



Science and  
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# Longitudinal Tomography in a scaling FFA

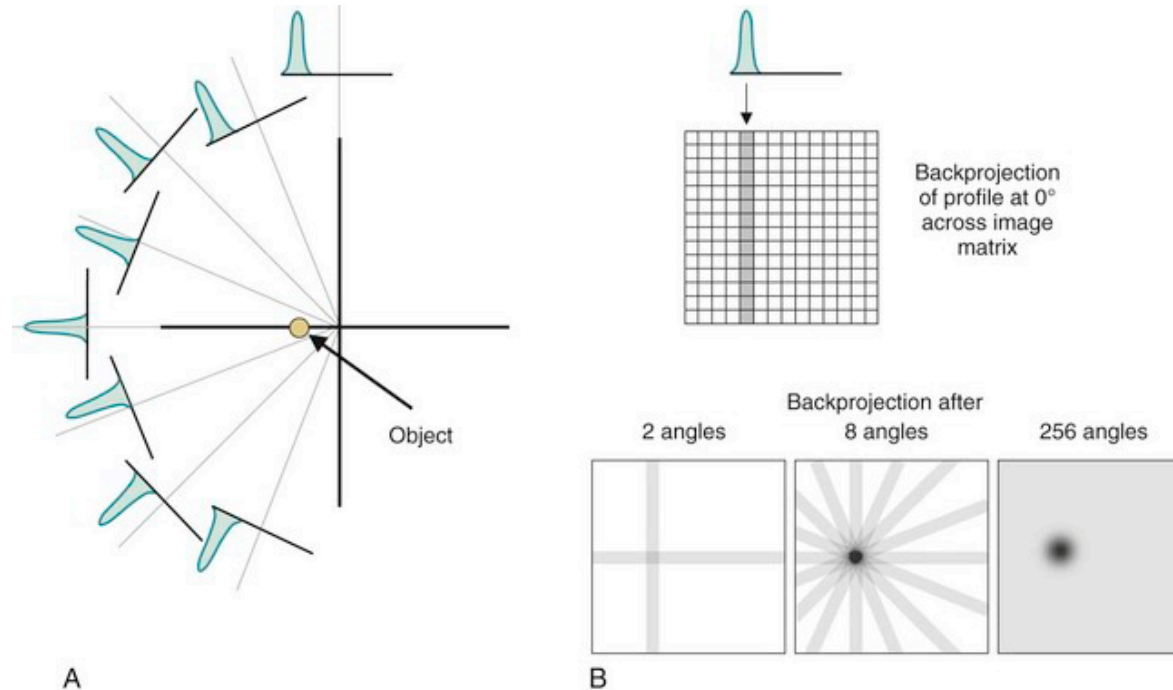
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Rutherford Appleton Laboratory  
FFA'19 workshop, PSI

\*On behalf of the teams at RAL and KURNS

# Tomography

- Back projection is the technique “by which the contents of the bins of a 1D histogram are redistributed over the 2D array of cells (pixels) which comprise the reconstruction image”\*.
- Typically the object is fixed and the viewing angle rotated.



# Longitudinal tomography

- In longitudinal tomography the “object” rotates whereas the projection is along a fixed line (the time axis).
- Conventional algorithms normally assume a rigid, circular motion of the 2D distribution.
- The **hybrid iterative algorithm** developed by S. Hancock considers how each point in longitudinal phase space are projected into the bins of a particular profile by tracking a number of test particles. Large amplitude synchrotron motion is then taken into account.

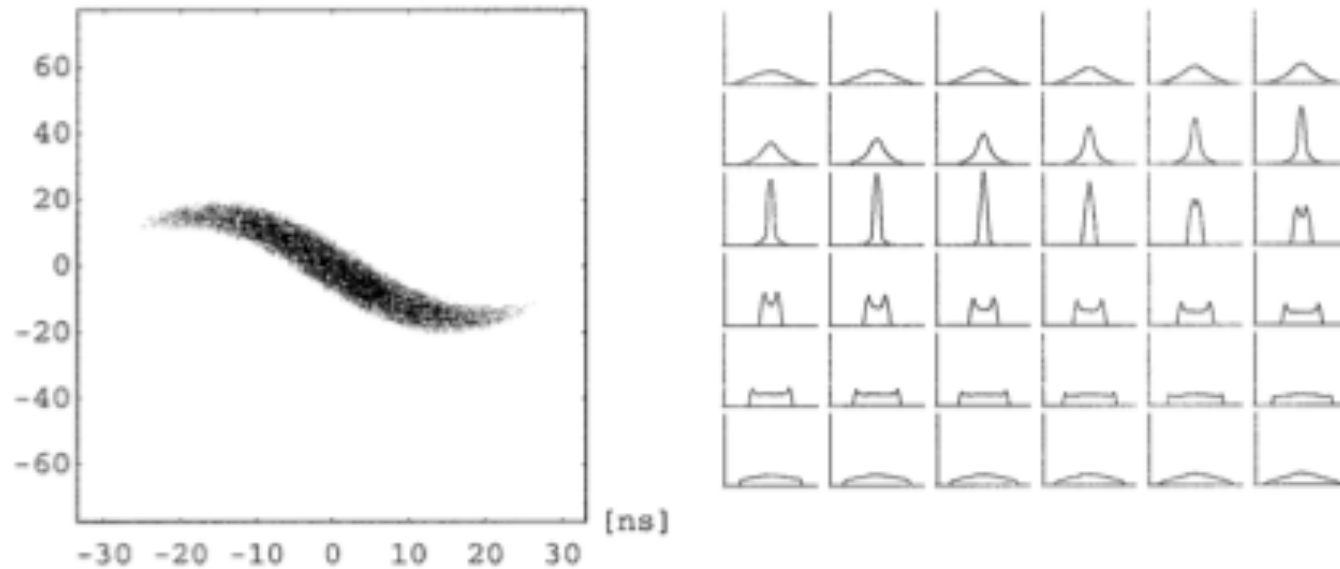


Figure 2: The initial phase space distribution and thirty-six of its projections at intervals of  $5^\circ$  of synchrotron phase. The latter constitute the set of “measured” data.

