Electron model of Vertical FFA

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KYUSHU UNIVERSITY

Outline

- **Introduction**
- **Ring design**
  - Optics
  - Magnet
- **Field measurement**
  - Overview
  - Measurement
- **Summary**

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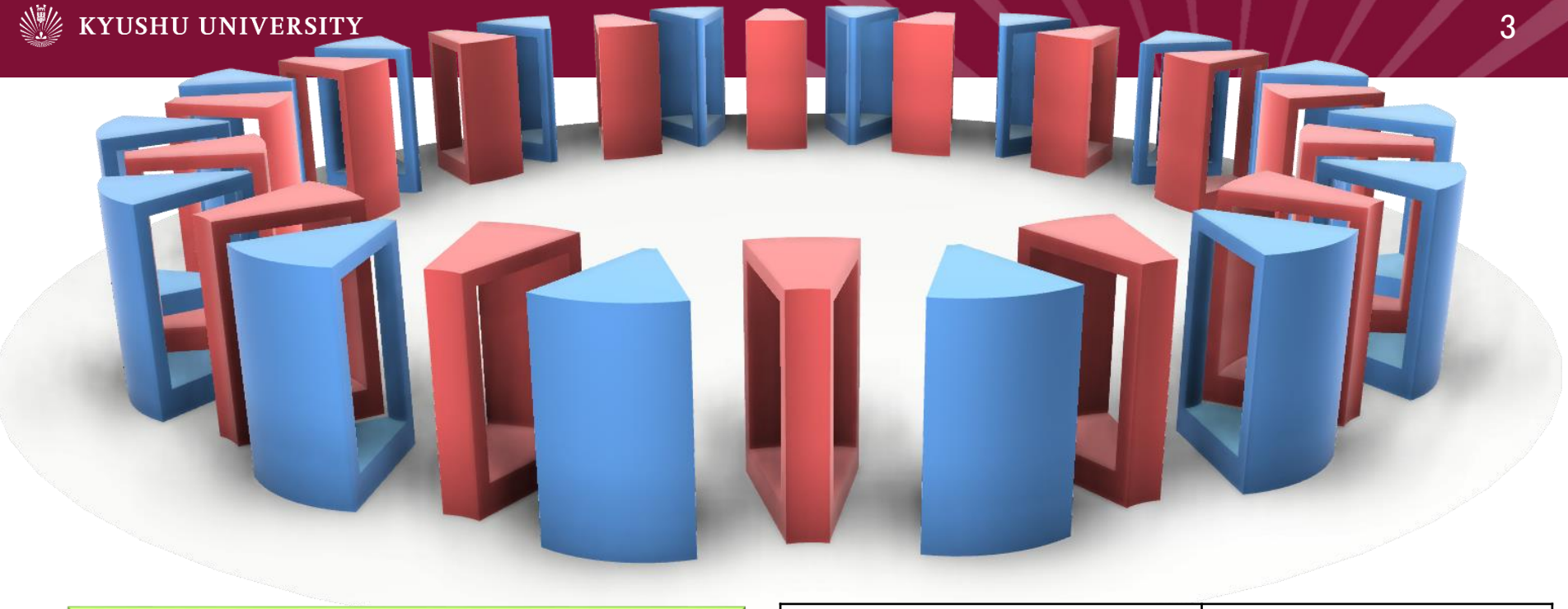


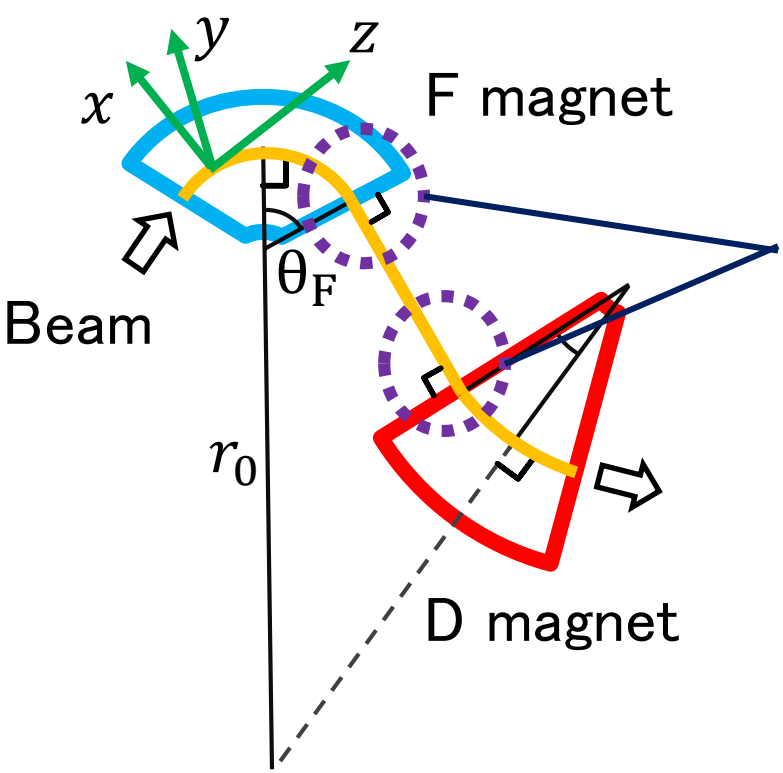
Electron VFFA (prototype)

Focusing system	F-D Singlet
Number of cell	16
Magnet	Sector
Injection energy	20 [keV]
Maximum energy	100 [keV]
Radius	1.0 [m]

Development and Proof-of-Principle Experiment is goal in our study.

Electron VFFA (prototype)

Focusing system	F-D Singlet
Magnet	Sector



Advantage

1. Edge angle is 0.
2. Vertical kick is negligible.
   →B_x and B_z of fringing field = 0
3. Design orbit would be closed.

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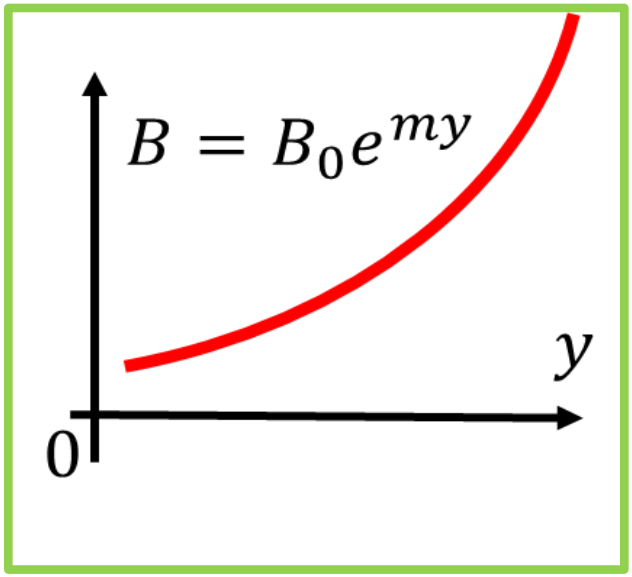
Beam optics of VFFA

Magnetic field becomes

$B = B_0 e^{my}$ m : field gradient



Expanding

$$B = B_0 \left[\underbrace{1}_{\text{Normal}} + \overbrace{\underbrace{my}_{\text{Skew}}}^{\text{Linear}} + \overbrace{\underbrace{\frac{1}{2}(my)^2 + \cdots}_{\text{Skew}}}^{\text{Non linear}} \right]$$

Magnetic field of VFFA includes skew component.

Optics design

Calculation method	Numerical analysis	Linear approximation and transfer matrix
Accuracy	High	Depends on approximation model
Speed	Slow	Fast
Ring parameter optimization	Difficult	Easy

Calculation methods including skew field are being examined.

The purpose of this section is to establish linear approximation and transfer matrix methods for VFFA.

Difficulty in transfer matrix derivation

Equation of motion in VFFA

$$x'' + \frac{x}{\rho(s)^2} + \frac{1}{B\rho(s)}\left(\frac{\partial B_y}{\partial y}\right)y = 0$$

$$y'' - \frac{1}{B\rho(s)}\left(\frac{\partial B_x}{\partial x}\right)x = 0$$

Normal

Skew

Horizontal motion and vertical motion are coupled.
Therefore, derivation of transfer matrix becomes difficult.

New method is required in transfer matrix derivation

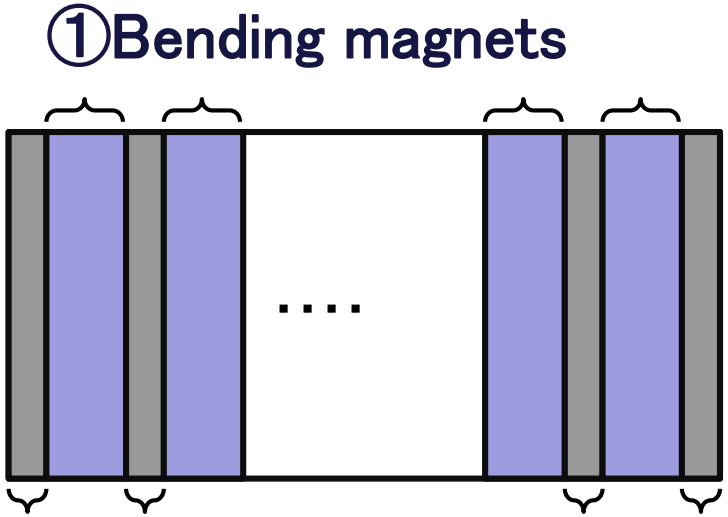
New method of transfer matrix derivation

VFFA Magnet

approximation

$$x'' + \frac{x}{\rho(s)^2} + \frac{1}{B\rho(s)}\left(\frac{\partial B_y}{\partial y}\right) y = 0$$

$$y'' - \frac{1}{B\rho(s)}\left(\frac{\partial B_x}{\partial x}\right) x = 0$$



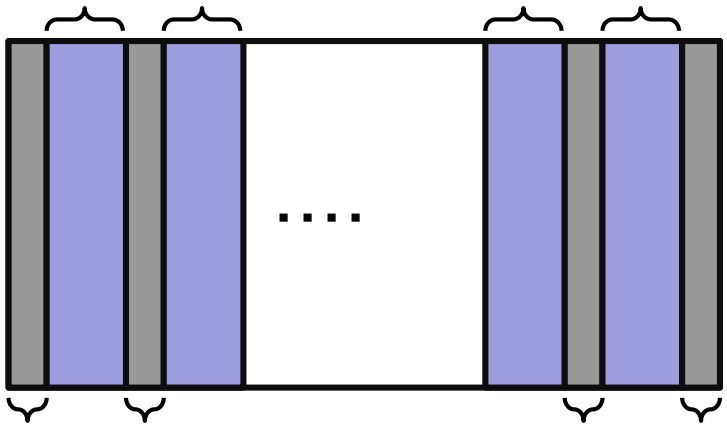
VFFA magnet is approximated to a series of bending magnet and thin skew Q magnet.

