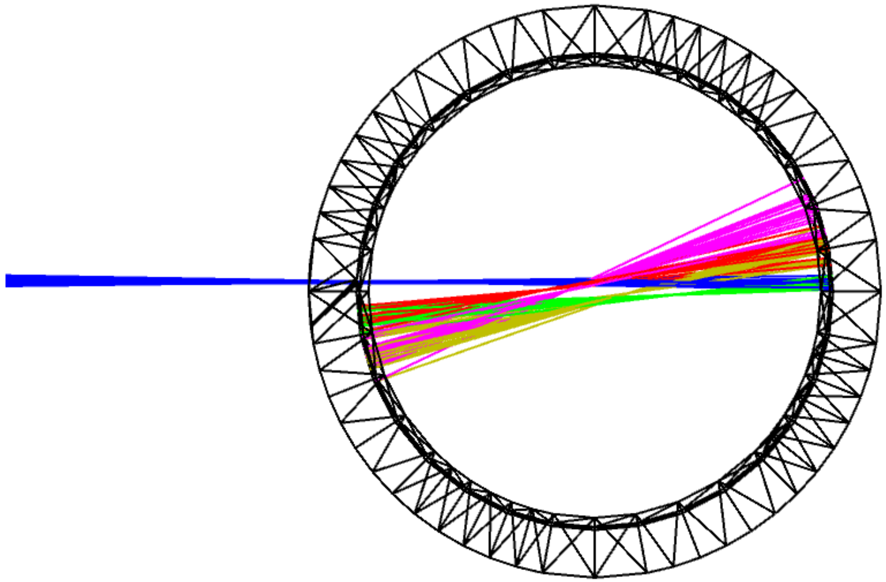
Self formula:

$$\frac{1}{f} = \frac{1}{x + \frac{z_R^2}{x - f}} + \frac{1}{y}$$

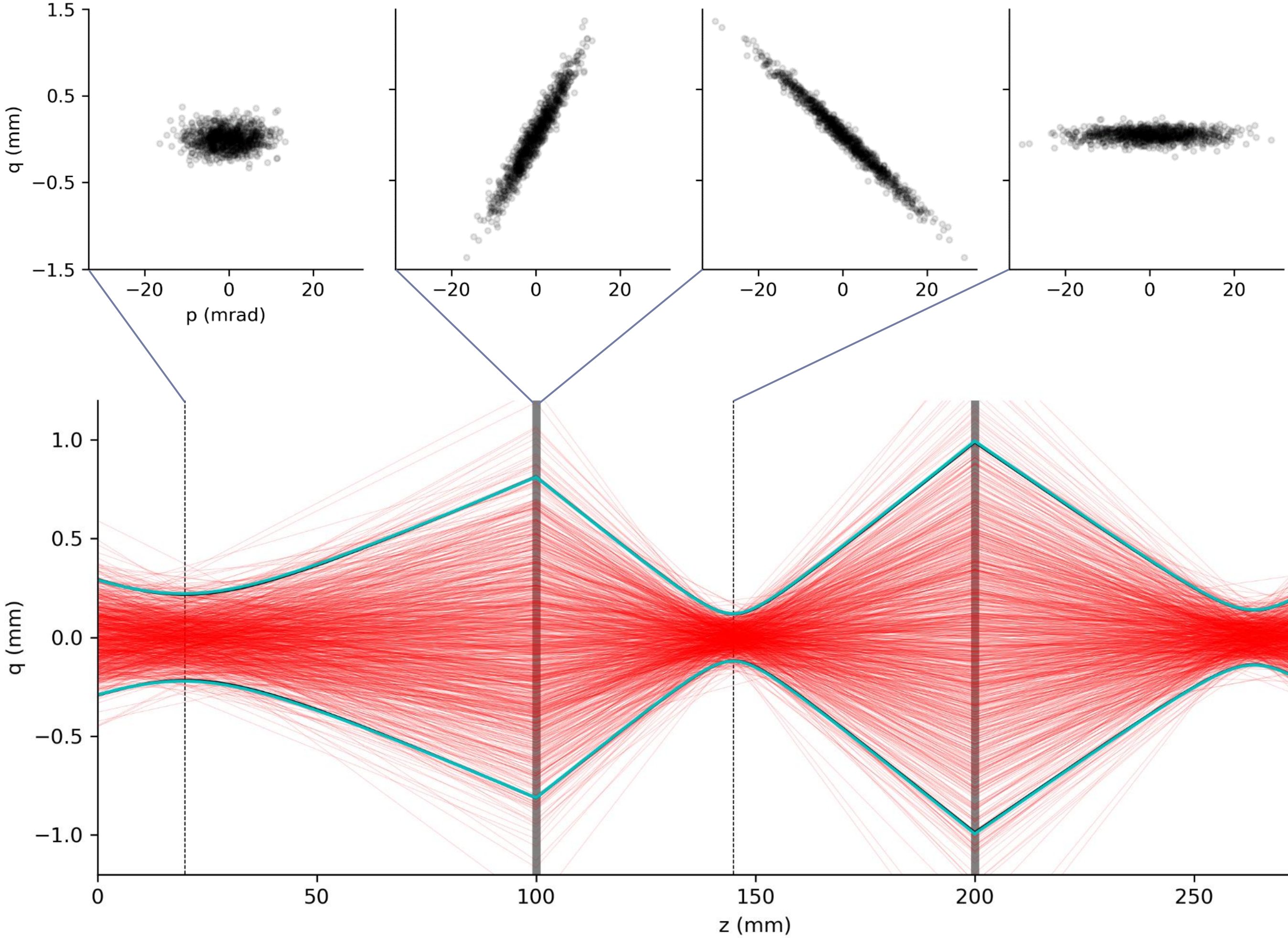


Ray tracing

- Fast and simple
- Geometrical optics
- Is it accurate enough?



Gaussian beam in the phase space



$$\sigma(q) = \frac{1}{2} w_0$$

$$\sigma(p) = \frac{1}{2} \frac{\lambda}{\pi w_0}$$

Diffraction integrals

Fresnel integral in 1D

$$E(x',z)=\frac{1}{i\sqrt{\lambda z}}\exp\left(\frac{2\pi i z}{\lambda}\right)\exp\left(\frac{\pi i}{\lambda z}x'^2\right)\int_{-\infty}^{+\infty}E(x,0)\exp\left(\frac{\pi i}{\lambda z}x^2\right)\exp\left(\frac{-2\pi i}{\lambda z}xx'\right)dx$$