# Knowledge transfer

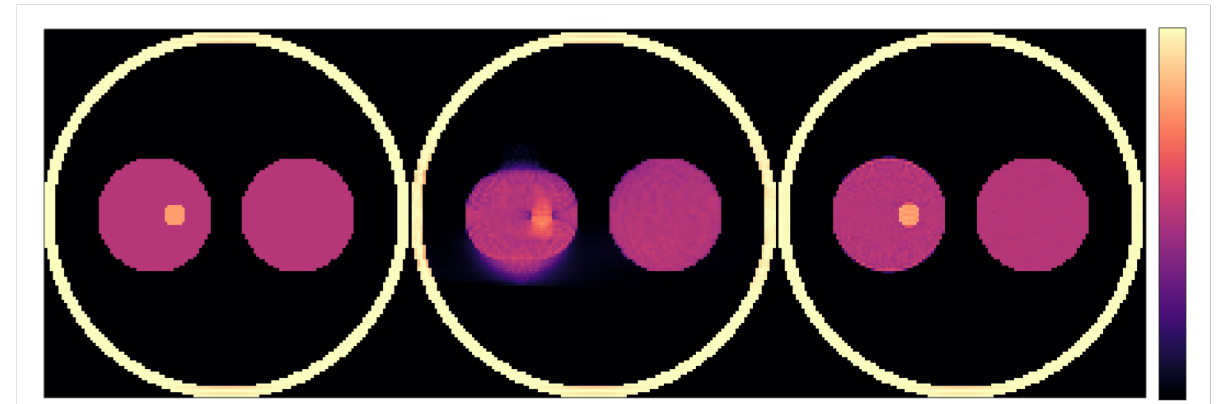
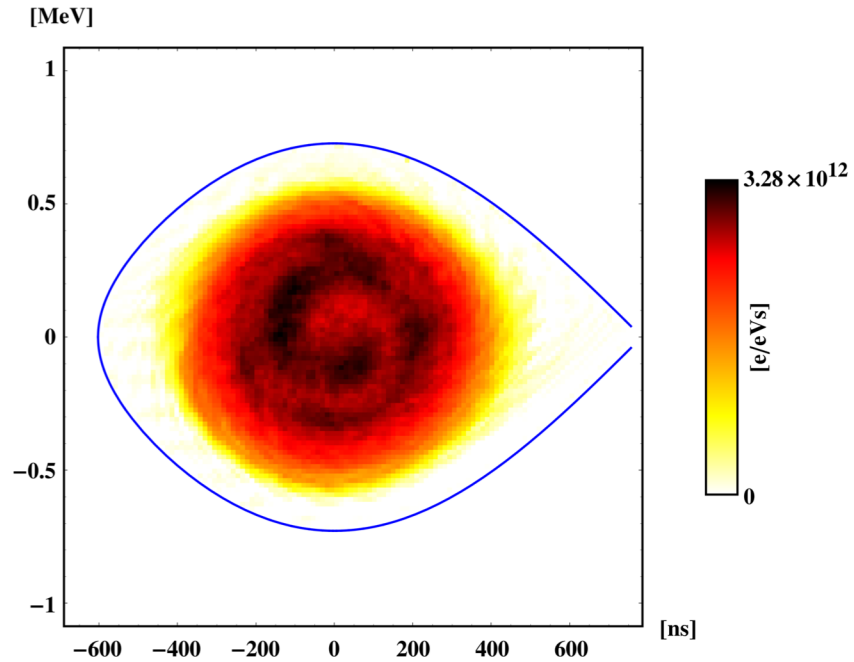


Figure 6-2: Vertical cross-section of images on: (a) Image reconstructed with SART and no motion happening, (b) image reconstructed with SART and motion is happening, but no compensation is applied, (c) image reconstructed with SART and motion happening, with motion compensation. The colour scale is linear attenuation coefficient in the range [0-1]

A. Biguri, “Iterative Reconstruction and Motion Compensation in Computed Tomography on GPUS”, PhD Thesis 2017.



# CERN tomography code

(<http://tomograp.web.cern.ch/tomograp/>)

- Starting with the bunch monitor profiles  $b(\phi, m)$ , we want to find the phase space distribution  $x(\phi, \delta)$  such that

$$A[\phi, \delta, m]x = b$$

- Find the operator A by tracking test particles starting from cells covering the bucket using

$$\begin{aligned}\Delta E_{i,m+1} &= \Delta E_{i,m} + q [V_{rf}(\phi_{0,m+1} + \Delta\phi_{i,m+1}) - V_{rf}(\phi_{0,m+1}) + V_{self}(\phi_{0,m+1} + \Delta\phi_{i,m+1})], \\ \Delta\phi_{i,m+1} &= \Delta\phi_{i,m} - 2\pi h (\eta_{0,m} \beta_{0,m}^{-2} \Delta E_{i,m} / E_{0,m}).\end{aligned}$$

- Repeat back projection  $b(\phi, m) \rightarrow x(\phi, \delta)$  and forward projection  $x(\phi, \delta) \rightarrow b(\phi, m)$  steps until the discrepancy between the measured and calculated bunch profiles converges.

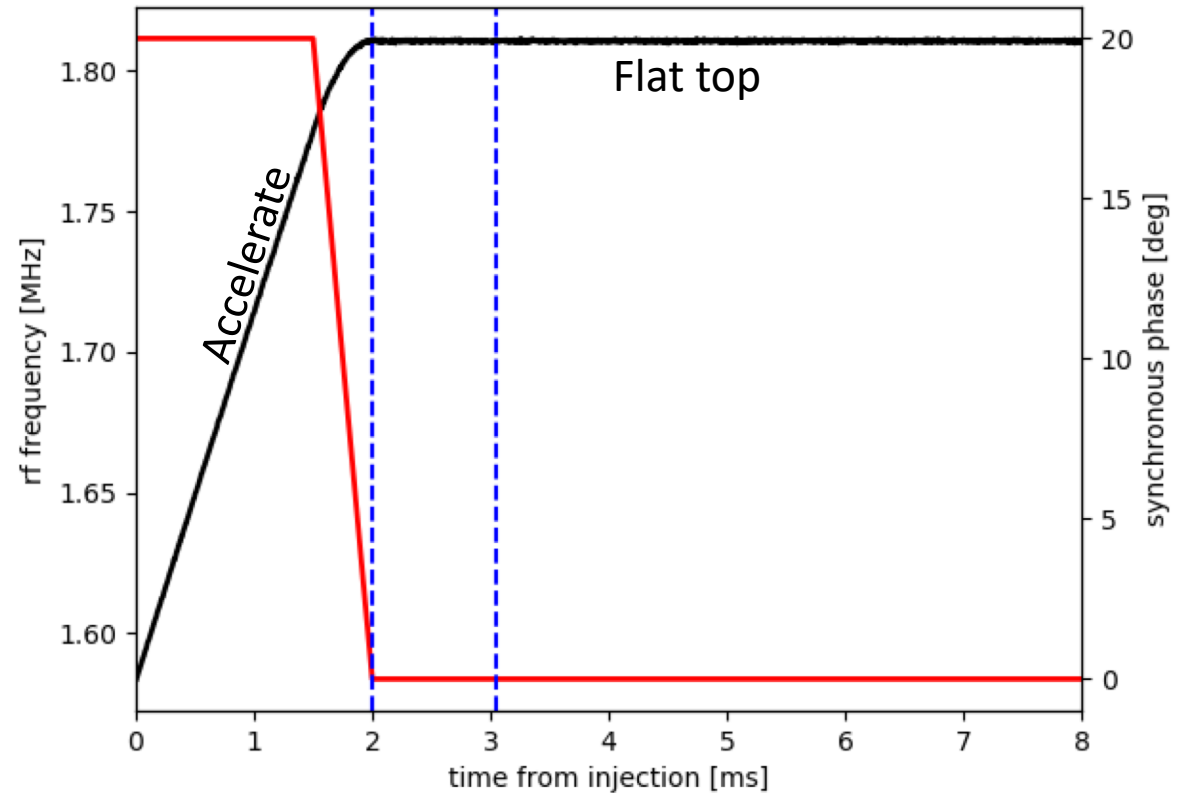
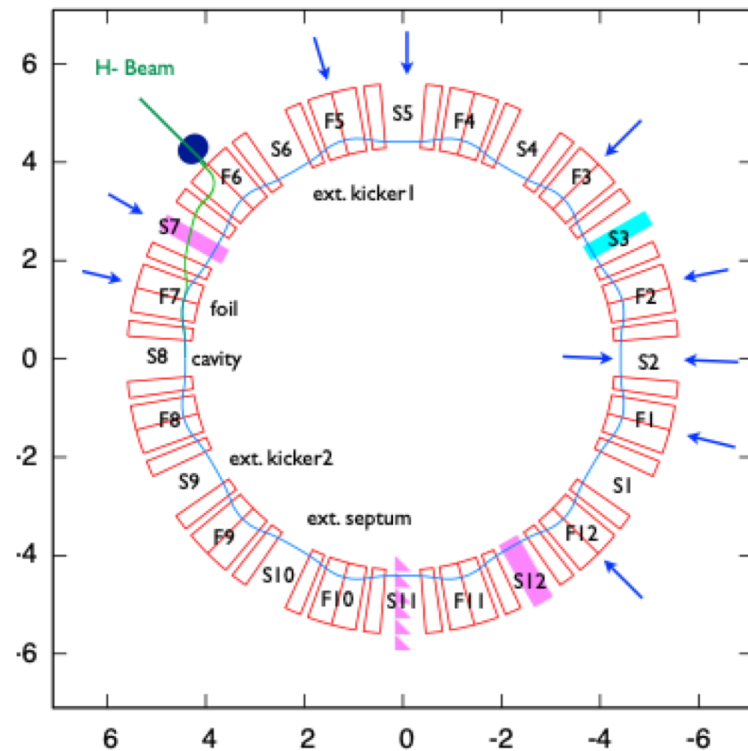
Add more physics here if required (e.g. effect of foil crossing).

