

HIGGS PAIR PRODUCTION AT NLO QCD

Michael Spira (PSI)

- I Introduction
- II Higgs Boson Pair Production
- III Calculation
- IV Conclusions

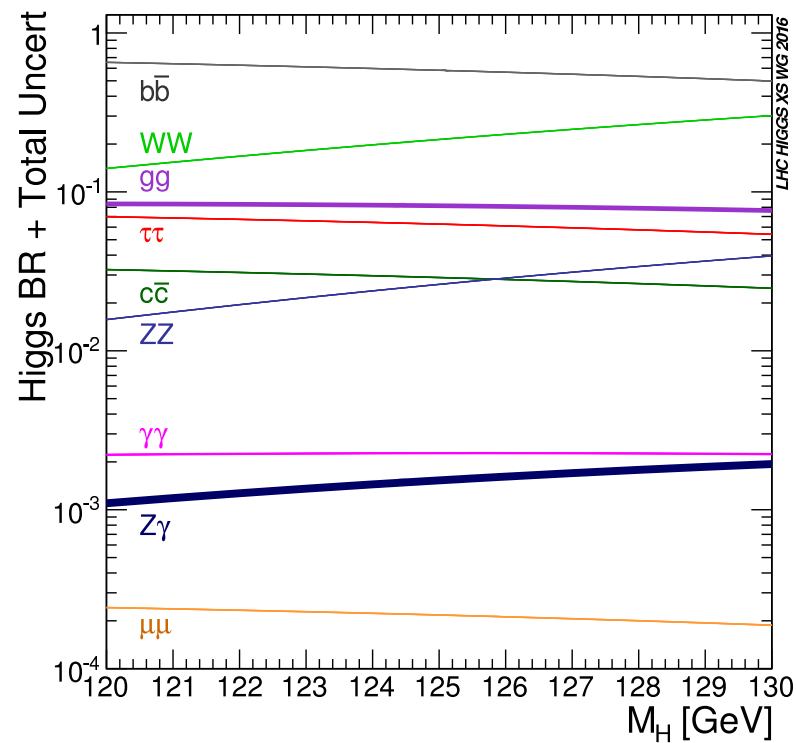
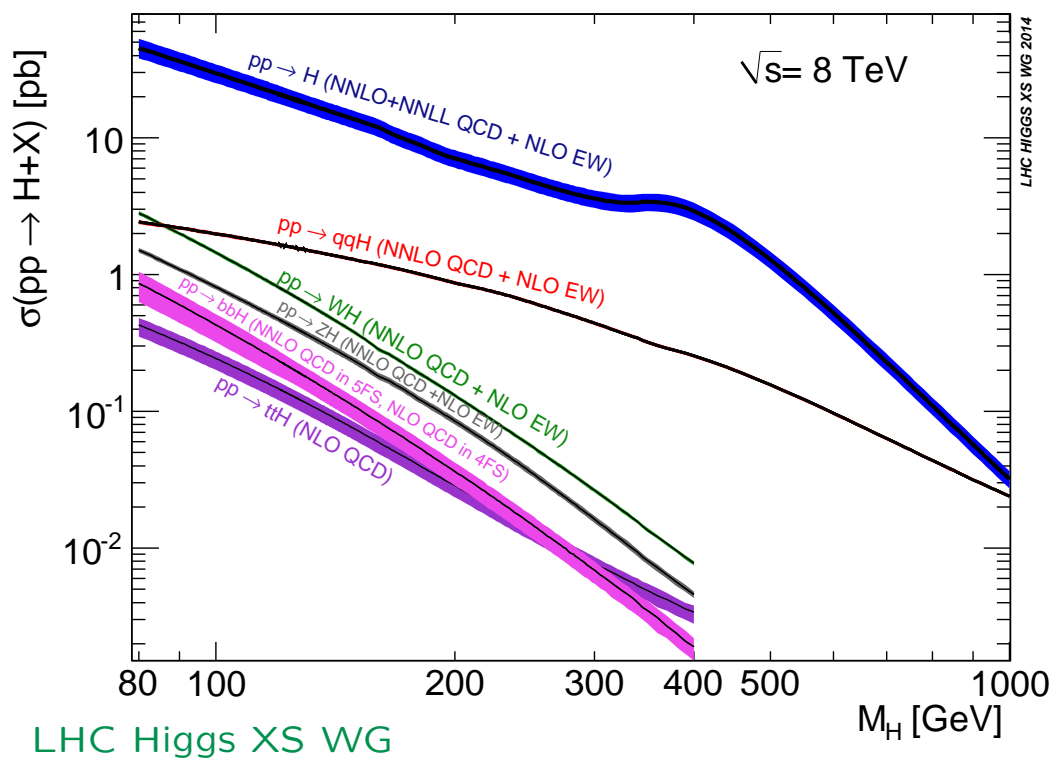
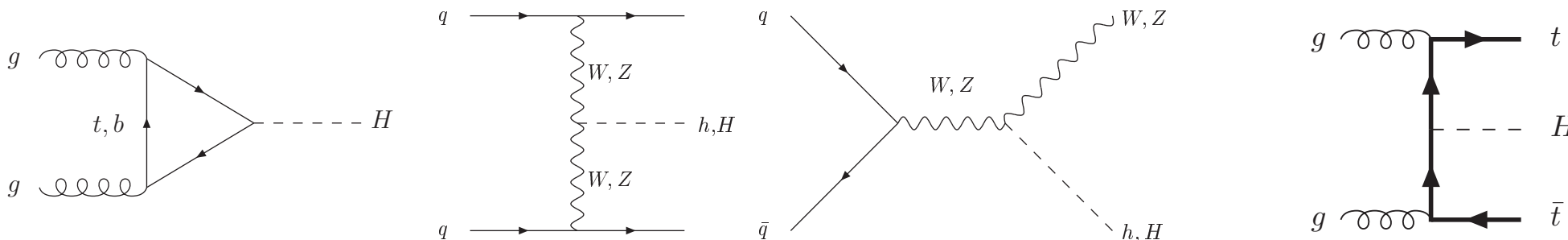
I INTRODUCTION

- SM very successful ← precision data [LEP, Tevatron, LHC]
- open problems: – mechanism of electroweak symmetry breaking
 - unification of forces
 - space-time structure @ short distances
- LHC: fundamental discoveries: Higgs boson(s?)
 - Supersymmetry ?
 - Extra space dimensions ?
- electroweak symmetry breaking: two classes of realization:
 - standard Higgs mechanism [SM, SUSY, . . .]
 - strong elw. symmetry breaking [TC, LH, Higgsless, ED, . . .]

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● Higgs Boson Production



- Discovery: LHC [Tevatron]