New method of transfer matrix derivation

$$x'' + \frac{x}{\rho(s)^2} = 0$$

$$y'' = 0$$

①Bending magnets



$$x'' - \frac{x''}{\rho(s)^2} + \frac{1}{B\rho(s)}\left(\frac{\partial B_y}{\partial y}\right) y = 0$$

$$y'' - \frac{y''}{B\rho(s)}\left(\frac{\partial B_x}{\partial x}\right) x = 0$$

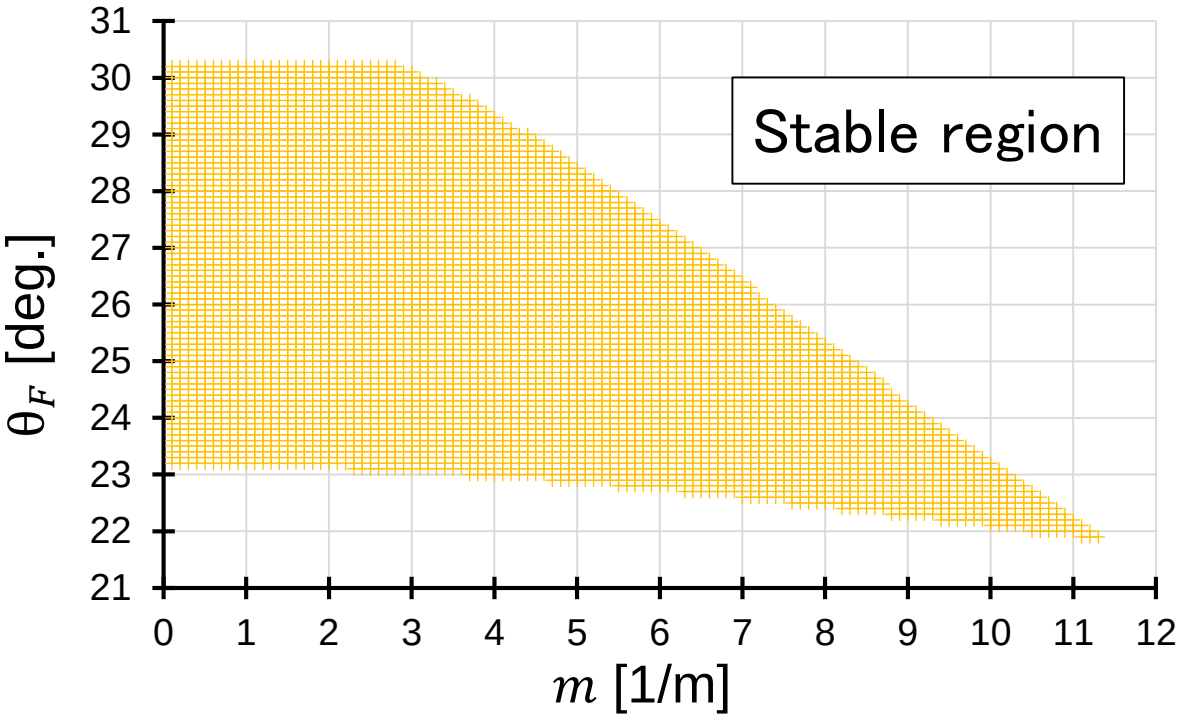
②Thin lens of skew Q magnet

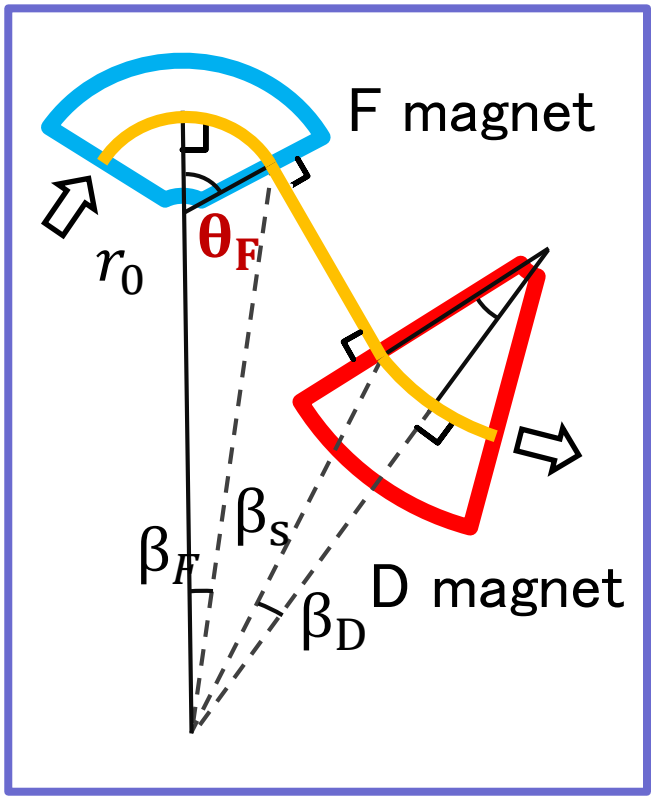
These transfer matrixes can be derived.

Transfer matrix of VFFA can be derived.

Calculation

- Parameters
  m and bending angle (θ_F)
- Number of division is 50





Stable region is derived by linear approximation method.

Comparison of new method and numerical analysis

Hard edge model in numerical analysis.

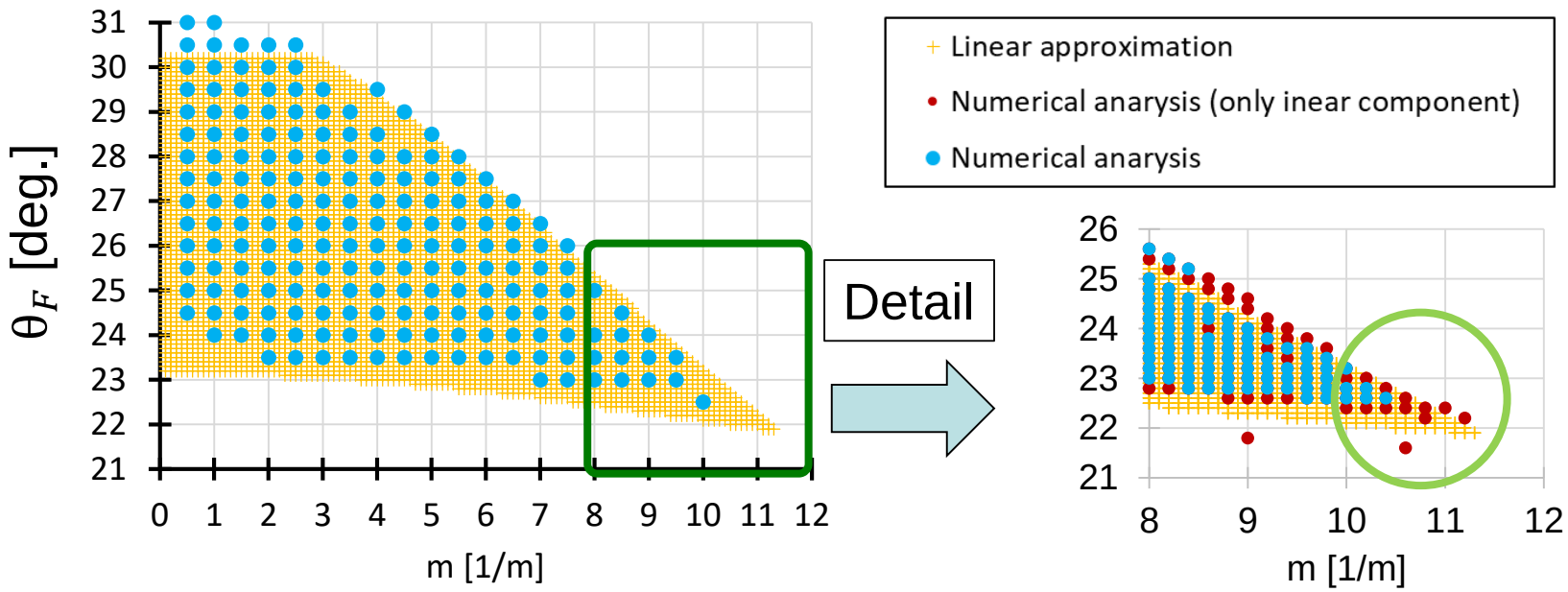
Magnetic field

$$B_x \simeq -B_0 e^{my} \sin(mx)$$
$$B_y \simeq B_0 e^{my} \cos(mx)$$

or

Only linear field

$$B_x \simeq -B_0 my$$
$$B_y \simeq B_0 (1 + mx)$$



Result of linear approximation method shows good agreement with numerical analysis, but, not to large region of m.

Conclusion of optics design including skew field.

- Derivation method of transfer matrix is proposed.
- Stable region was derived and compared by two calculation methods.
  (Linear approximation method and numerical analysis)

1. In sector VFFA, linear approximation and transfer matrix method can be applied for optics design.
2. In high region of m, this method is thought that is not adequate.