$$\eta_0 = \frac{1}{k+1} - \frac{1}{\gamma^2}$$

It's a synchrotron code, so set "Bdot"

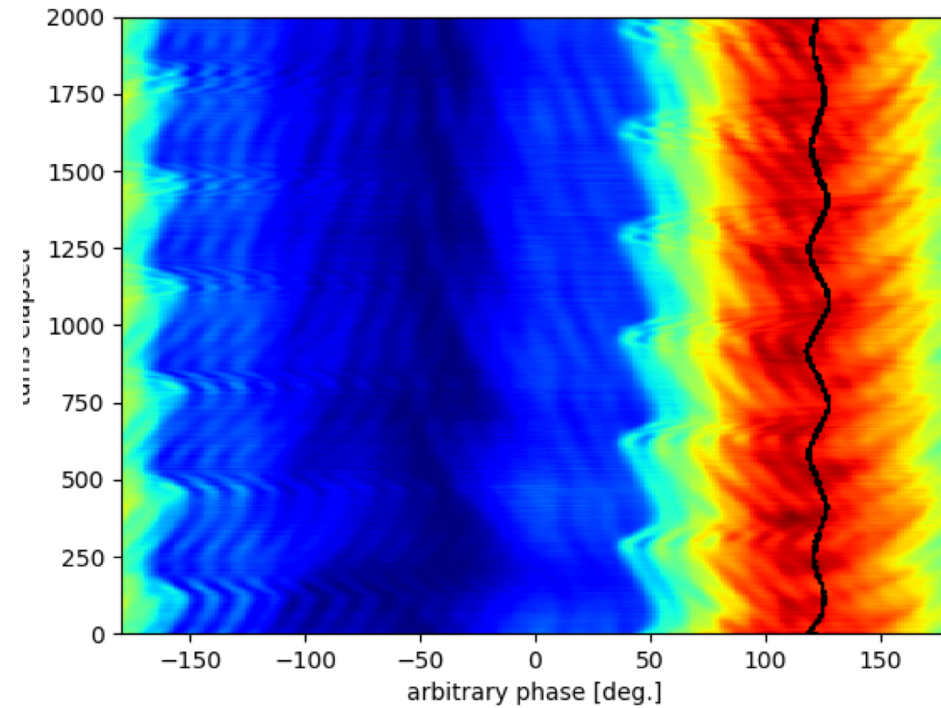
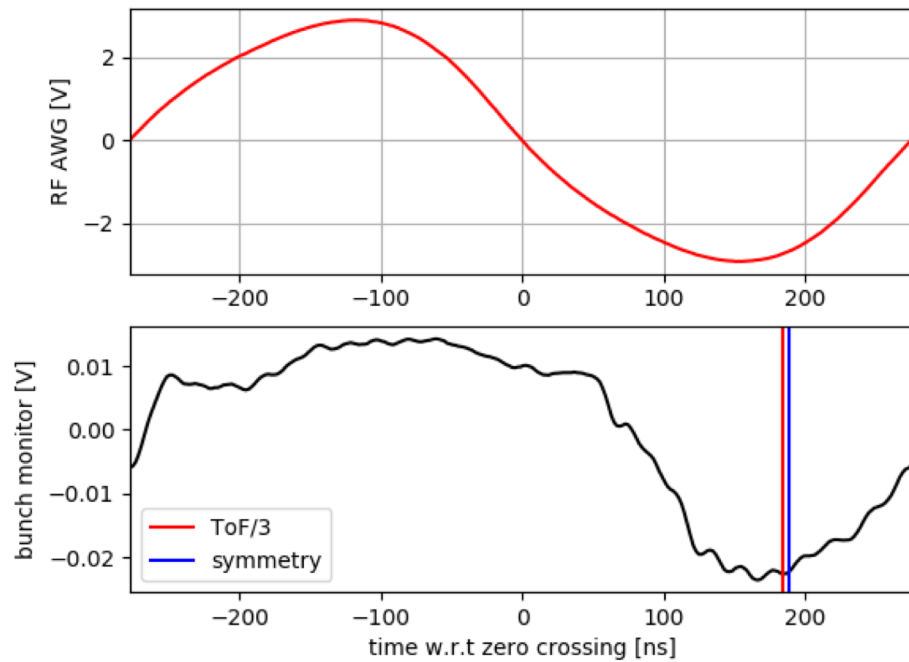
$$\dot{B} = \frac{V_0 \sin\phi_s}{2\pi R\rho}$$