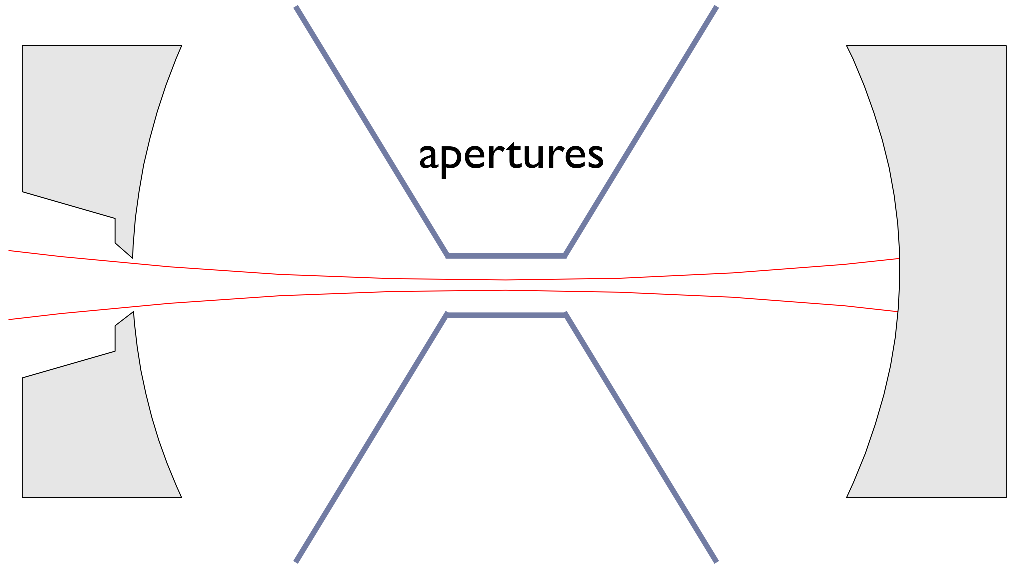
Diffraction integrals

Fresnel integral in 1D

$$E(x',z) = \frac{1}{i\sqrt{\lambda z}} \exp\left(\frac{2\pi i z}{\lambda}\right) \exp\left(\frac{\pi i}{\lambda z} x'^2\right) \int_{-\infty}^{+\infty} E(x,0) \exp\left(\frac{\pi i}{\lambda z} x^2\right) \exp\left(\frac{-2\pi i}{\lambda z} x x'\right) dx$$

- Scalar approximation
- Breaks down at high angles
- Can be numerically unstable

Figures of merit – losses in the cavity



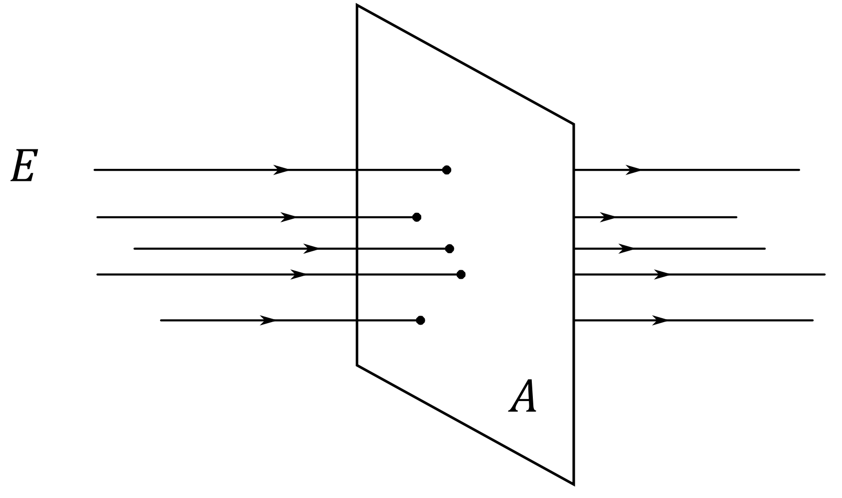
$$L_H \approx \frac{h_x}{2\pi R_x}$$

$$L_M = 1 - R_M$$

$$E_{\text{tot}} = \frac{E_0}{1-R} = \frac{E_0}{L}$$

$$L \approx L_M + L_H + L_A$$

Figures of merit – average fluence



$$F = \frac{\partial E}{\partial A}$$

$$P = 1 - \exp\left(-\frac{F}{F_s}\right)$$

For $F < F_s$: $P \propto F$

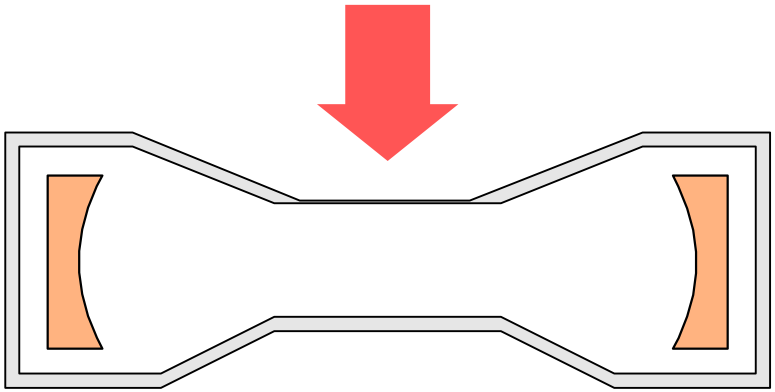
Figures of merit – signal-to-noise ratio