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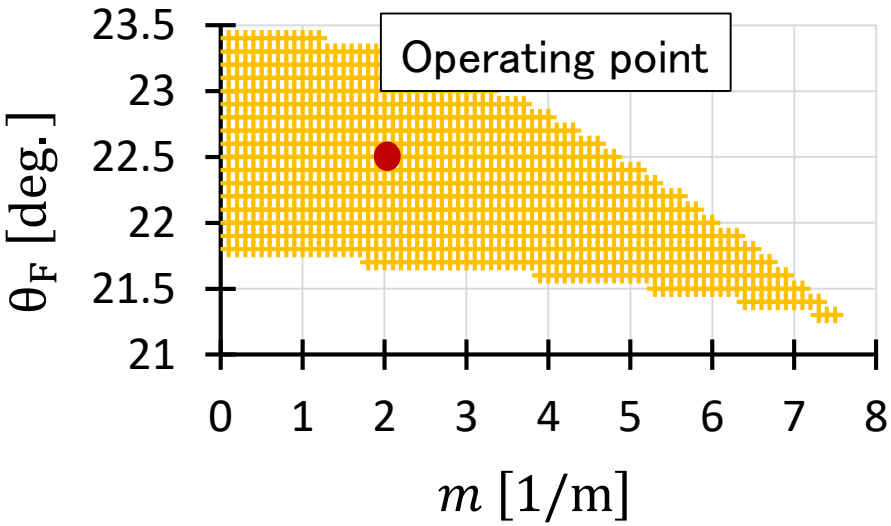
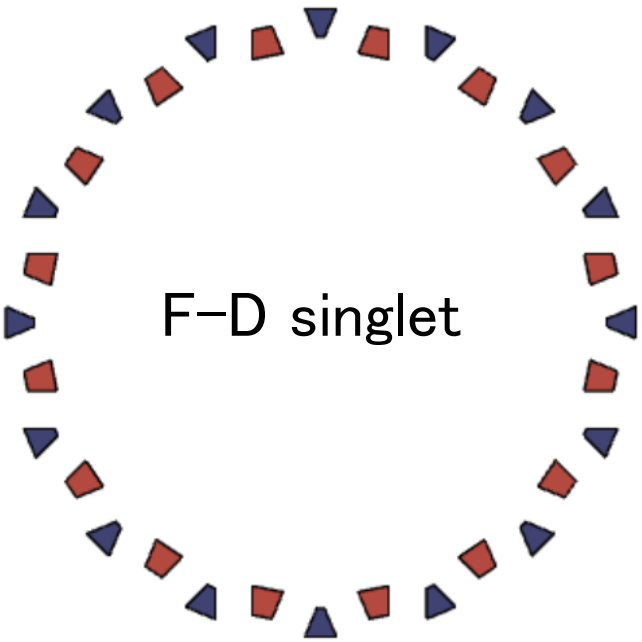
Parameters of Electron VFFA





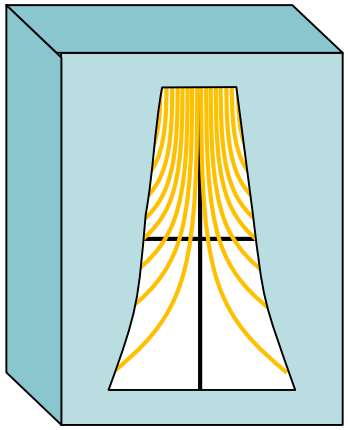
particle	electron
Number of cell	16
Radius	1.0 [m]
θ_F	22.5 [deg.]
θ_D	11.25 [deg.]
m	2.0 [1/m]
Orbit excursion	0.1 [m]

Injection energy	20 [keV]
Maximum energy	100 [keV]
B (F)	60.8 [gauss]
B (D)	22.2 [gauss]
FD ratio	2.73
RF gap Voltage	200 [Vp]

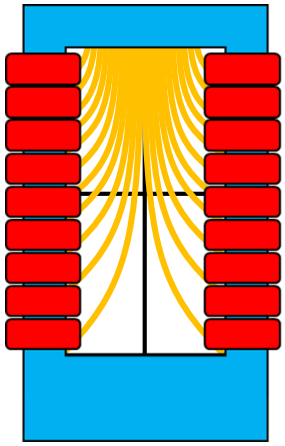
Requirements of magnet design

①Magnetic pole shape : Simple
②m value : Adjustable

Two design candidates of magnet



Magnetic pole shape
(Ordinary type)



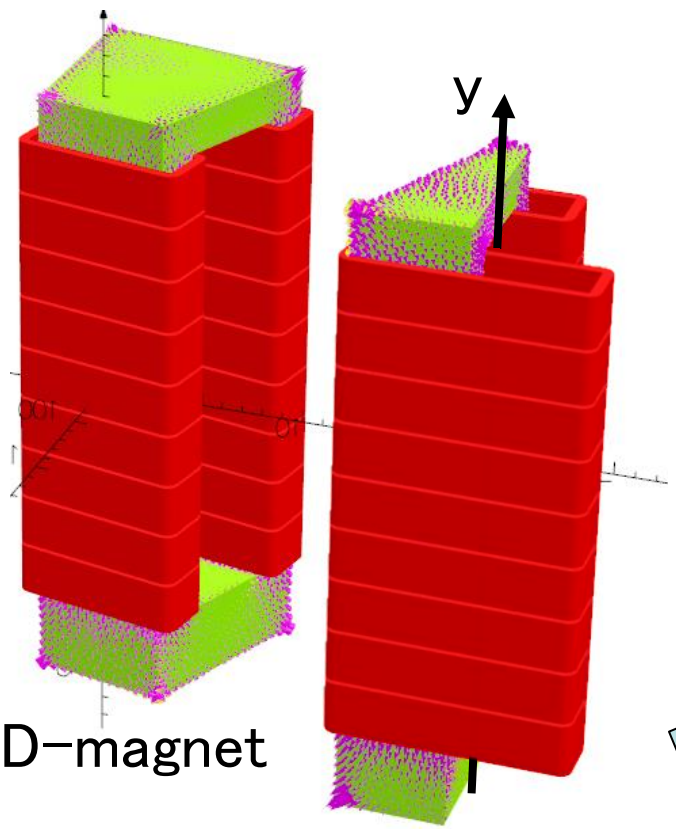
Multi coil type
(Proposed by Prof. Mori)

Meets the needs

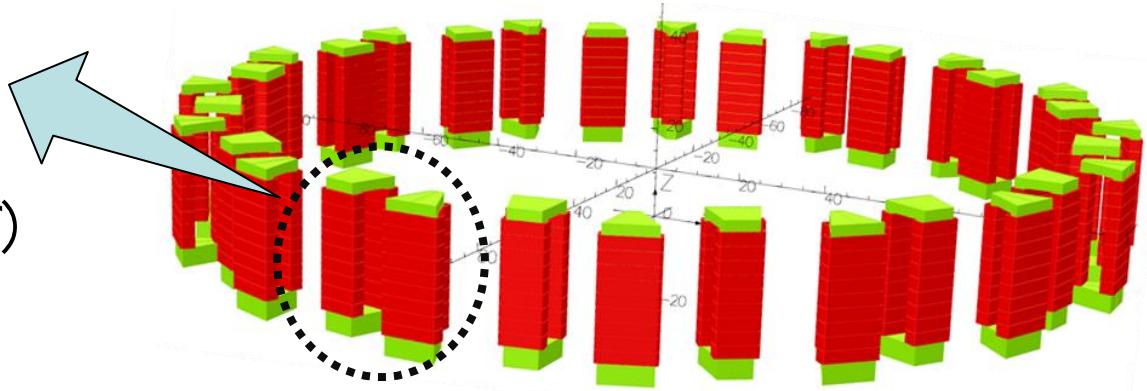
- m is adjustable
- Simple shape

We adopted.

Calculation system in OPERA-3D

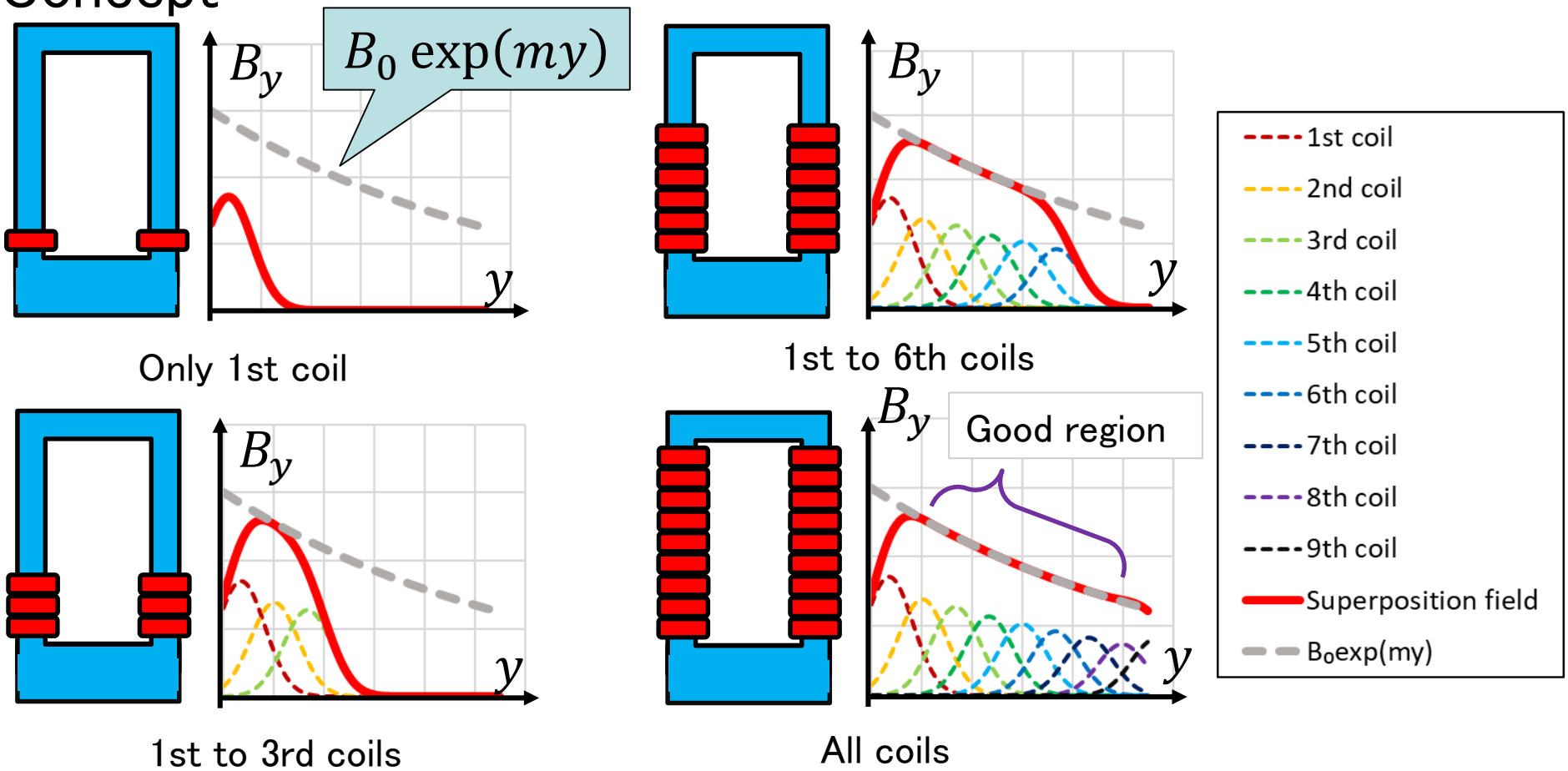


- Advantage of multi-coil type are examined with OPERA-3D.
- m is calculated on y-axis center of F-magnet. (Left figure shows)