# KURNS setup



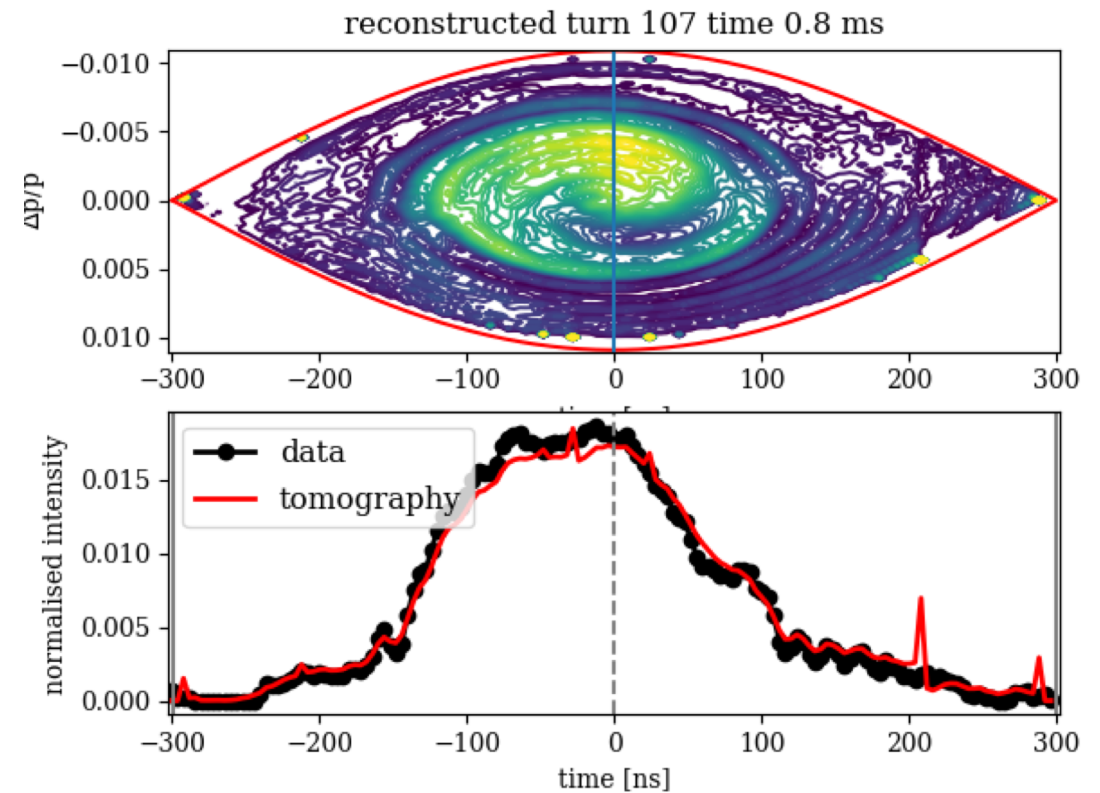
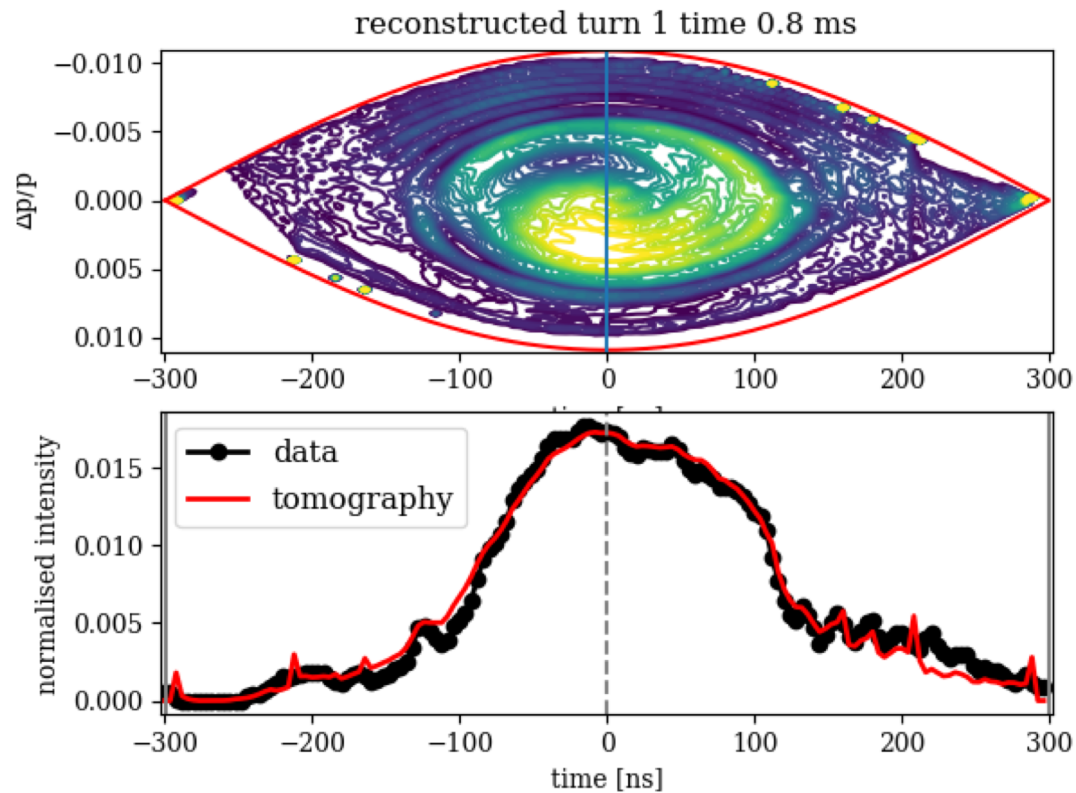
Flexible RF – pattern generated by AWG.  
Use bunch monitor in S12. Typically 4ns time step.

# Establishing the phase



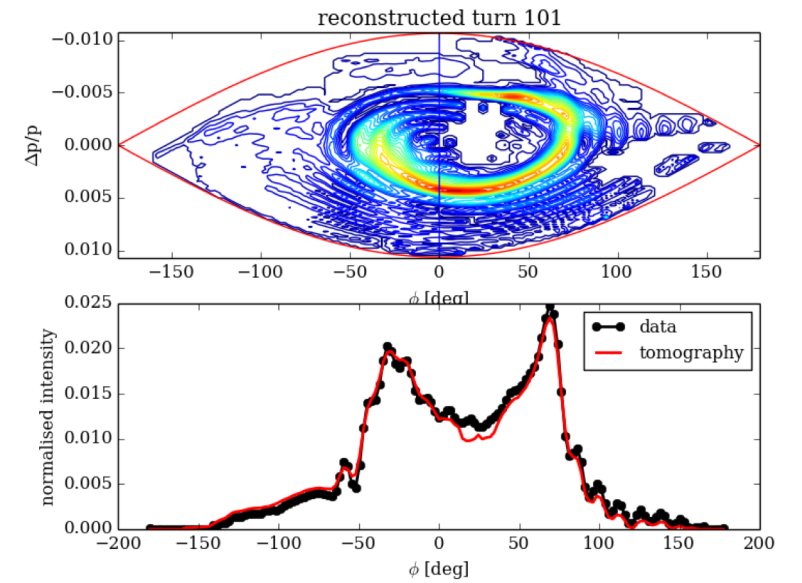
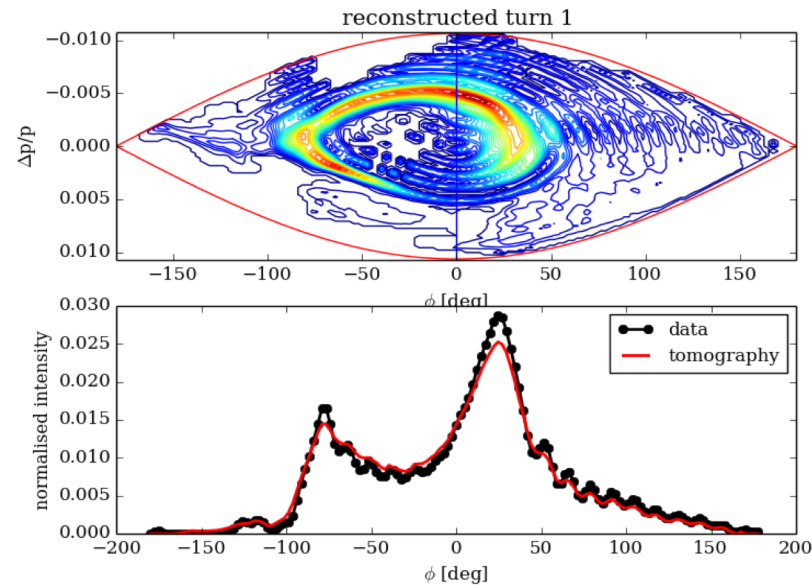
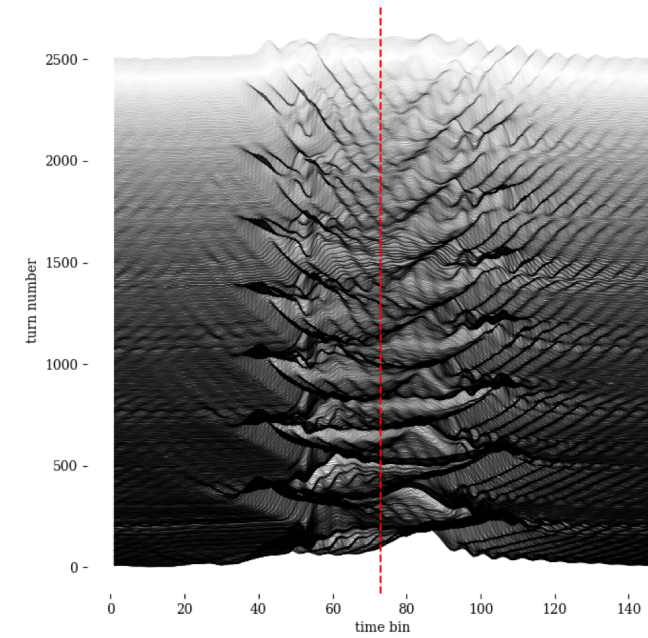
Map time to phase using the RF signal zero-crossing.