$$\text{SNR} = \frac{S}{\sqrt{B}}$$

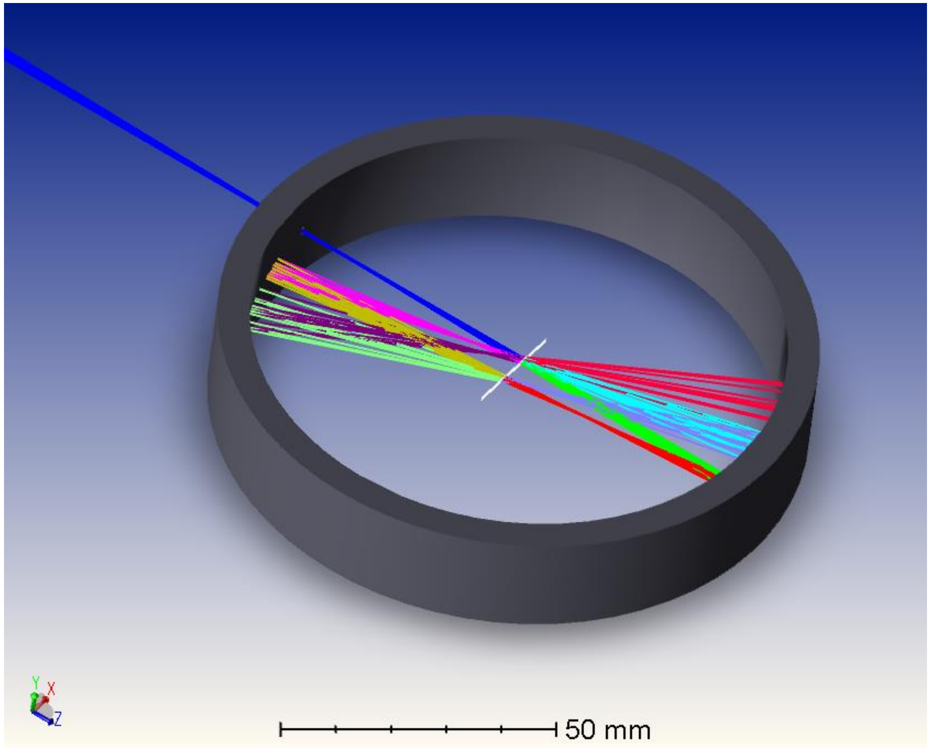
$$S \propto FA$$

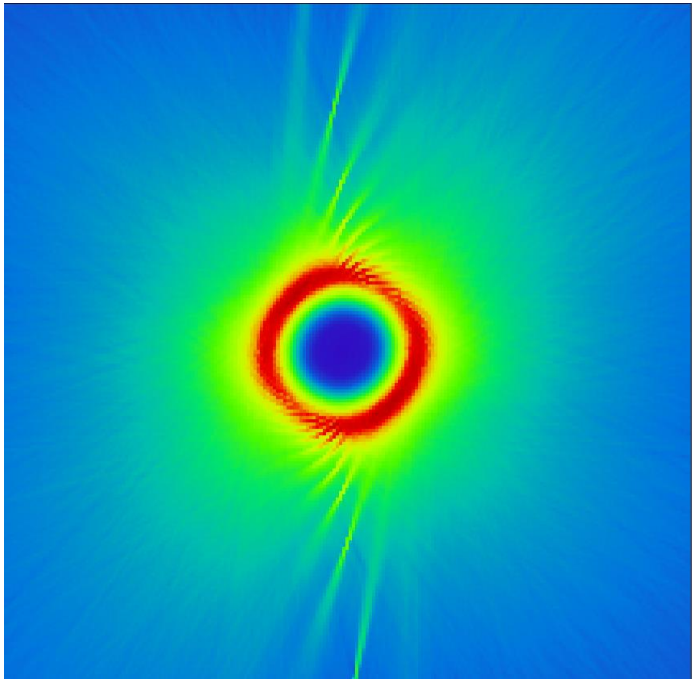
$$B \propto A$$

$$\text{SNR} \propto F\sqrt{A}$$

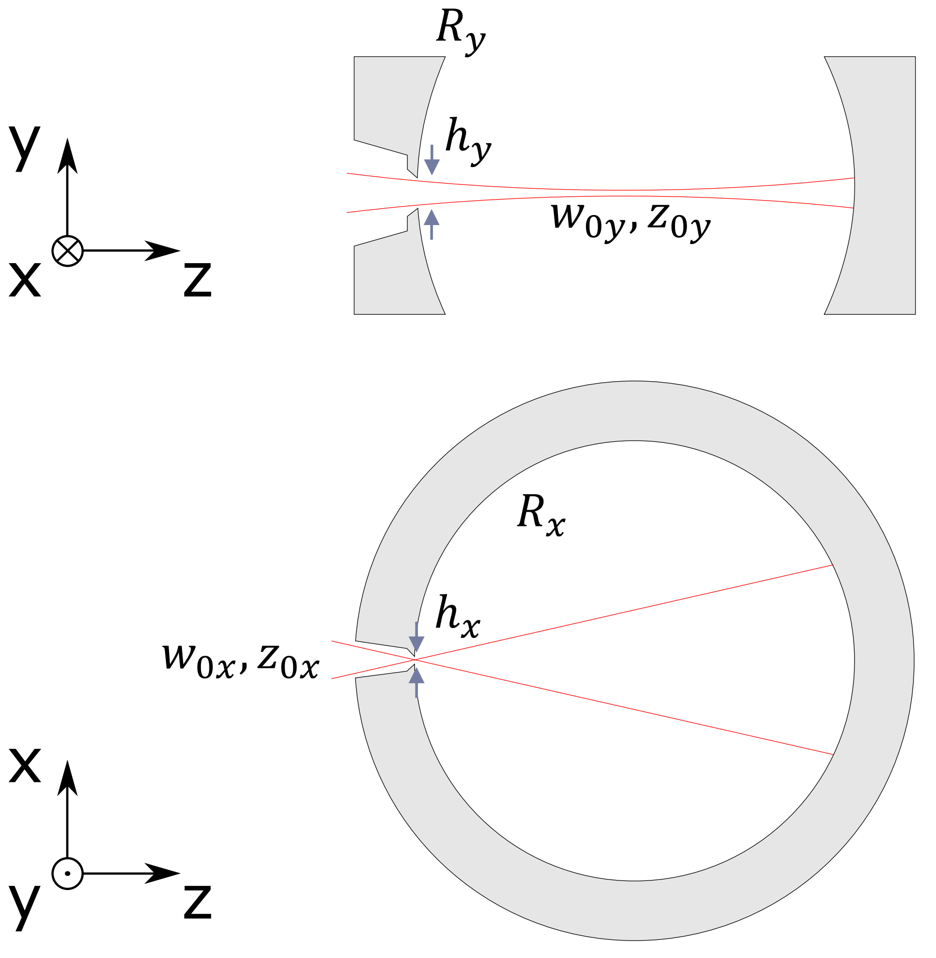


Toroidal cavity

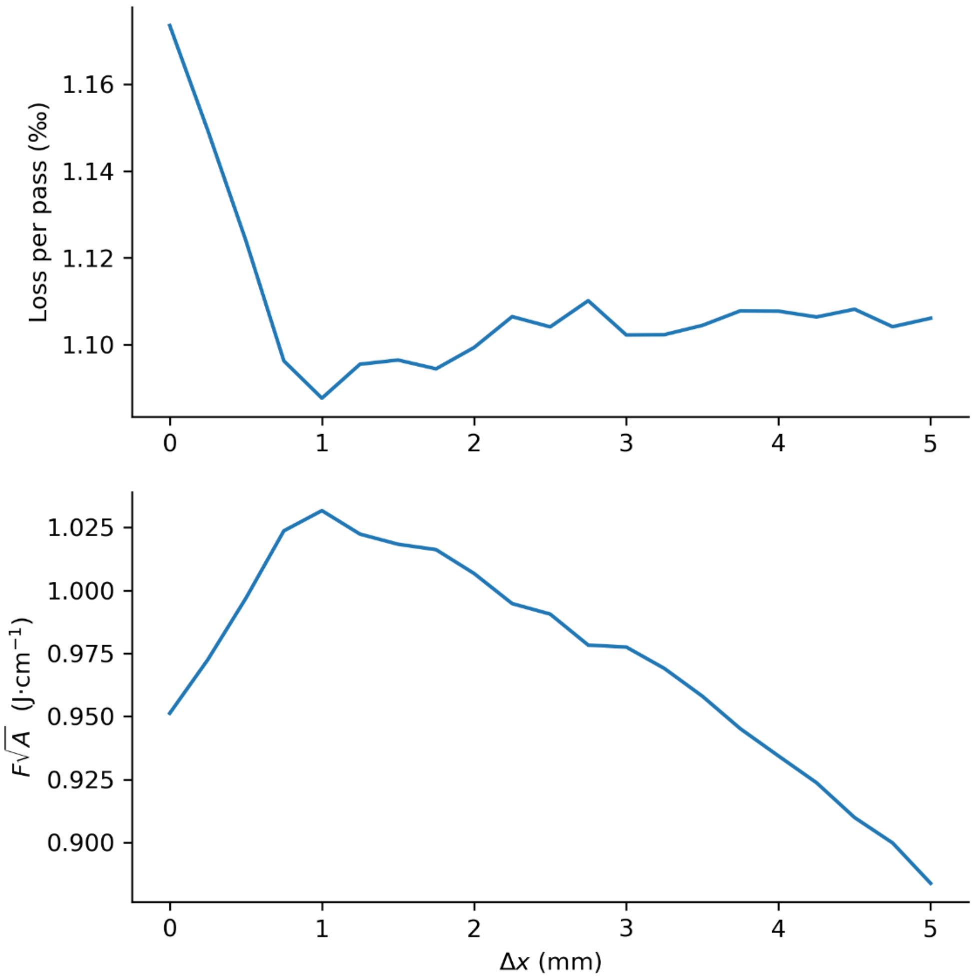


