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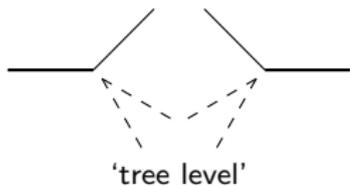
LTP seminar

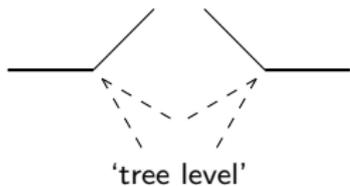
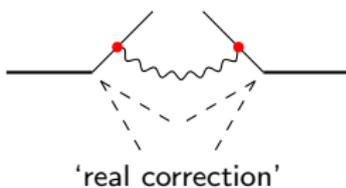
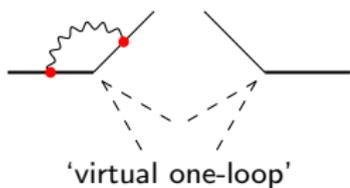
# Higher order corrections for muons

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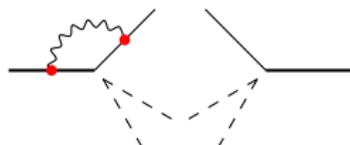
20<sup>TH</sup> MAY 2019

 $\alpha^0$   
(LO)

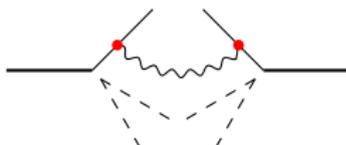
 $\alpha^0$   
(LO) $\alpha^1$   
(NLO)



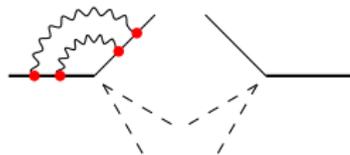
'tree level'

 $\alpha^0$   
(LO)

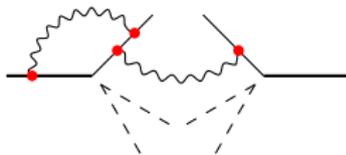
'virtual one-loop'



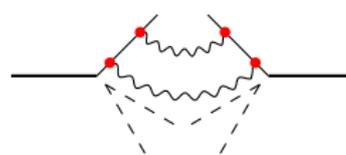
'real correction'

 $\alpha^1$   
(NLO)

'virtual two-loop'



'real-virtual'



'double real'

 $\alpha^2$   
(NNLO)



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- ⇒ minimal requirement:
- NLO for  $\mu \rightarrow e\nu\bar{\nu}\gamma$ ,  $\mu \rightarrow e\nu\bar{\nu}ee$  (PiBeta, MEG, Mu3e)
  - NNLO for  $\mu \rightarrow e\nu\bar{\nu}$  (TWIST, MEG<sub>J</sub>)
  - at the very least NNLO for  $\mu e \rightarrow \mu e$  and  $\mu p \rightarrow \mu p$  (MUonE, MUSE)

$$\mu \rightarrow e\nu\bar{\nu} + \gamma$$

and

$$\mu \rightarrow e\nu\bar{\nu} + e^+e^-$$

[Pruna, Signer, YU 16, Pruna, Signer, YU 17, YU 17]

$E_\gamma > 10 \text{ MeV}$  and  $\theta > 30^\circ$

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- assuming  $m_e = 0$ :  $\mathcal{B}_{\text{PSU}}^{m_e=0} = (4.35_{\text{LO}} + 0.06_{\text{NLO}}) \cdot 10^{-3}$
- $3.7\sigma \rightarrow \sim 1\sigma$