# Example reconstruction

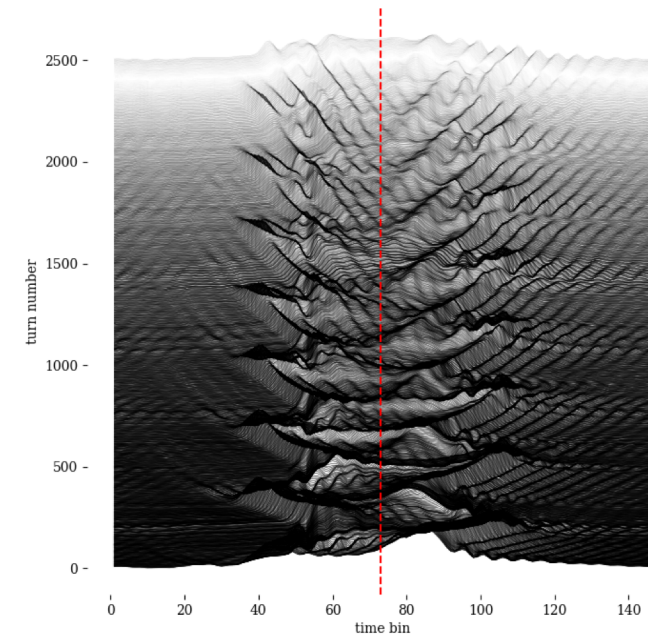


- $\phi = 20$  deg for 0.5ms, linearly decreases to zero for 0.3 ms. Tomography in flat top.

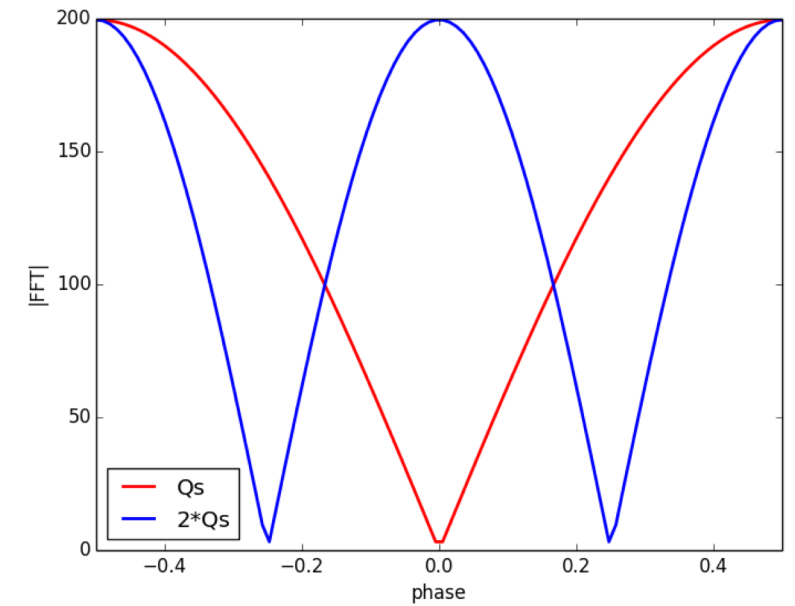
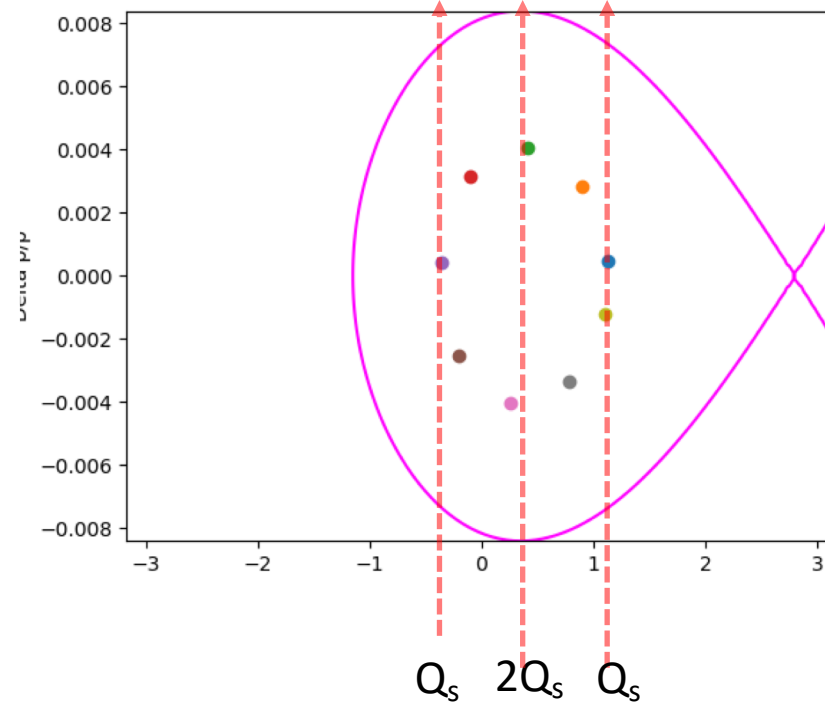
# Example with bunch mismatch



