

A multi-pass optical cavity for the HyperMu experiment

Mirosław Marszałek

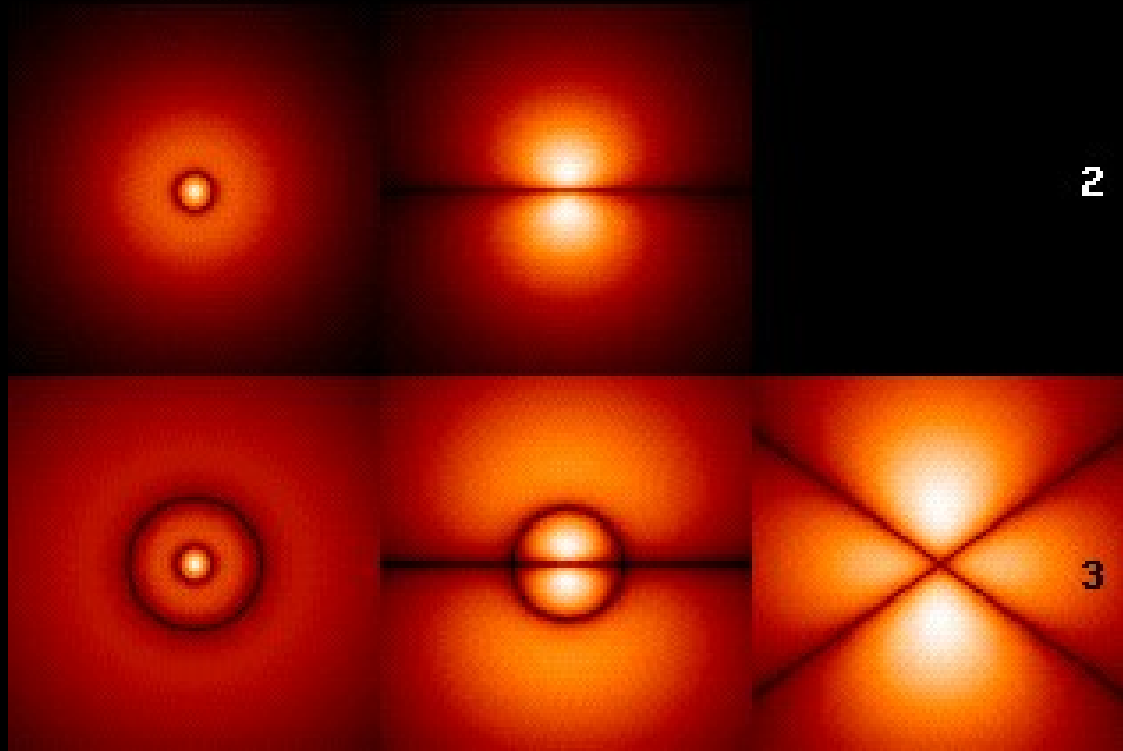
On behalf of the CREMA collaboration

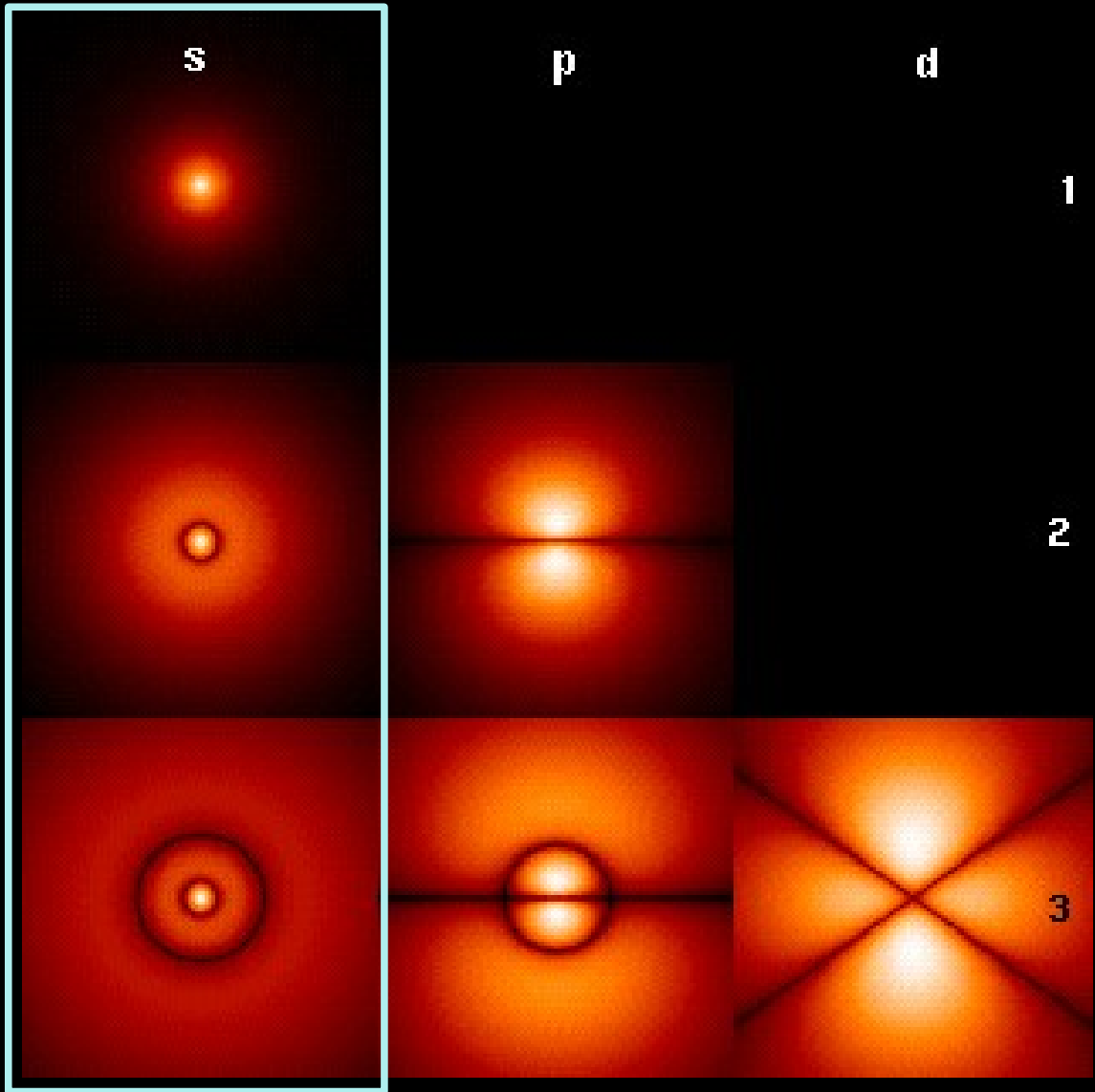
s

p

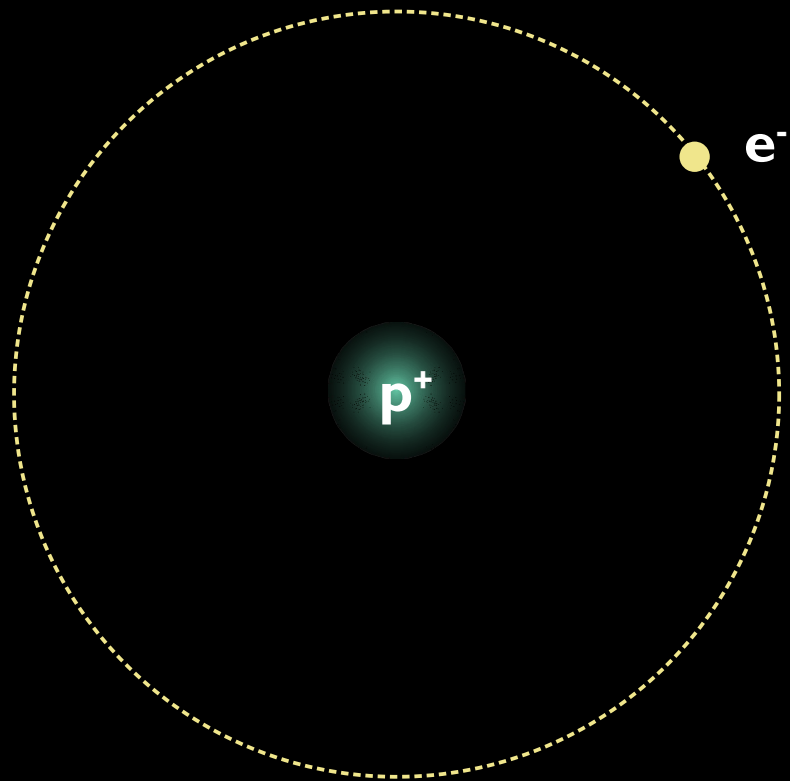
d

1

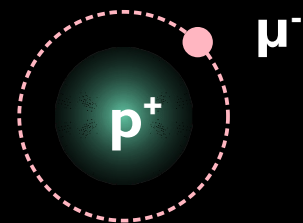




Electronic hydrogen



Muonic hydrogen

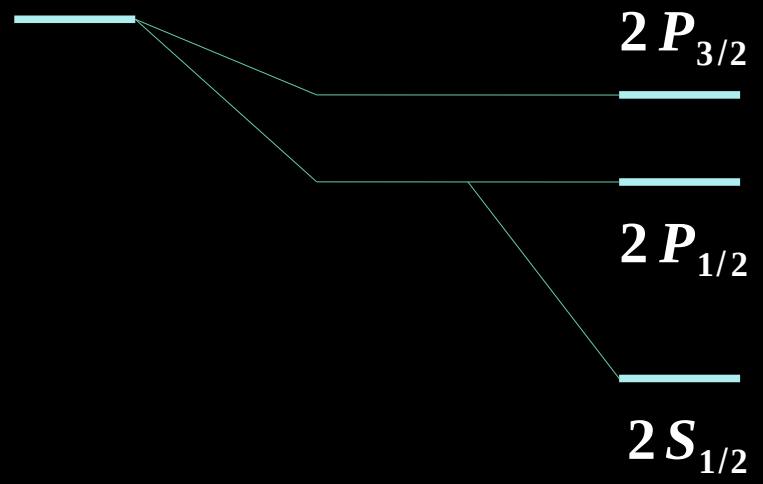


$$\Delta E \propto |\Psi(\mathbf{0})|^2 \propto m_r^3$$

$$m_\mu \approx 200 m_e$$

$$\Delta E_\mu \sim 10^7 \Delta E_e$$

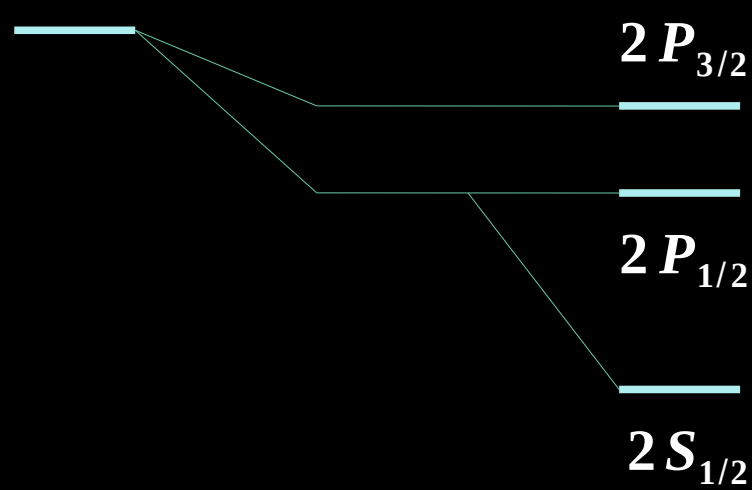
$n=2$



$n=1$



$n=2$

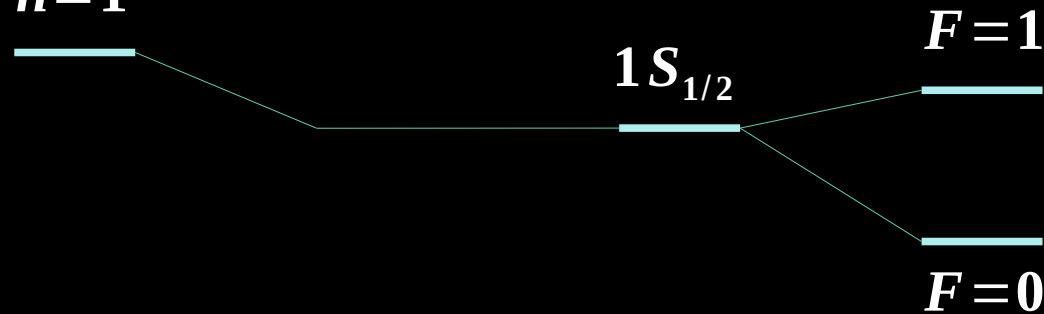


Lamb shift → **charge radius**

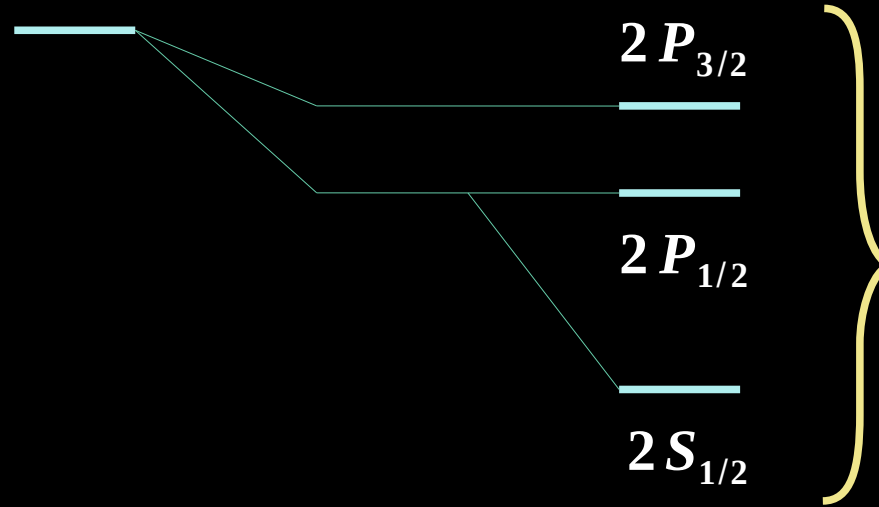
Pohl et al., *Nature* 466, 213 (2010)

Antognini et al., *Science* 339, 417 (2013)

$n=1$



$n=2$

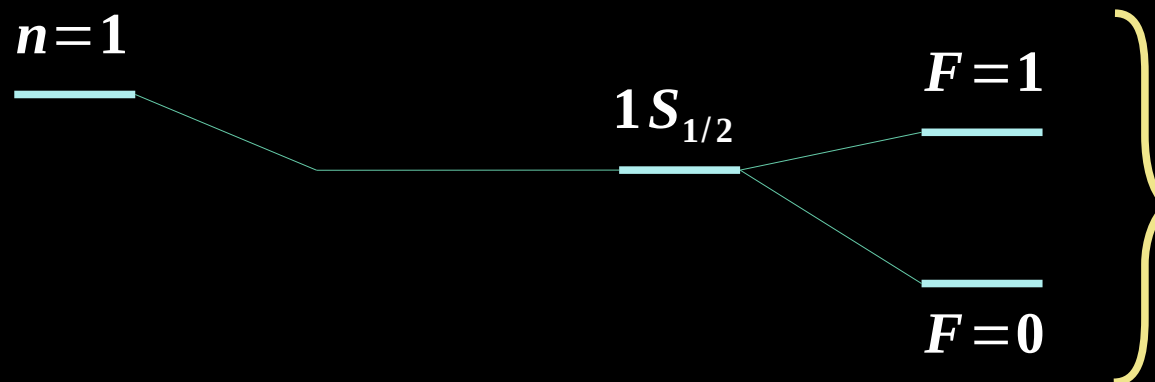


Lamb shift → **charge radius**

Pohl et al., *Nature* 466, 213 (2010)

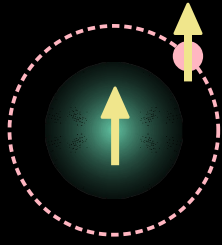
Antognini et al., *Science* 339, 417 (2013)