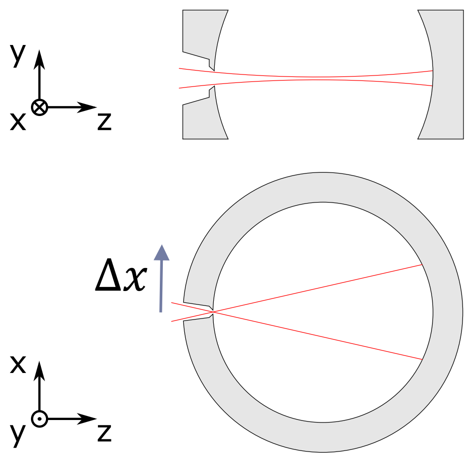


Relevant parameters

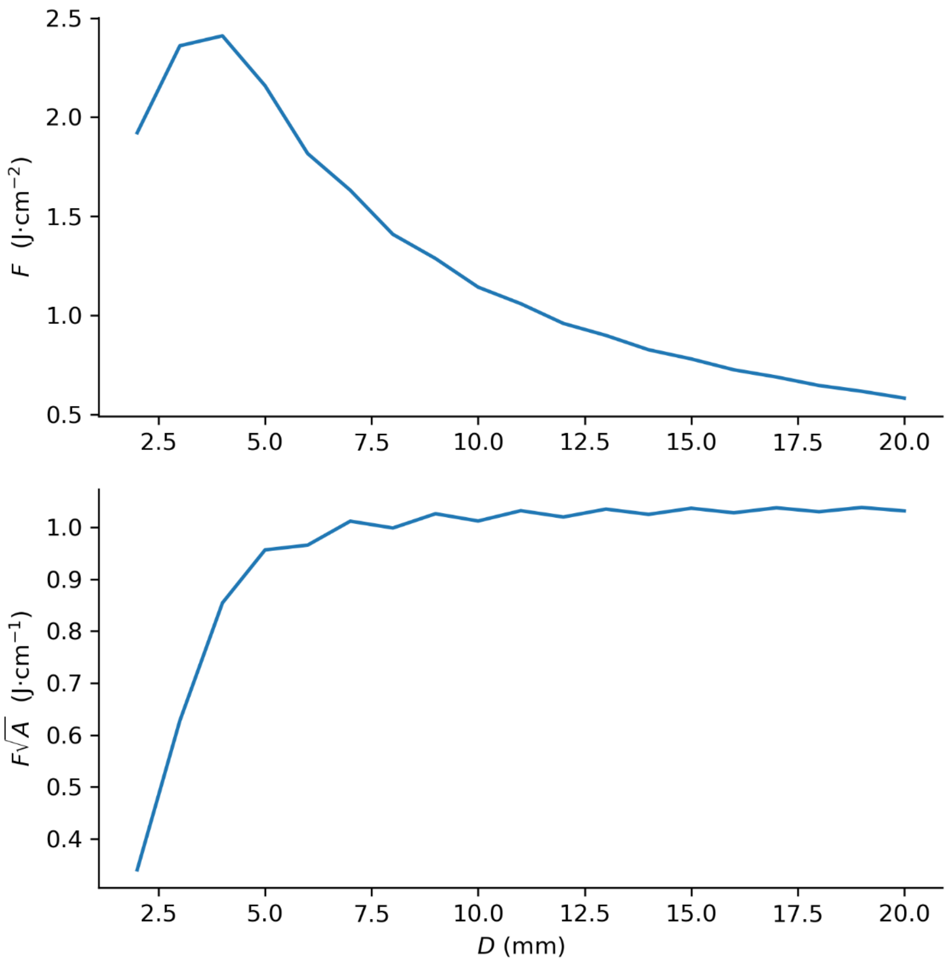


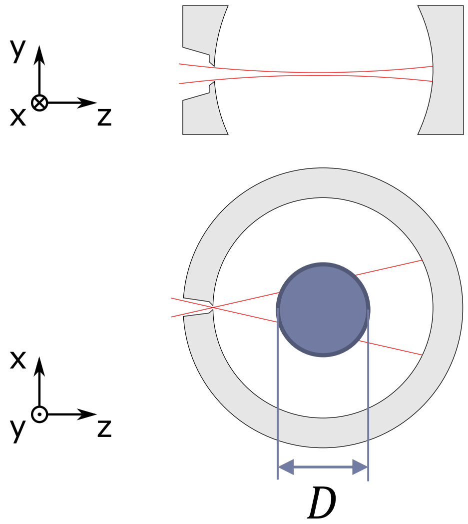
Horizontal offset





Target diameter





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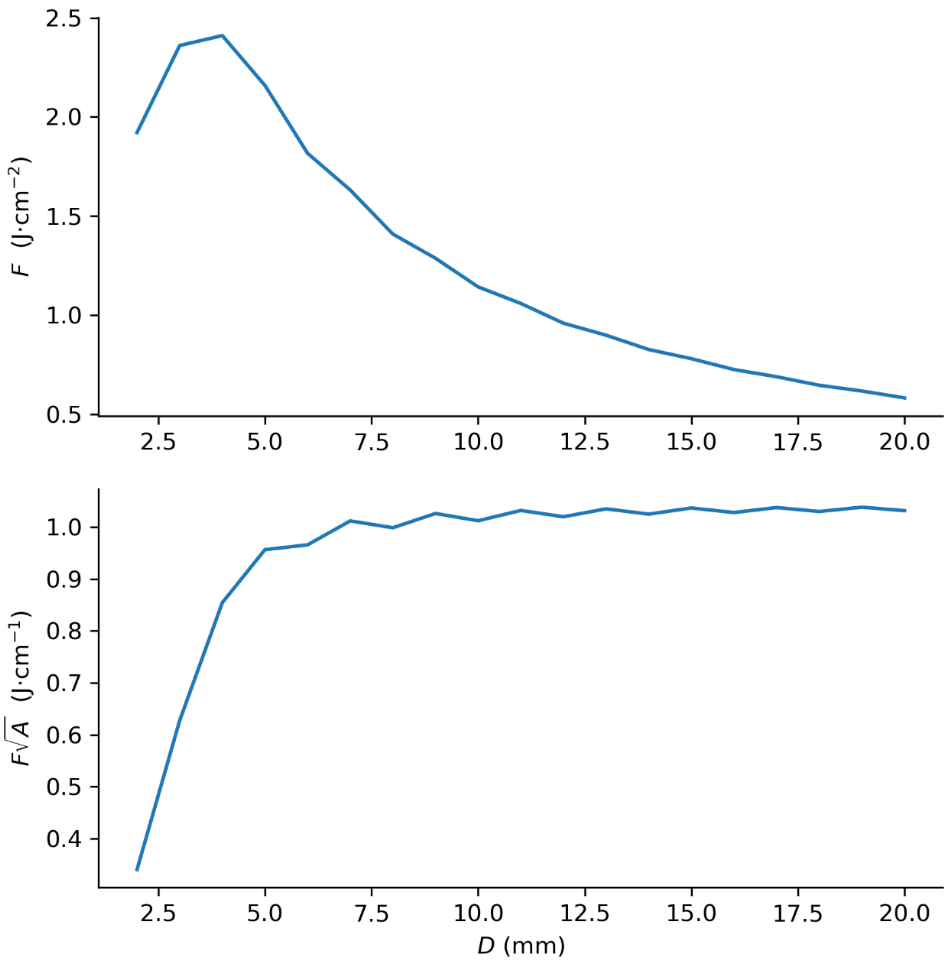
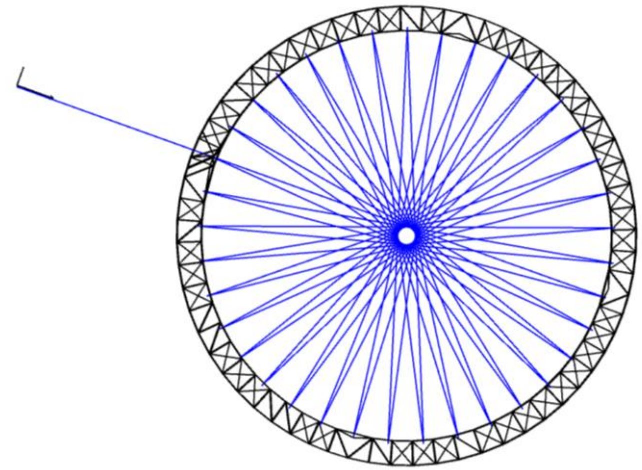