# Synchrotron tune vs amplitude



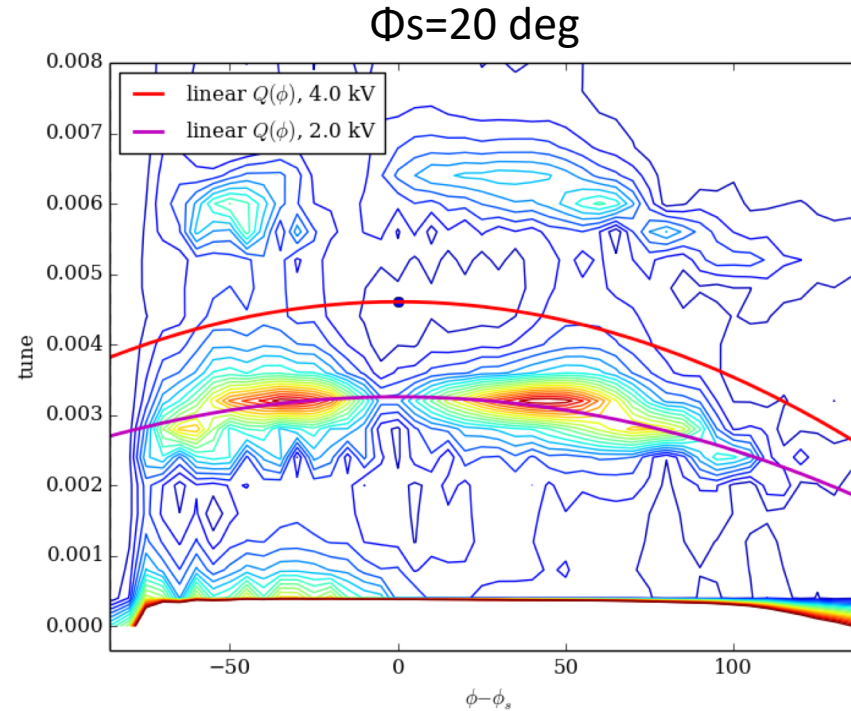
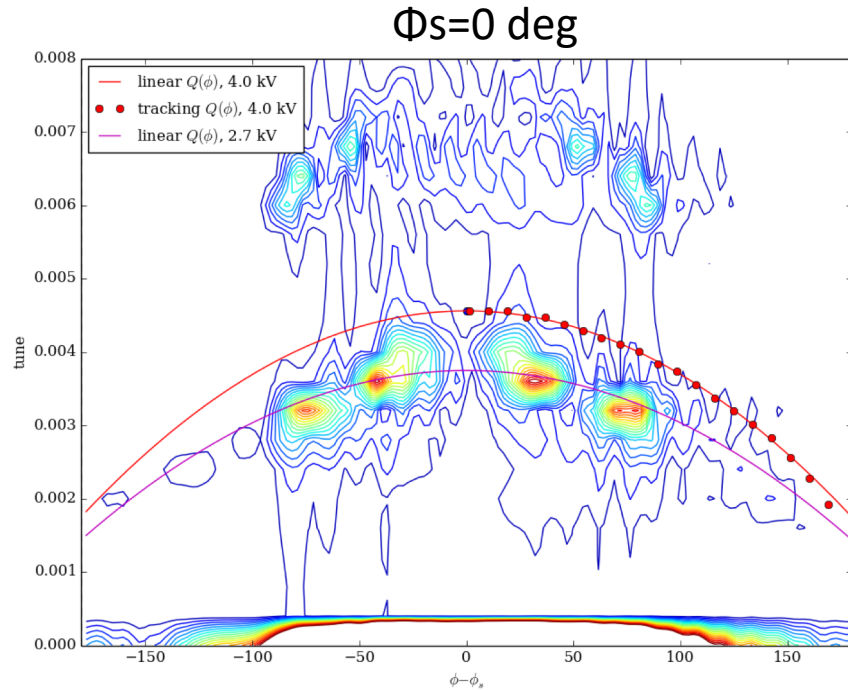
Idealised picture



- Calculate FFT of turn-by-turn data at each phase.



# Measured frequency maps



- Contour plot shows  $|\text{FFT}(\text{phase})|$ . Resolution is 1/2500.
- Compare with expected tune variation with amplitude.

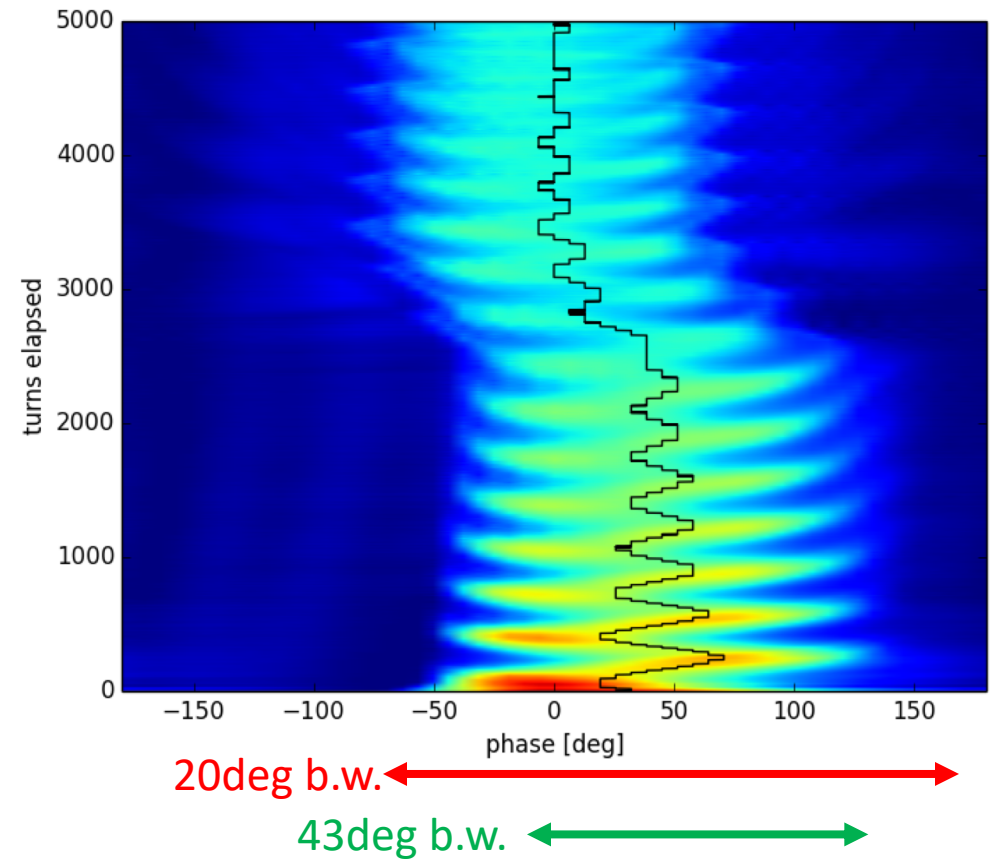
SY Lee

$$Q_s(\hat{\phi}) = Q_{s0} \left( 1 - \frac{1}{16} \left( 1 + \frac{5}{3} \tan^2 \phi_s \right) \hat{\phi}^2 \right)$$