$n=1$



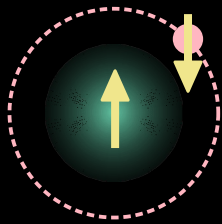
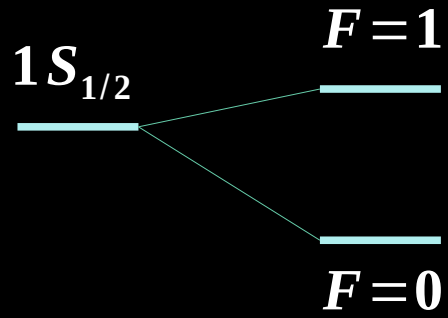
HFS → **Zemach radius**

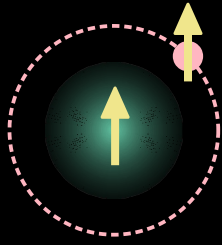
In progress!



$$E_F \propto \langle \mu_p \cdot \mu_n \rangle |\Psi(0)|^2$$

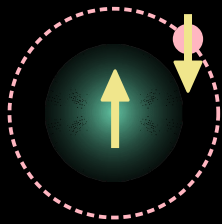
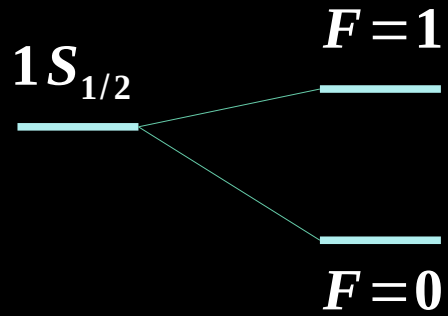
$$\Delta E_{\text{HFS}} = E_F$$

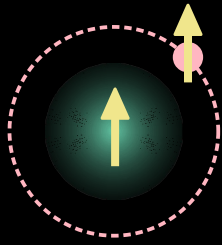




$$E_F \propto \langle \mu_p \cdot \mu_u \rangle |\Psi(0)|^2$$

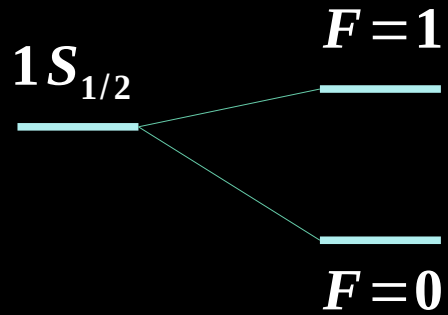
$$\Delta E_{\text{HFS}} = E_F (1 + \Delta E_{\text{QED}} + \Delta E_{\text{TPE}} + \dots)$$



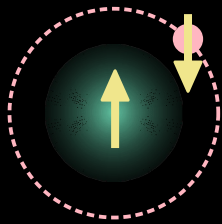
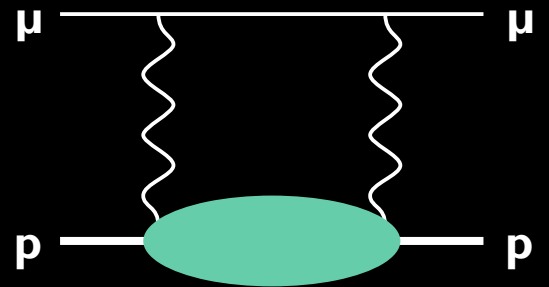


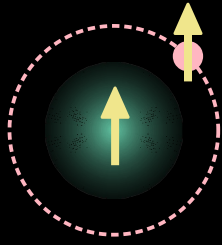
$$E_F \propto \langle \mu_p \cdot \mu_\mu \rangle |\Psi(0)|^2$$

$$\Delta E_{\text{HFS}} = E_F (1 + \Delta E_{\text{QED}} + \Delta E_{\text{TPE}} + \dots)$$



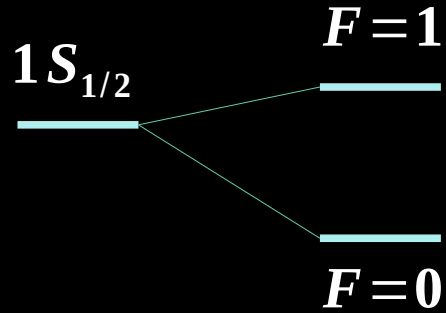
Two-photon exchange





$$E_F \propto \langle \mu_p \cdot \mu_n \rangle |\Psi(0)|^2$$

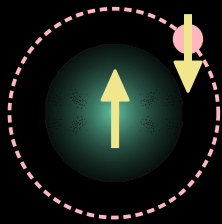
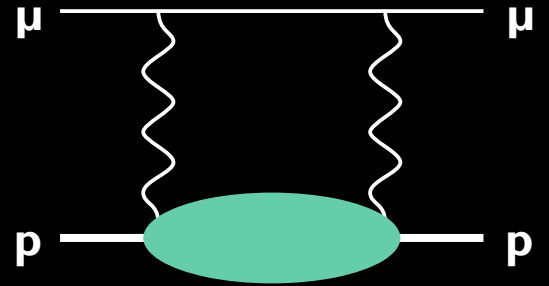
$$\Delta E_{\text{HFS}} = E_F (1 + \Delta E_{\text{QED}} + \Delta E_{\text{TPE}} + \dots)$$



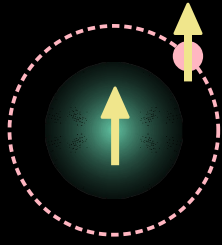
$$\Delta E_{\text{TPE}} = \Delta E_Z + \Delta E_{\text{recoil}} + \Delta E_{\text{pol}}$$

$$\Delta E_Z = -2Z\alpha m_r R_Z$$

Two-photon exchange

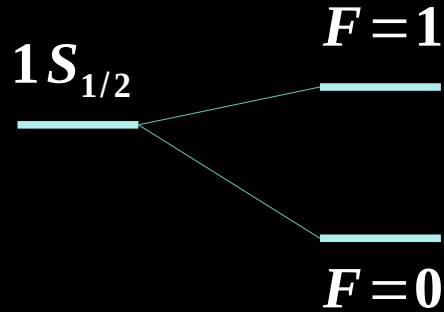


$$R_Z = \int d^3 r |r| \int d^3 r' \rho_E(\mathbf{r}-\mathbf{r}') \rho_M(\mathbf{r}')$$



$$E_F \propto \langle \mu_p \cdot \mu_n \rangle |\Psi(0)|^2$$

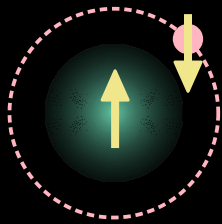
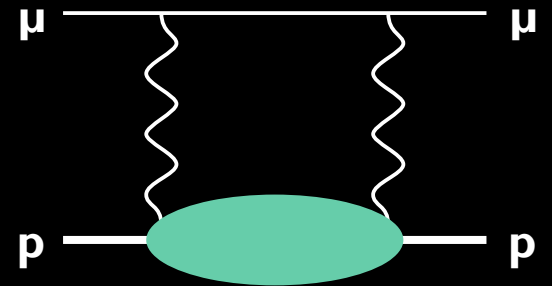
$$\Delta E_{\text{HFS}} = E_F (1 + \Delta E_{\text{QED}} + \Delta E_{\text{TPE}} + \dots)$$



$$\Delta E_{\text{TPE}} = \Delta E_Z + \Delta E_{\text{recoil}} + \Delta E_{\text{pol}}$$

$$\Delta E_Z = -2Z\alpha m_r R_Z$$

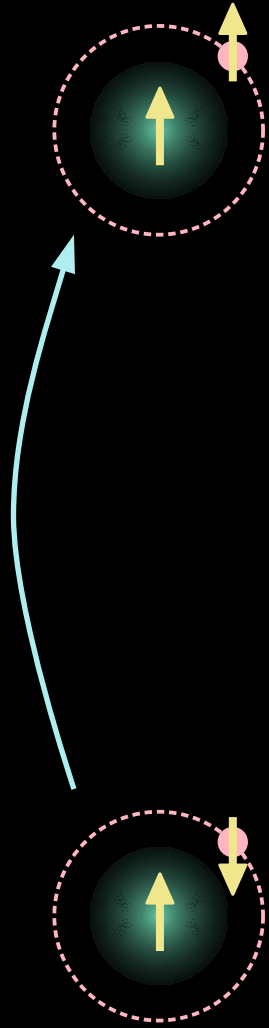
Two-photon exchange



$$R_Z = \int d^3 r |r| \int d^3 r' \rho_E(\mathbf{r}-\mathbf{r}') \rho_M(\mathbf{r}')$$

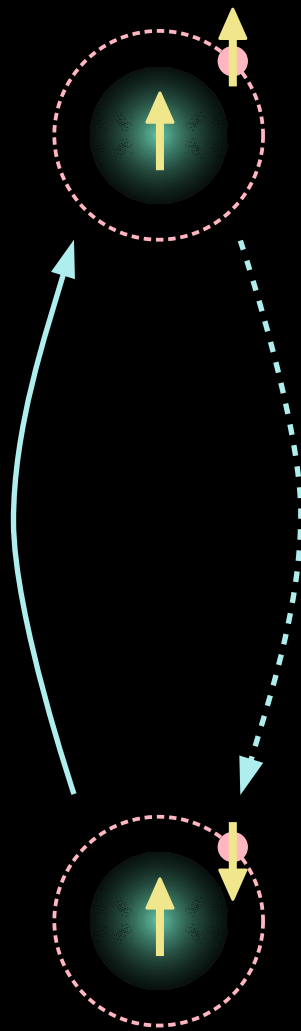
$$R_Z = -\frac{4}{\pi} \int_0^\infty \frac{dQ}{Q^2} \left(G_E(Q^2) \frac{G_M(Q^2)}{1 + K_p} - 1 \right)$$

6.8 μm laser



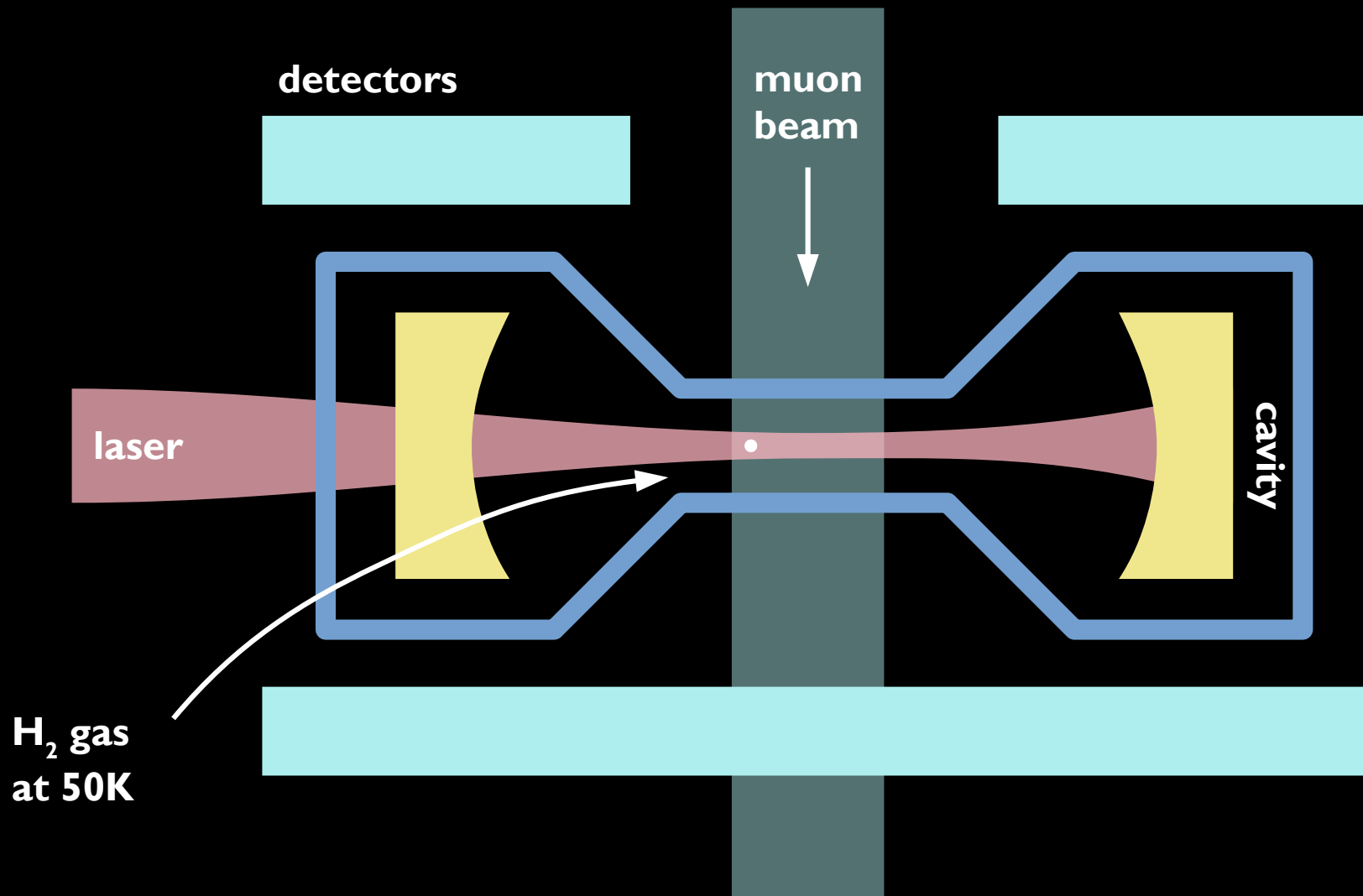
- **Magnetic dipole transition**
- **Small atom cross-section**

