Electron model of Vertical FFA

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Outline

- Introduction
- Ring design
  - Optics
  - Magnet
- Field measurement
  - Overview
  - Measurement
- Summary
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Electron VFFA (prototype)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Focusing system</td>
<td>F-D Singlet</td>
</tr>
<tr>
<td>Number of cell</td>
<td>16</td>
</tr>
<tr>
<td>Magnet</td>
<td>Sector</td>
</tr>
<tr>
<td>Injection energy</td>
<td>20 [keV]</td>
</tr>
<tr>
<td>Maximum energy</td>
<td>100 [keV]</td>
</tr>
<tr>
<td>Radius</td>
<td>1.0 [m]</td>
</tr>
</tbody>
</table>

Development and Proof-of-Principle Experiment is goal in our study.
Electron VFFA (prototype)

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Advantage
1. Edge angle is 0.
2. Vertical kick is negligible.
   \( \rightarrow B_x \) and \( B_z \) of fringing field = 0
3. Design orbit would be closed.
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Beam optics of VFFA

Magnetic field becomes

\[ B = B_0 e^{m y} \]

\[ m : \text{field gradient} \]

Expanding

Linear

Non linear

\[ B = B_0 \left[ 1 + my + \frac{1}{2} (my)^2 + \cdots \right] \]

Normal  Skew

Magnetic field of VFFA includes skew component.
Optics design

<table>
<thead>
<tr>
<th>Calculation method</th>
<th>Numerical analysis</th>
<th>Linear approximation and transfer matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accuracy</td>
<td>High</td>
<td>Depends on approximation model</td>
</tr>
<tr>
<td>Speed</td>
<td>Slow</td>
<td>Fast</td>
</tr>
<tr>
<td>Ring parameter optimization</td>
<td>Difficult</td>
<td>Easy</td>
</tr>
</tbody>
</table>

Calculation methods including skew field are being examined.

The purpose of this section is to establish linear approximation and transfer matrix methods for VFFA.
Difficulty in transfer matrix derivation

Equation of motion in VFFA

\[
x'' + \frac{x}{\rho(s)^2} + \frac{1}{B\rho(s)} \left( \frac{\partial B_y}{\partial y} \right) y = 0
\]

\[
y'' - \frac{1}{B\rho(s)} \left( \frac{\partial B_x}{\partial x} \right) x = 0
\]

Horizontal motion and vertical motion are coupled. Therefore, derivation of transfer matrix becomes difficult.

New method is required in transfer matrix derivation
New method of transfer matrix derivation

VFFA magnet is approximated to a series of bending magnet and thin skew Q magnet.

\[
\begin{align*}
x'' + \frac{x}{\rho(s)^2} + \frac{1}{B\rho(s)} \left( \frac{\partial B_y}{\partial y} \right) y &= 0 \\
y'' - \frac{1}{B\rho(s)} \left( \frac{\partial B_x}{\partial x} \right) x &= 0
\end{align*}
\]
New method of transfer matrix derivation

\[ x'' + \frac{x}{\rho(s)^2} = 0 \]
\[ y'' = 0 \]

\[
\begin{align*}
x'' &= \frac{x''}{\rho(s)^2} + \frac{1}{B\rho(s)} \frac{\partial B_y}{\partial y} y = 0 \\
y'' &= \frac{1}{B\rho(s)} \frac{\partial B_x}{\partial x} \frac{\partial B_y}{\partial y} x = 0
\end{align*}
\]

These transfer matrixes can be derived.

Transfer matrix of VFFA can be derived.
Calculation

- Parameters $m$ and bending angle ($\theta_F$)
- Number of division is 50

Stable region is derived by linear approximation method.
Comparison of new method and numerical analysis

Hard edge model in numerical analysis.

Magnetic field

\[ B_x \simeq -B_0 e^{my} \sin(mx) \]
\[ B_y \simeq B_0 e^{my} \cos(mx) \]

or

Only linear field

\[ B_x \simeq -B_0 my \]
\[ B_y \simeq B_0 (1 + mx) \]

Result of linear approximation method shows good agreement with numerical analysis, but, not to large region of m.
Conclusion of optics design including skew field.