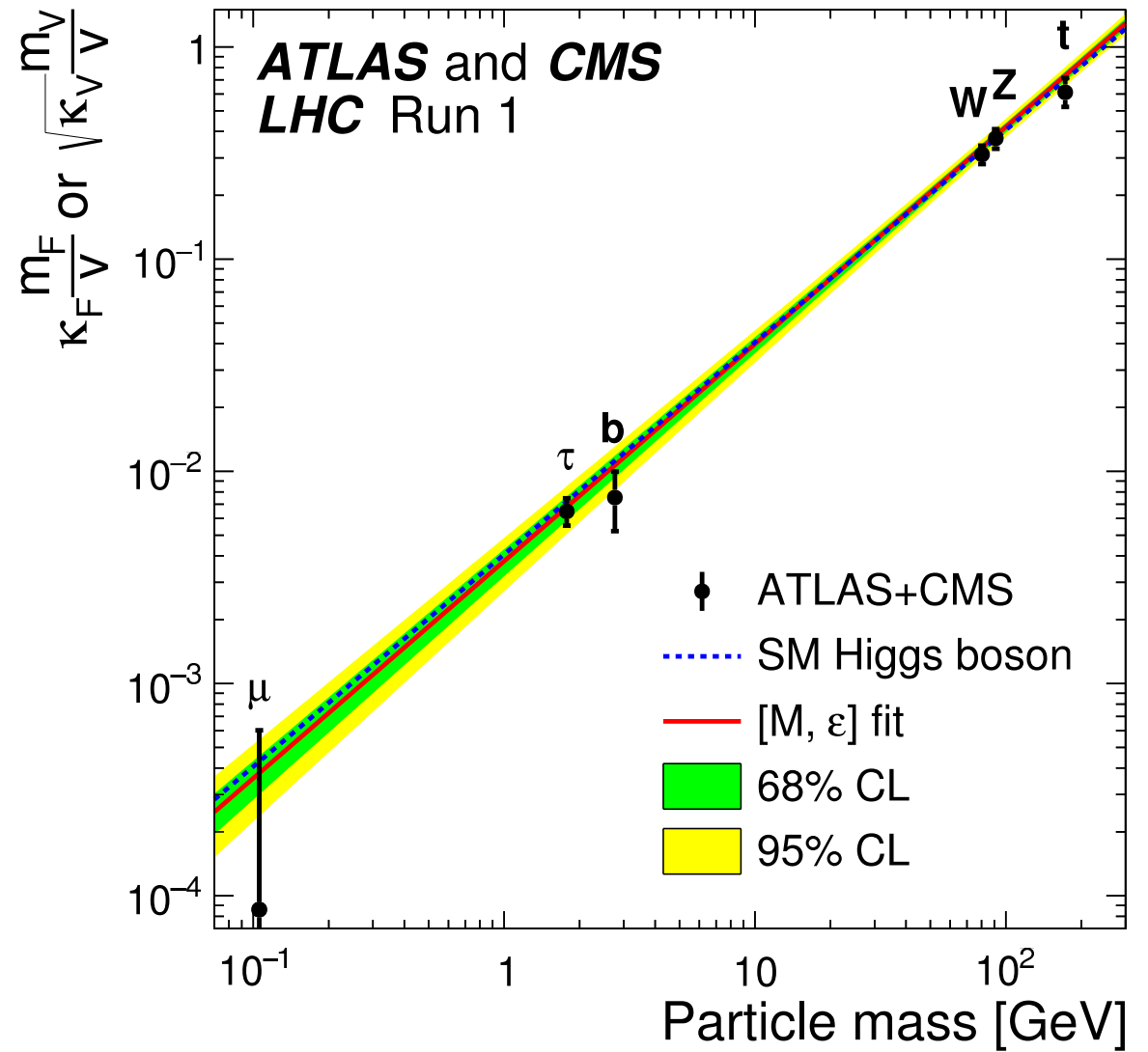
→ Higgs mass

couplings

spin

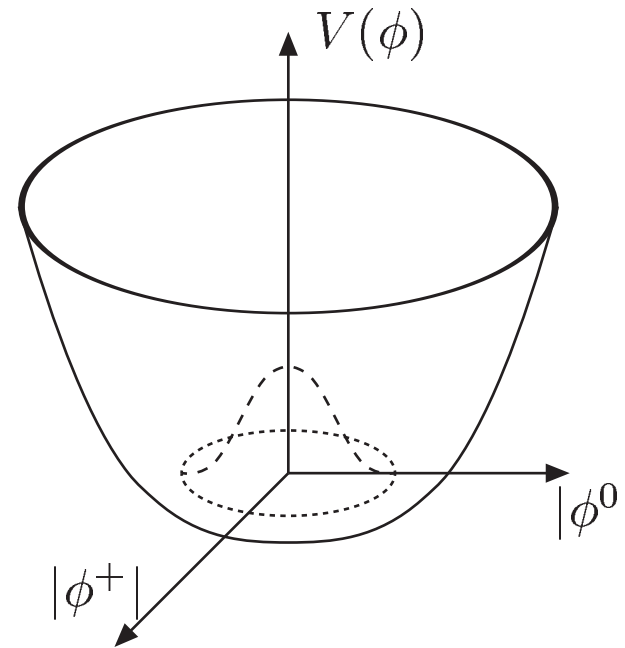
CP

$\lambda ?$



- Higgs potential:

$$\begin{aligned} \phi &= \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \\ &= \underbrace{\exp \left[i \vec{\Theta} \frac{\vec{\tau}}{2} \right]}_{\Rightarrow W^\pm, Z} \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix} \end{aligned}$$



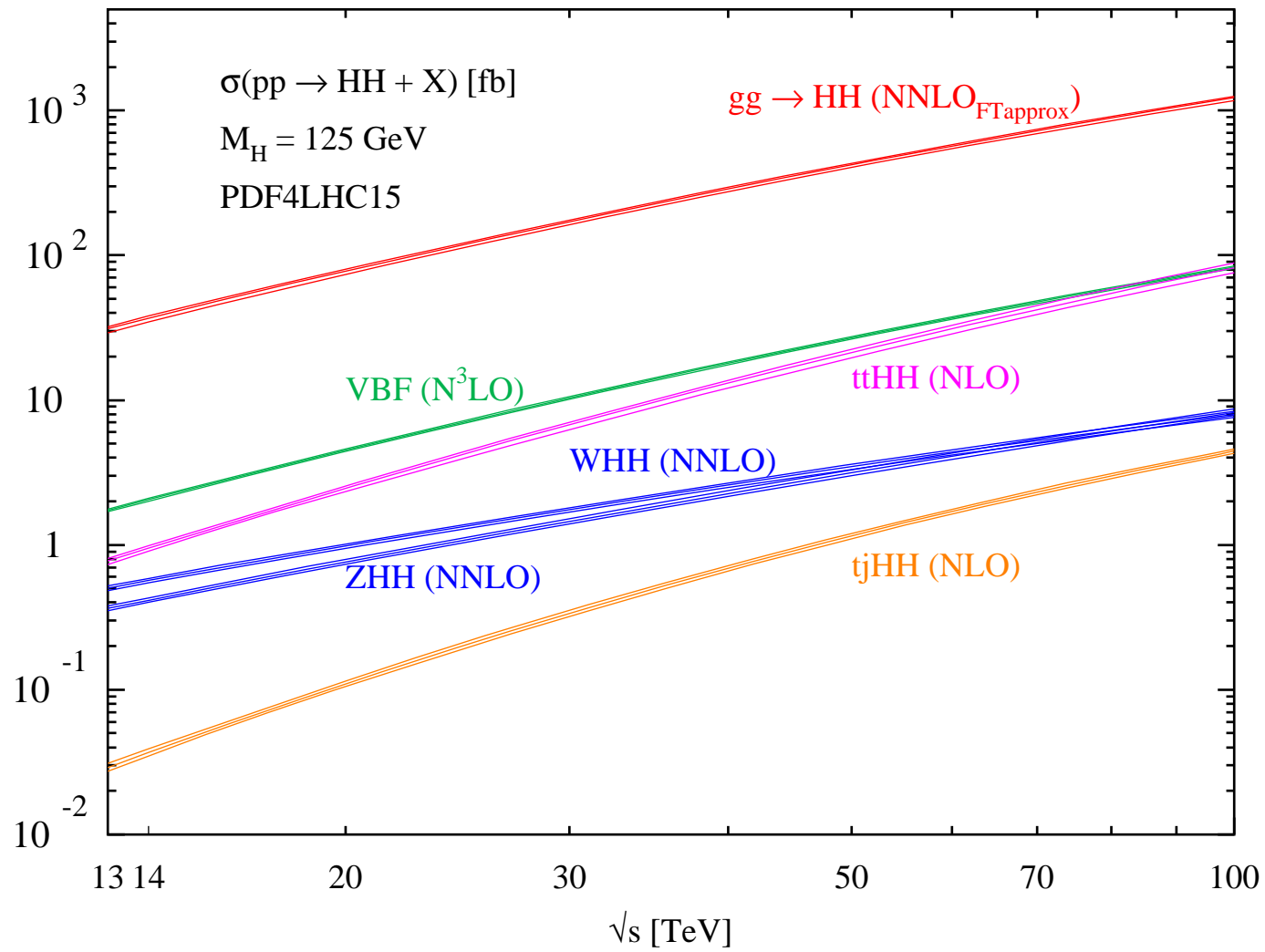
Higgs
Englert, Brout
Guralnik, Hagen, Kibble

$$\begin{aligned} V(\phi) &= \frac{\lambda}{2} \left[|\phi|^2 - \frac{v^2}{2} \right]^2 \\ &= \frac{M_H^2}{2} H^2 + \frac{M_H^2}{2v} H^3 + \frac{M_H^2}{8v^2} H^4 \end{aligned}$$