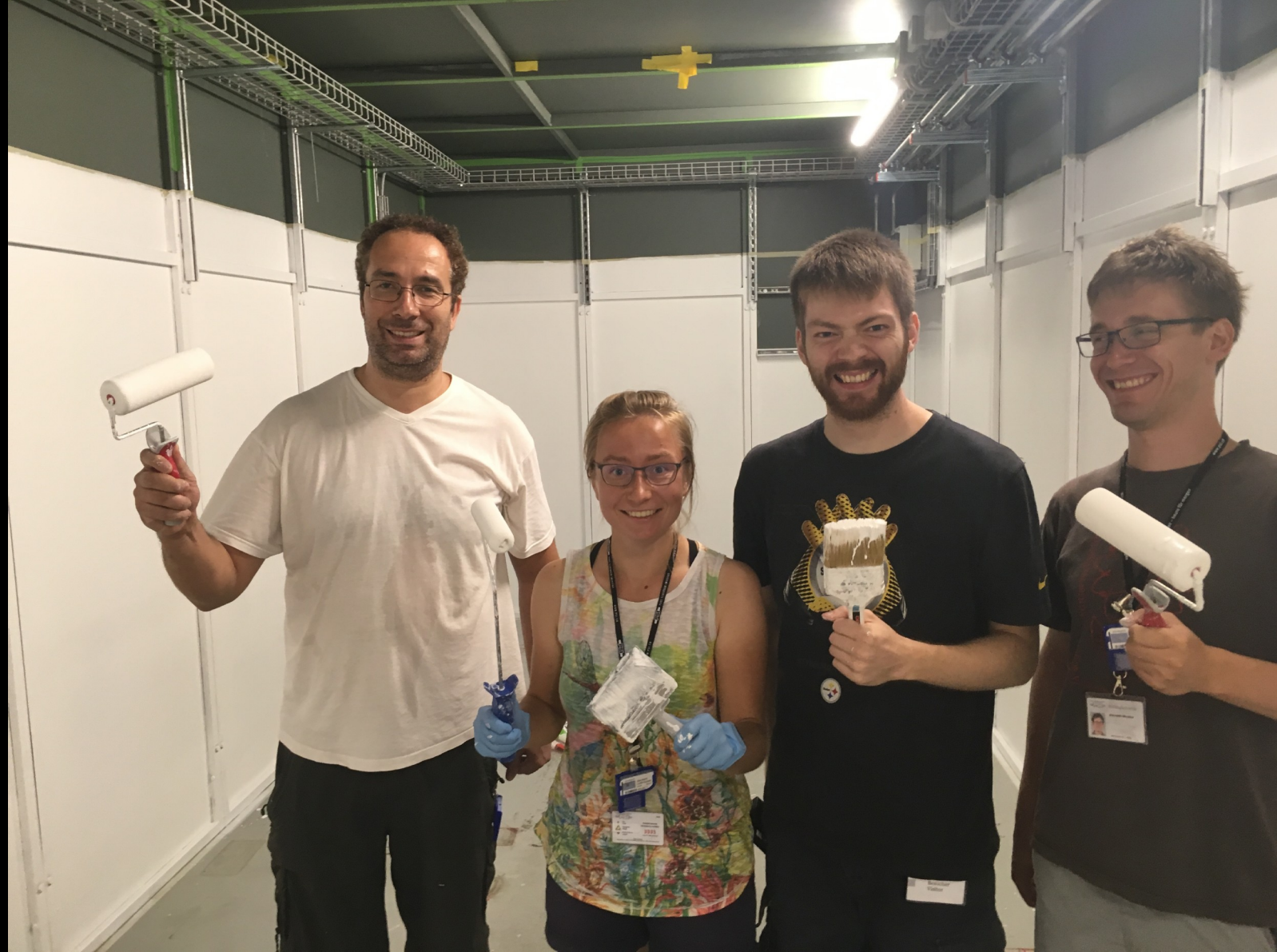
6.8 μm laser

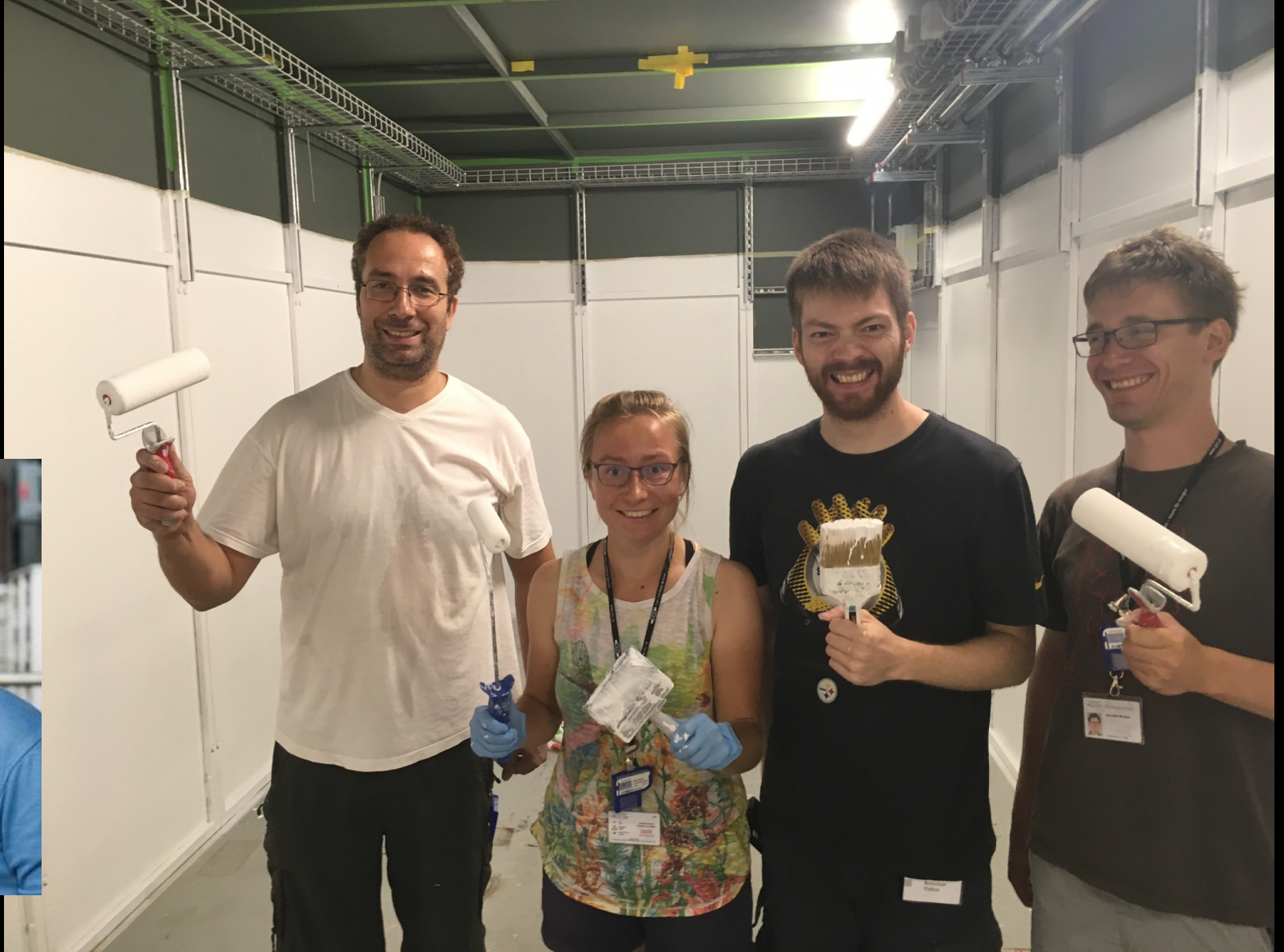
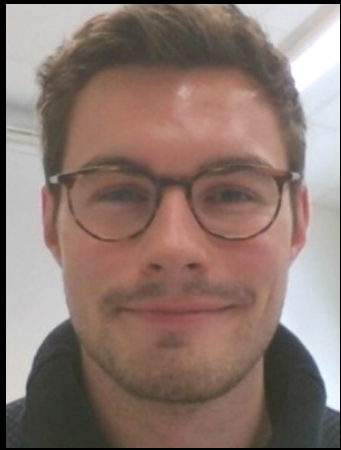


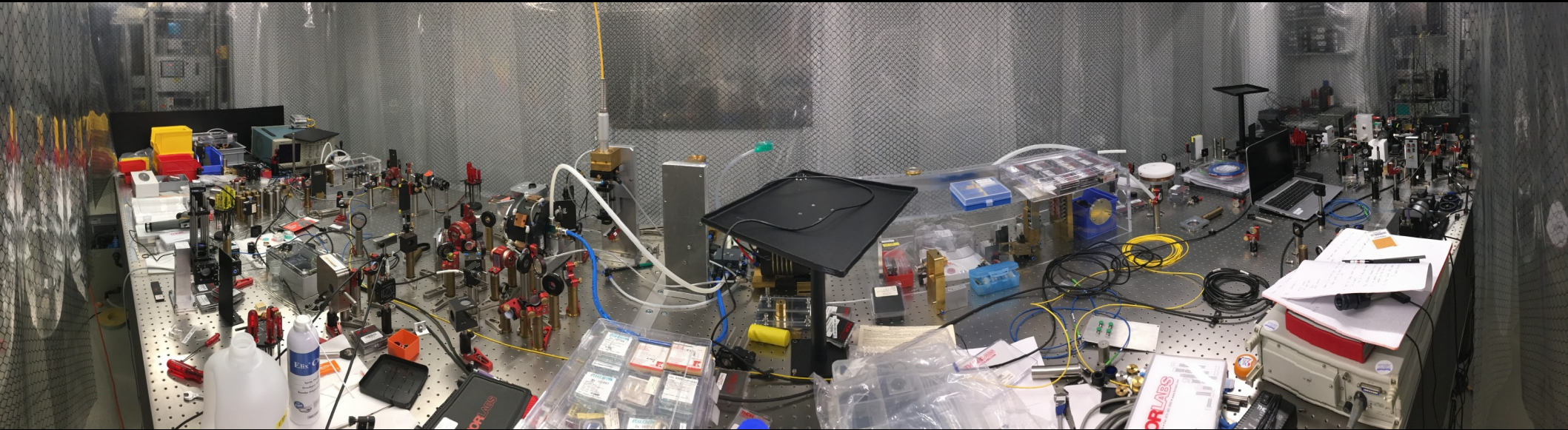
+ E_{kin}

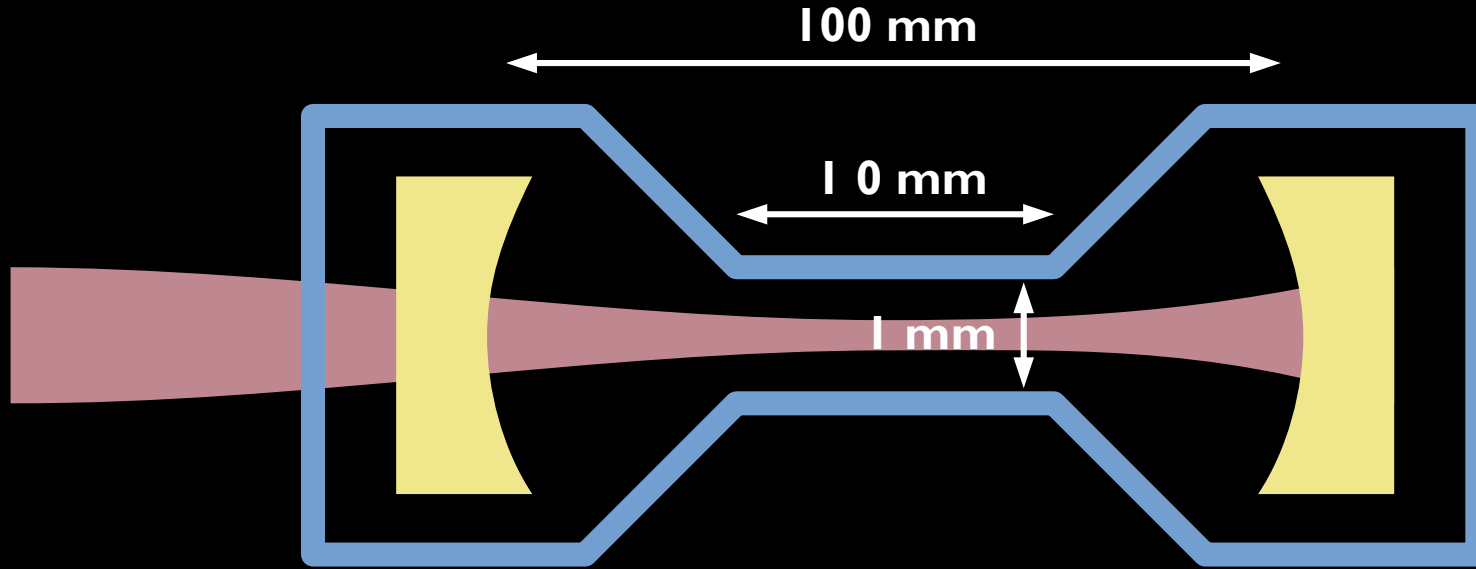
- **Magnetic dipole transition**
- **Small atom cross-section**
- **Non-radiative decay**







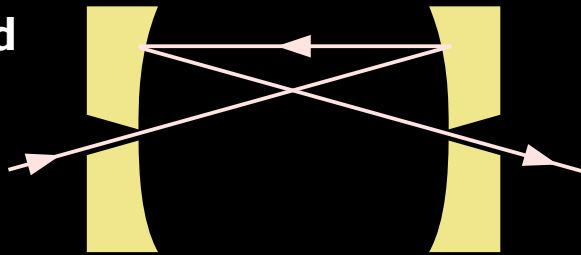




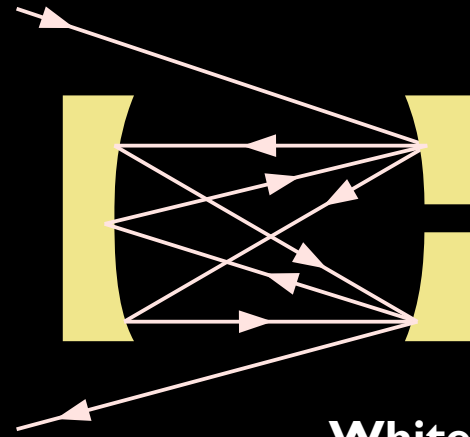
- **Operation at 50 K**
- **Large illuminated volume**
- **Large laser fluence required (2 J/cm^2)**
- **~ 500 reflections through a 1 mm gap**