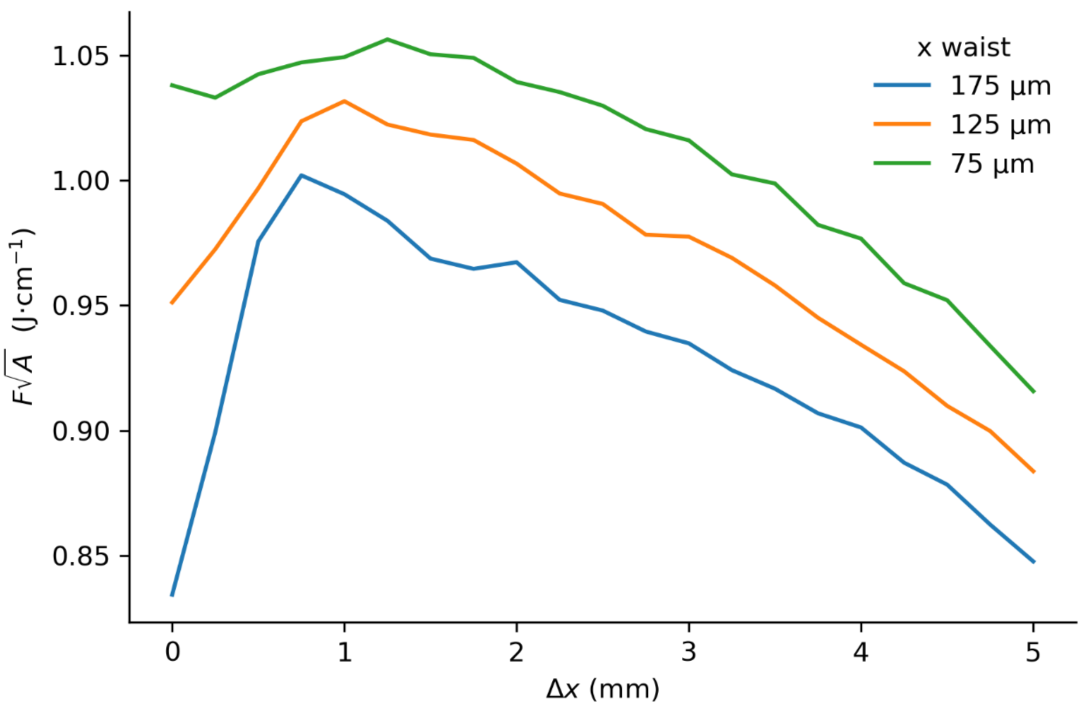
For large r

$$F \propto r^{-1}$$

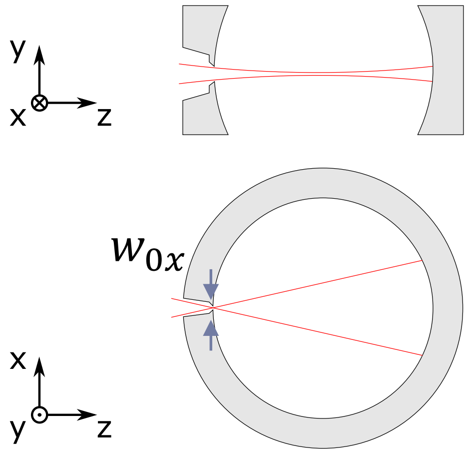
$$\sqrt{A} \propto r$$

$$F\sqrt{A} \propto 1$$

Beam waist

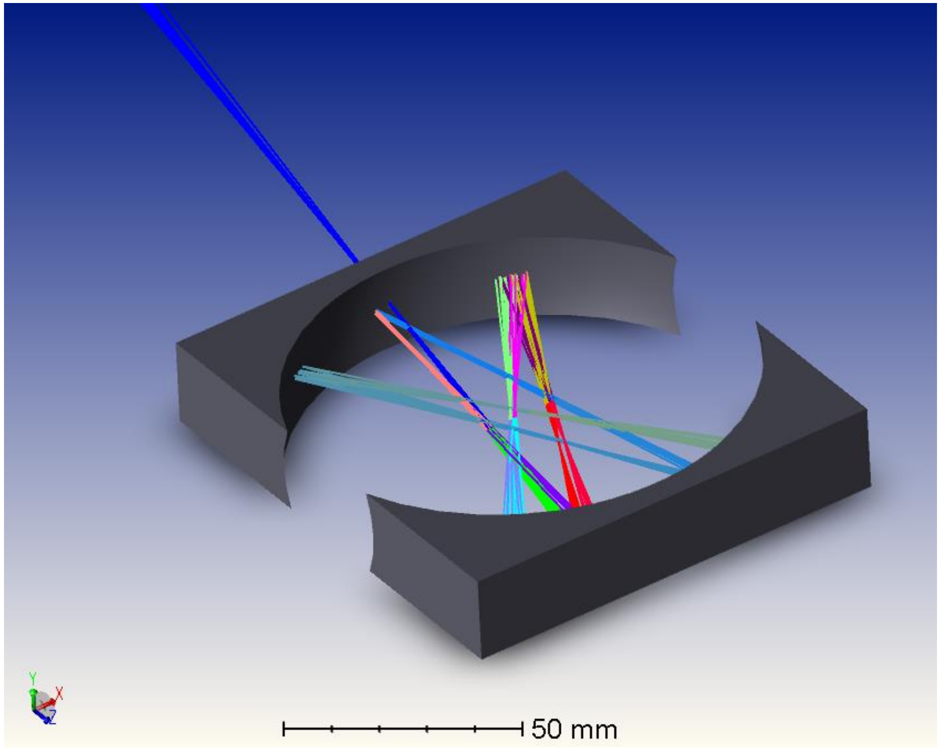


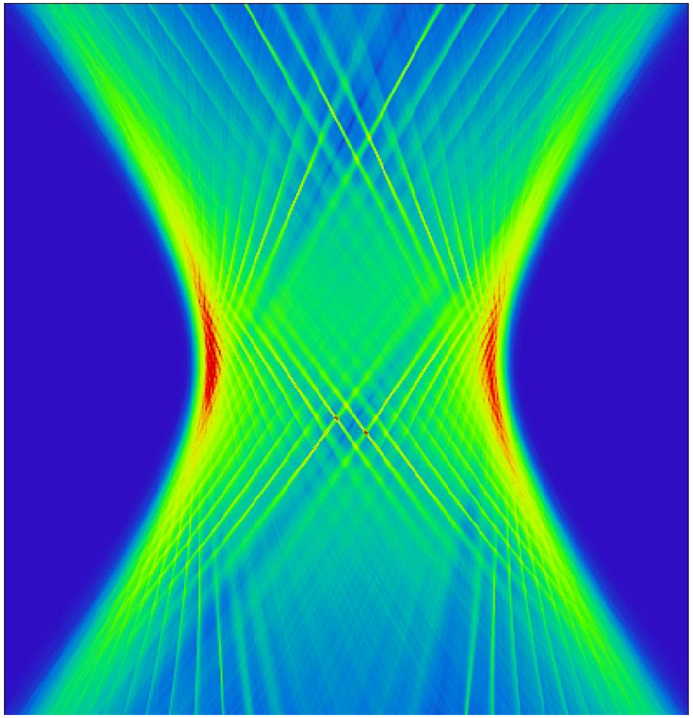
x waist
175 µm
125 µm
75 µm
1.05
1.00
0.95
0.90
0.85
$F\sqrt{A}$ (J·cm^{-1})
0
1
2
3
4
5
Δx (mm)