Overall view of ring

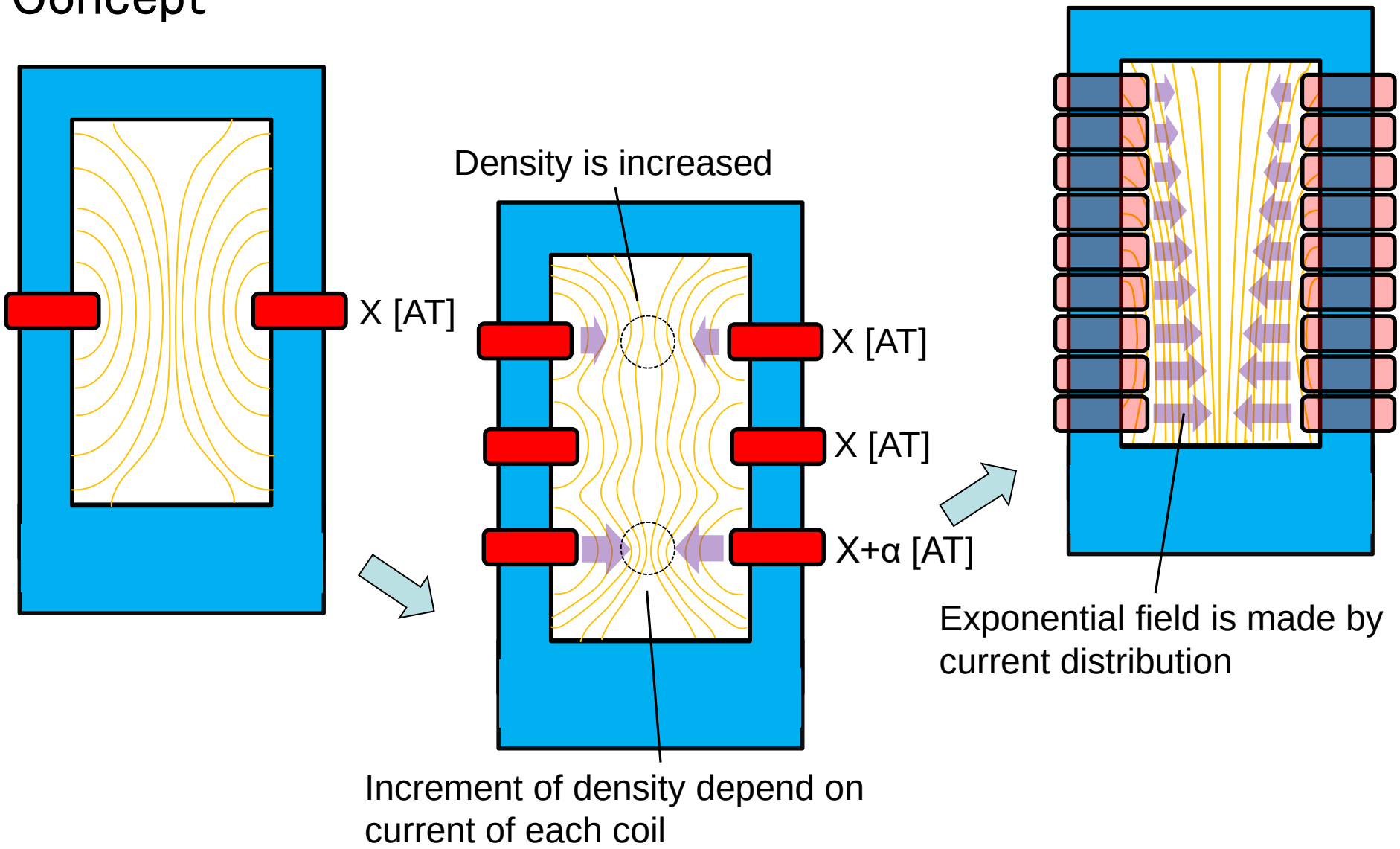
Concept



- Magnetic field is optimized to exponential field by multi coils.

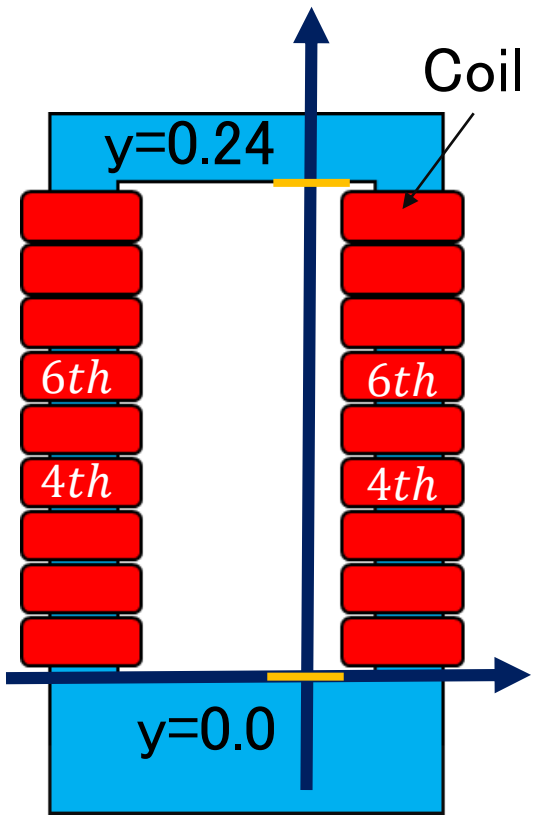
  ⇨ It is required to confirm about relationship between magnetic field and current.

Concept

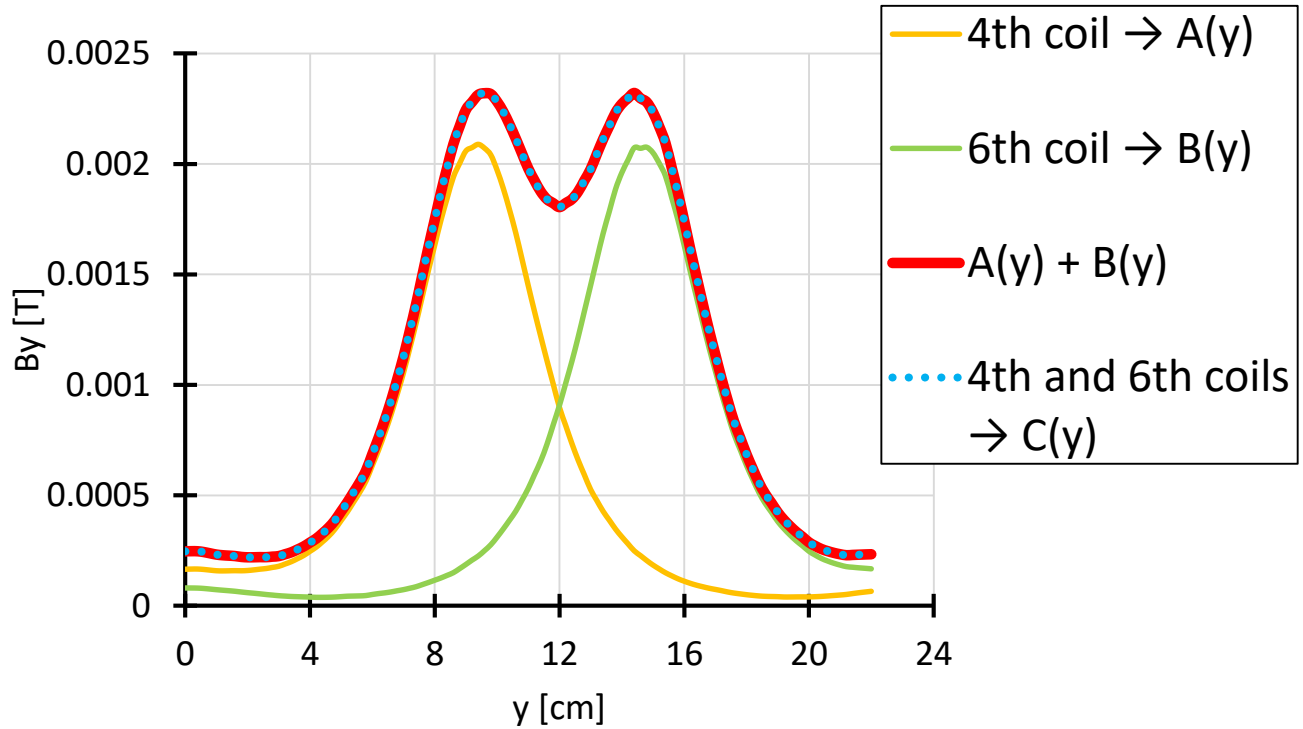

X [AT]
Density is increased
X [AT]
X [AT]
X+α [AT]
Exponential field is made by current distribution
Increment of density depend on current of each coil