Multi-pass cells

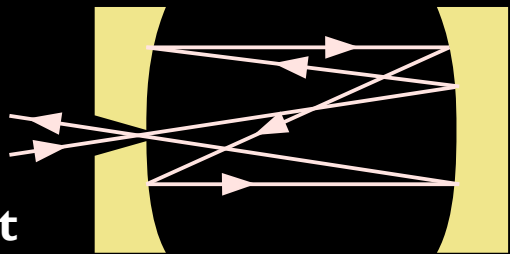
Pfund



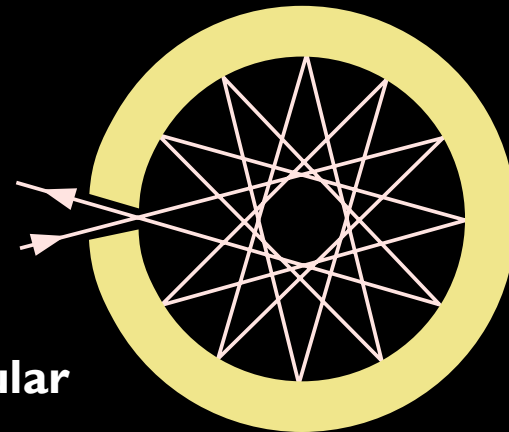
White



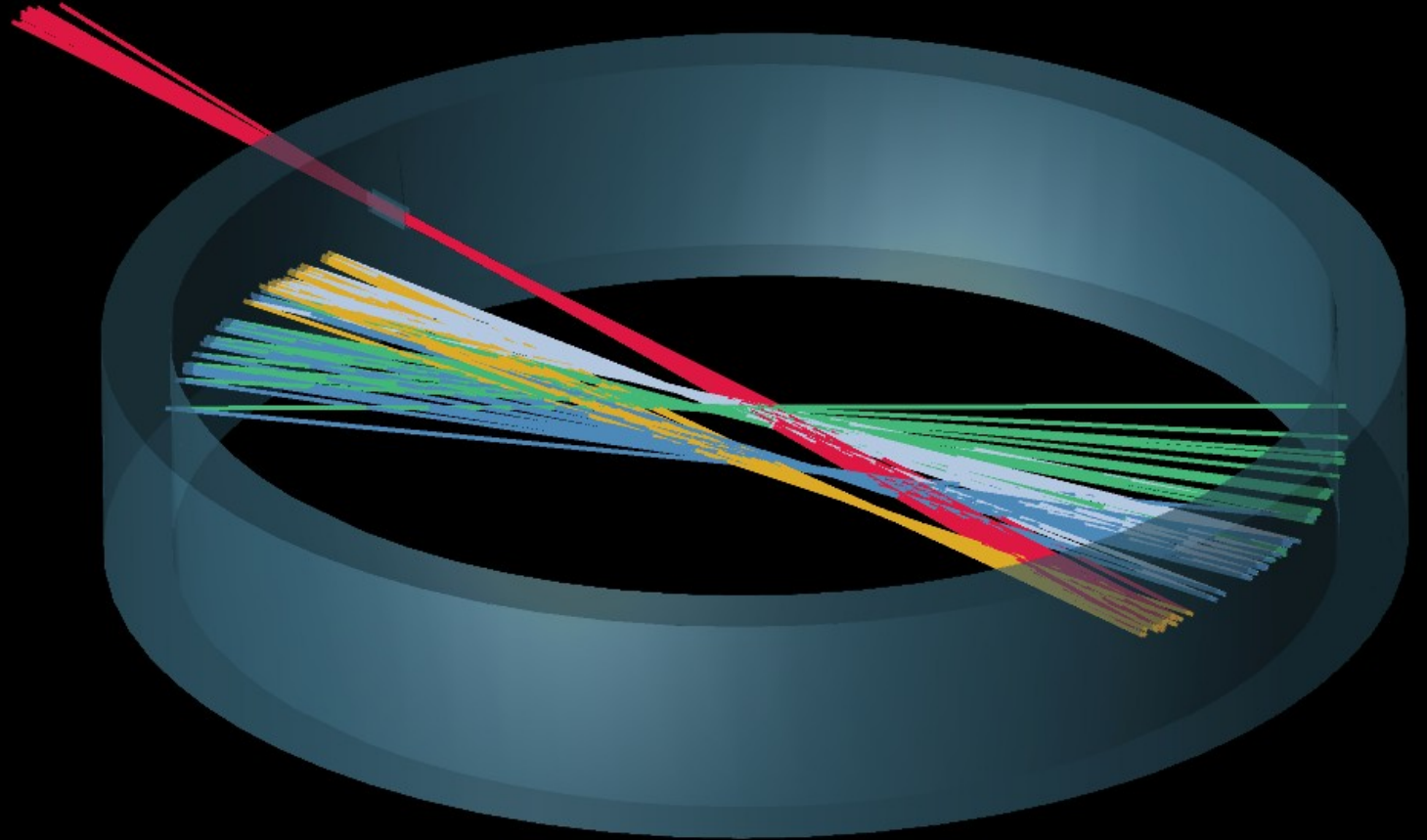
Herriott



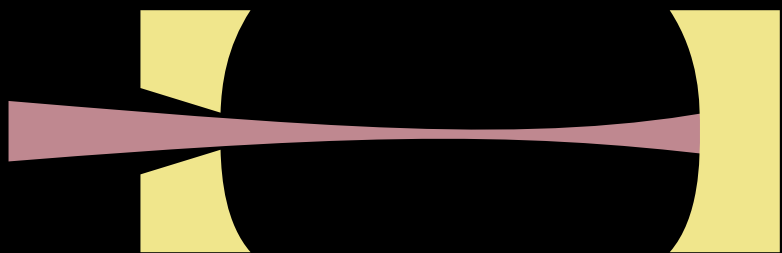
Circular



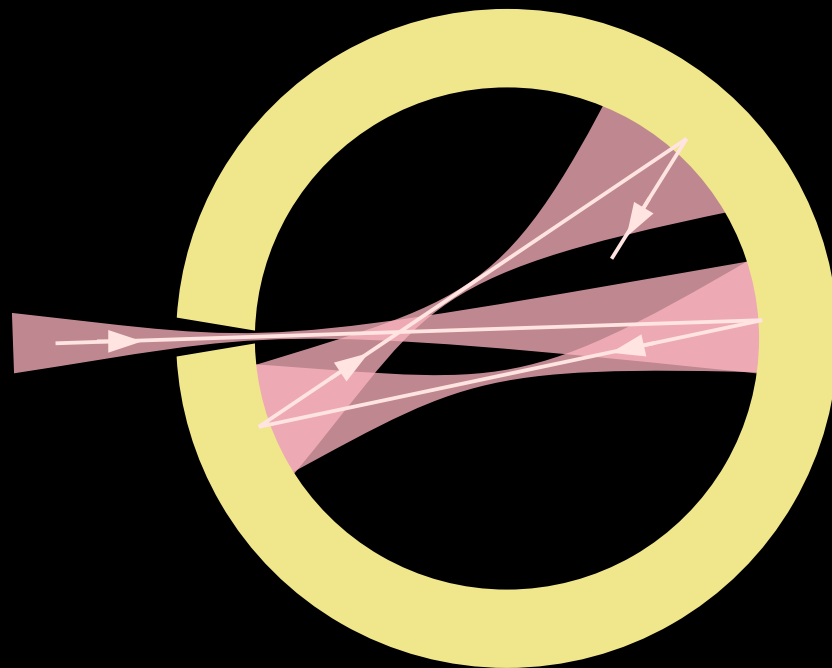
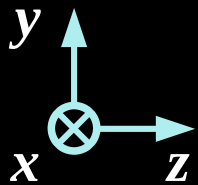
Design No. 1.



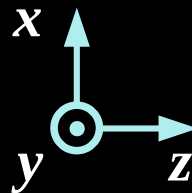
Design No. 1.



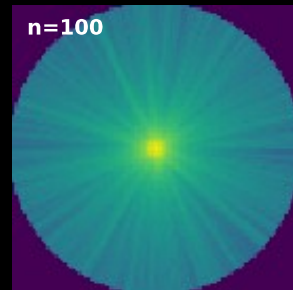
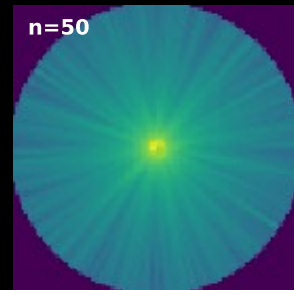
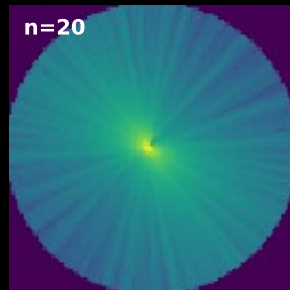
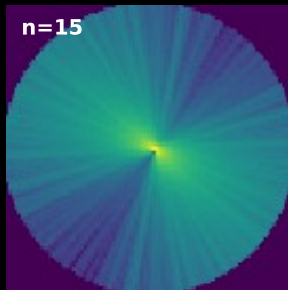
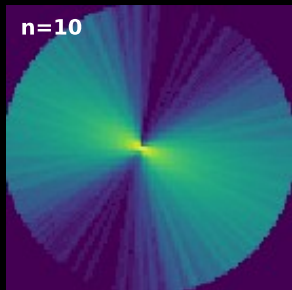
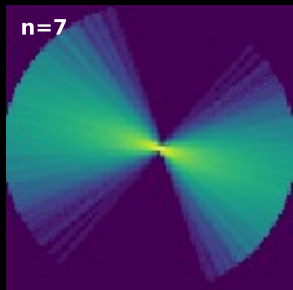
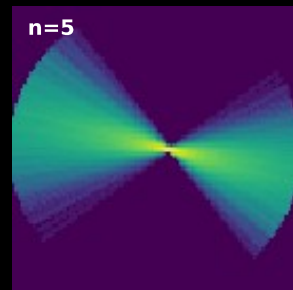
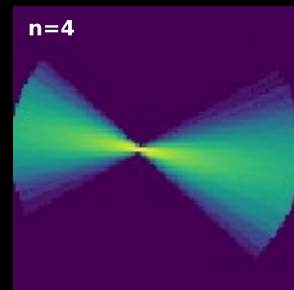
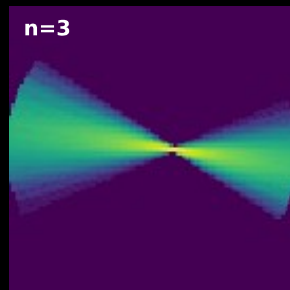
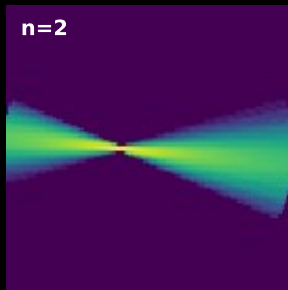
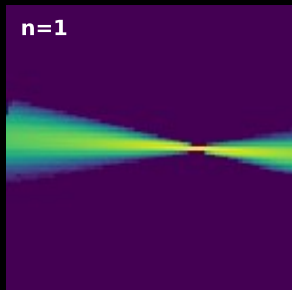
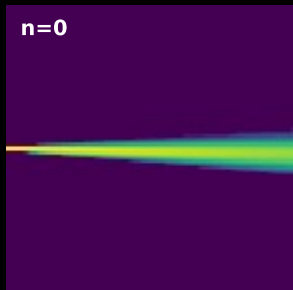
$$L < 2R_y$$

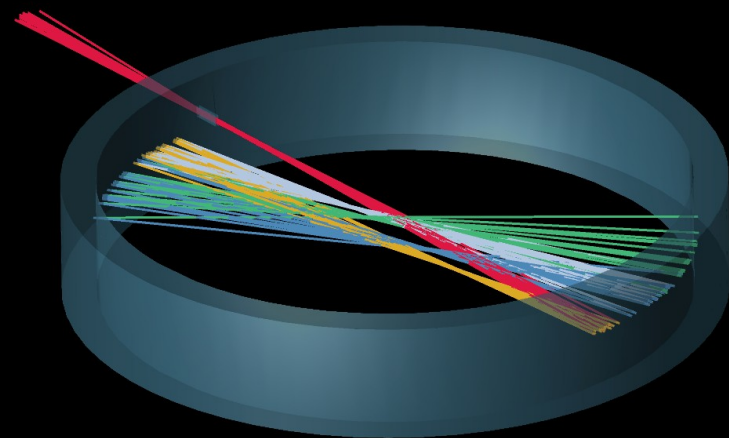
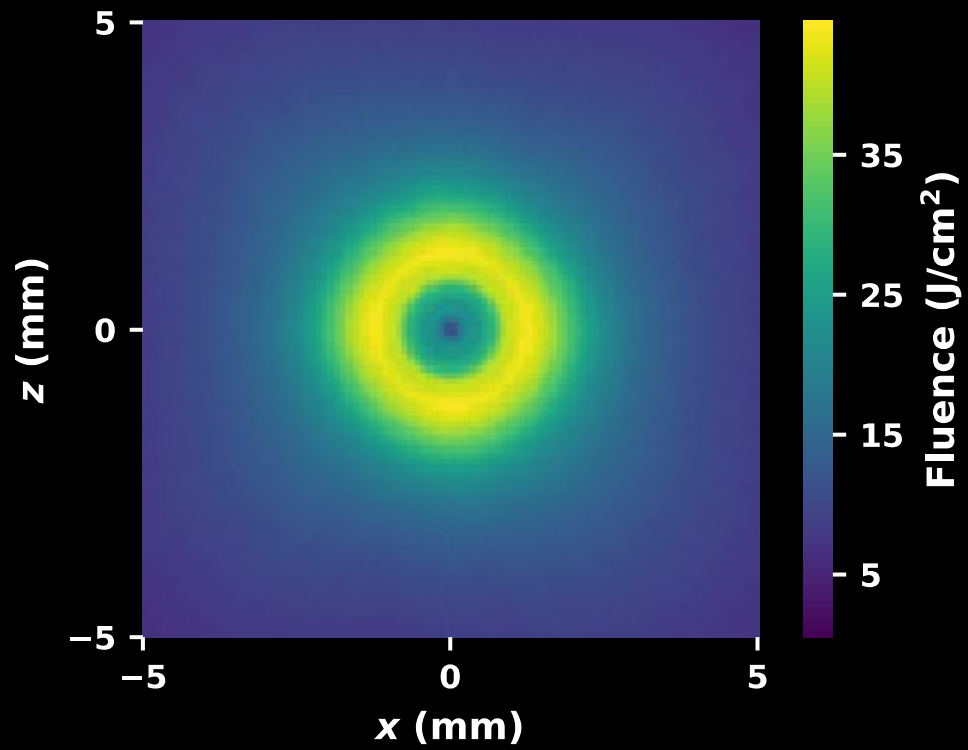


$$L = 2R_x$$



Ray tracing



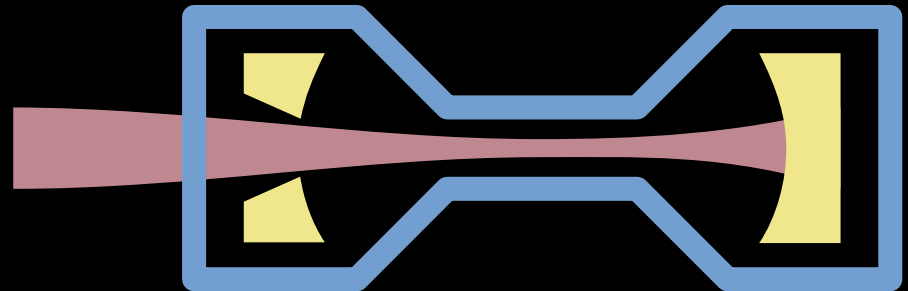


What to optimize

- Excitation probability

$$P = 1 - \exp\left(-\frac{F}{F_s}\right)$$

$$F \propto \frac{E_0}{\mathcal{L}}$$

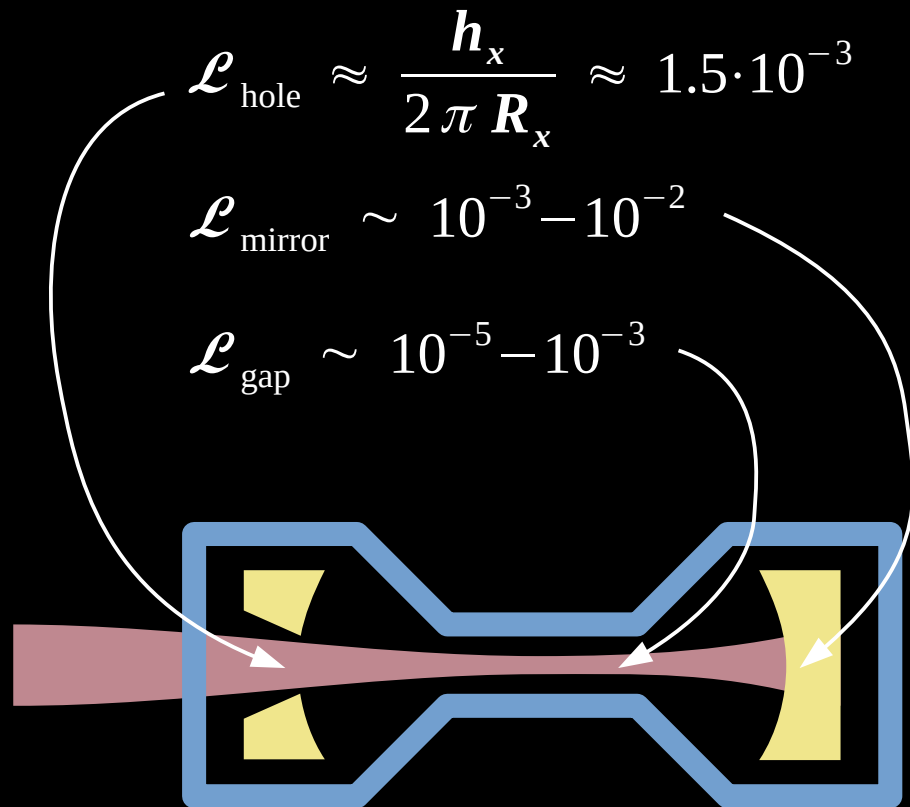


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What to optimize

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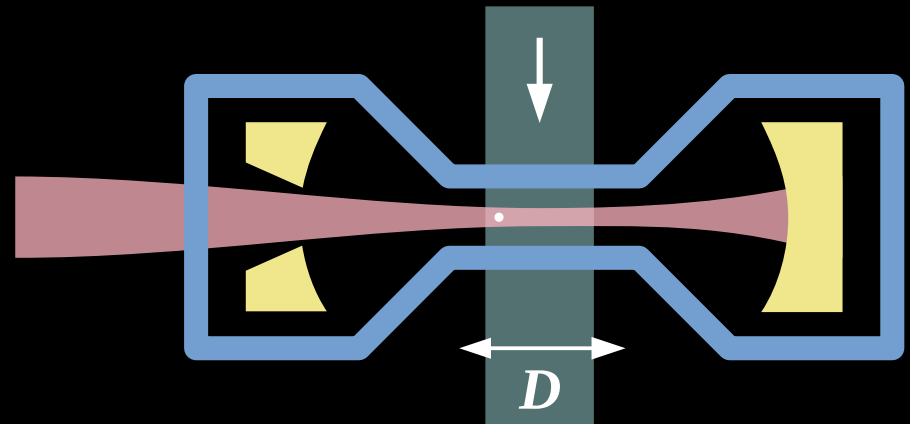
$$F \propto \frac{E_0}{\mathcal{L}}$$