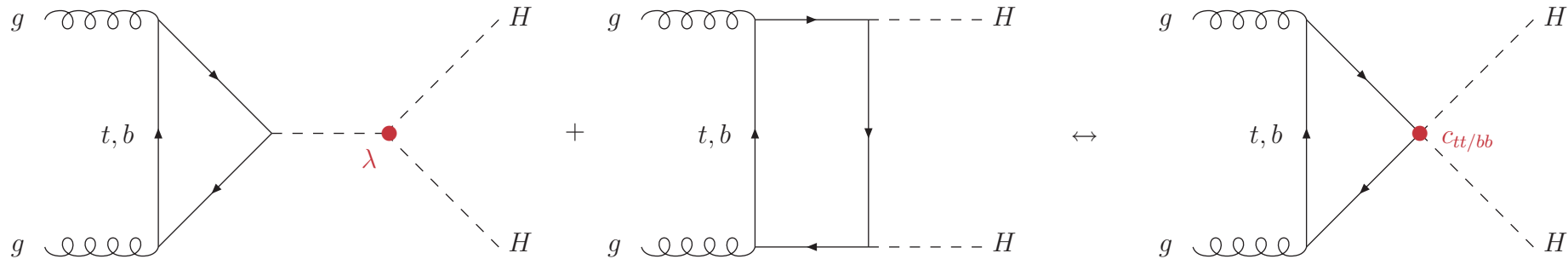
⇒ **one** scalar Higgs boson

$$v = 1/\sqrt{\sqrt{2}G_F} \approx 246 \text{ GeV}$$

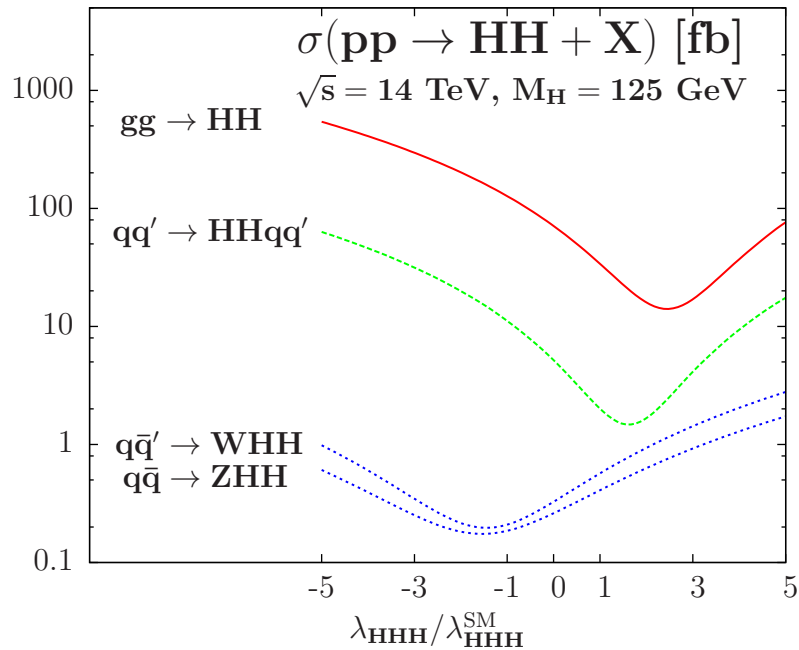
II HIGGS PAIR PRODUCTION



$gg \rightarrow HH$



- threshold region: sensitive to λ
- large M_{HH} : sensitive to $c_{tt/bb}$ [e.g. boosted Higgs pairs]

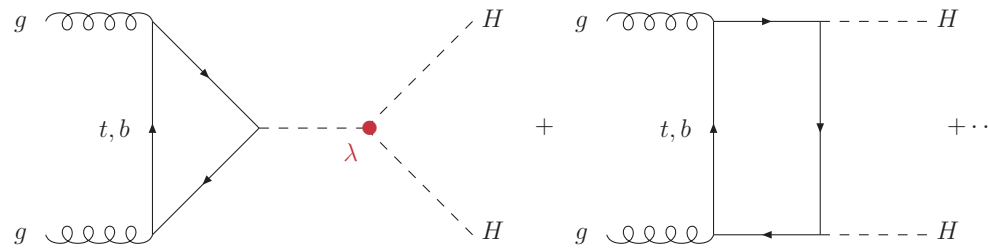


$$gg \rightarrow HH : \frac{\Delta\sigma}{\sigma} \sim -\frac{\Delta\lambda}{\lambda}$$

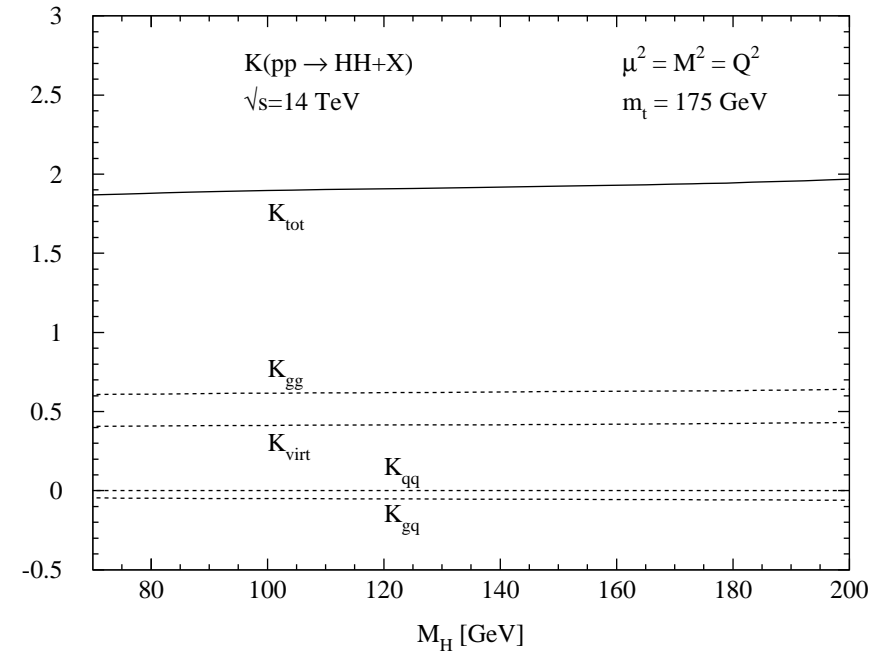
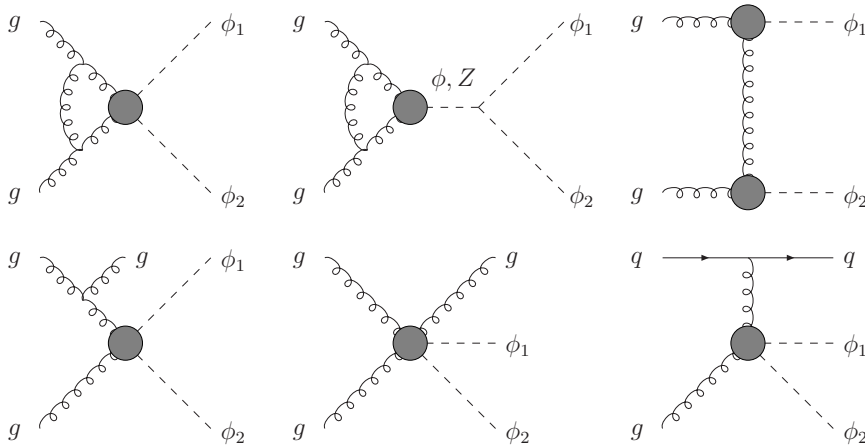
[decreasing with M_{HH}^2]

$gg \rightarrow HH$

SM



- third generation dominant $\rightarrow t, b$
- 2-loop QCD corrections: $\sim 90 - 100\%$
 $[M_H^2 \ll 4m_t^2, \quad \mu = M_{HH}]$



Dawson, Dittmaier, S.

- 2-loop QCD corrections:

$$\sigma = \sigma_0 + \frac{\sigma_1}{m_t^2} + \dots + \frac{\sigma_4}{m_t^8}$$

Grigo, Hoff, Melnikov, Steinhauser

[refinement: full LO at diff. level]