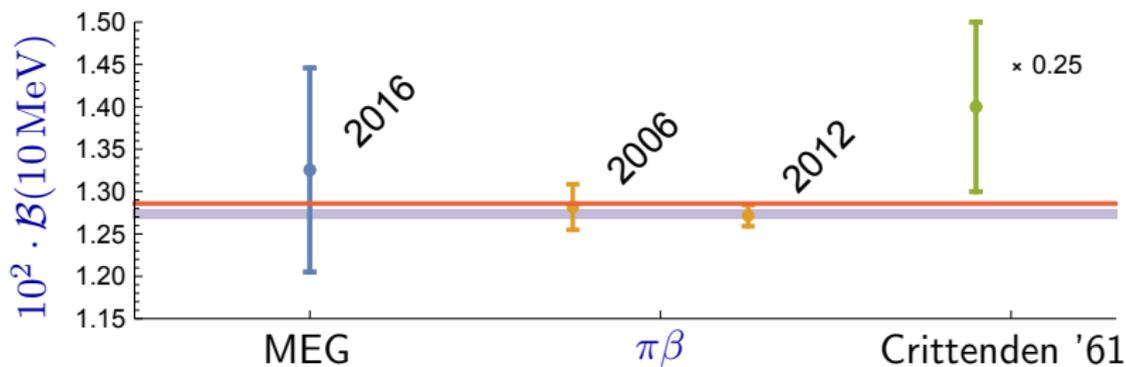
$\Rightarrow$  details matter!!

Relate all data using NLO Monte Carlo to  $E_\gamma > 10 \text{ MeV}$

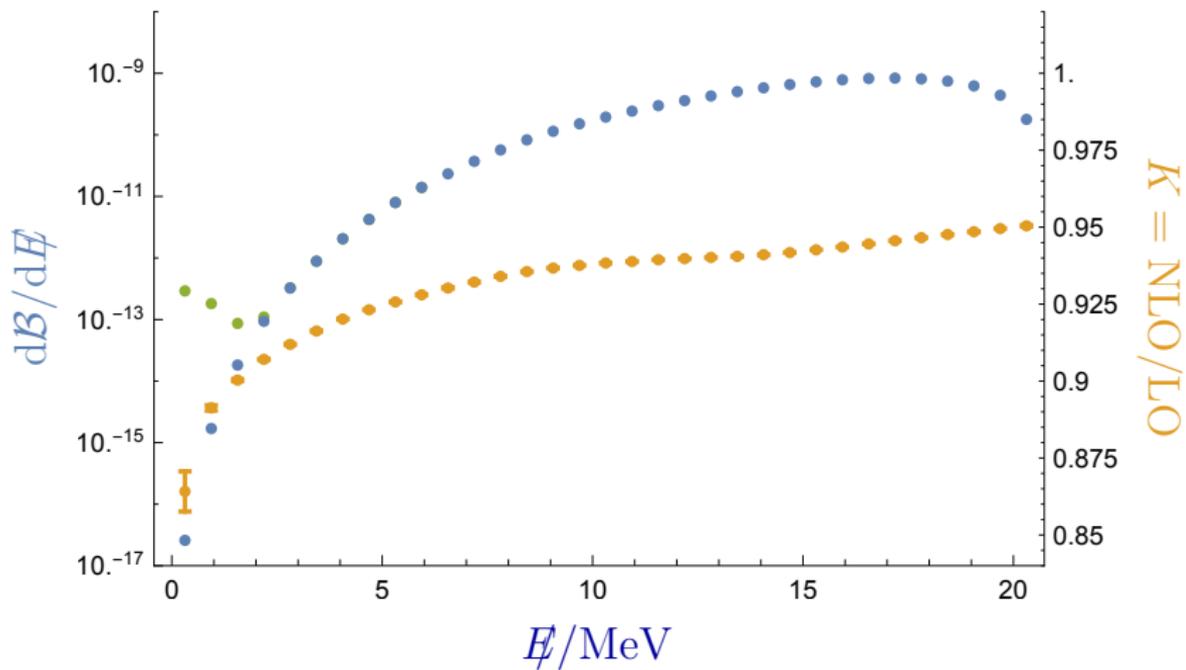
- Compute kinematic acceptance  $\epsilon$

$$\mathcal{B}(10 \text{ MeV}) = \frac{\mathcal{B}_{\text{PSU}}(10 \text{ MeV})}{\underbrace{\mathcal{B}_{\text{PSU}}(\text{exp. cuts})}_{\epsilon}} \mathcal{B}_{\text{exp}}(\text{exp. cuts})$$