- Derivation method of transfer matrix is proposed.
- Stable region was derived and compared by two calculation methods.
  (Linear approximation method and numerical analysis)

1. In sector VFFA, linear approximation and transfer matrix method can be applied for optics design.
2. In high region of m, this method is thought that is not adequate.
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Parameters of Electron VFFA

- Injection energy: 20 [keV]
- Maximum energy: 100 [keV]
- B (F): 60.8 [gauss]
- B (D): 22.2 [gauss]
- FD ratio: 2.73
- RF gap Voltage: 200 [Vp]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>particle</td>
<td>electron</td>
</tr>
<tr>
<td>Number of cell</td>
<td>16</td>
</tr>
<tr>
<td>Radius</td>
<td>1.0 [m]</td>
</tr>
<tr>
<td>$\theta_F$</td>
<td>22.5 [deg.]</td>
</tr>
<tr>
<td>$\theta_D$</td>
<td>11.25 [deg.]</td>
</tr>
<tr>
<td>$m$</td>
<td>2.0 [1/m]</td>
</tr>
<tr>
<td>Orbit excursion</td>
<td>0.1 [m]</td>
</tr>
</tbody>
</table>
Requirements of magnet design

① Magnetic pole shape : Simple
② m value : Adjustable

Two design candidates of magnet

Magnetic pole shape (Ordinary type)

Multi coil type (Proposed by Prof. Mori)

Meets the needs

• m is adjustable
• Simple shape

We adopted.
Calculation system in OPERA-3D

- Advantage of multi-coil type are examined with OPERA-3D.
- $m$ is calculated on $y$-axis center of F-magnet. (Left figure shows)
• Magnetic field is optimized to exponential field by multi coils.

It is required to confirm about relationship between magnetic field and current.
Density is increased increment of density depend on current of each coil

Exponential field is made by current distribution
First approach

Superposition of magnetic fields of two coils

- The figure indicates that summation of A(y) and B(y) gives C(y).
- **Superposition** field well reproduces C(y).
Second approach

Relations of magnetic field and current of coil
(only 5th coil is excited)

Field strength is proportional to current.
If magnet has infinite length, it is conceivable that current of each coil is determined to exponential.

I calculated magnetic field with following formula.

- **Coil position**
  \[
  y_n \rightarrow \left( L_G + \frac{L_W}{2} \right) n + L_W(n - 1)
  \]

- **Current**
  \[
  I_n \rightarrow I_0 \times \exp(my_n)
  \]

\(I_0\) : Base current [AT]
Magnetic field Calculation (1)

Current was given by

\[ I_n = 25 \times \exp(-2y_n) \]

<table>
<thead>
<tr>
<th>coils</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
<th>6th</th>
<th>7th</th>
<th>8th</th>
<th>9th</th>
</tr>
</thead>
<tbody>
<tr>
<td>current [AT]</td>
<td>24.3</td>
<td>23.1</td>
<td>21.9</td>
<td>20.7</td>
<td>19.7</td>
<td>18.7</td>
<td>17.7</td>
<td>16.8</td>
<td>15.9</td>
</tr>
</tbody>
</table>

- \( m \) value remains constant at ‘−2.0’ at region A. (Good region)
  \[ m = -2 \pm 0.2 \ [1/m] \]
- \( m \) value largely deviates from −2.0 in region B

Field of both ends have affected by magnetic pole.
Third approach

Contribution of individual coils

Fields generated by 1st and 9th coils are larger than those by the other coils

Correction of current is necessary
Magnetic field Calculation (2)

Correction was carried out for 1st and 9th coil currents.