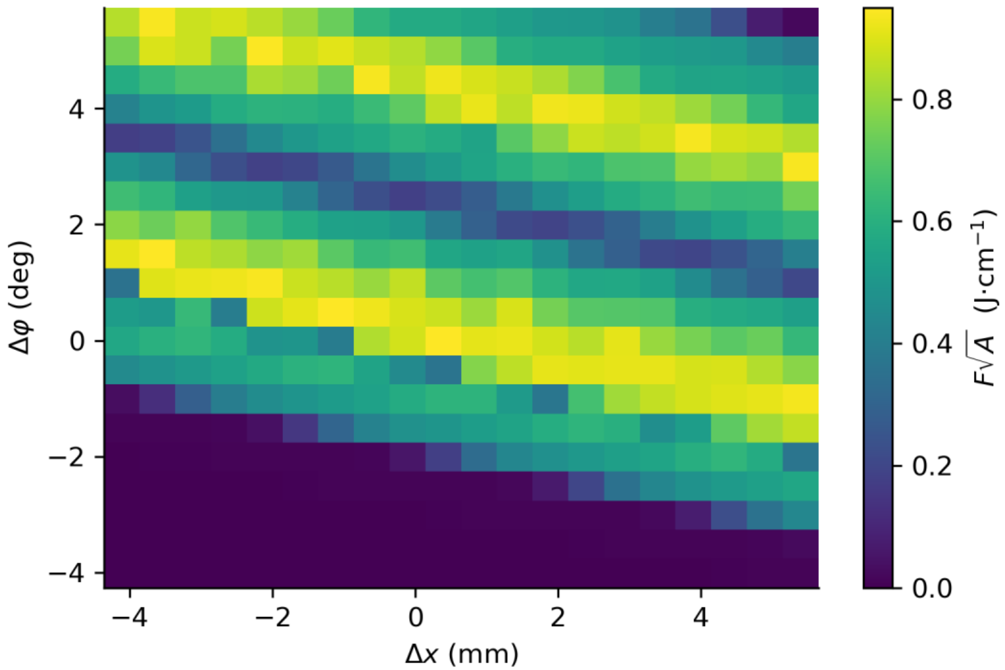
y
x
z
w_{0x}
x
y
z

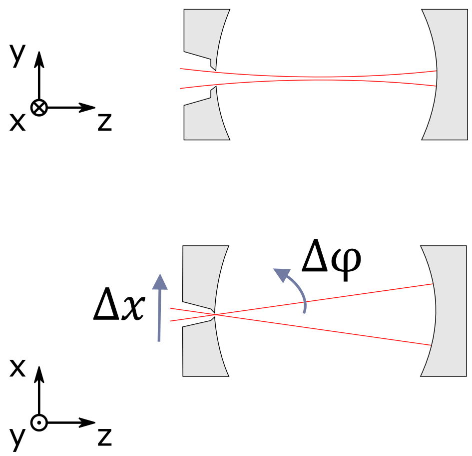
Cylindrical cavity



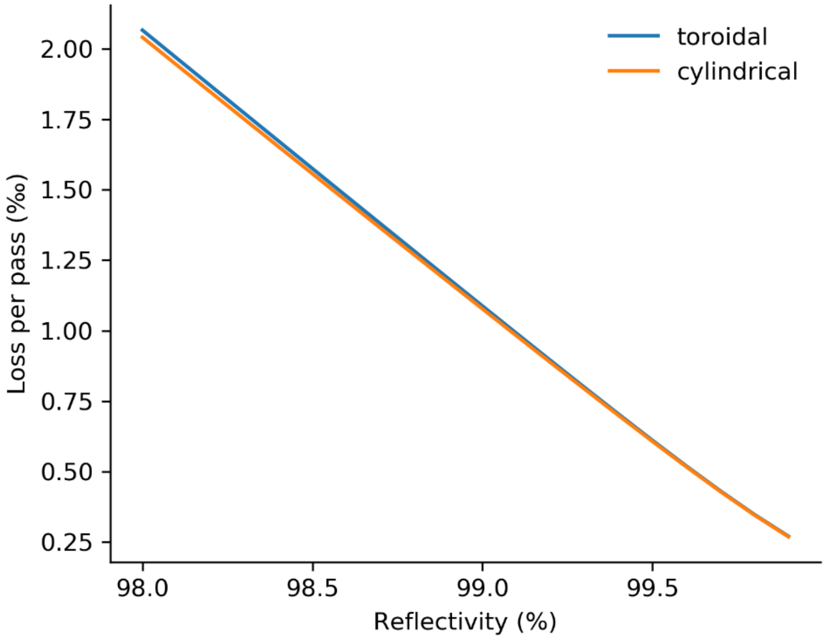


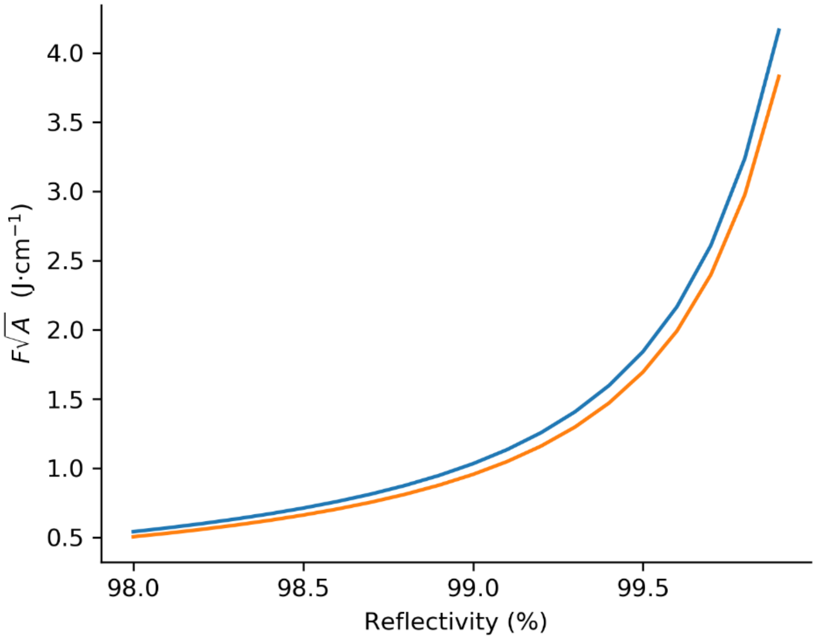
Offset + tilt