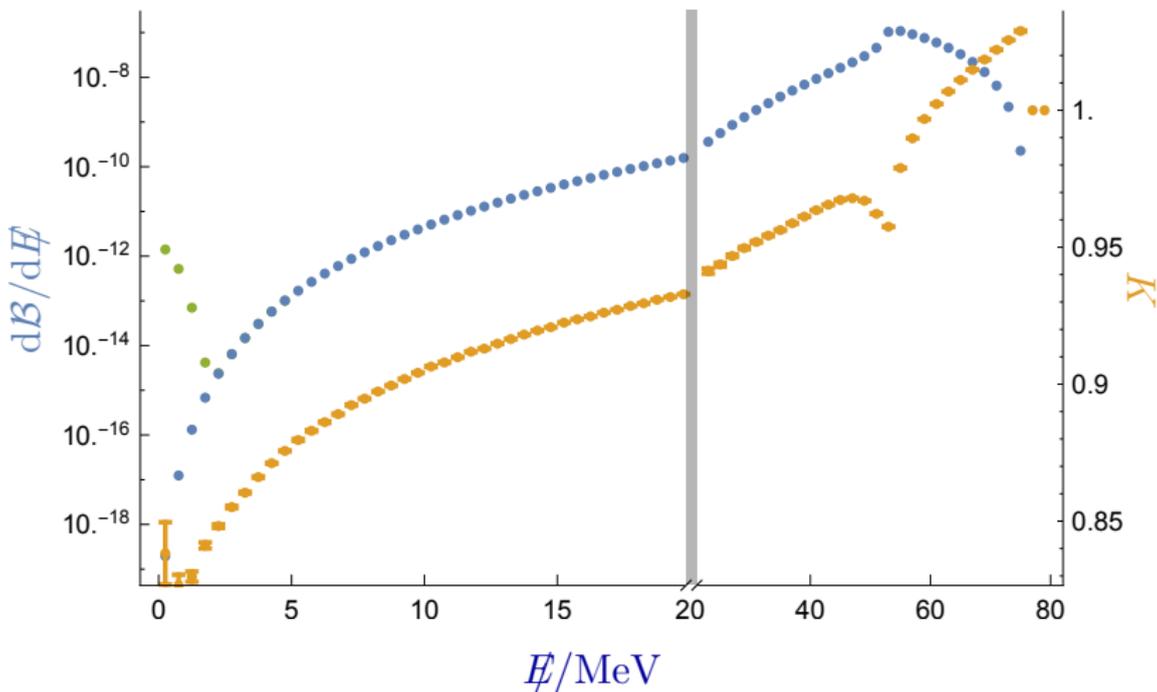
- $\epsilon_{\text{MEG}} \approx 2 \cdot 10^5$ ,  $\epsilon_{\pi\beta} \approx 3$
- combined experimental  $\bar{\mathcal{B}}(10 \text{ MeV}) = 1.27(1) \cdot 10^{-2}$



exactly one photon  $E_\gamma > 40\text{MeV}$  in the detector.  $\mathcal{B}_{\text{NP}} \simeq 4.2 \cdot 10^{-13}$