- NLO mass effects @ NLO in real corrections: $\sim -10\%$

Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Torrielli, Vryonidou, Zaro

→ sizeable virtual mass effects

- NNLO QCD corrections: $\sim 20\%$

$$[M_H^2 \ll 4m_t^2]$$

de Florian, Mazzitelli

Grigo, Melnikov, Steinhauser

- soft gluon resummation: $\sim 10\%$

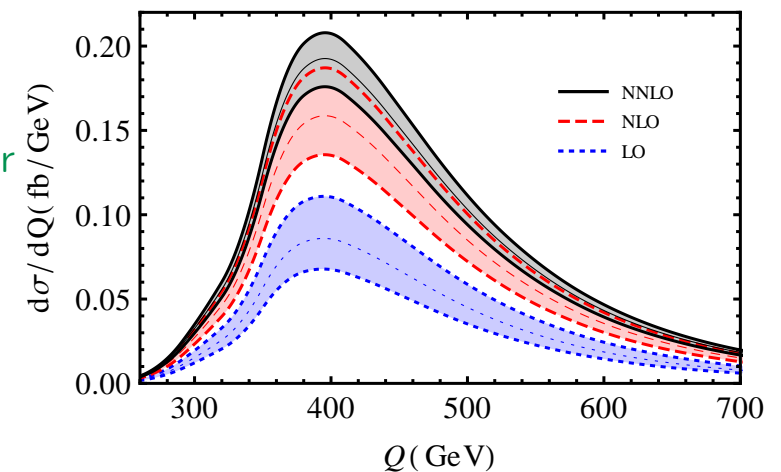
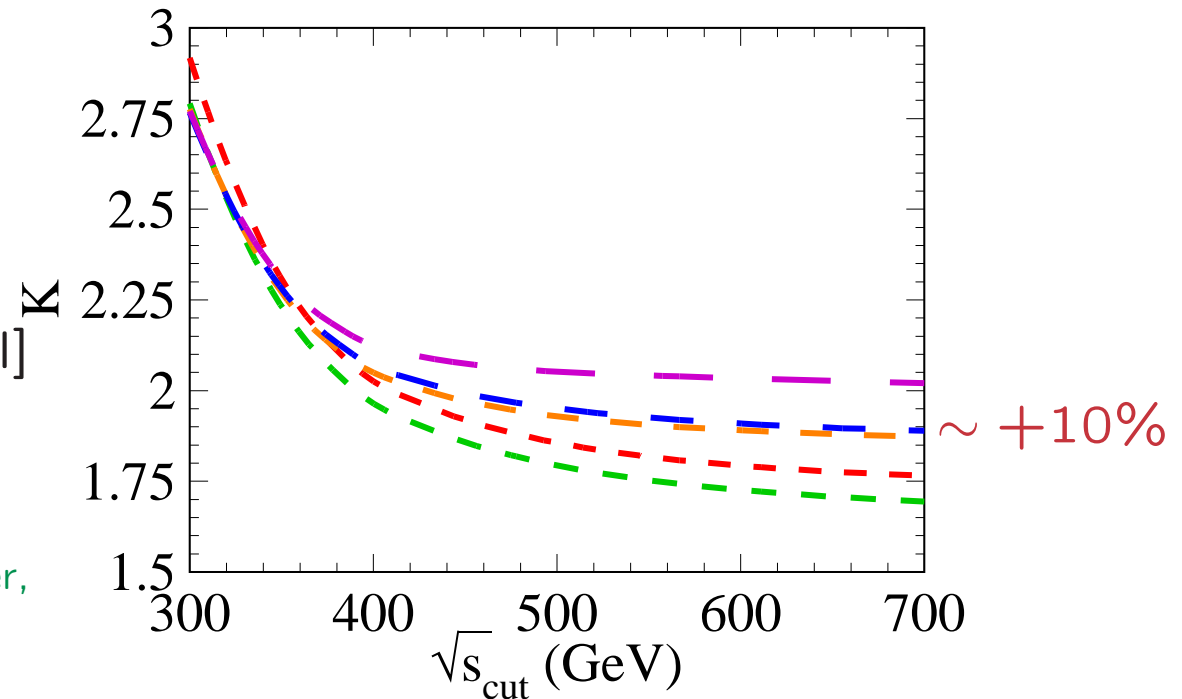
$$[M_H^2 \ll 4m_t^2]$$

Shao, Li, Li, Wang
de Florian, Mazzitelli

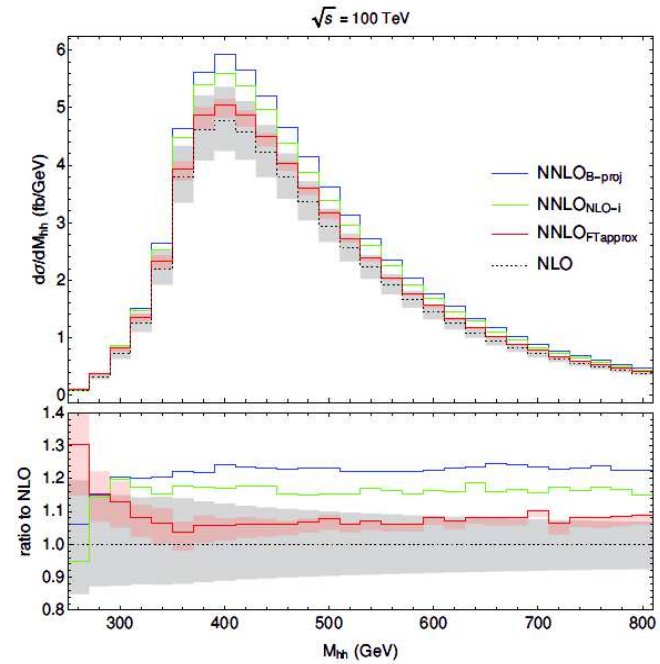
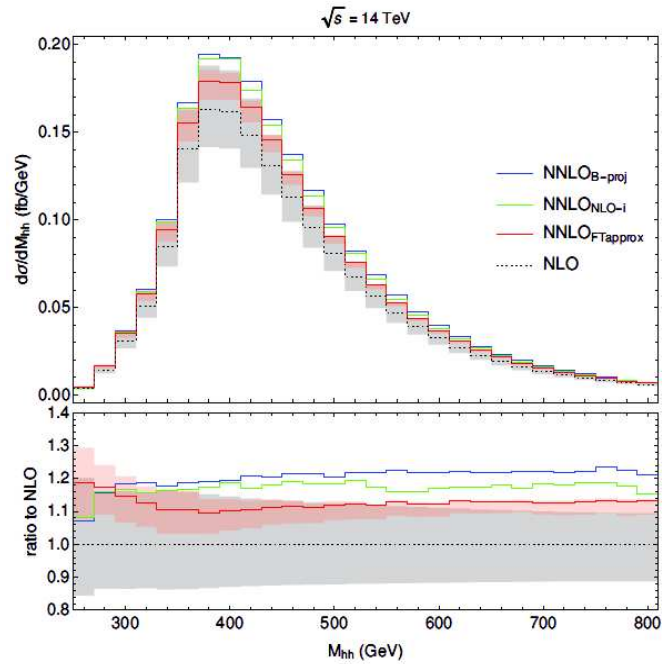
- NLO: small quark mass expansion

$$[Q^2 \gg m_t^2]$$

Davies, Mishima, Steinhauser, Wellmann



- NNLO Monte Carlo: inclusion of full top-mass effects @ NLO



Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli

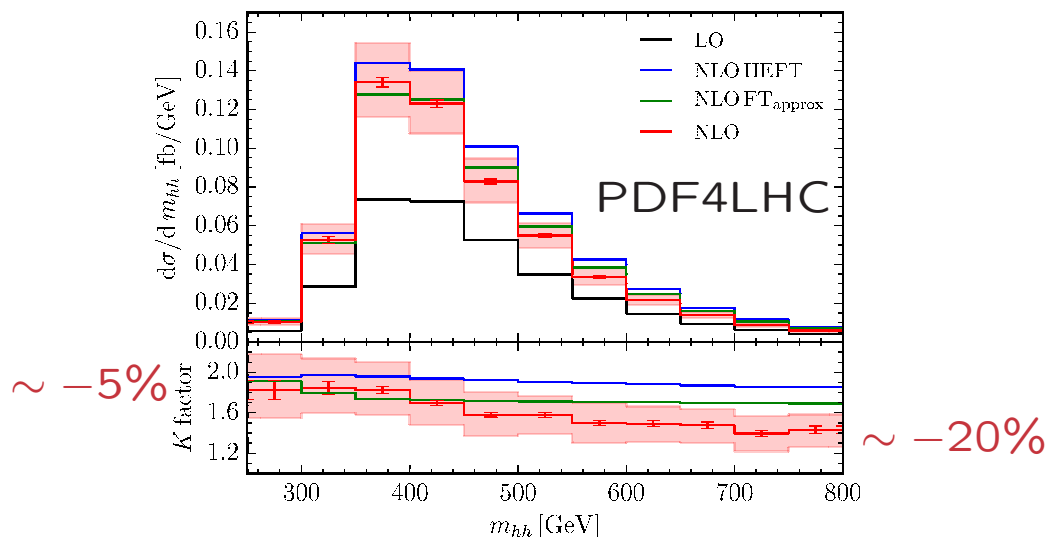
⇒ 20% effects beyond NLO

- NLO: matching to parton showers

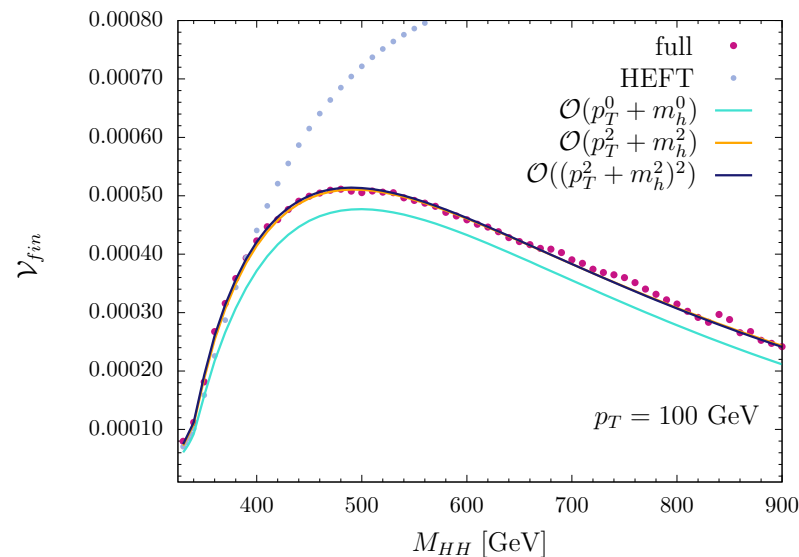
Heinrich, Jones, Kerner, Luisoni, Vryonidou