First approach

Superposition of magnetic fields of two coils



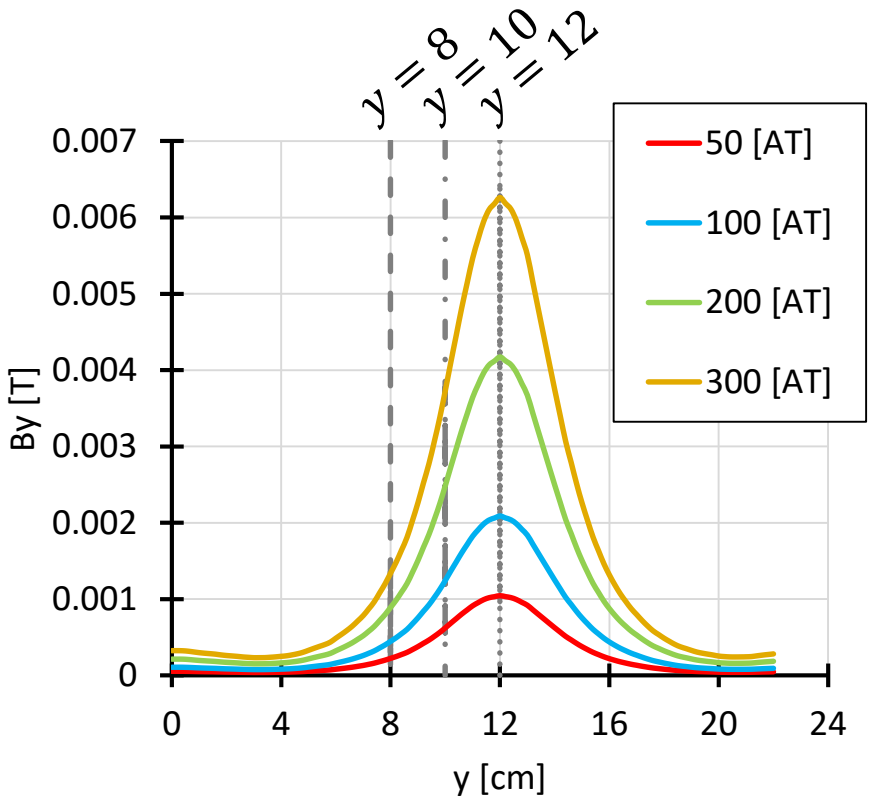
Magnet

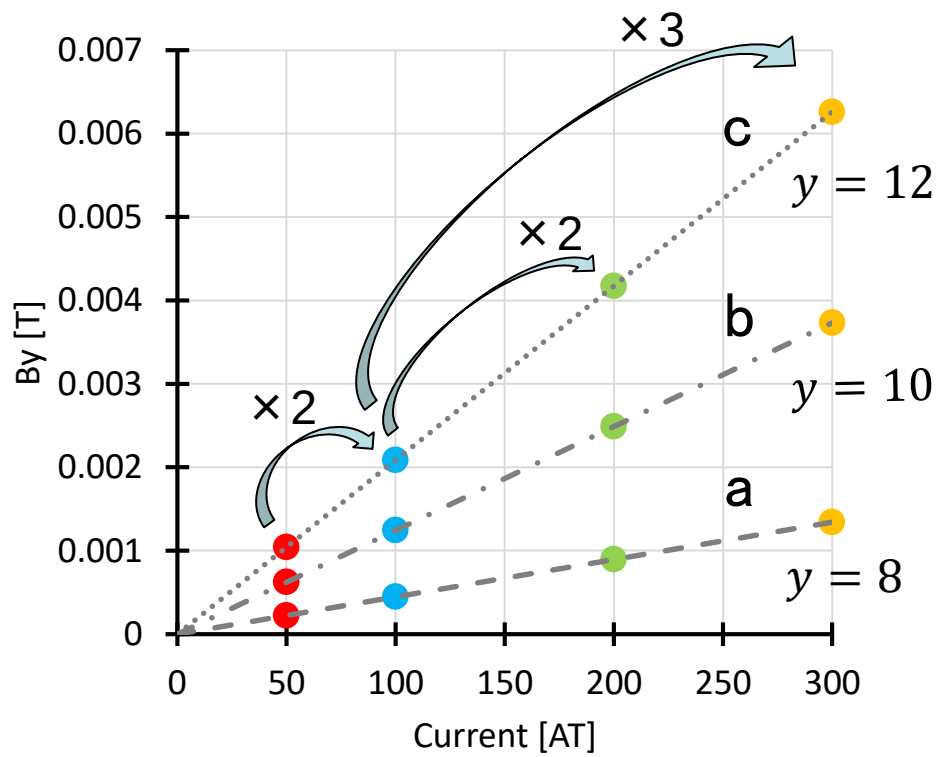


- The figure indicates that summation of A(y) and B(y) gives C(y).
- Superposition field well reproduces C(y).

Second approach

Relations of magnetic field and current of coil
(only 5th coil is excited)

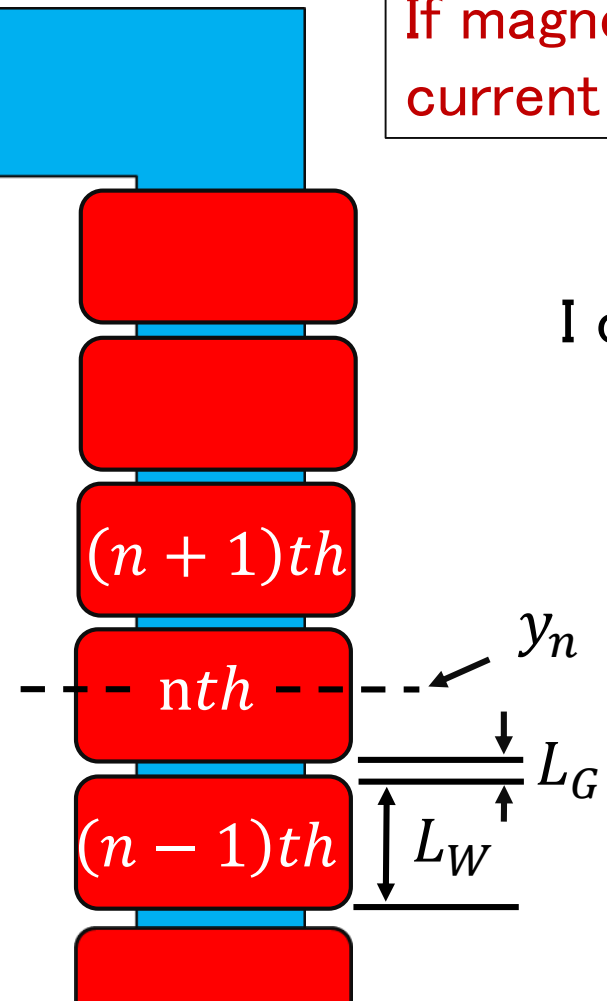




Field strength is proportional to current.

- First approach ・・・ Field superposition
- Second approach ・・・ Proportional to current

If magnet has infinite length, it is conceivable that current of each coil is determined to exponential.



I calculated magnetic field with following formula.

・Coil position

$$y_n \rightarrow \left(L_G + \frac{L_W}{2}\right)n + L_W(n-1)$$

・Current

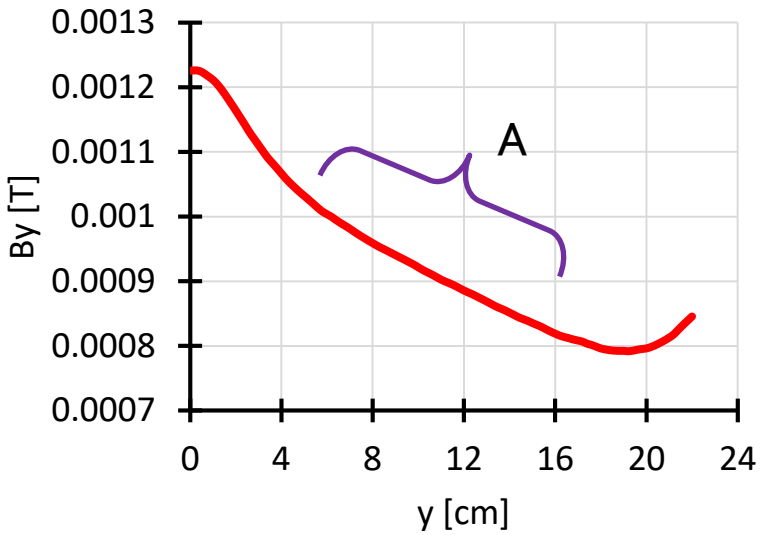
$$I_n \rightarrow I_0 \times \exp(m y_n)$$

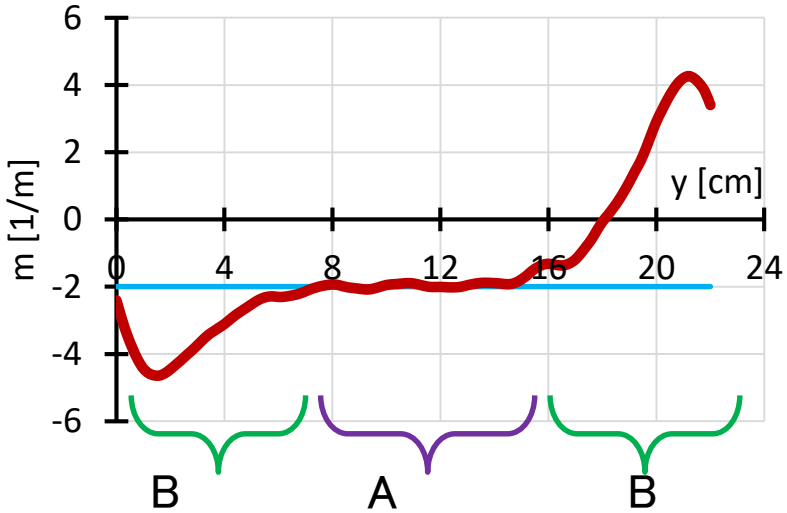
I_0 : Base current [AT]

Magnetic field Calculation (1)

Current was given by $I_n = 25 \times \exp(-2y_n)$

coils	1st	2nd	3rd	4th	5th	6th	7th	8th	9th
current [AT]	24.3	23.1	21.9	20.7	19.7	18.7	17.7	16.8	15.9



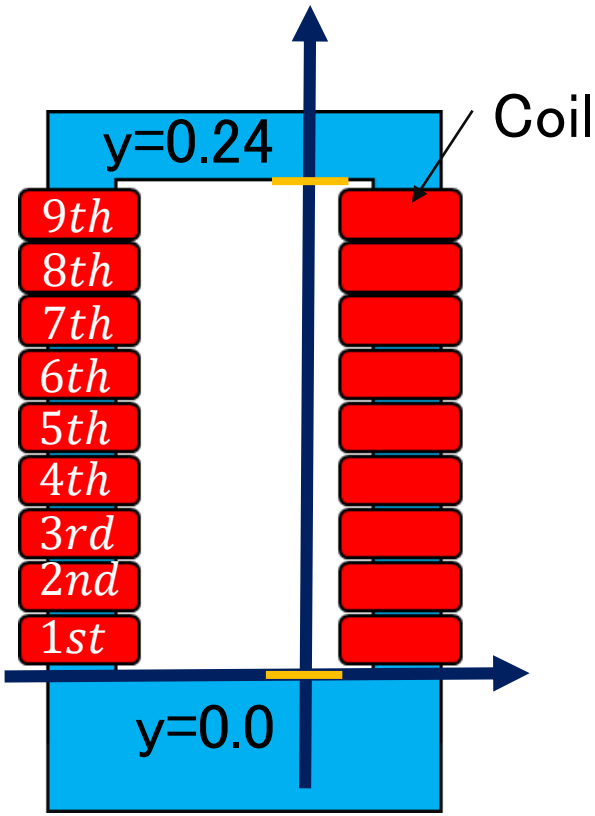


- m value remains constant at '−2.0' at region A. (Good region) $\rightarrow m = -2 \pm 0.2$ [1/m]
- m value largely deviates from -2.0 in region B