- Signal-to-noise ratio

$$S \propto FA$$

$$B \propto A$$

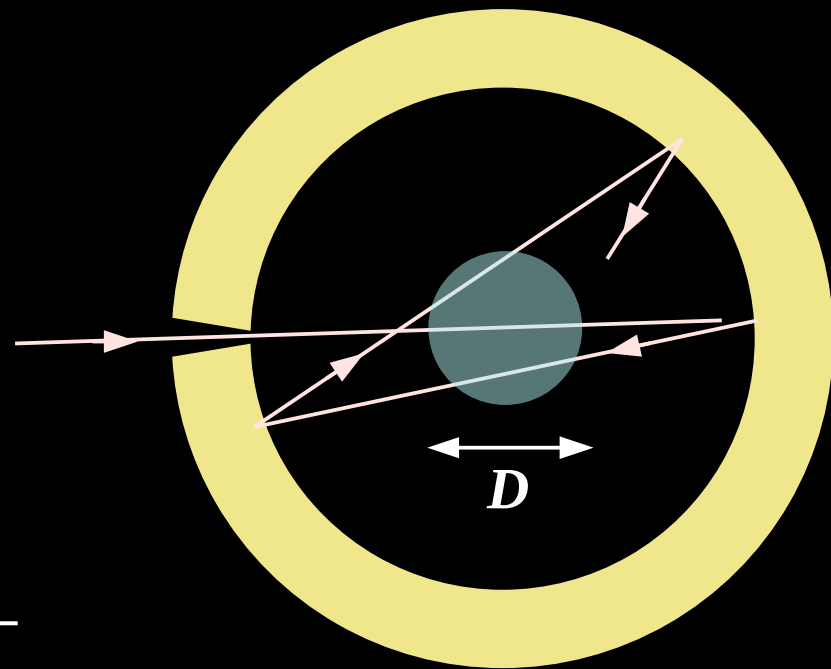
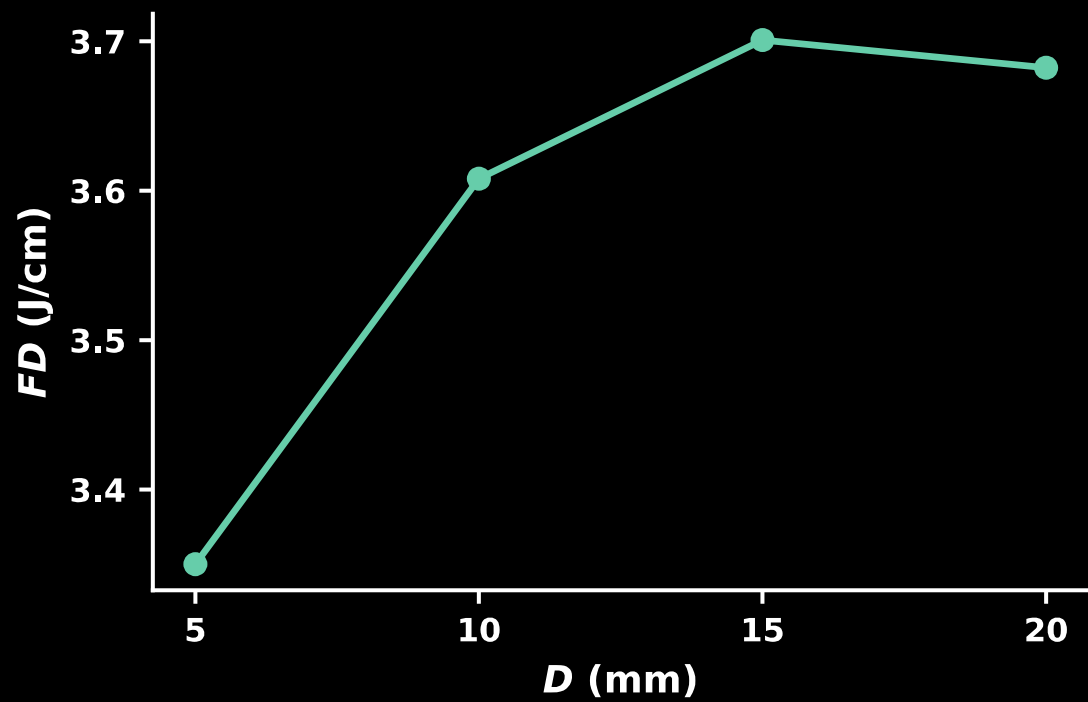
$$\text{SNR} = \frac{S}{\sqrt{B}} \propto F\sqrt{A} \propto FD$$



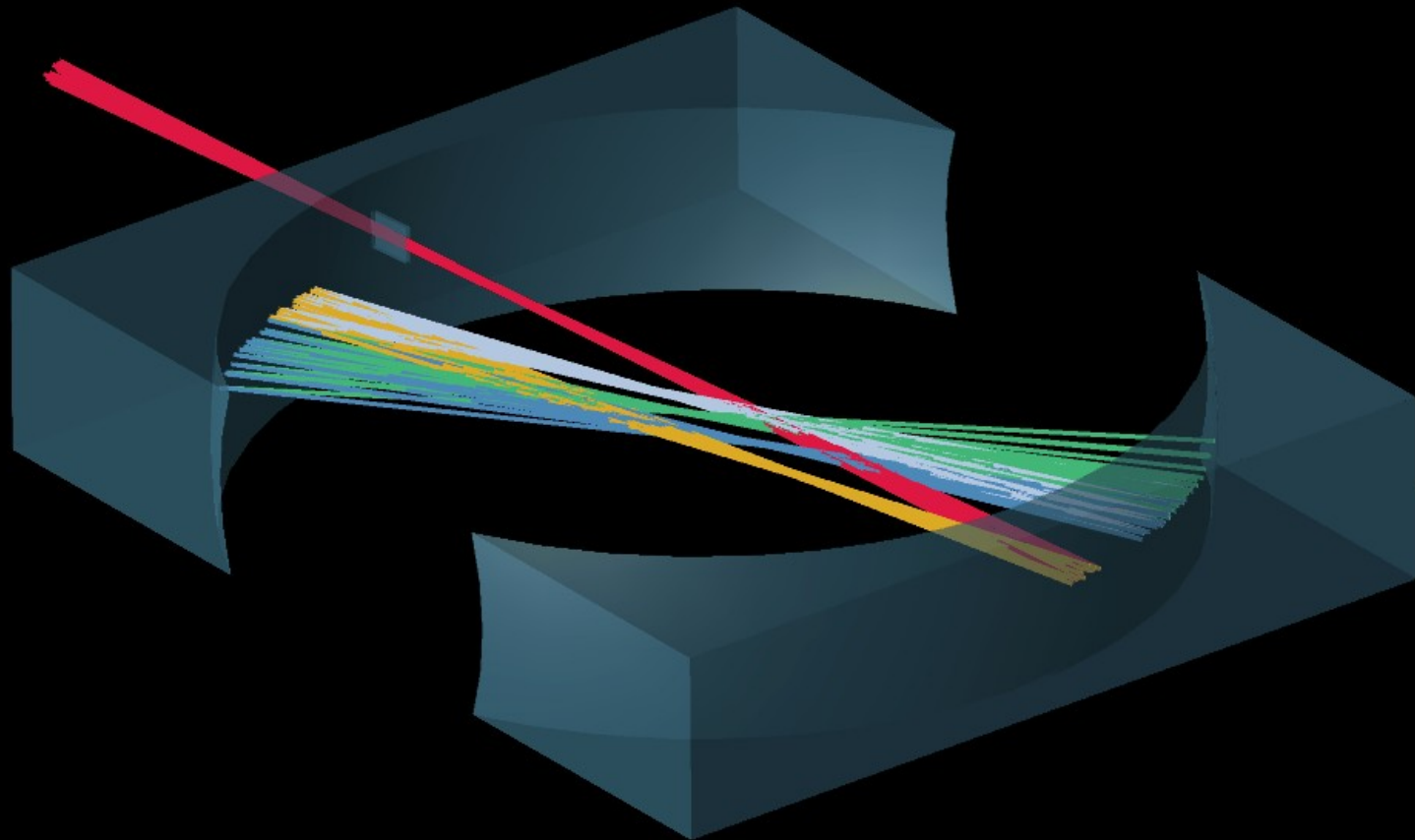
Recipe

for a multi-pass circular cavity

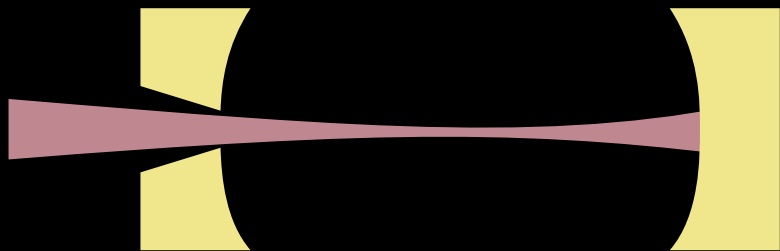
- **Preheat the oven to 50 K**
- **Make the hole as small as possible**
- **Make the beam as large as possible, but still fitting comfortably through the hole**
- **Adjust the input angle for best performance**



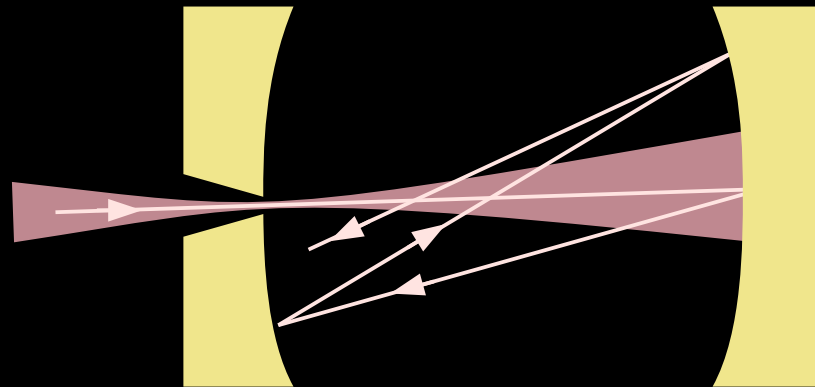
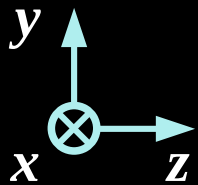
Design No. 2.



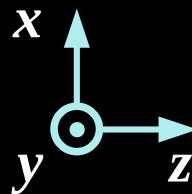
Design No. 2.

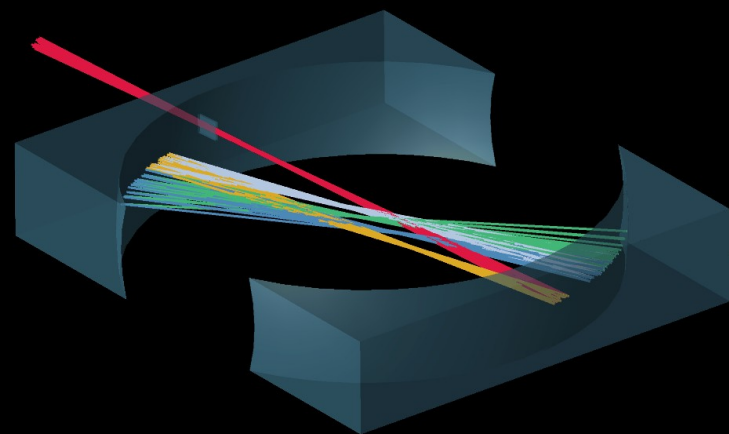
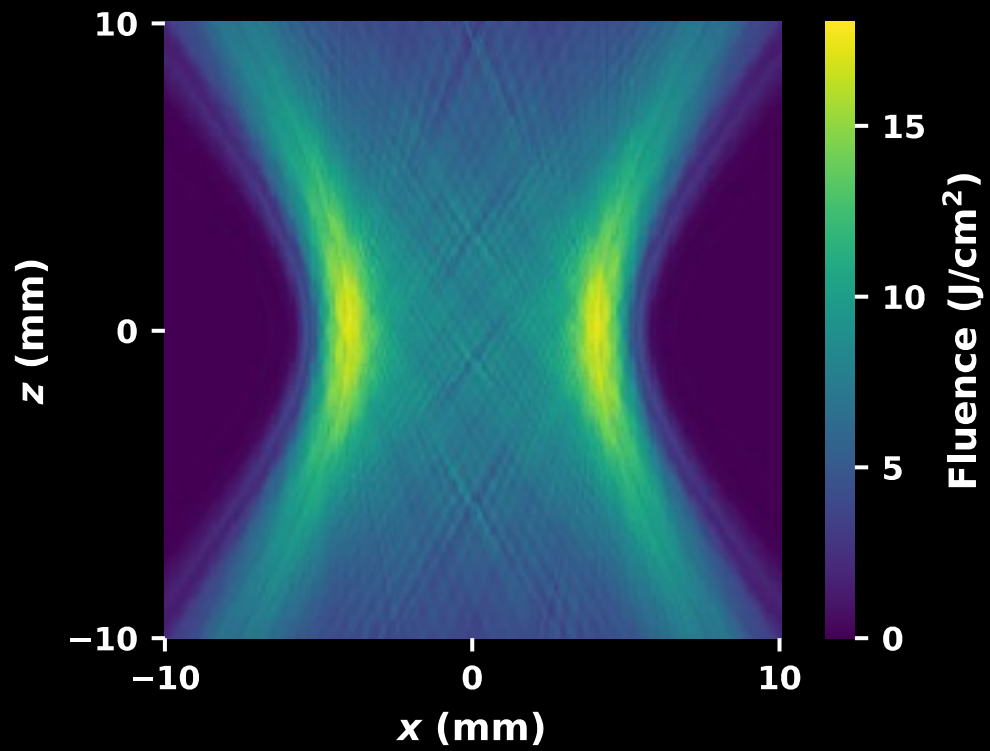


$$L < 2R_y$$

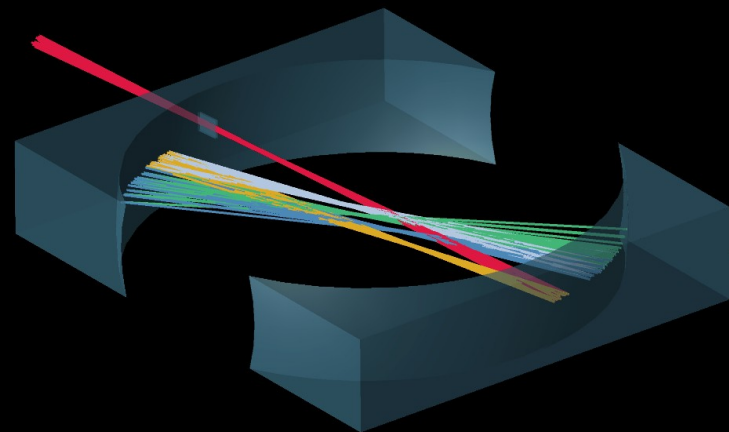
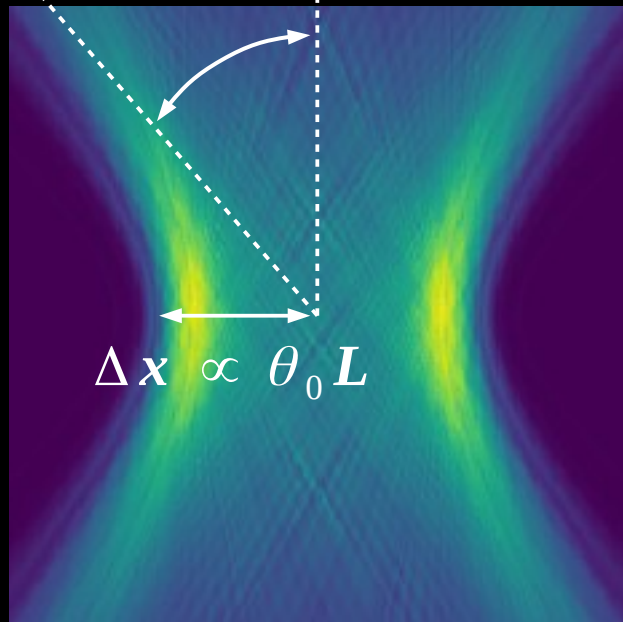