Comparing losses





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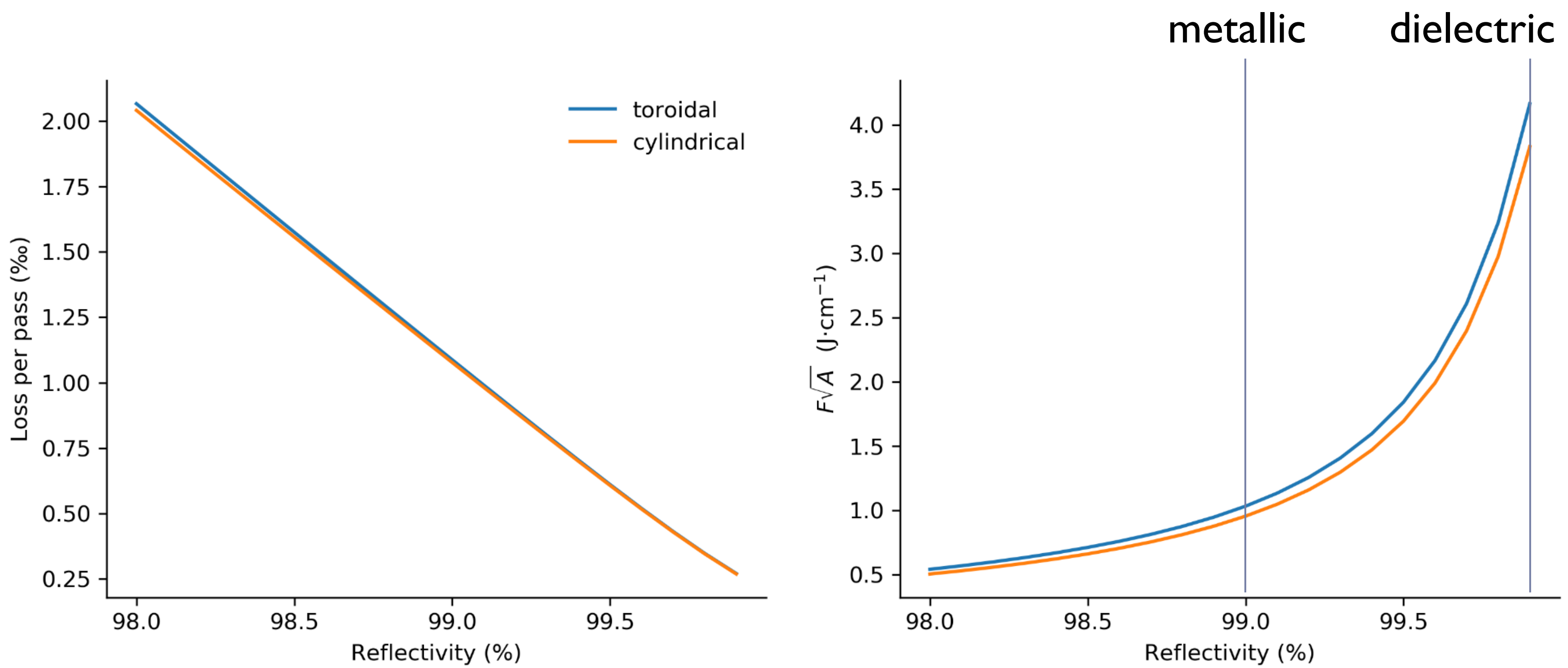
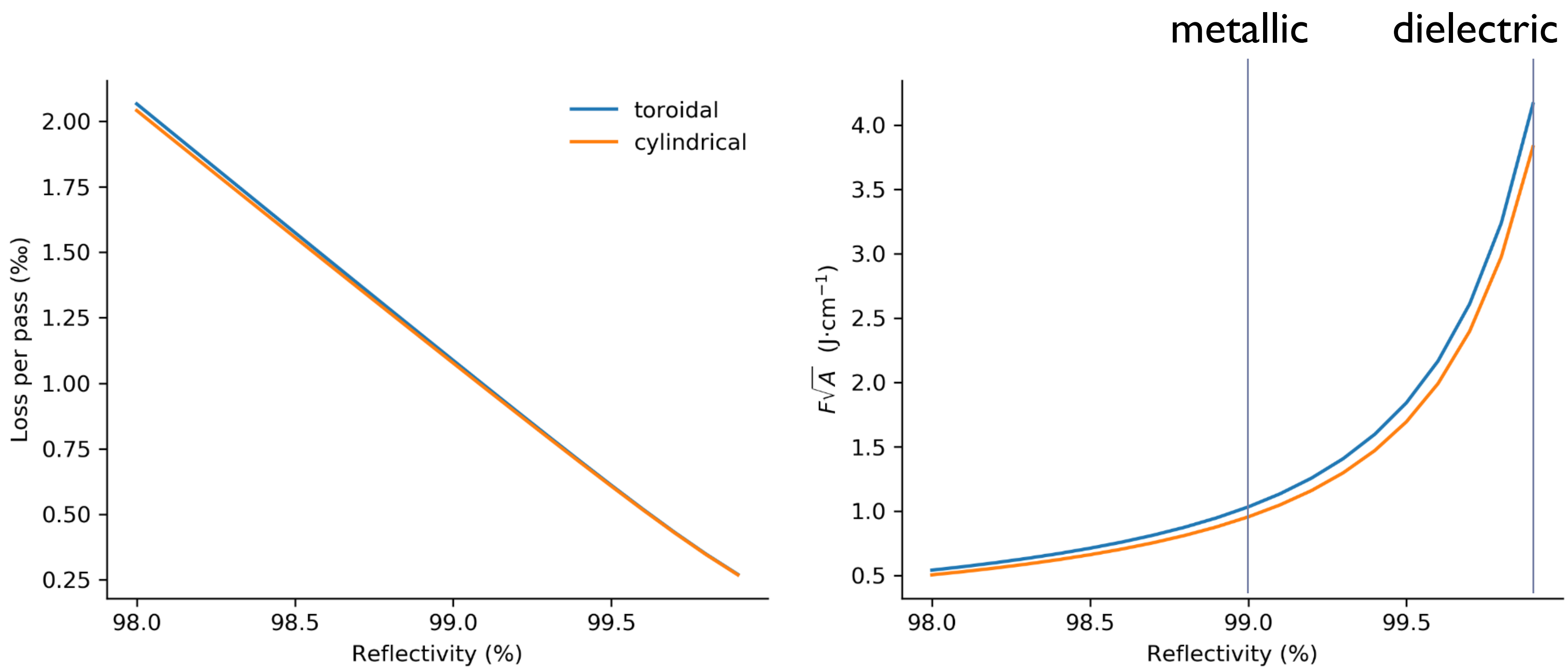


Comparing losses



The toroidal cavity is impossible to be coated with a dielectric!