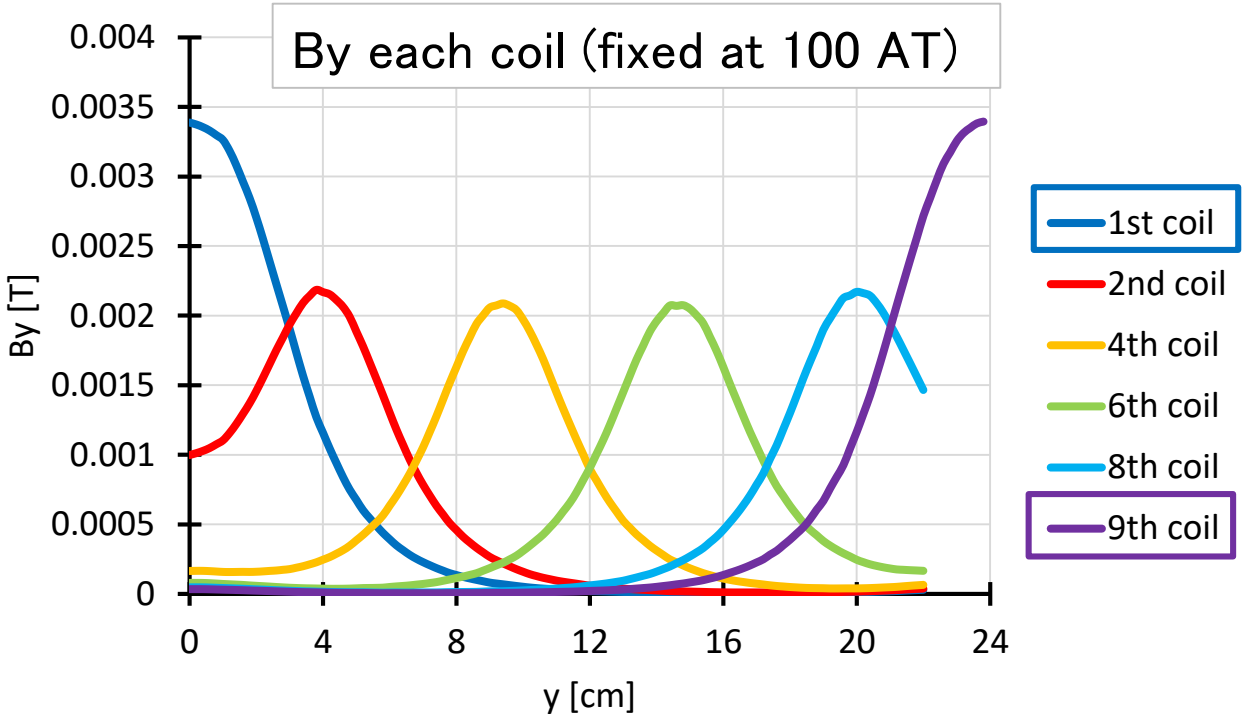
⇨ Field of both ends have affected by magnetic pole.

Third approach

Contribution of individual coils



Magnet



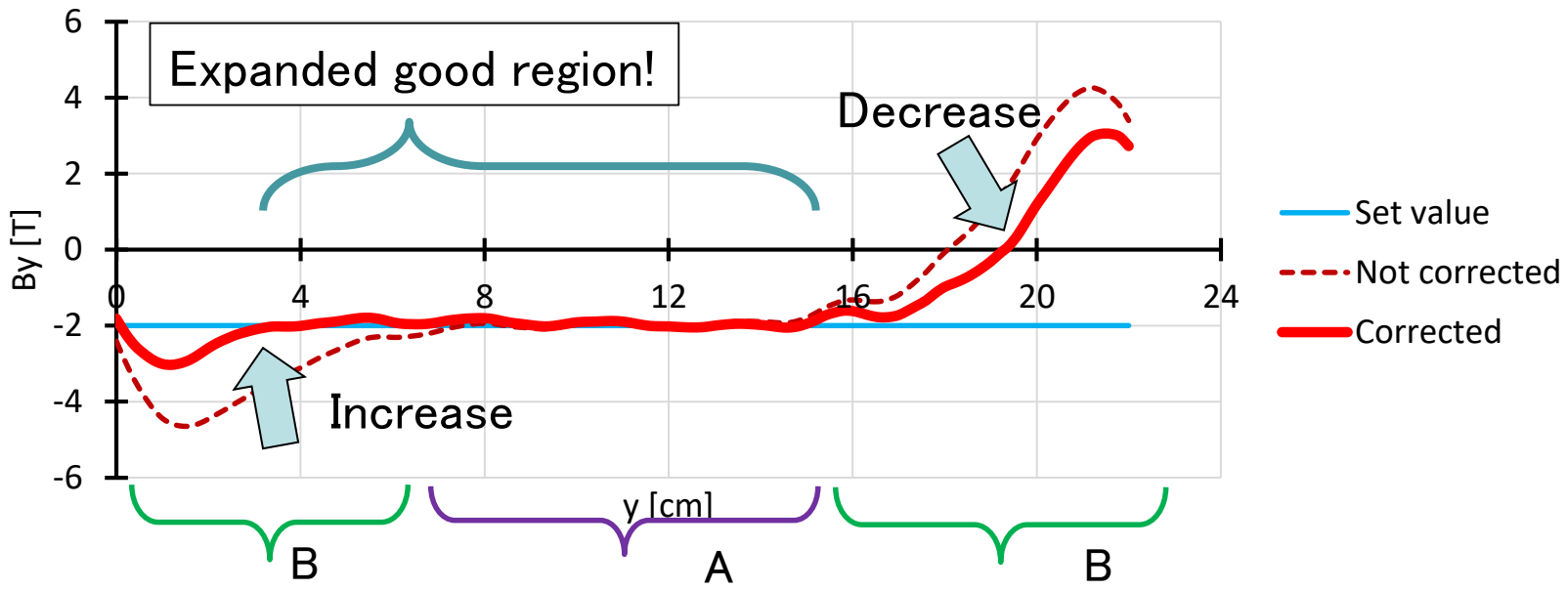
Fields generated by 1st and 9th coils are larger than those by the other coils

Correction of current is necessary

Magnetic field Calculation (2)

Correction was carried out for 1st and 9th coil currents.

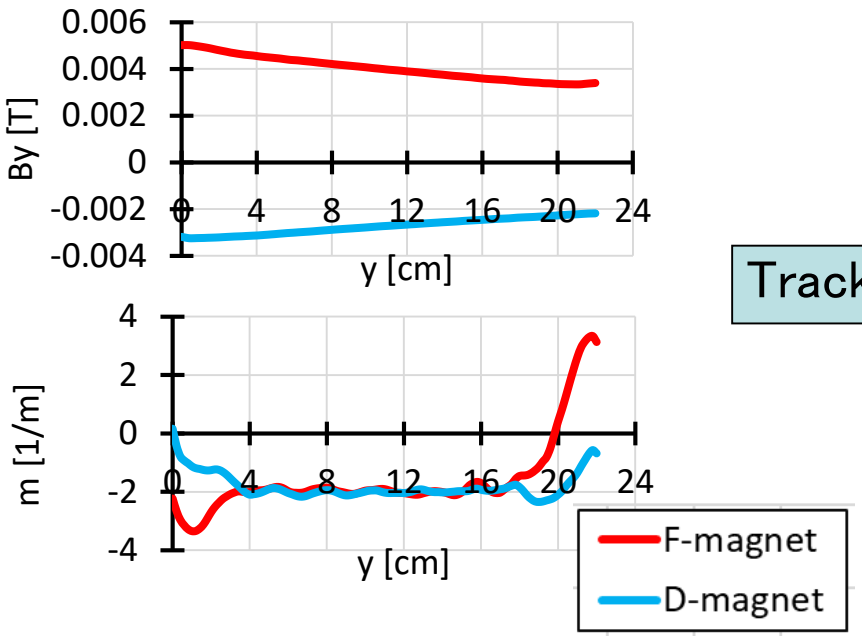
coils	1st	2nd	3rd	4th	5th	6th	7th	8th	9th
Original [AT]	24.3	23.1	21.9	20.7	19.7	18.7	17.7	16.8	15.9
Corrected [AT]	21.4	23.1	21.9	20.7	19.7	18.7	17.7	16.8	14.0



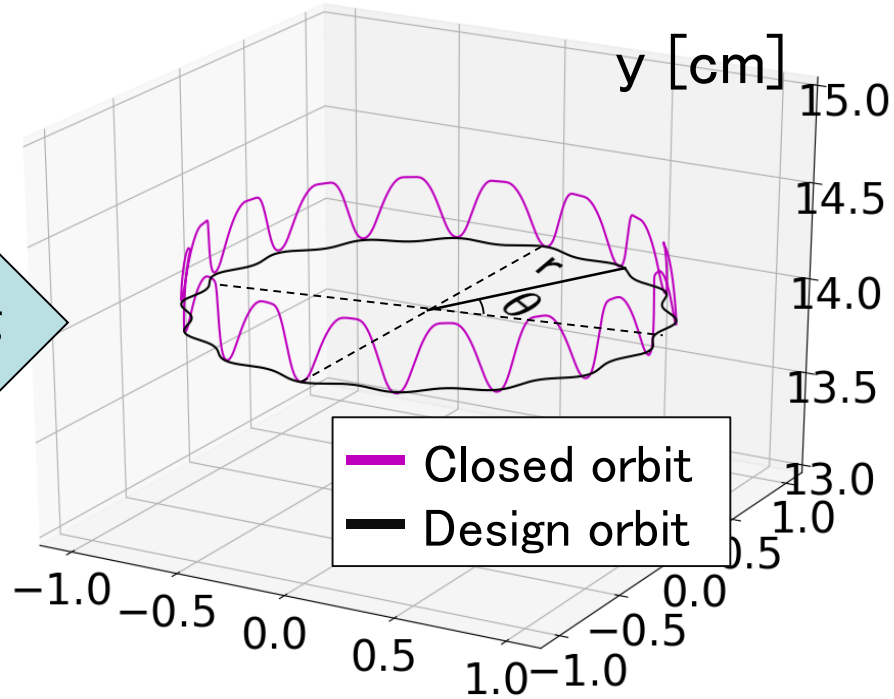
Good region for m value can be expanded by adjusting the individual coil currents.

Beam tracking (1)

D−magnet field can be corrected with similar method for F−magnet.



Tracking



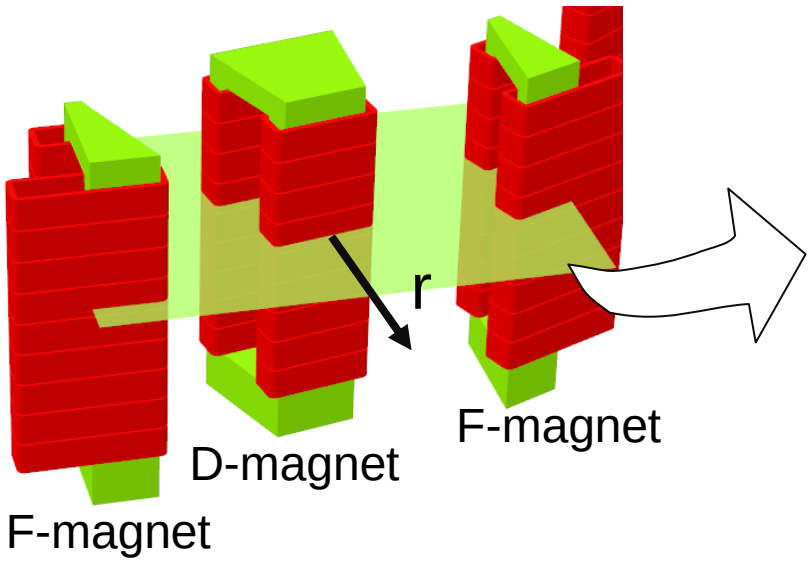
- 2000 turn have been achieved in 20 [keV].
- Component of vertical direction is contained in oscillation.