<table>
<thead>
<tr>
<th>coils</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
<th>6th</th>
<th>7th</th>
<th>8th</th>
<th>9th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original [AT]</td>
<td>24.3</td>
<td>23.1</td>
<td>21.9</td>
<td>20.7</td>
<td>19.7</td>
<td>18.7</td>
<td>17.7</td>
<td>16.8</td>
<td>15.9</td>
</tr>
<tr>
<td>Corrected [AT]</td>
<td>21.4</td>
<td>23.1</td>
<td>21.9</td>
<td>20.7</td>
<td>19.7</td>
<td>18.7</td>
<td>17.7</td>
<td>16.8</td>
<td>14.0</td>
</tr>
</tbody>
</table>

By [T]

Expanded good region!

Good region for m value can be expanded by adjusting the individual coil currents.
• 2000 turn have been achieved in 20 [keV].
• Component of vertical direction is contained in oscillation.
→ Trajectory of horizontal direction has been almost reproduced on design orbit.
→ Oscillation of vertical direction is occurred.
Beam tracking (2)

Magnetic field on the closed orbit contains Br component.

Br is contained on closed orbit. In order to eliminate oscillation of vertical direction,

It is required to reduce Br with additional field clamps
Br was reduced by field clamps. But, Br at some region has been increased (such as the A).

In order to obtain stability of beam, optimization of position and shape of field clamp is needed.
Conclusion of Magnet design

• The following result was confirmed.
  ① Superposition of field.
  ② Field strength is proportional to current.
  ③ Field of both ends have affected by magnetic pole.

In current distribution magnet, m value can be optimized by adjusting the individual coil currents.

• In tracking,
  Vertical oscillation is occurred by fringing field.
  Therefore, optimization of position and shape of field clamp is needed.
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Purpose of measurement

Conclusion of Preceding chapter of Magnet

The following result was confirmed.
① Superposition of field.
② Field strength is proportional to current.
③ Field of both ends have affected by magnetic pole.

In current distribution magnet, m value can be optimized by adjusting the individual coil currents.

These result was given by only simulation.

Purpose of measurement is demonstration of simulation result.
**Measurement condition and system**

- **Magnet type**: Rectangular
- **Coil**: Winding wire (Φ2.0, Cu)
- **Number of coils**: 9
- **Current**: 12 to 24 [AT]
- **Field measurement**: Hall device
Measurement condition and system

Overall view in measurement

Overall view in OPERA-3D

Measured field was compared to calculated field by OPERA-3D.
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Result of measurement

As for superposition of field

As for proportionality of current.

Dashed line : Result of OPERA–3D
Solid line : Result of measurement

Simulation results with OPERA–3D agree with measurements
Measurement

- Demonstrate optimize method of magnetic field with multi coils

<table>
<thead>
<tr>
<th></th>
<th>1st coil</th>
<th>2nd coil</th>
<th>3rd coil</th>
<th>4th coil</th>
<th>5th coil</th>
<th>6th coil</th>
<th>7th coil</th>
<th>8th coil</th>
<th>9th coil</th>
</tr>
</thead>
<tbody>
<tr>
<td>No corection</td>
<td>4.07</td>
<td>3.78</td>
<td>3.52</td>
<td>3.31</td>
<td>3.03</td>
<td>2.70</td>
<td>2.61</td>
<td>2.44</td>
<td>2.30</td>
</tr>
<tr>
<td>Corection</td>
<td>2.07</td>
<td>3.78</td>
<td>3.47</td>
<td>3.22</td>
<td>2.95</td>
<td>2.73</td>
<td>2.58</td>
<td>2.42</td>
<td>0.69</td>
</tr>
</tbody>
</table>

Optimize method of magnetic field with multi coils was demonstrated
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Summary

• Linear approximation and transfer matrix method can be applied for optics design of sector type VFFA.
• In case of multi-coil type magnet, m value can be optimized by adjusting the individual coil currents.

Future plan

1. Field measurement of sector type magnet.
2. Design of field clamps.
   We will be carried out.

Acknowledgement

We would like to appreciate Mr. Waga, Mr. Takuichiro and Mr. Tanaka for their contributions on the field measurement of the magnet.
Thank you for your attention