# Is the RF voltage lower?

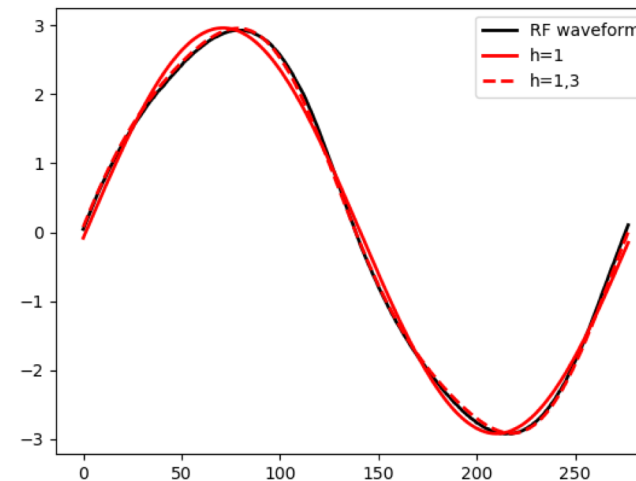
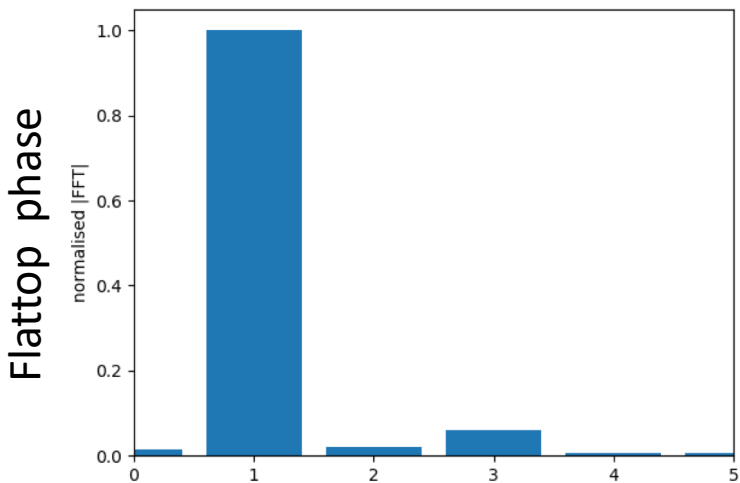
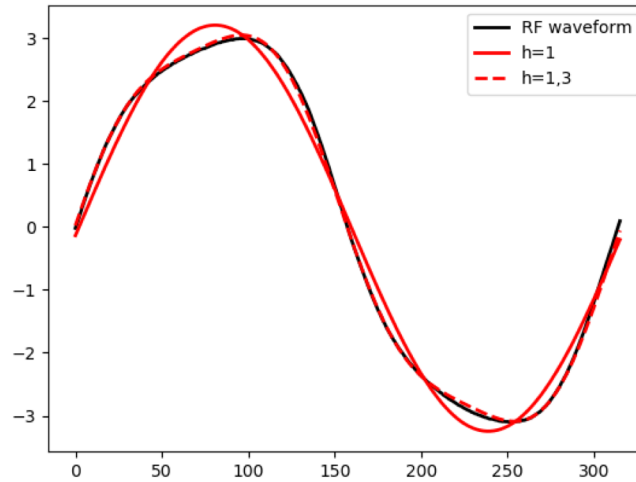
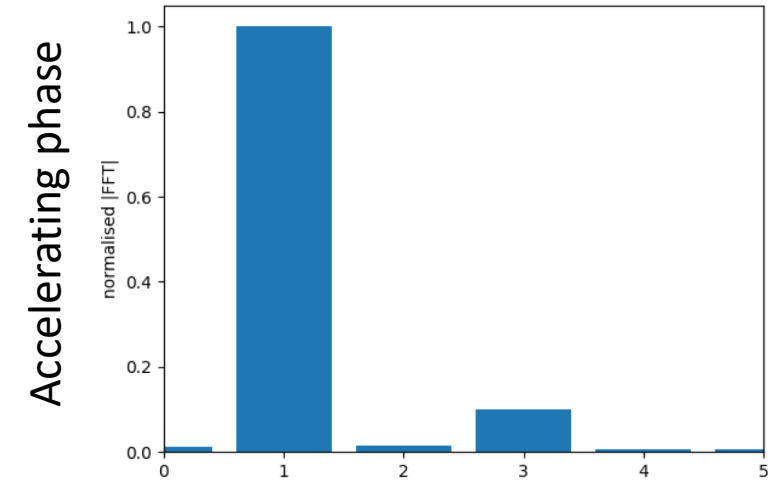
- Energy gain per turn  $V_0 \sin \phi_s$  given by RF frequency variation.
- Design parameters  $V_0=4\text{kV}$ ,  $\phi_s=20$  deg. If voltage is 2kV,  $\phi_s=43.2$  deg to maintain energy gain.
- However the bucket width at this higher  $\phi_s$  is not consistent with phase occupied by bunch.

$\Phi_s$ [deg]	$\Phi_u$ [deg]	$\pi-\phi_s$ [deg]
20	-65.8	160
43.2	-7.0	136.8



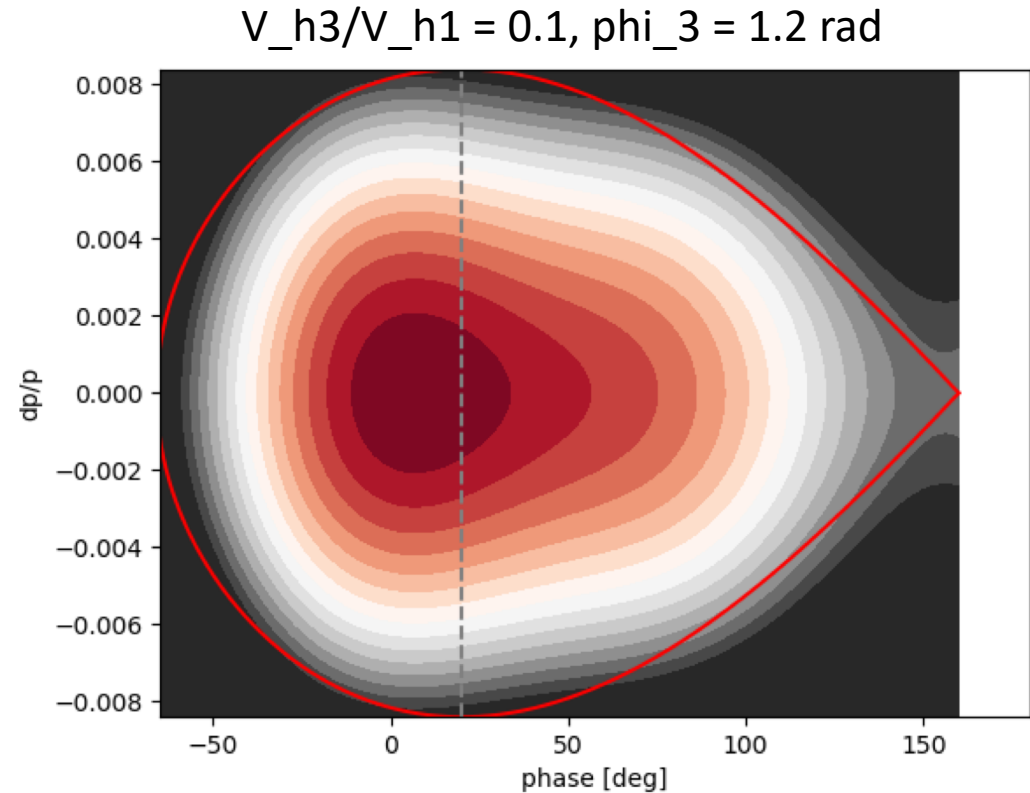
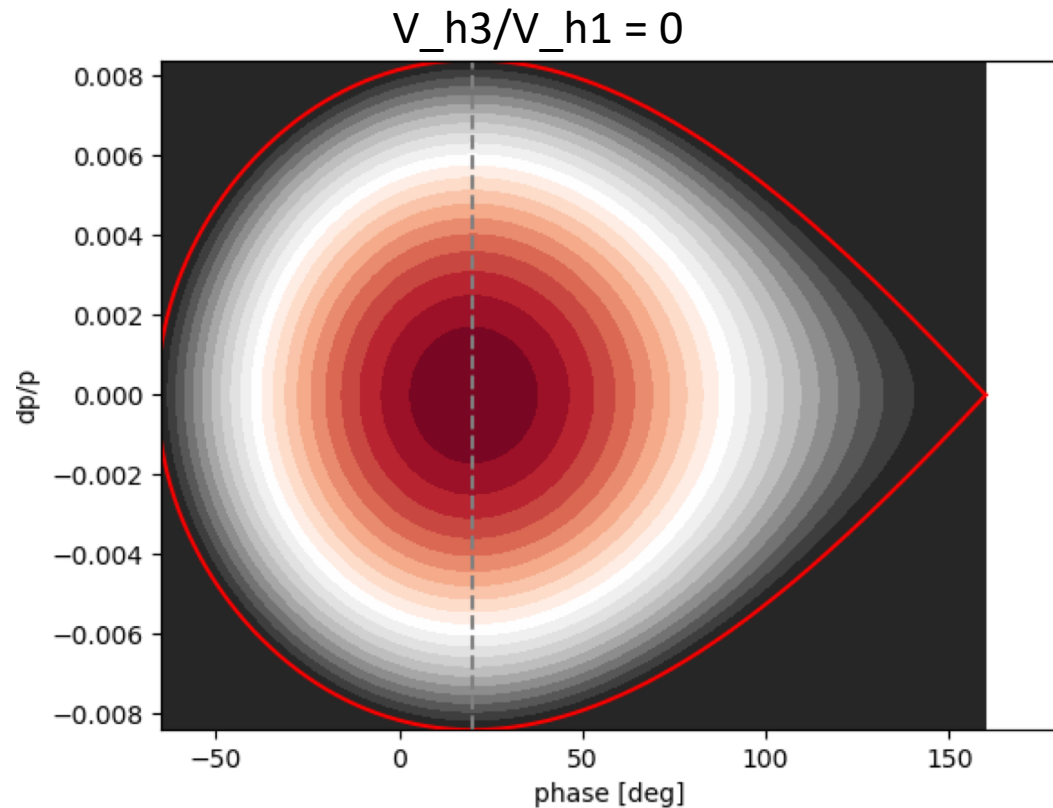