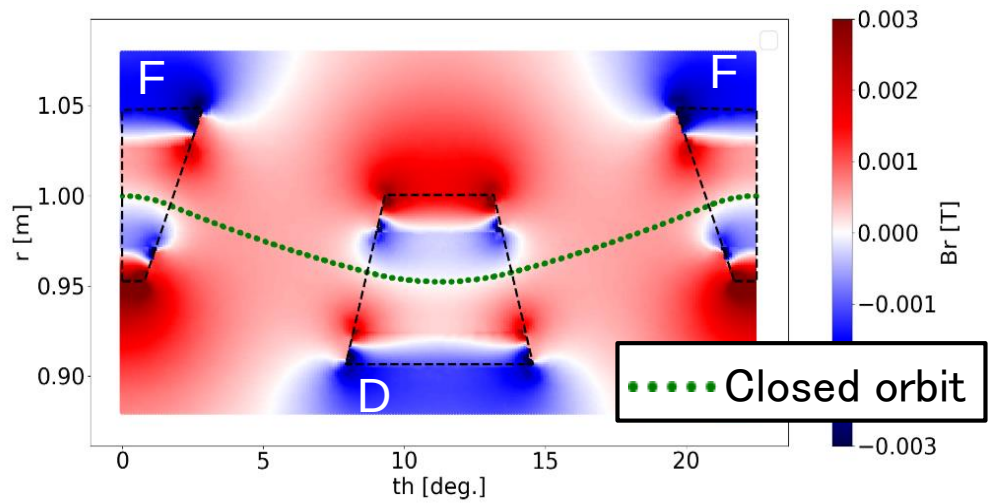
  → Trajectory of horizontal direction has been almost reproduced on design orbit.

  → Oscillation of vertical direction is occurred.

Beam tracking (2)

Magnetic field on the closed orbit contains Br component.



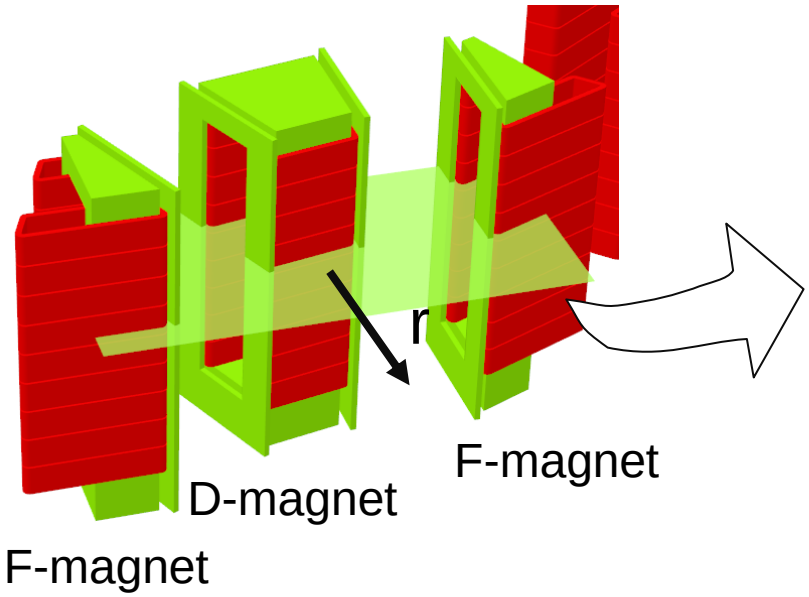


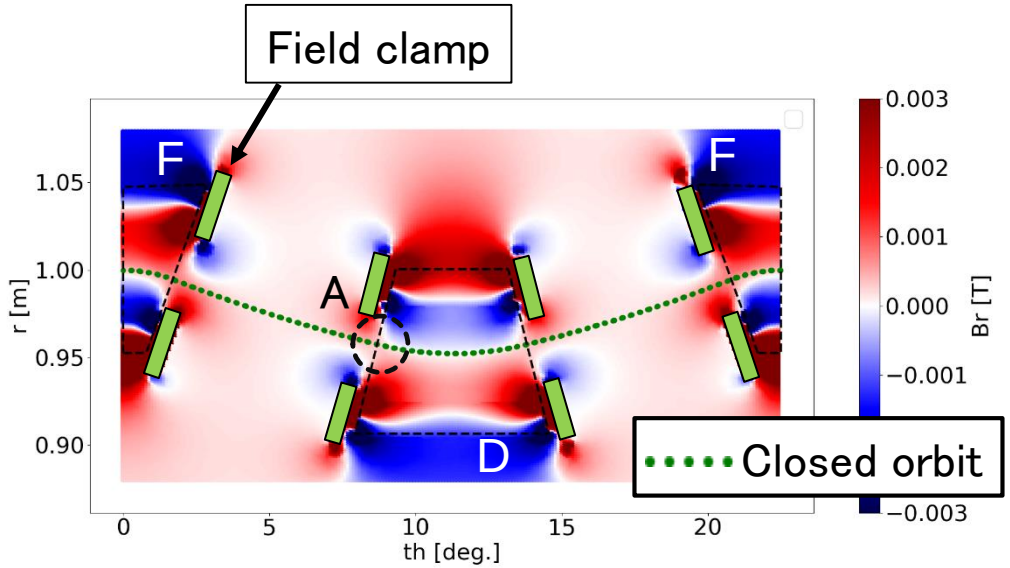
Br is contained on closed orbit. In order to eliminate oscillation of vertical direction,

It is required to reduce Br with additional field clamps

Beam tracking (3)

Effect of field clamp





Br was reduced by field clamps. But, Br at some region has been increased (such as the A).

In order to obtain stability of beam, optimization of position and shape of field clamp is needed.

Conclusion of Magnet design

- The following result was confirmed.
  ① Superposition of field.
  ② Field strength is proportional to current.
  ③ Field of both ends have affected by magnetic pole.

In current distribution magnet,
m value can be optimized by adjusting the individual coil currents.

- In tracking,
  Vertical oscillation is occurred by fringing field.
  Therefore, optimization of position and shape of field clamp is needed.

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Purpose of measurement

Conclusion of Preceding chapter of Magnet

The following result was confirmed.

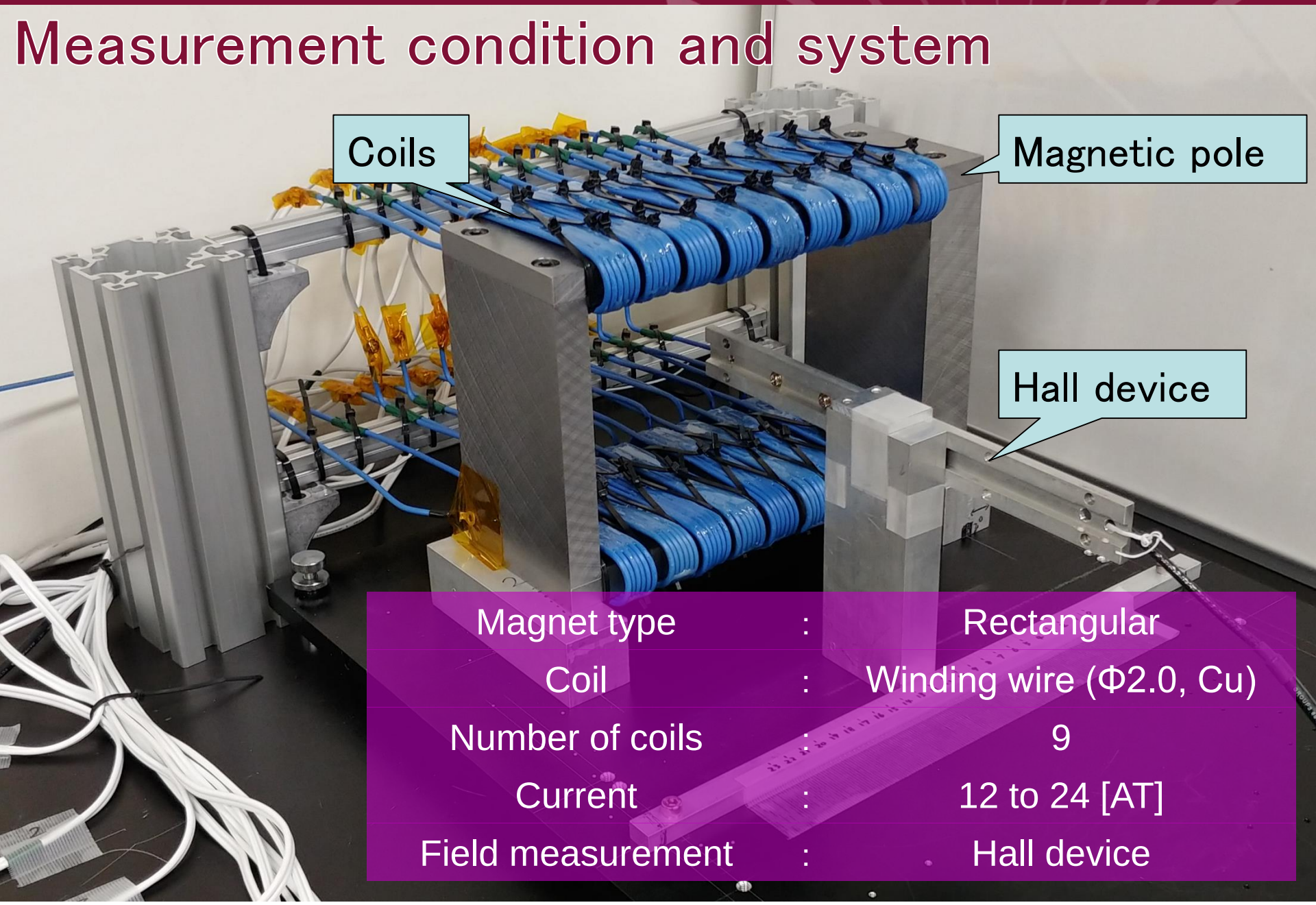
① Superposition of field.
② Field strength is proportional to current.
③ Field of both ends have affected by magnetic pole.

In current distribution magnet,
m value can be optimized by adjusting the individual coil currents.

These result was given by only simulation.