$$L < 2R_x$$

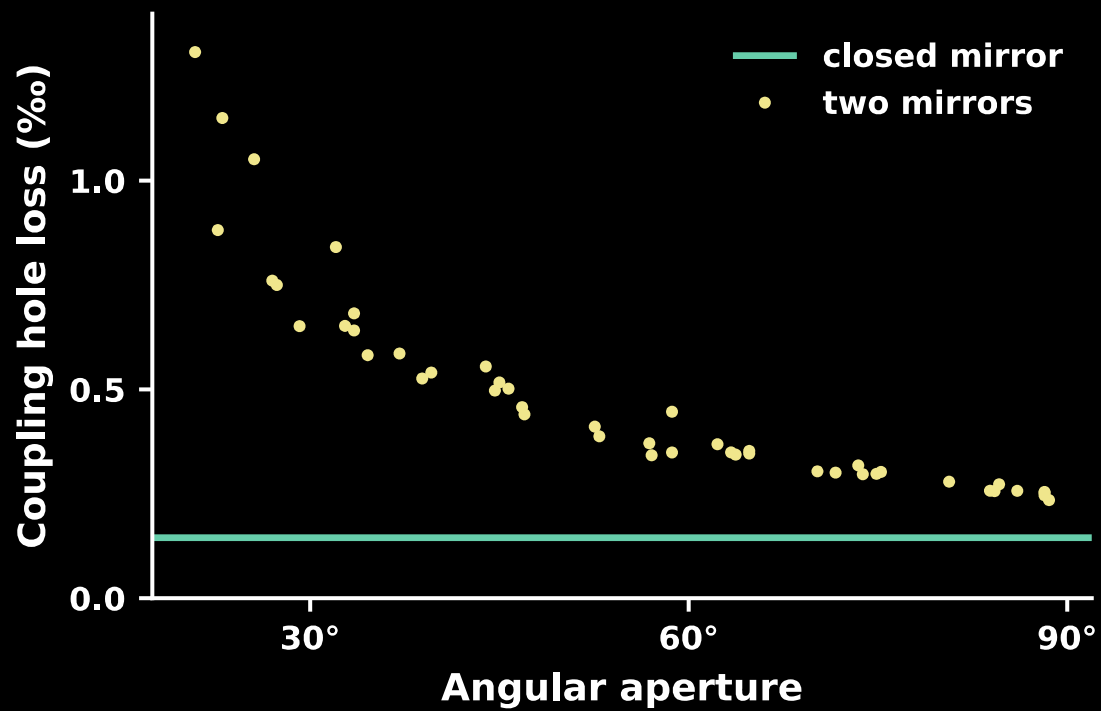
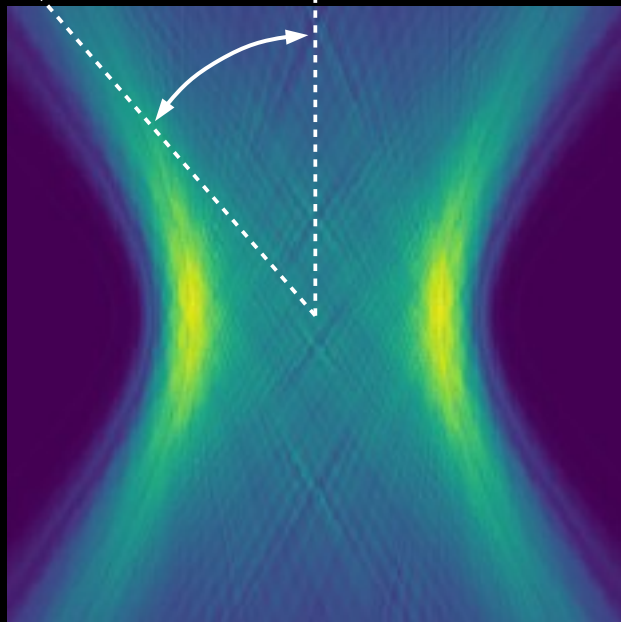


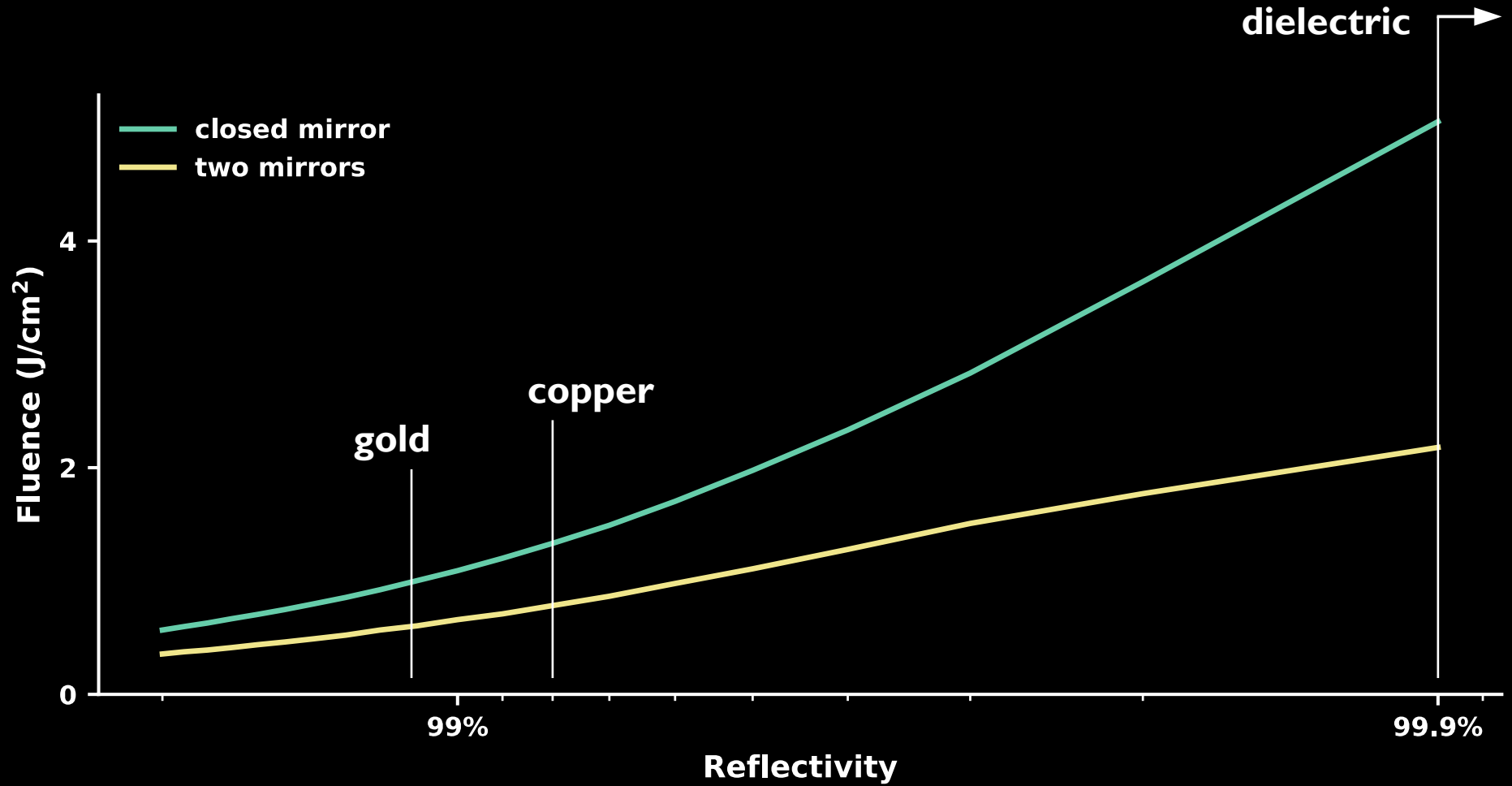


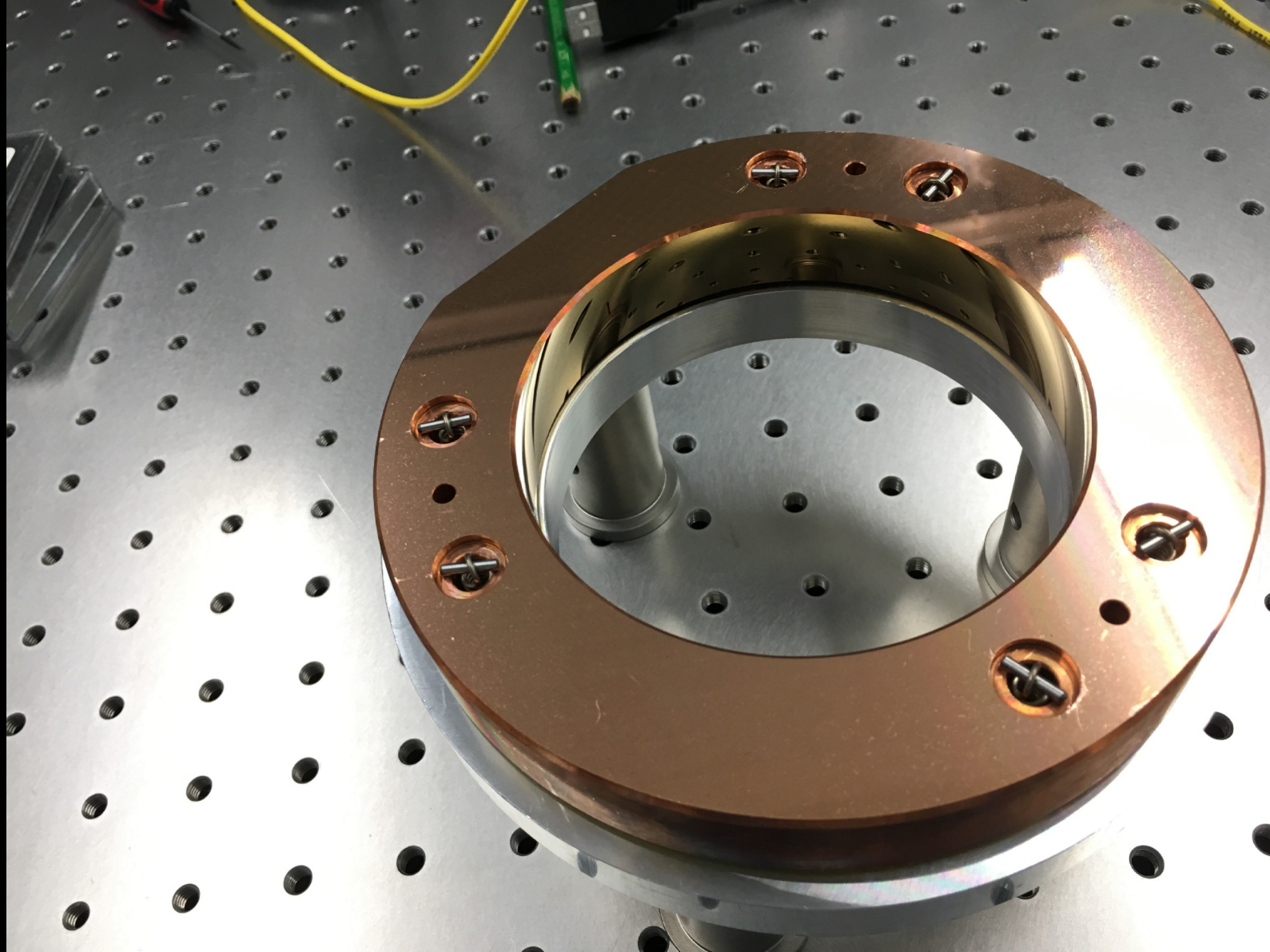
$$\alpha \propto \frac{\theta_0}{\sqrt{1 - \frac{L}{2R_x}}}$$