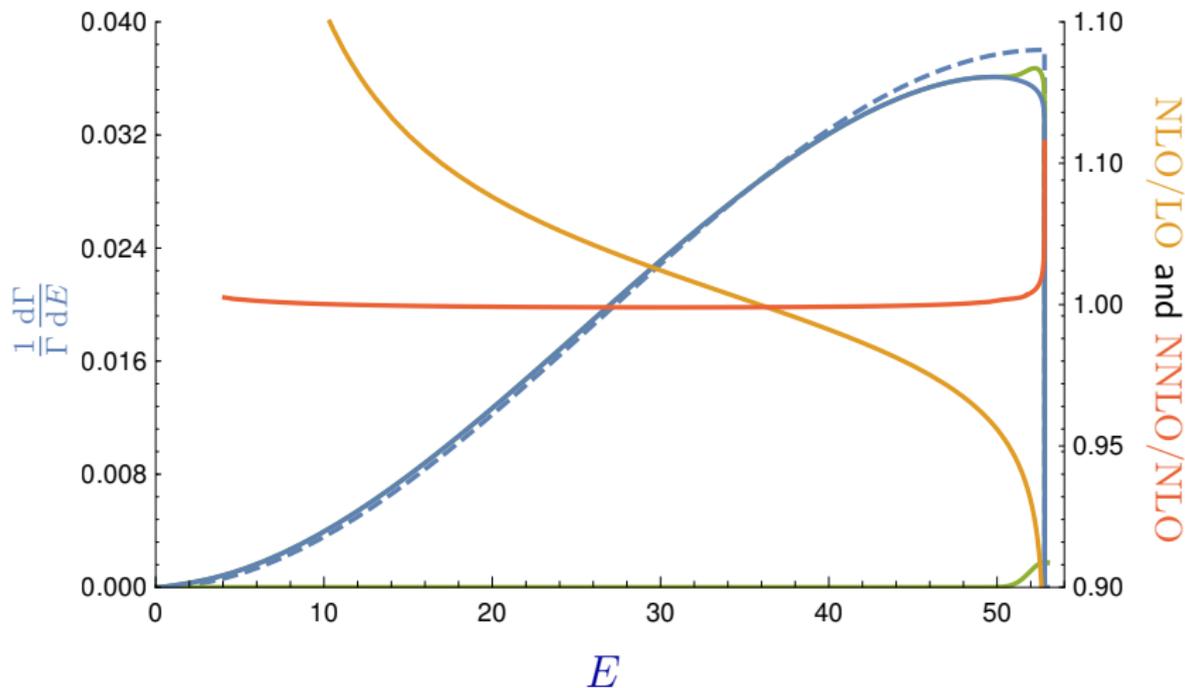
assuming  $E > 10\text{MeV}$  and geometry cuts.  $\mathcal{B}_{\mu \rightarrow 3e} \simeq 10^{-12}$