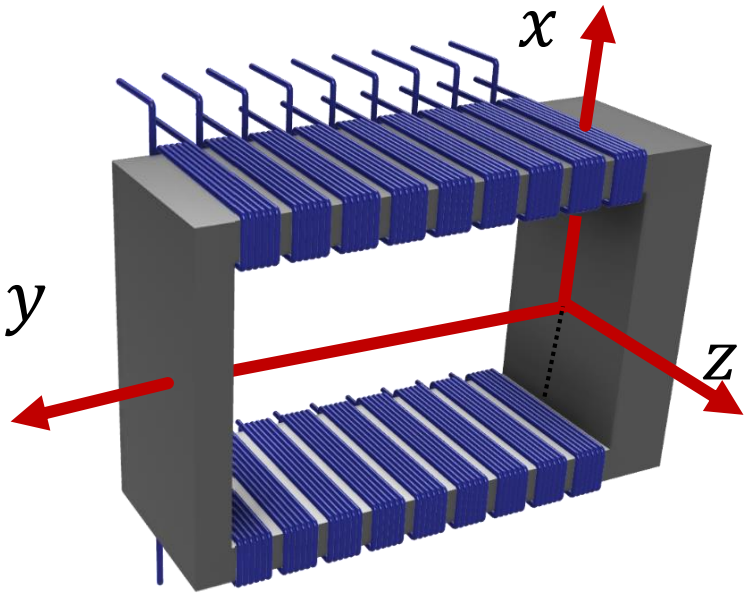
Purpose of measurement is demonstration of simulation result.

Measurement condition and system

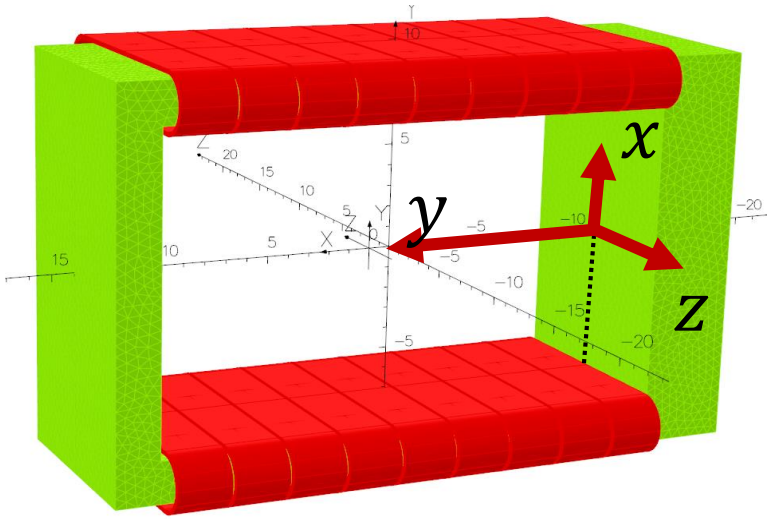


Magnet type	:	Rectangular
Coil	:	Winding wire (Φ2.0, Cu)
Number of coils	:	9
Current	:	12 to 24 [AT]
Field measurement	:	Hall device

Measurement condition and system



Overall view in measurement



Overall view in OPERA−3D

Measured field was compared to calculated field by OPERA-3D.

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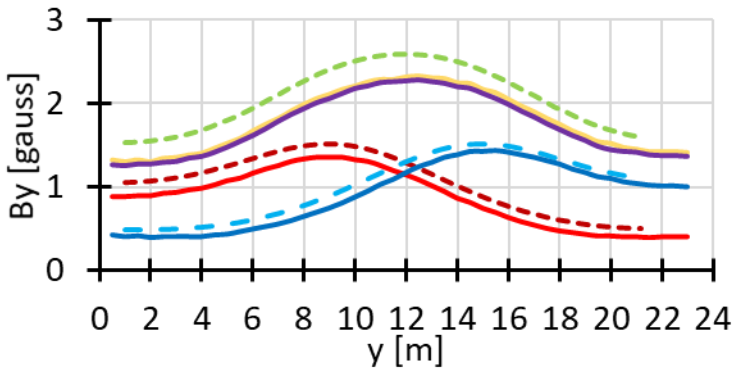
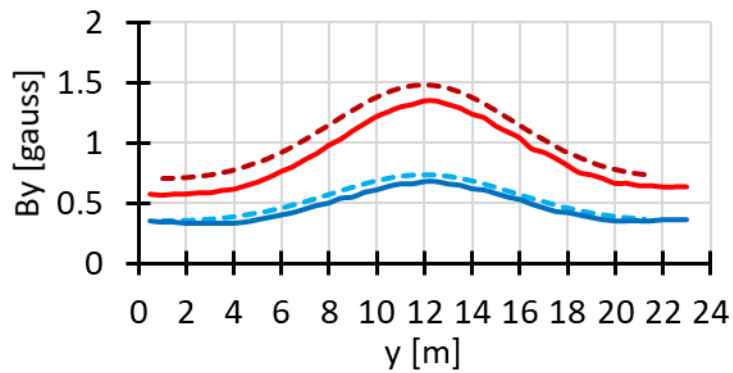
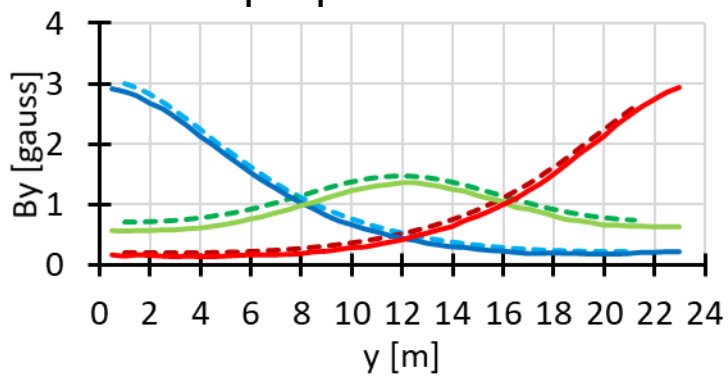
Result of measurement



As for superposition of field



As for proportionality of current.



As for effect by magnetic pole

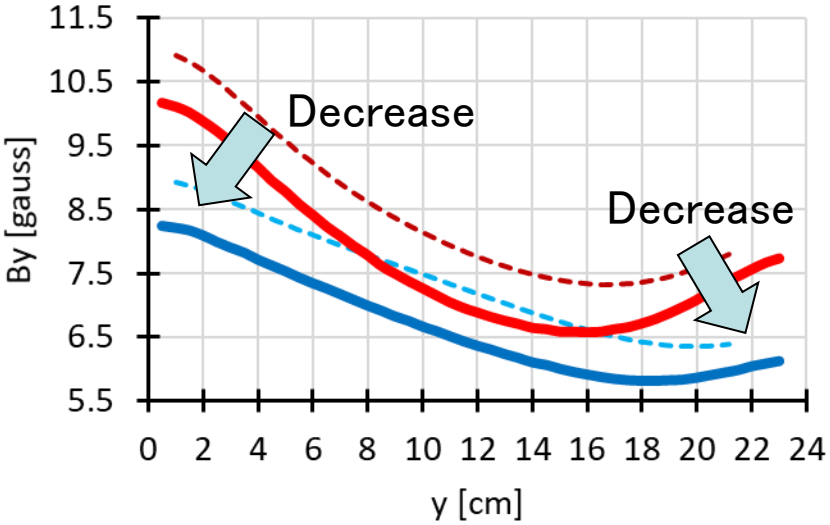
Dashed line : Result of OPERA-3D
Solid line : Result of measurement

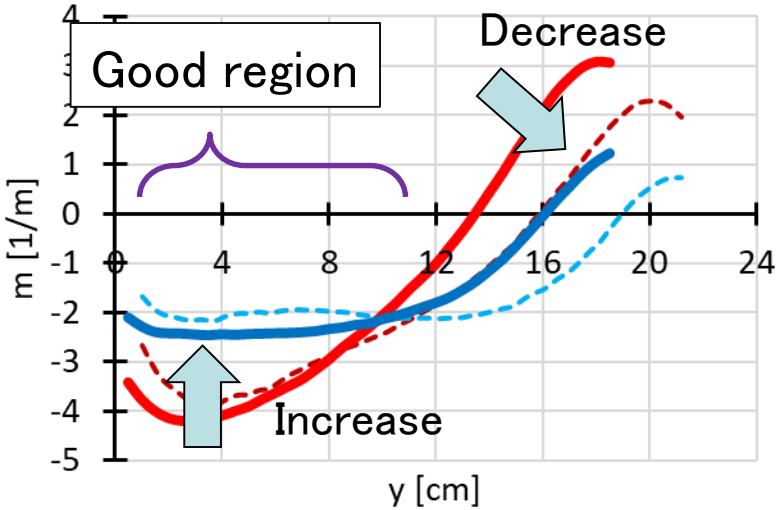
Simulation results with OPERA-3D agree with measurements

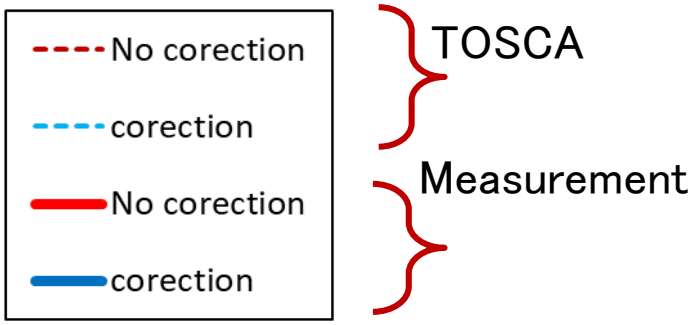
Measurement

- Demonstrate optimize method of magnetic field with multi coils

		1st coil	2nd coil	3rd coil	4th coil	5th coil	6th coil	7th coil	8th coil	9th coil
curent [A]	No corecti	4.07	3.78	3.52	3.31	3.03	2.70	2.61	2.44	2.30
	Corection	2.07	3.78	3.47	3.22	2.95	2.73	2.58	2.42	0.69







Optimize method of magnetic field with multi coils was demonstrated

Outline

- **Introduction**
- **Ring design**
  - Optics
  - Magnet
- **Field measurement**
  - Overview
  - Measurement
- **summary**

Summary

- Linear approximation and transfer matrix method can be applied for optics design of sector type VFFA.
- In case of multi-coil type magnet, m value can be optimized by adjusting the individual coil currents.

Future plan

1. Field measurement of sector type magnet.
2. Design of field clamps.
3. Proposal of linear approximation method including effect of fringing field.

We will be carried out.

Acknowledgement

We would like to appreciate Mr. Waga, Mr. Takuichiro and Mr. Tanaka for their contributions on the field measurement of the magnet.

Thank you for your attention