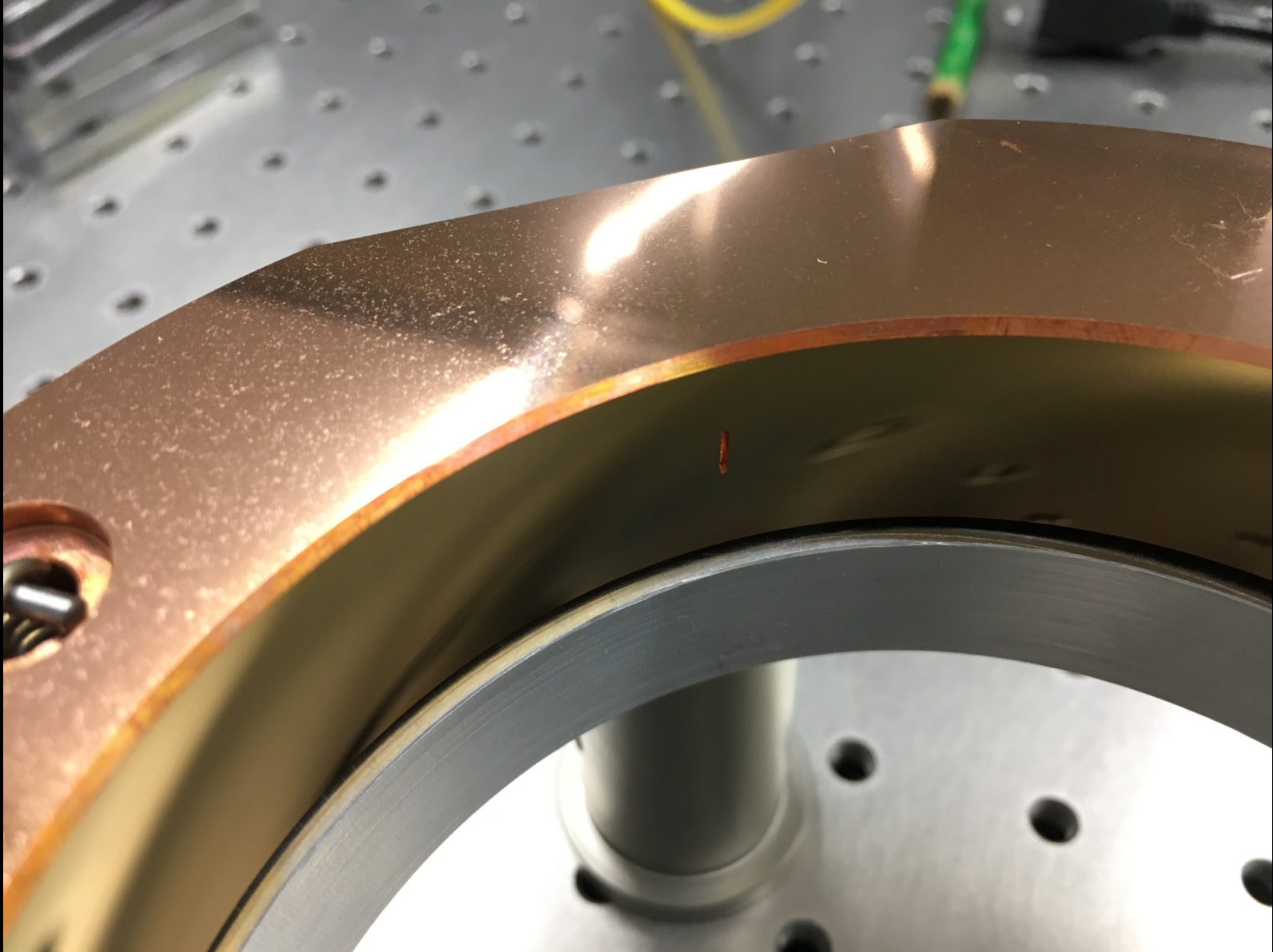


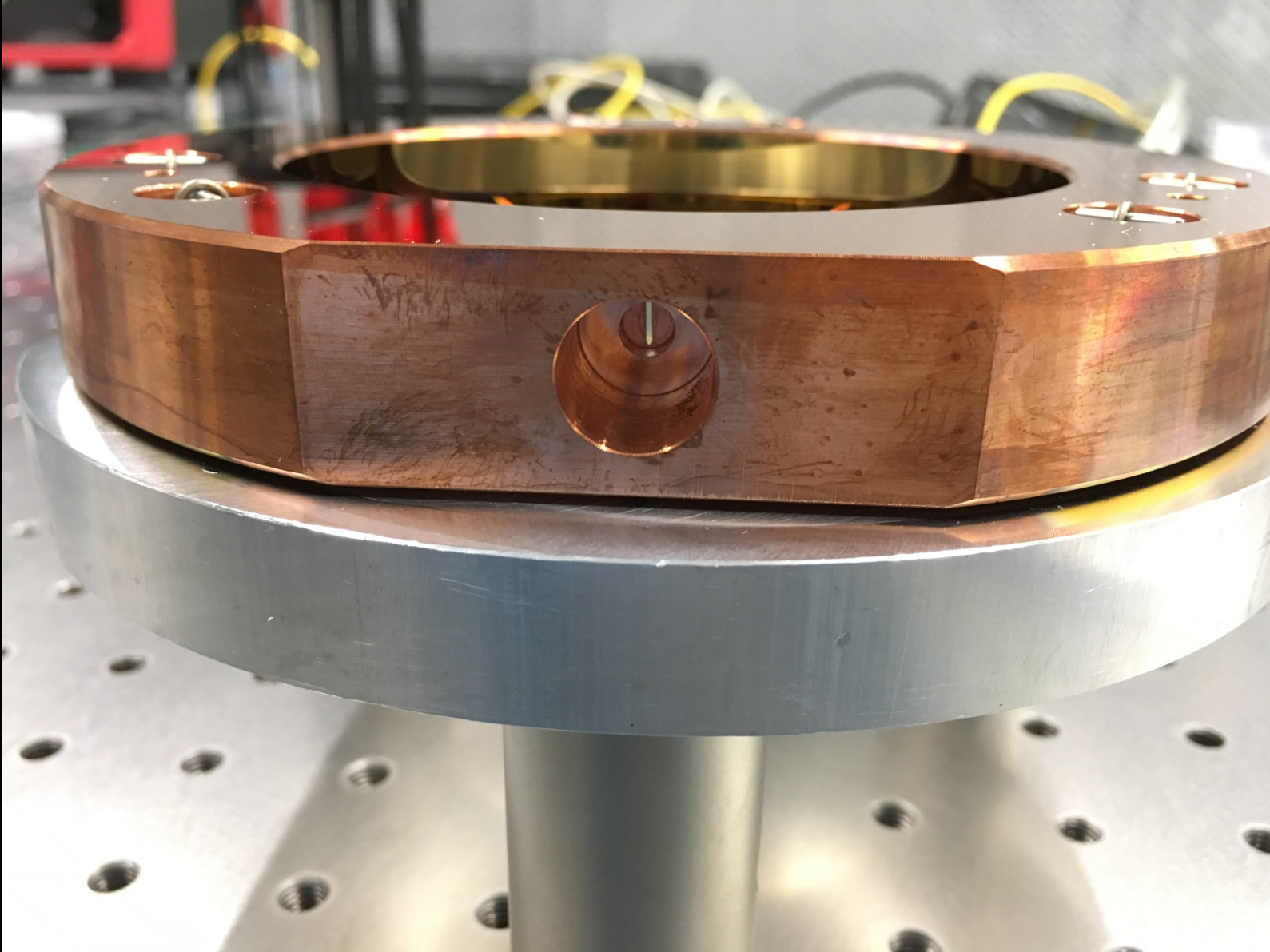
$$\alpha \propto \frac{\theta_0}{\sqrt{1 - \frac{L}{2R_x}}}$$