Full NLO calculation: top only

Numerical integration, sector decomposition, tensor reduction, contour deformation ($M_H = 125$ GeV, $m_t = 173$ GeV)



Borowka, Greiner, Heinrich, Jones, Kerner
Schlenk, Schubert, Zirke



Boncianni, Degrassi, Giardino, Gröber

- 14 TeV: ($m_t = 173$ GeV) $\sigma_{NLO} = 32.91(10)_{-12.6\%}^{+13.6\%} fb$
 $\sigma_{NLO}^{HTL} = 38.75_{-15\%}^{+18\%} fb$ (\leftarrow HPAIR)

\Rightarrow -15% mass effects on top of LO

• new expansion/extrapolation methods:

(i) $1/m_t^2$ expansion + conformal mapping + Padé approximants

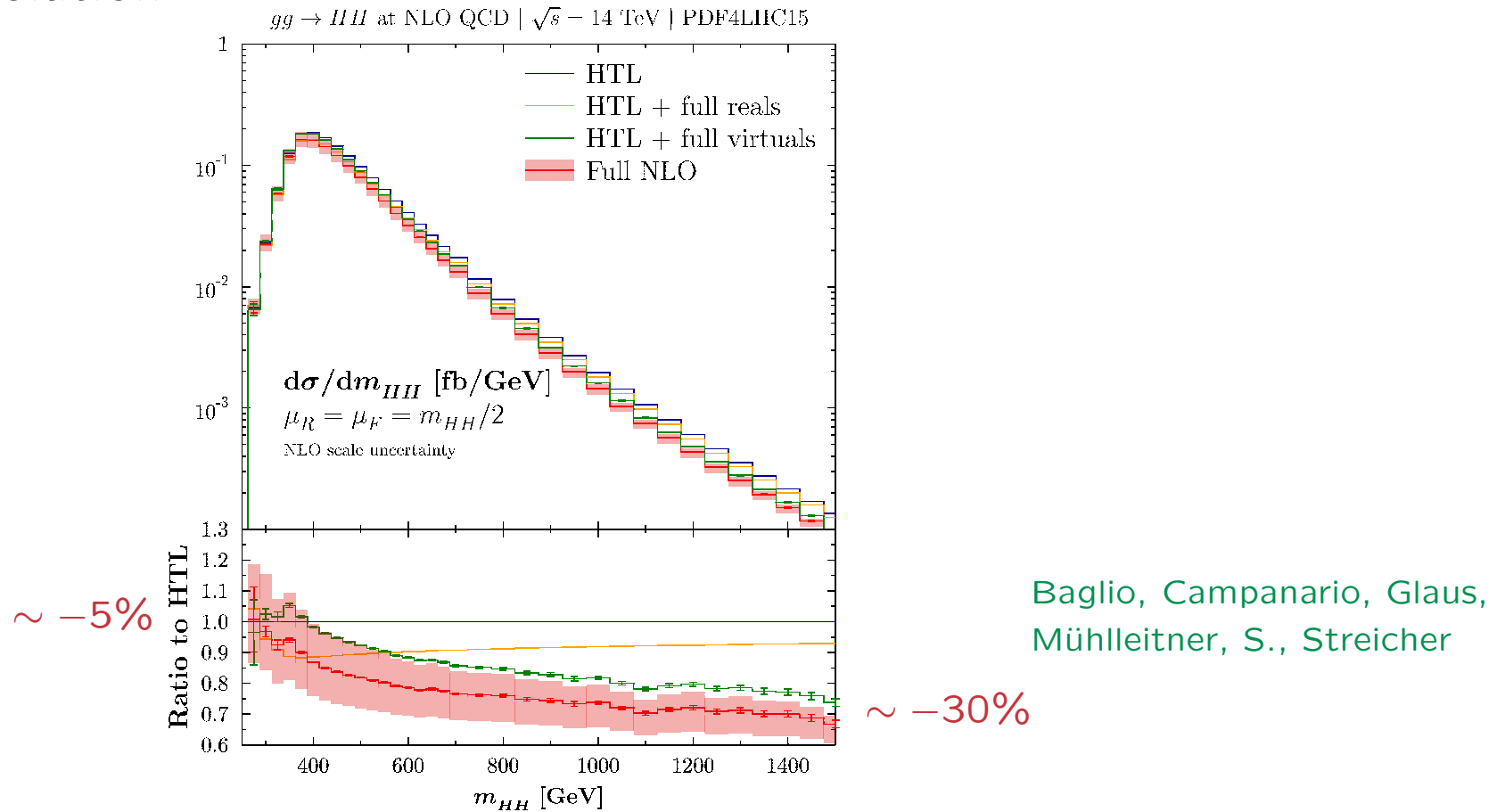
Gröber, Maier, Rauh

(ii) p_T^2 expansion

Boncianni, Degrassi, Giardino, Gröber

Full NLO calculation: top only

Numerical integration, IR subtraction, no tensor reduction, Richardson extrapolation



- 14 TeV: ($m_t = 172.5$ GeV) $\sigma_{NLO} = 32.78(7)_{-12.5\%}^{+13.5\%}$ fb
 $\sigma_{NLO}^{HTL} = 38.66_{-15\%}^{+18\%}$ fb (← HPAIR)

⇒ -15% mass effects on top of LO

III CALCULATION

$$\sigma_{\text{NLO}}(pp \rightarrow HH + X) = \sigma_{\text{LO}} + \Delta\sigma_{\text{virt}} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{q\bar{q}}$$

$$\sigma_{\text{LO}} = \int_{\tau_0}^1 d\tau \frac{d\mathcal{L}^{gg}}{d\tau} \hat{\sigma}_{\text{LO}}(Q^2 = \tau s)$$

$$\Delta\sigma_{\text{virt}} = \frac{\alpha_s(\mu_R)}{\pi} \int_{\tau_0}^1 d\tau \frac{d\mathcal{L}^{gg}}{d\tau} \hat{\sigma}_{\text{LO}}(Q^2 = \tau s) C$$

$$\Delta\sigma_{gg} = \frac{\alpha_s(\mu_R)}{\pi} \int_{\tau_0}^1 d\tau \frac{d\mathcal{L}^{gg}}{d\tau} \int_{\tau_0/\tau}^1 \frac{dz}{z} \hat{\sigma}_{\text{LO}}(Q^2 = z\tau s) \left\{ -z P_{gg}(z) \log \frac{\mu_F^2}{\tau s} \right. \\ \left. + d_{gg}(z) + 6[1 + z^4 + (1 - z)^4] \left(\frac{\log(1 - z)}{1 - z} \right)_+ \right\}$$