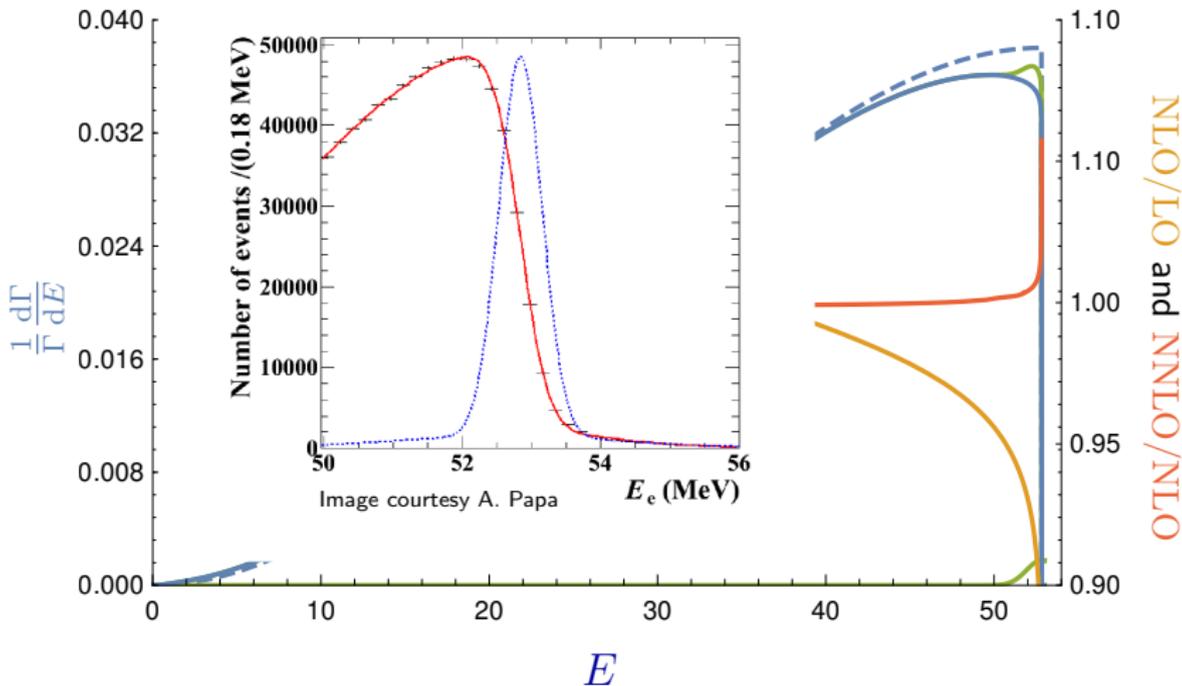
$$\mu \rightarrow e\nu\bar{\nu}$$

[Engel, Gnendiger, Signer, YU 18, Banerjee, Engel, Signer, YU soon]

Majoron with  $m_J \sim 0$ ,  $\mathcal{B}_{\mu \rightarrow e J} \sim 2 \times 10^{-3}$



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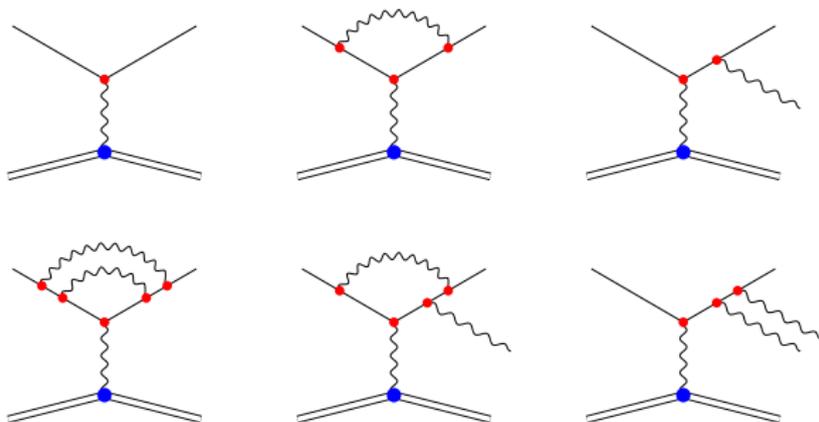


$$\mu e \rightarrow \mu e$$

and

$$\mu p \rightarrow \mu p$$

- leptonic NNLO done [Bucoveanu, Spiesberger 18]
- second calculation using new methods in progress [Banerjee, Engel, Signer, YU soon-ish]
- hadronic correction  $\Rightarrow$  someone else's problem



- recent proposal to measure  $a_{\mu}^{\text{HLO}}$  with  $\mu e$  scattering
- LTP Colloquium Graziano Venanzoni  
28<sup>th</sup> March
- requires theoretical uncertainties below  $10^{-5} \Rightarrow$  at least NNLO (electronic contribution almost done)
  - $\mathcal{O}(\alpha^3 \log^3 \frac{m_e^2}{m_{\mu}^2}) \gg 10^{-5}$
  - NNLO not enough  $\Rightarrow$  resummation and exploratory N<sup>3</sup>LO studies



## details matter!

- $\mu \rightarrow e\nu\bar{\nu}\gamma$  and  $\mu \rightarrow e\nu\bar{\nu}ee$  at NLO
- input into PiBeta reanalysis  $\Rightarrow$  current best value
- $\mu \rightarrow e\nu\bar{\nu}$  at NNLO: large corrections at the endpoints
- $\mu e \rightarrow \mu e$  and  $\mu p \rightarrow \mu p$  at NNLO: ongoing