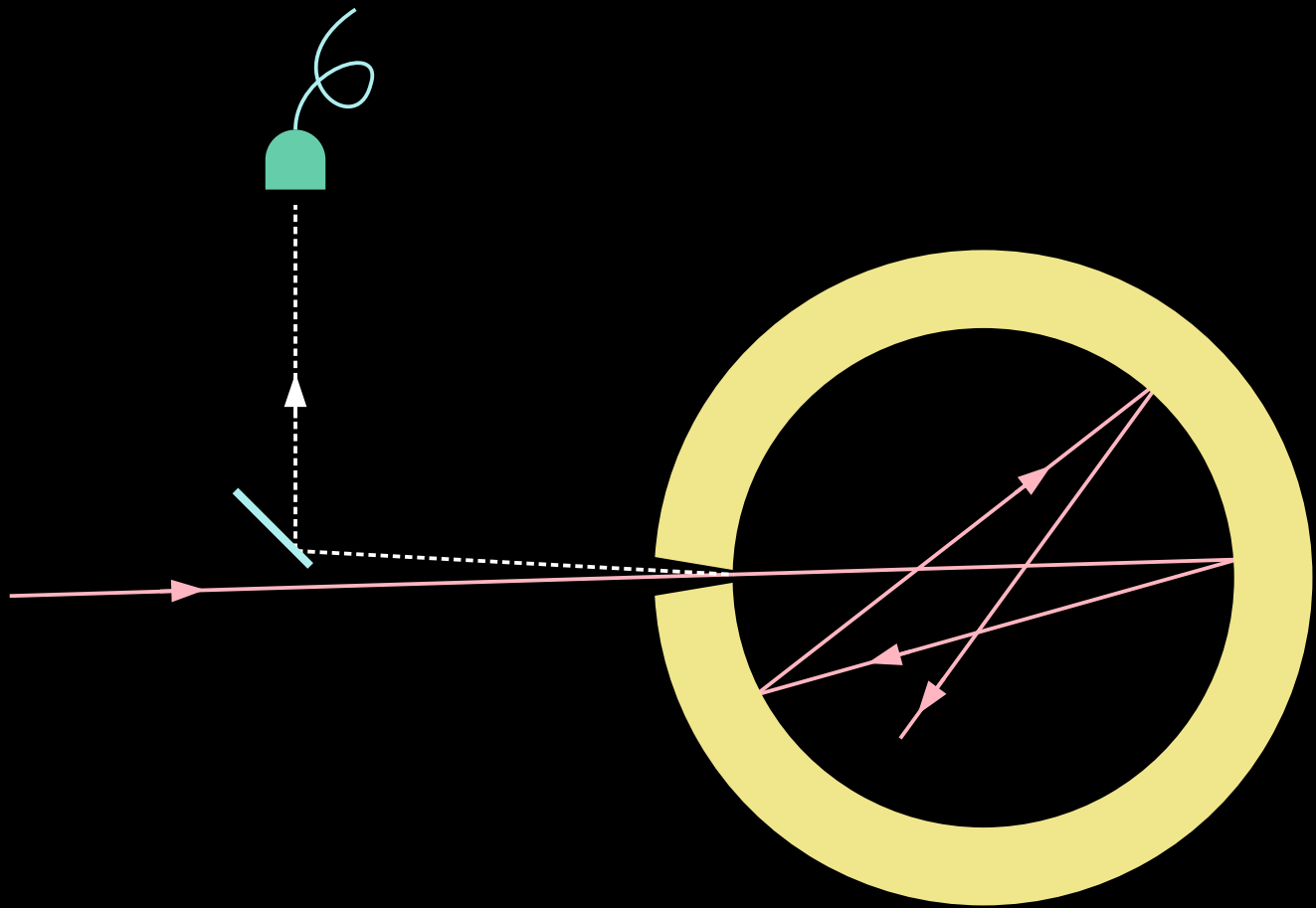


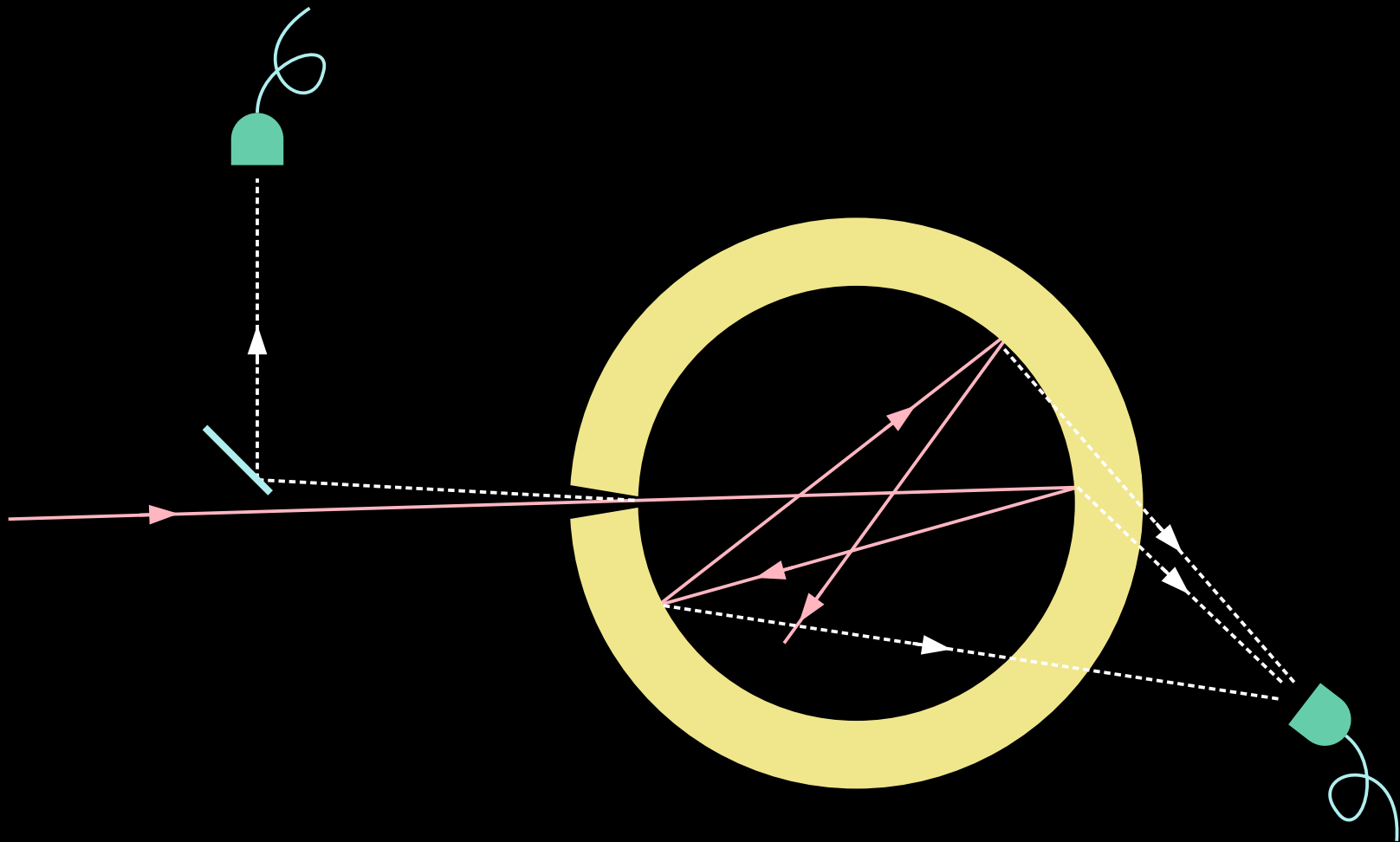


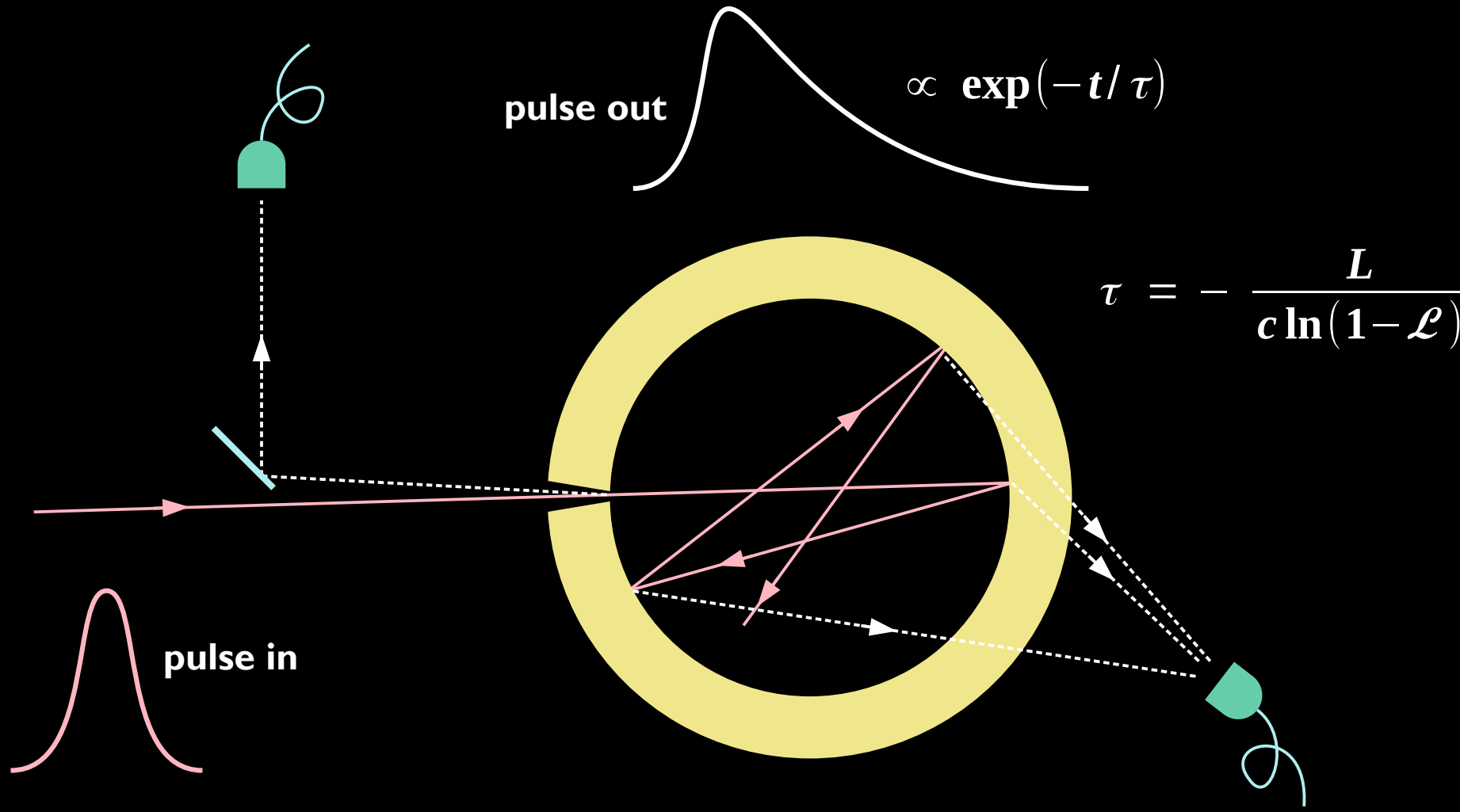










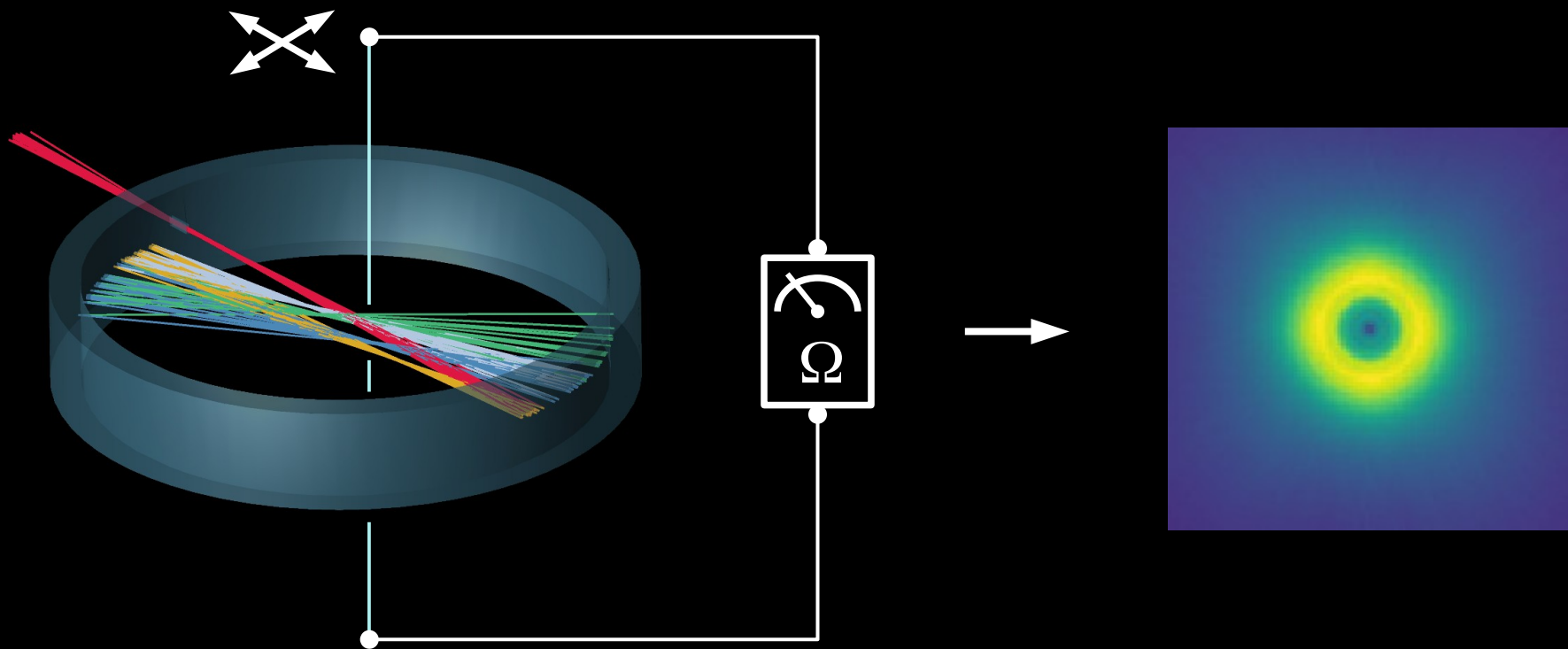


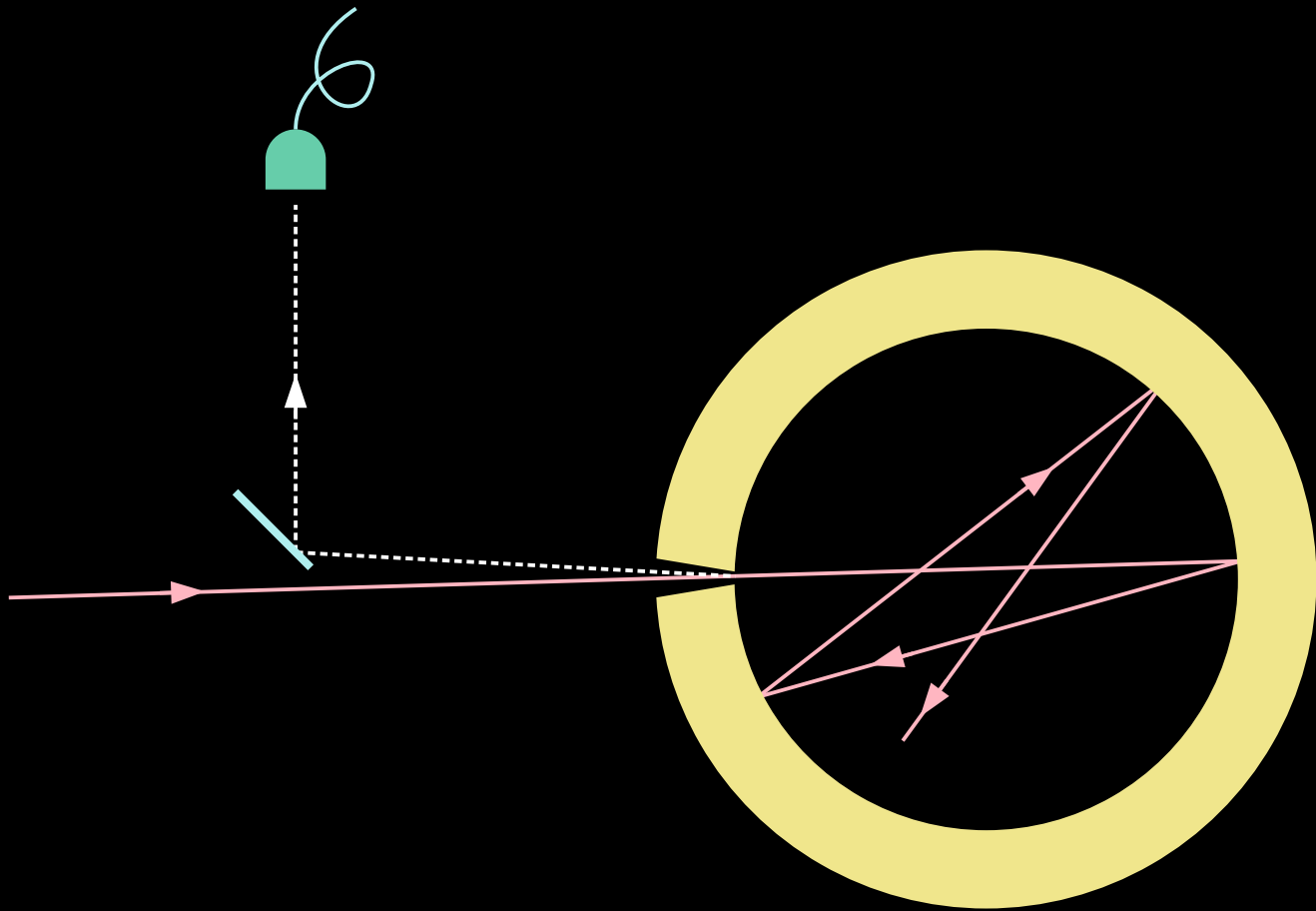
pulse out

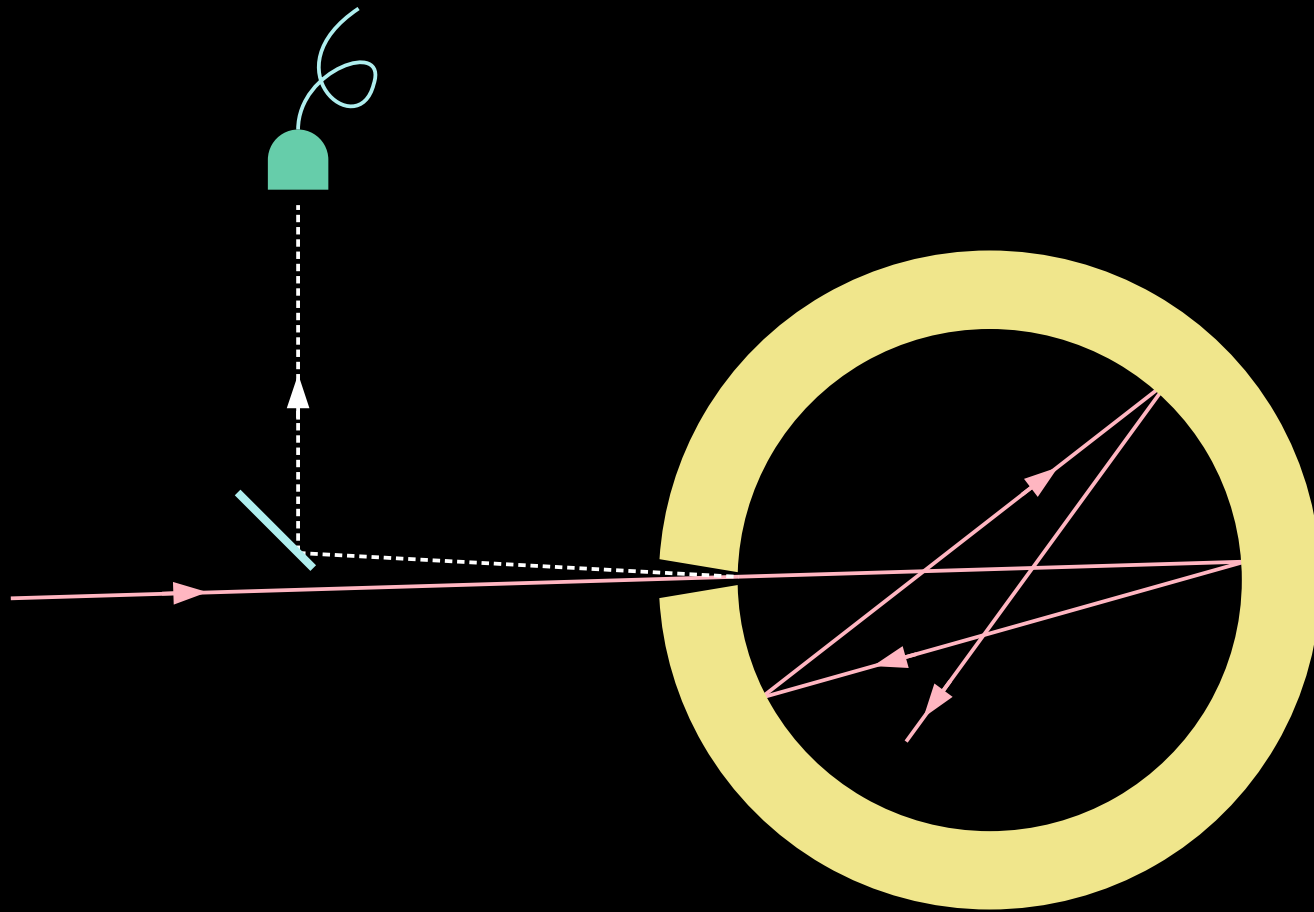
$$\propto \exp(-t/\tau)$$

$$\tau = -\frac{L}{c \ln(1-\mathcal{L})}$$

pulse in



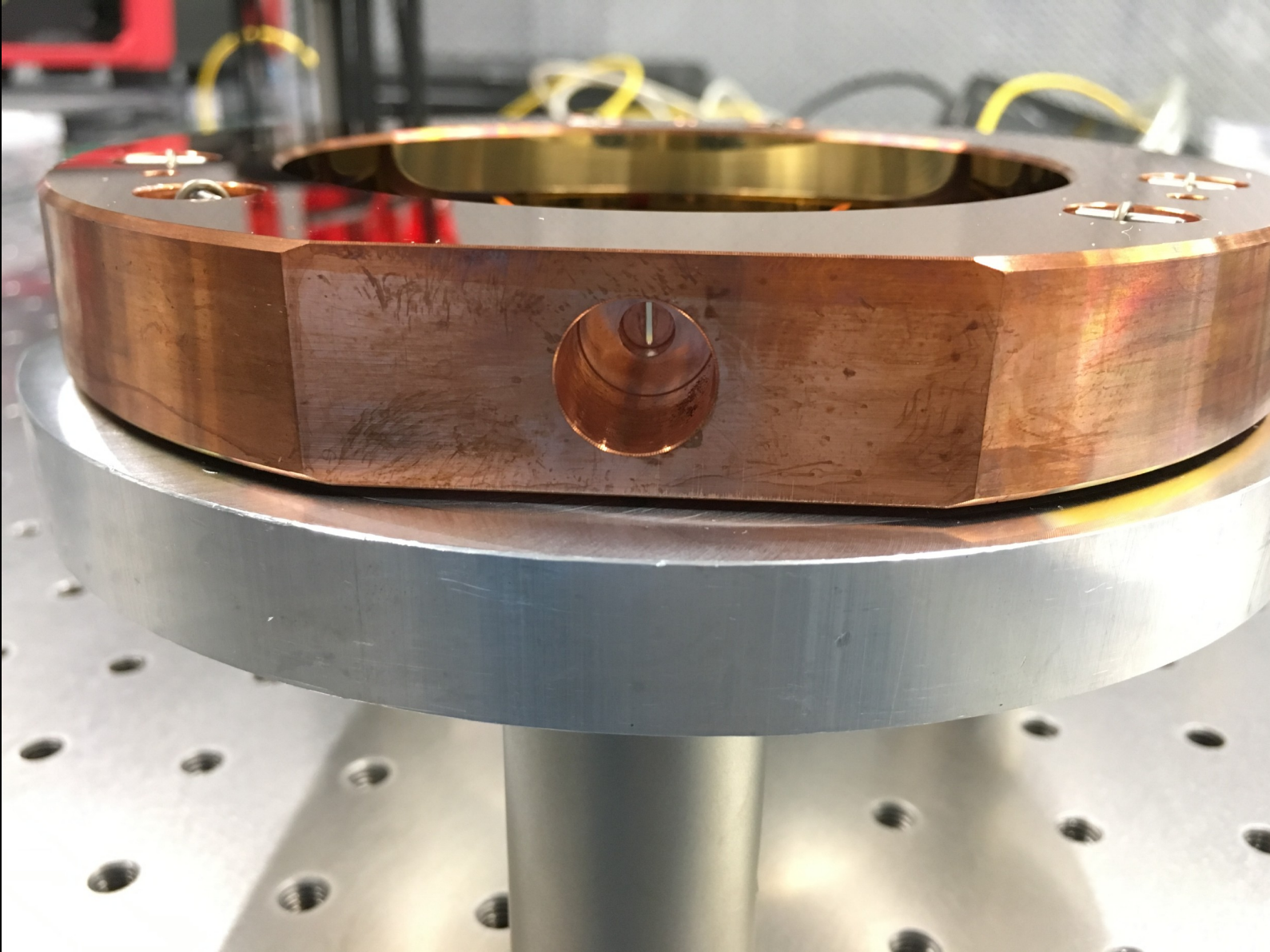




$R = 99.9 \%$

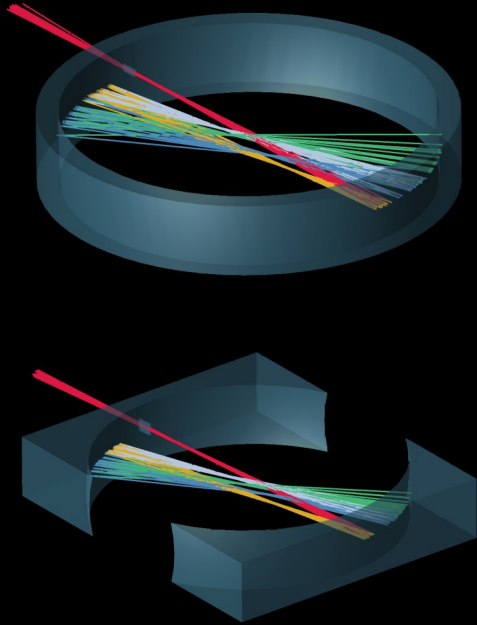
$\mathcal{L}_{\text{hole}} = 1.5 \text{ ‰}$

$L_{\text{eff}} = 40 \text{ m}$

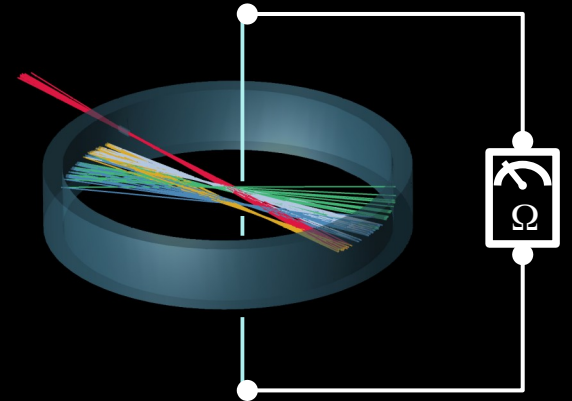
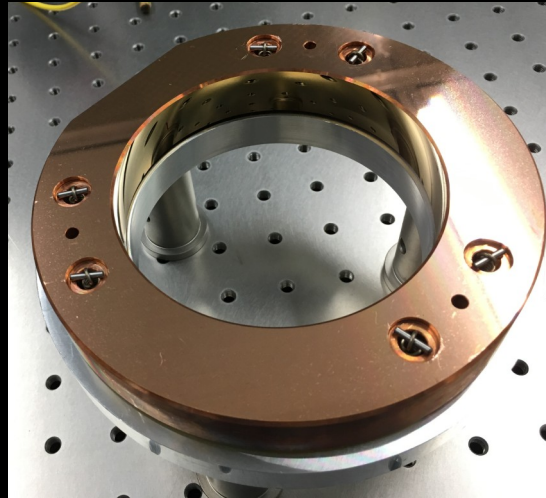


Summary

We have the first prototypes



Two plausible design forms



We have some ideas
how to test them