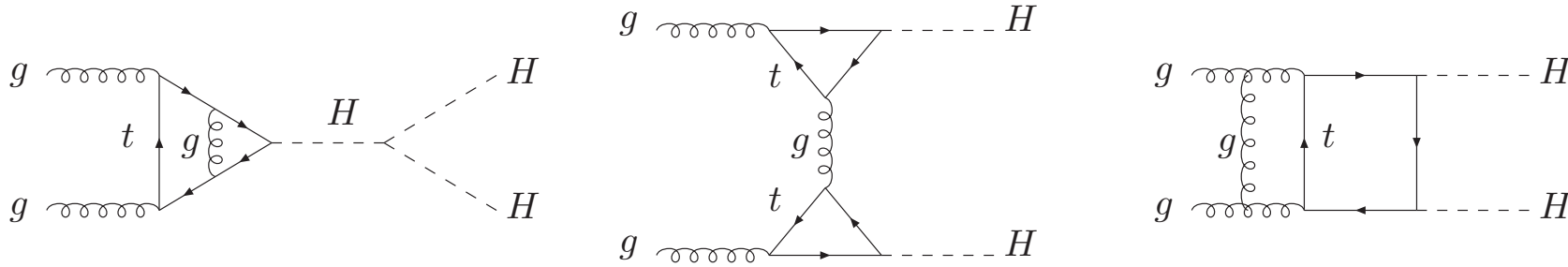
$$\Delta\sigma_{gq} = \frac{\alpha_s(\mu_R)}{\pi} \int_{\tau_0}^1 d\tau \sum_{q, \bar{q}} \frac{d\mathcal{L}^{gq}}{d\tau} \int_{\tau_0/\tau}^1 \frac{dz}{z} \hat{\sigma}_{\text{LO}}(Q^2 = z\tau s) \left\{ -\frac{z}{2} P_{gq}(z) \log \frac{\mu_F^2}{\tau s(1 - z)^2} + d_{gq}(z) \right\}$$

$$\Delta\sigma_{q\bar{q}} = \frac{\alpha_s(\mu_R)}{\pi} \int_{\tau_0}^1 d\tau \sum_q \frac{d\mathcal{L}^{q\bar{q}}}{d\tau} \int_{\tau_0/\tau}^1 \frac{dz}{z} \hat{\sigma}_{\text{LO}}(Q^2 = z\tau s) d_{q\bar{q}}(z)$$

$$C \rightarrow \pi^2 + \frac{11}{2} + C_{\Delta\Delta} + \frac{33 - 2N_F}{6} \log \frac{\mu_R^2}{Q^2}, \quad d_{gg} \rightarrow -\frac{11}{2}(1 - z)^3, \quad d_{gq} \rightarrow \frac{2}{3}z^2 - (1 - z)^2, \quad d_{q\bar{q}} \rightarrow \frac{32}{27}(1 - z)^3$$

(i) virtual corrections

47 gen. box diags, 8 triangle diags (\leftarrow single Higgs), 1PR ($\leftarrow H \rightarrow Z\gamma$)



- full diagram w/o tensor reduction \rightarrow 6-dim. Feynman integral (2 FF)
- UV-singularities: end-point subtractions

$$\int_0^1 dx \frac{f(x)}{(1-x)^{1-\epsilon}} = \int_0^1 dx \frac{f(1)}{(1-x)^{1-\epsilon}} + \int_0^1 dx \frac{f(x) - f(1)}{(1-x)^{1-\epsilon}} = \frac{f(1)}{\epsilon} + \int_0^1 dx \frac{f(x) - f(1)}{1-x} + \mathcal{O}(\epsilon)$$

- IR-sing.: IR-subtraction (based on struc. of integr. and rel. to HTL)
- thresholds: $Q^2 \geq 0, 4m_t^2 \rightarrow$ IBP \rightarrow reduction of power of denominator [$m_t^2 \rightarrow m_t^2(1 - ih)$]

$$\int_0^1 dx \frac{f(x)}{(a+bx)^3} = \frac{f(0)}{2a^2b} - \frac{f(1)}{2b(a+b)^2} + \int_0^1 dx \frac{f'(x)}{2b(a+bx)^2}$$

- renormalization: α_s : $\overline{\text{MS}}$, 5 flavours
 m_t : on-shell
- PS-integration \rightarrow 7-dim. integrals for $d\sigma/dQ^2$
- subtraction of HTL \rightarrow IR-finite mass effects [adding back HTL results \leftarrow HPAIR]
- extrapolation to NWA ($h \rightarrow 0$): Richardson extrapolation

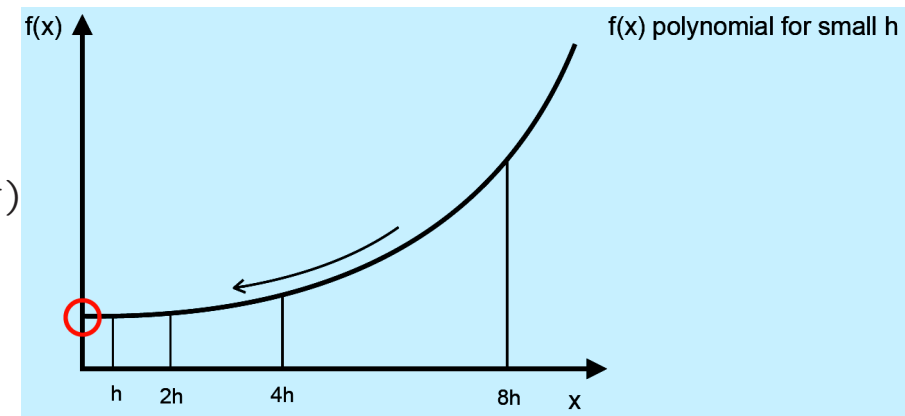
$$M_2 = 2f(h) - f(2h) = f(0) + \mathcal{O}(h^2)$$

$$M_4 = \{8f(h) - 6f(2h) + f(4h)\}/3 = f(0) + \mathcal{O}(h^3)$$

$$M_8 = \{64f(h) - 56f(2h) + 14f(4h) - f(8h)\}/21 = f(0) + \mathcal{O}(h^4)$$

etc.

[$h \geq 0.025$]



(ii) real corrections

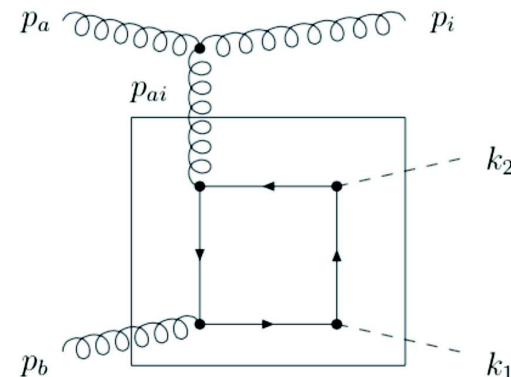
- full matrix elements generated with FeynArts and FormCalc
- matrix elements in HTL involving full LO sub-matrix elements subtracted \rightarrow IR-, COLL-finite [adding back HTL results \leftarrow HPAIR]

$$\sum \overline{|\mathcal{M}_{gg}|^2} = \sum \overline{|\tilde{\mathcal{M}}_{LO}|^2} \frac{24\pi^2 \alpha_s}{Q^4 \pi} \left\{ \frac{s^4 + t^4 + u^4 + Q^8}{stu} - 4 \frac{\epsilon}{1 - \epsilon} Q^2 \right\}$$

$$\sum \overline{|\mathcal{M}_{gq}|^2} = \sum \overline{|\tilde{\mathcal{M}}_{LO}|^2} \frac{32\pi^2 \alpha_s}{3Q^4 \pi} \left\{ \frac{s^2 + u^2}{-t} + \epsilon \frac{(s + u)^2}{t} \right\}$$

$$\sum \overline{|\mathcal{M}_{q\bar{q}}|^2} = \sum \overline{|\tilde{\mathcal{M}}_{LO}|^2} \frac{256\pi^2 \alpha_s}{9Q^4 \pi} (1 - \epsilon) \left\{ \frac{t^2 + u^2}{s} - \epsilon \frac{(t + u)^2}{s} \right\}$$