Mirror damage

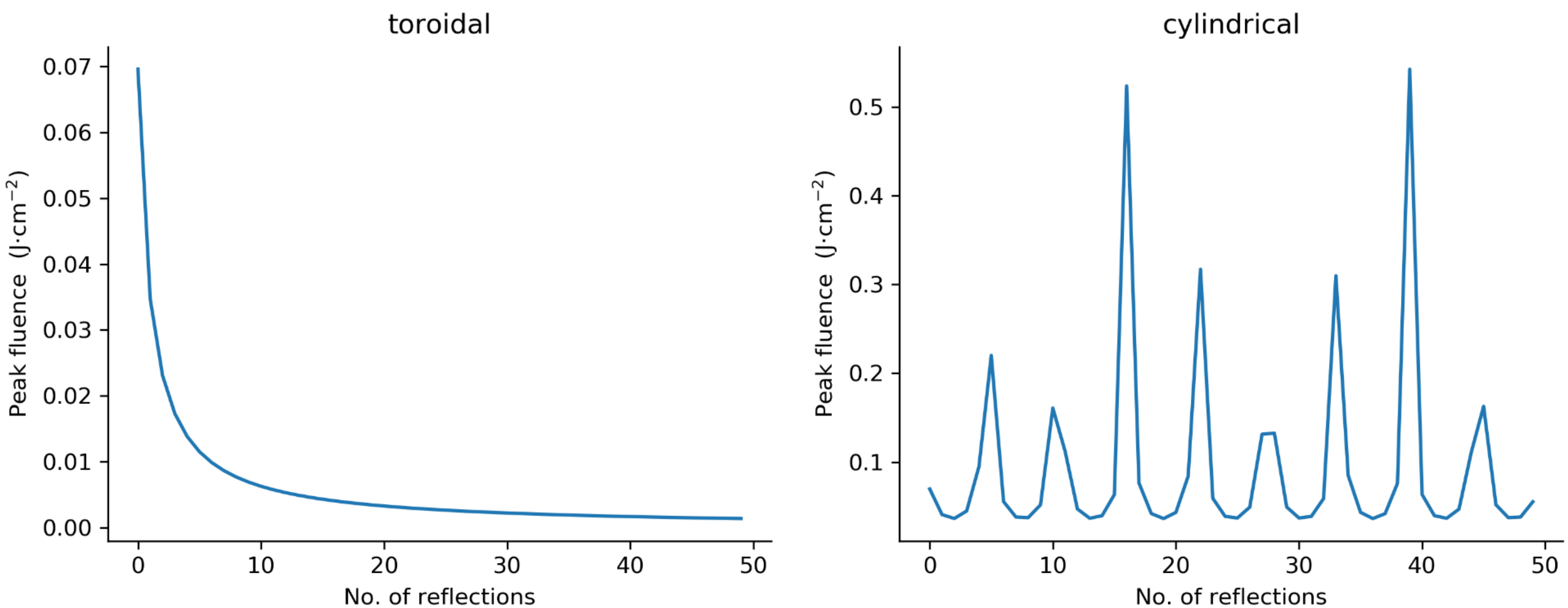
$$F_{peak} = \frac{2E_0}{\pi w_x w_y}$$

w_x, w_y - from Gaussian beam propagation

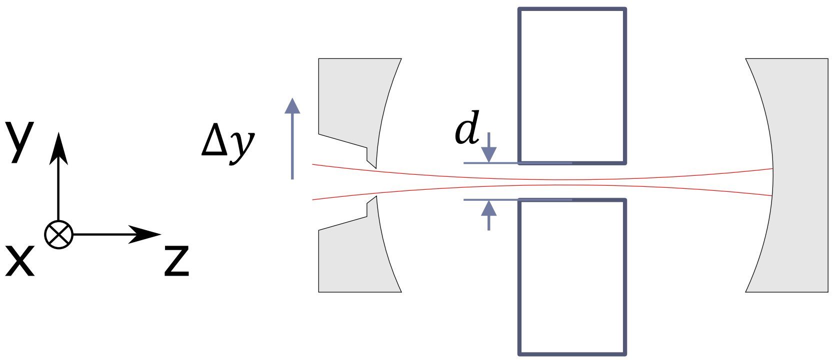
Mirror damage

$$F_{peak} = \frac{2E_0}{\pi w_x w_y}$$

w_x, w_y - from Gaussian beam propagation



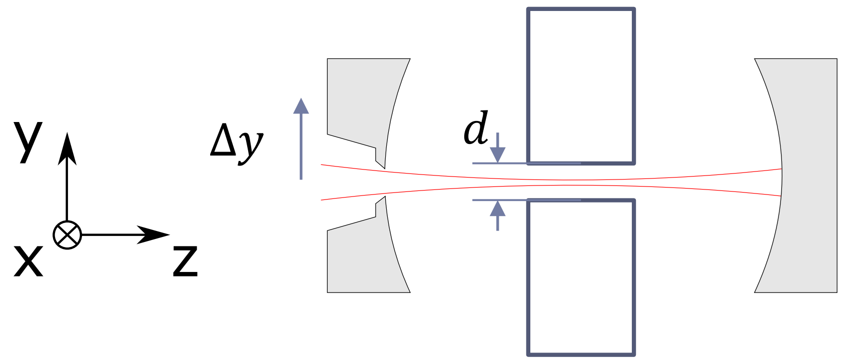
Diffraction analysis

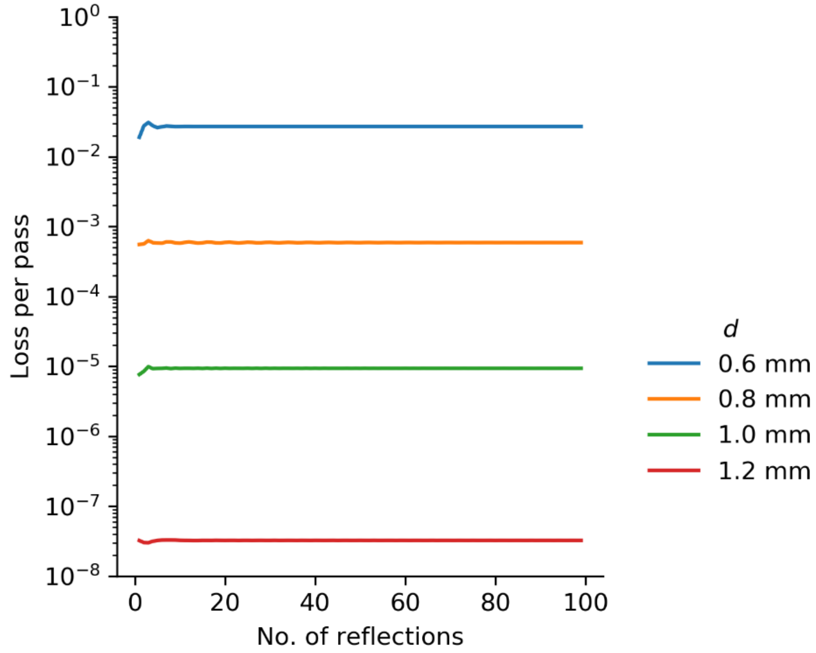


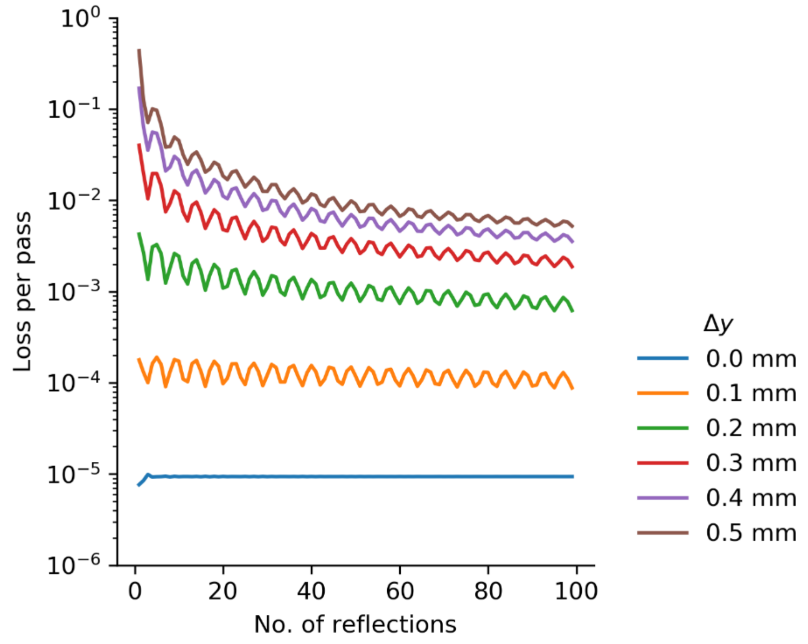
Diffraction analysis

$w_{0y} = 200\ \mu\text{m}$

$D = 10\ \text{mm}$







What comes next?

- We probably won't use the toroidal cavity, since it cannot have a good coating.
- It's time to order a test piece and verify the simulations.
- The final design will also depend on mechanical constraints (detectors, etc).