- PDFs: $\overline{\text{MS}}$ scheme, 5 flavours

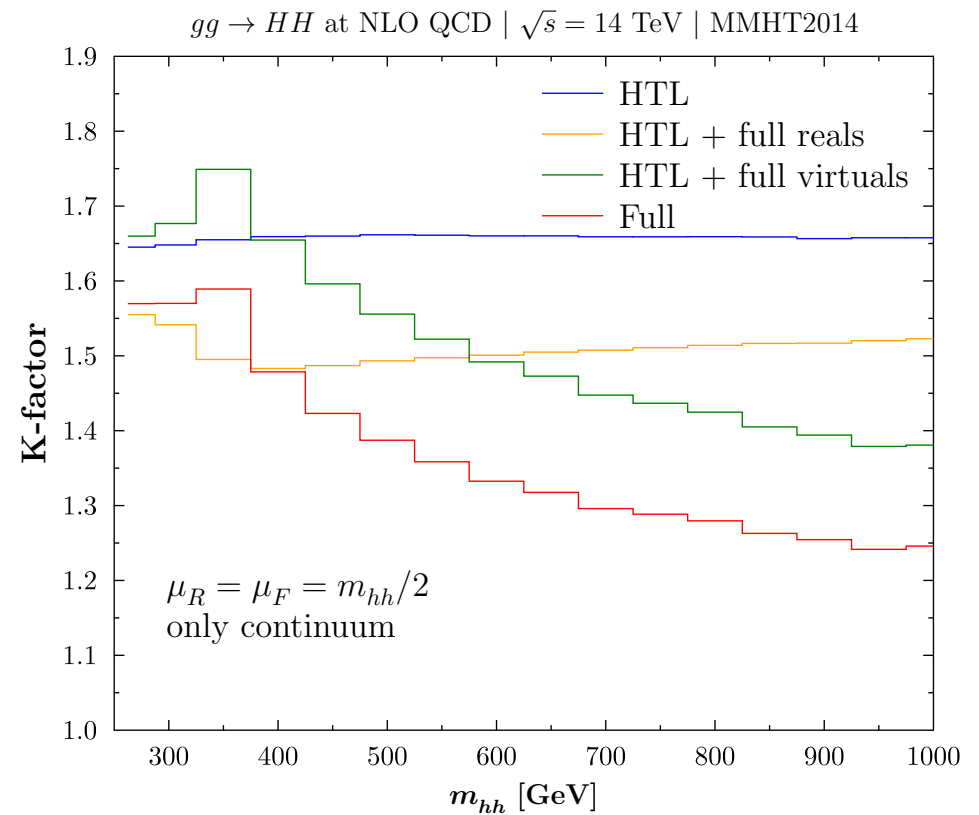
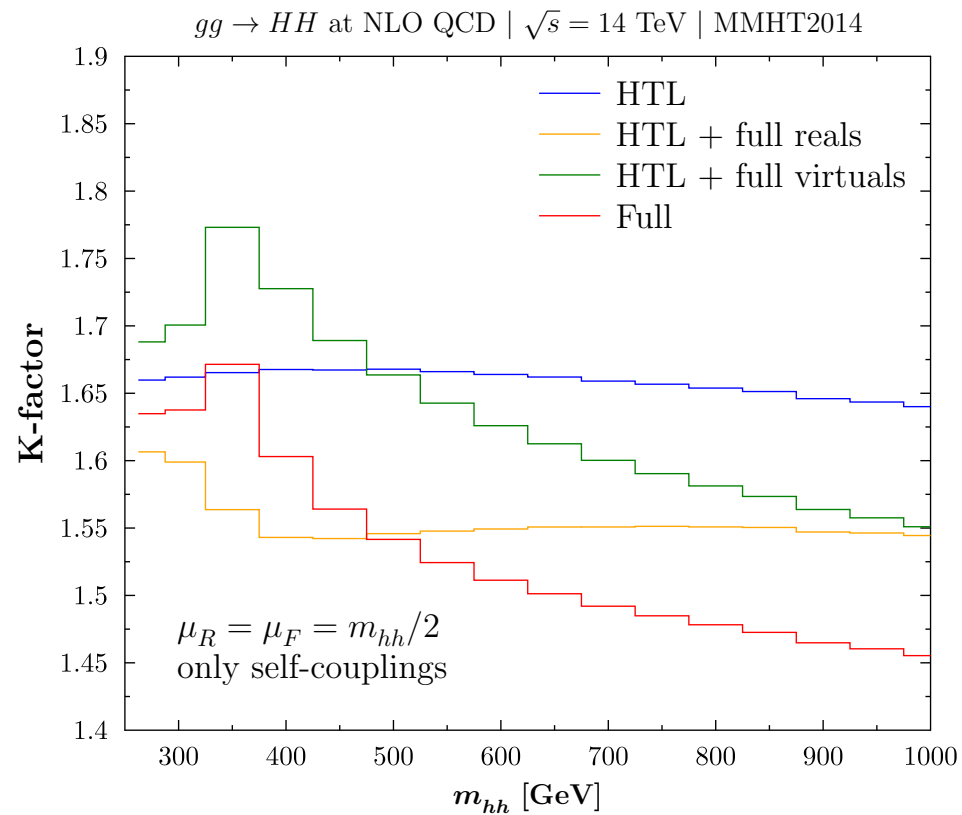


(iii) results

	PDF4LHC15	MMHT2014
σ_{LO}	19.80 fb	23.75 fb
σ_{NLO}^{HTL}	38.66 fb	39.34 fb
σ_{NLO}	32.78(7) fb	33.33(7) fb

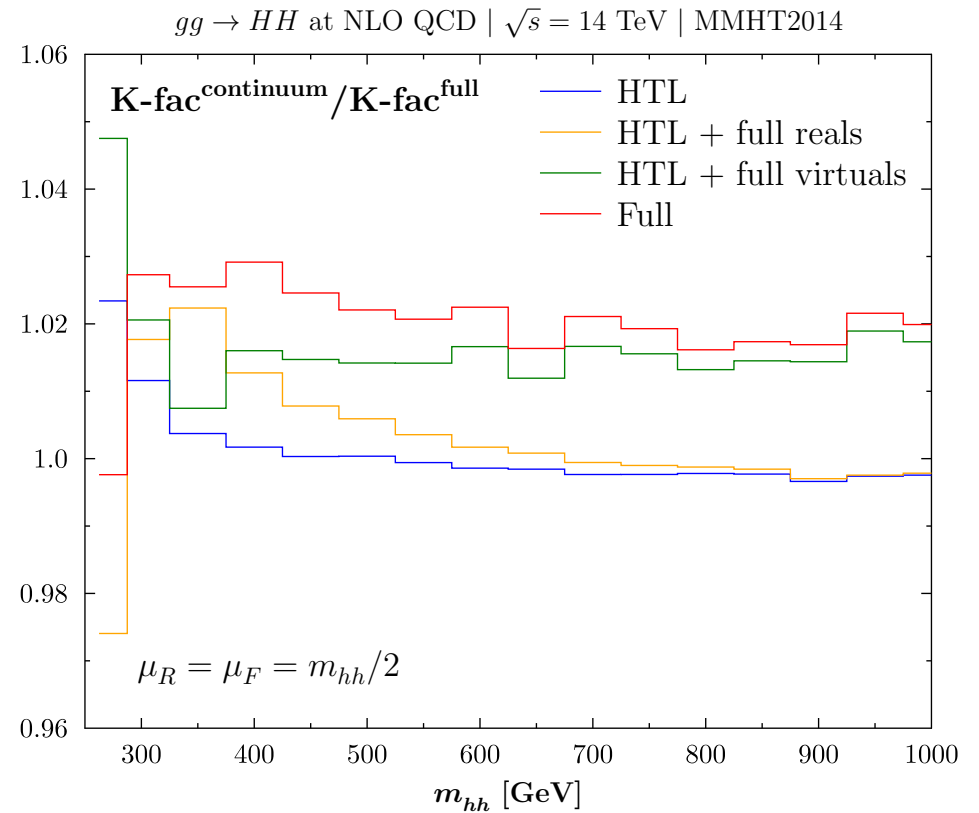
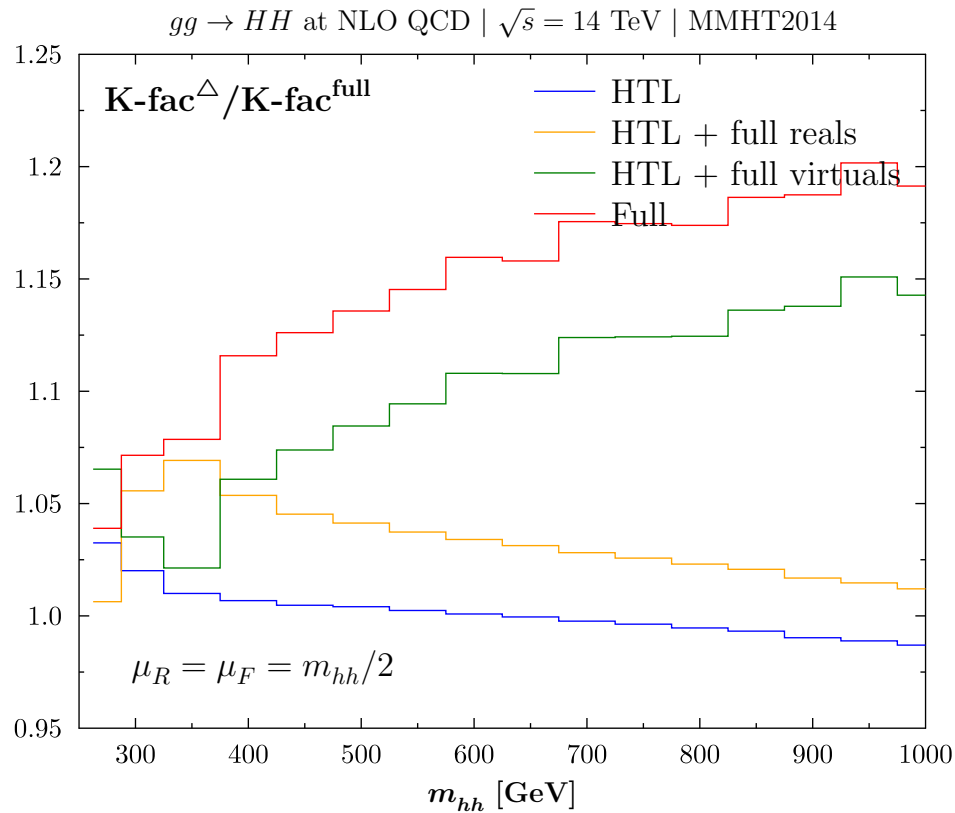
(iv) individual corrections

- take K-factors for triangle (\leftarrow single Higgs) and box diagrams separately: total K-factor approximated by single Higgs K-factor?



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(v) PDF + α_s uncertainties

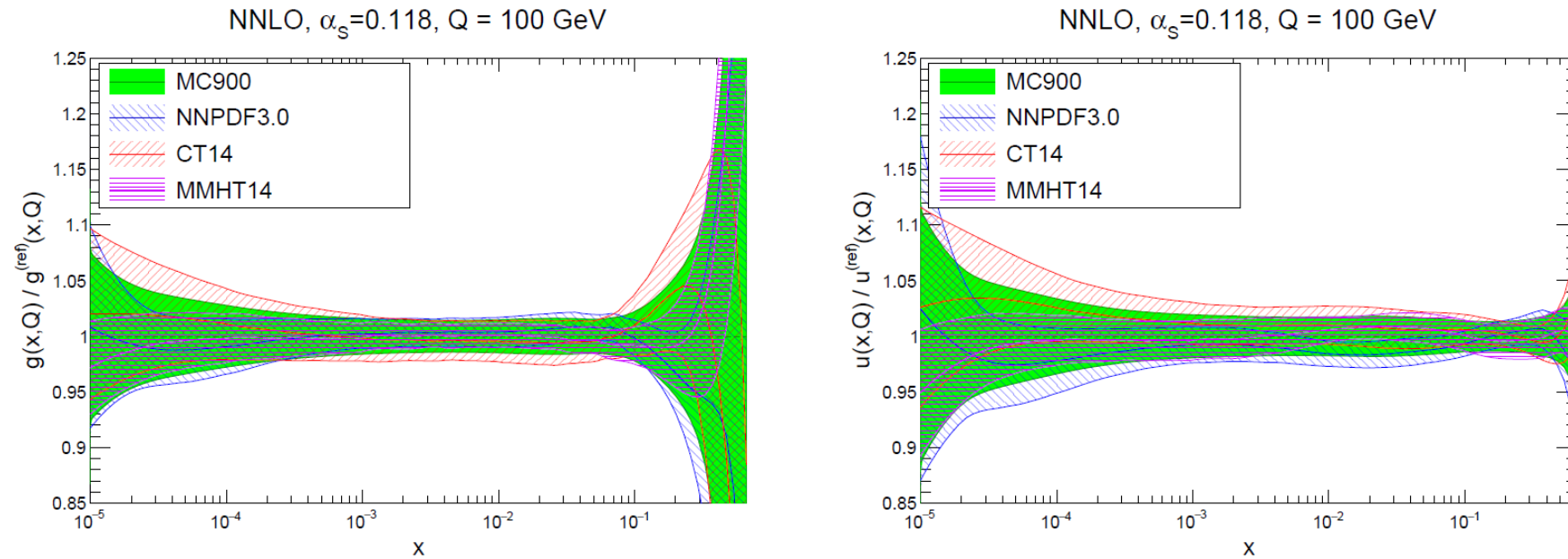


Figure 2: Comparison of the MC900 PDFs with the sets that enter the combination: CT14, MMHT14 and NNPDF3.0 at NNLO. We show the gluon and the up quark at $Q = 100$ GeV. Results are normalized to the central value of the prior set MC900.

- $\delta\sigma/\sigma \sim \pm 3.0\%$ @ LHC

(vi) uncertainties due to m_t

- transform $m_t \rightarrow \overline{m}_t(\mu)$ ($\overline{\text{MS}}$)

→ modification of mass CT

- use $m_t, \overline{m}_t(\overline{m}_t)$ and scan $Q/4 < \mu < Q \rightarrow$ uncertainty = envelope:

$$\left. \frac{d\sigma(gg \rightarrow HH)}{dQ} \right|_{Q=300 \text{ GeV}} = 0.031(1)_{-33\%}^{+4\%} \text{ fb/GeV},$$

$$\left. \frac{d\sigma(gg \rightarrow HH)}{dQ} \right|_{Q=400 \text{ GeV}} = 0.1609(4)_{-13\%}^{+0\%} \text{ fb/GeV},$$

$$\left. \frac{d\sigma(gg \rightarrow HH)}{dQ} \right|_{Q=600 \text{ GeV}} = 0.03204(9)_{-30\%}^{+0\%} \text{ fb/GeV},$$

$$\left. \frac{d\sigma(gg \rightarrow HH)}{dQ} \right|_{Q=1200 \text{ GeV}} = 0.000435(4)_{-34\%}^{+0\%} \text{ fb/GeV}$$

- **preliminary** interpolation:

$$\sigma(gg \rightarrow HH) = 32.78_{-21\%}^{+6\%} \text{ fb} \quad (\text{very preliminary})$$

(vi) uncertainties due to m_t for single Higgs

- transform $m_t \rightarrow \overline{m}_t(\mu)$ ($\overline{\text{MS}}$)

→ modification of mass CT

- use $m_t, \overline{m}_t(\overline{m}_t)$ and scan $Q/4 < \mu < Q \rightarrow$ uncertainty = envelope:

$$\sigma(gg \rightarrow H)|_{M_H=125 \text{ GeV}} = 42.17^{+0.4\%}_{-0.5\%} \text{ pb}$$

$$\sigma(gg \rightarrow H)|_{M_H=300 \text{ GeV}} = 9.85^{+7.5\%}_{-0.3\%} \text{ pb}$$

$$\sigma(gg \rightarrow H)|_{M_H=400 \text{ GeV}} = 9.43^{+0.1\%}_{-0.9\%} \text{ pb}$$

$$\sigma(gg \rightarrow H)|_{M_H=600 \text{ GeV}} = 1.97^{+0.0\%}_{-15.9\%} \text{ pb}$$

$$\sigma(gg \rightarrow H)|_{M_H=900 \text{ GeV}} = 0.230^{+0.0\%}_{-22.3\%} \text{ pb}$$

$$\sigma(gg \rightarrow H)|_{M_H=1200 \text{ GeV}} = 0.0402^{+0.0\%}_{-26.0\%} \text{ pb}$$

IV CONCLUSIONS

- Higgs pair production: first *direct* access to trilinear self-coupling
- NLO QCD corrs known $\rightarrow \Delta_{scale} \lesssim 10\%$ @ LHC
- NLO top mass effects: $\sim -15\%$ for σ_{tot} , $\sim -5 \dots -30\%$ for distr.
- sizeable uncertainties due to scheme and scale of top mass
- top mass effects beyond NLO: $\sim 5\%$? Grigo, Hoff, Steinhauser
- extension to BSM: bottom loops
- important to develop NLO event generators [\leftarrow backgrounds]

BACKUP SLIDES

$$\frac{d\mathcal{L}^{gg}}{d\tau} = \int_{\tau}^1 \frac{dx}{x} g(x, \mu_F^2) g\left(\frac{\tau}{x}, \mu_F^2\right)$$

$$\frac{d\mathcal{L}^{gq}}{d\tau} = \int_{\tau}^1 \frac{dx}{x} \left[q(x, \mu_F^2) g\left(\frac{\tau}{x}, \mu_F^2\right) + g(x, \mu_F^2) q\left(\frac{\tau}{x}, \mu_F^2\right) \right]$$

$$\frac{d\mathcal{L}^{q\bar{q}}}{d\tau} = \int_{\tau}^1 \frac{dx}{x} \left[q(x, \mu_F^2) \bar{q}\left(\frac{\tau}{x}, \mu_F^2\right) + \bar{q}(x, \mu_F^2) q\left(\frac{\tau}{x}, \mu_F^2\right) \right]$$