# RF waveform: harmonic composition



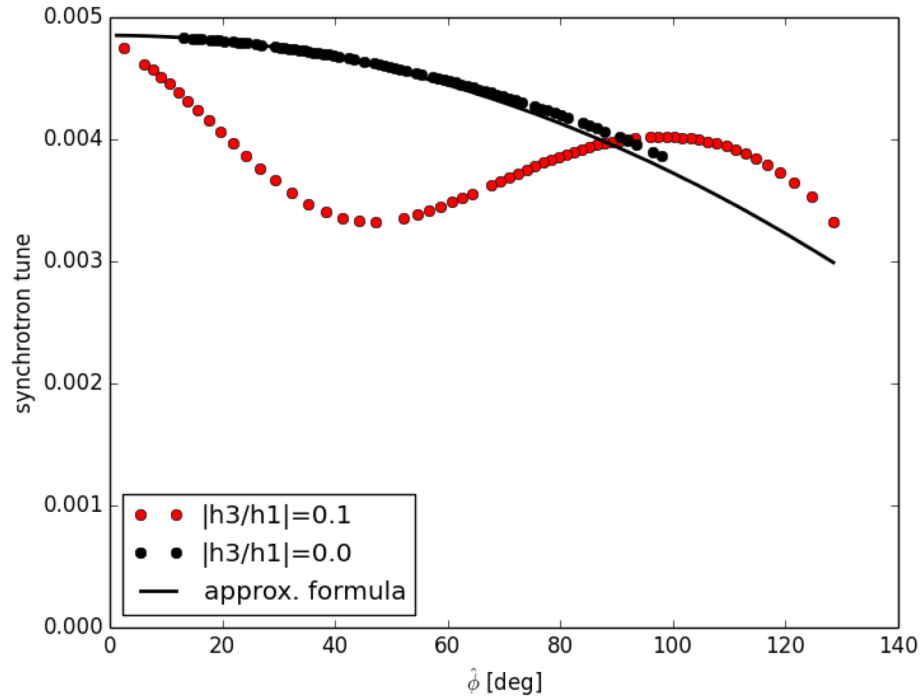
- FFT of RF waveform over 1000 turns taken during accelerating phase and flattop phase.
- There is a significant harmonic 3 component in both cases.
- During the accelerating phase  $h3/h1 \sim 0.1$ . During the flattop phase  $h3/h1 \sim 0.06$  in the cases examined.
- Will the beam see this or is it an artefact of the measurement?

# Bucket with multi-harmonic RF



Add harmonic 3 to potential term in Hamiltonian. Note: bucket area is not reduced.  
What is effect on synchronous phase and synchrotron period as a function of amplitude?

# Amplitude-dependent synchrotron tune

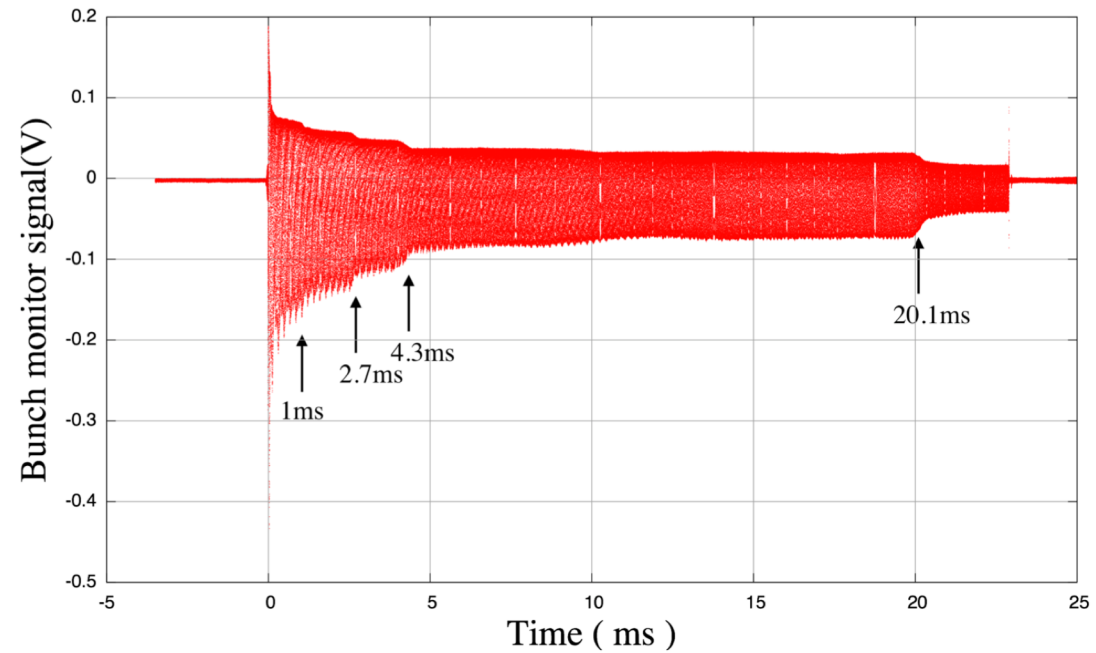
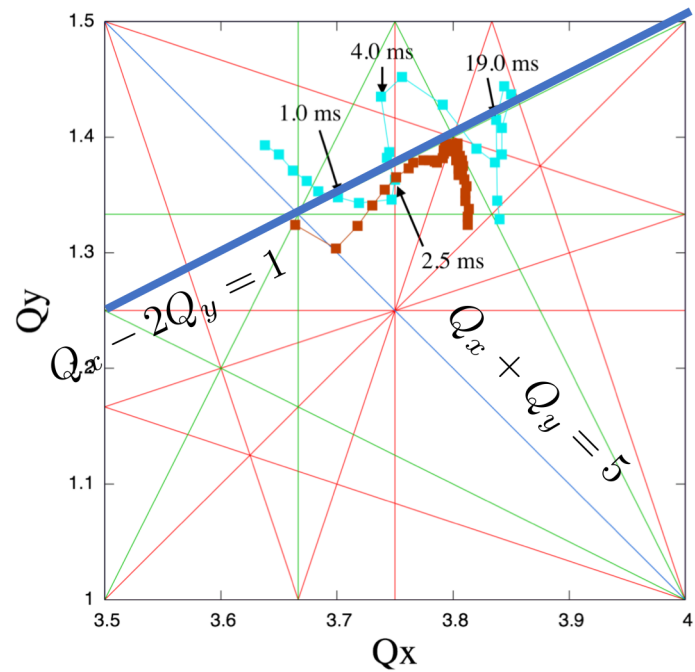


SY Lee

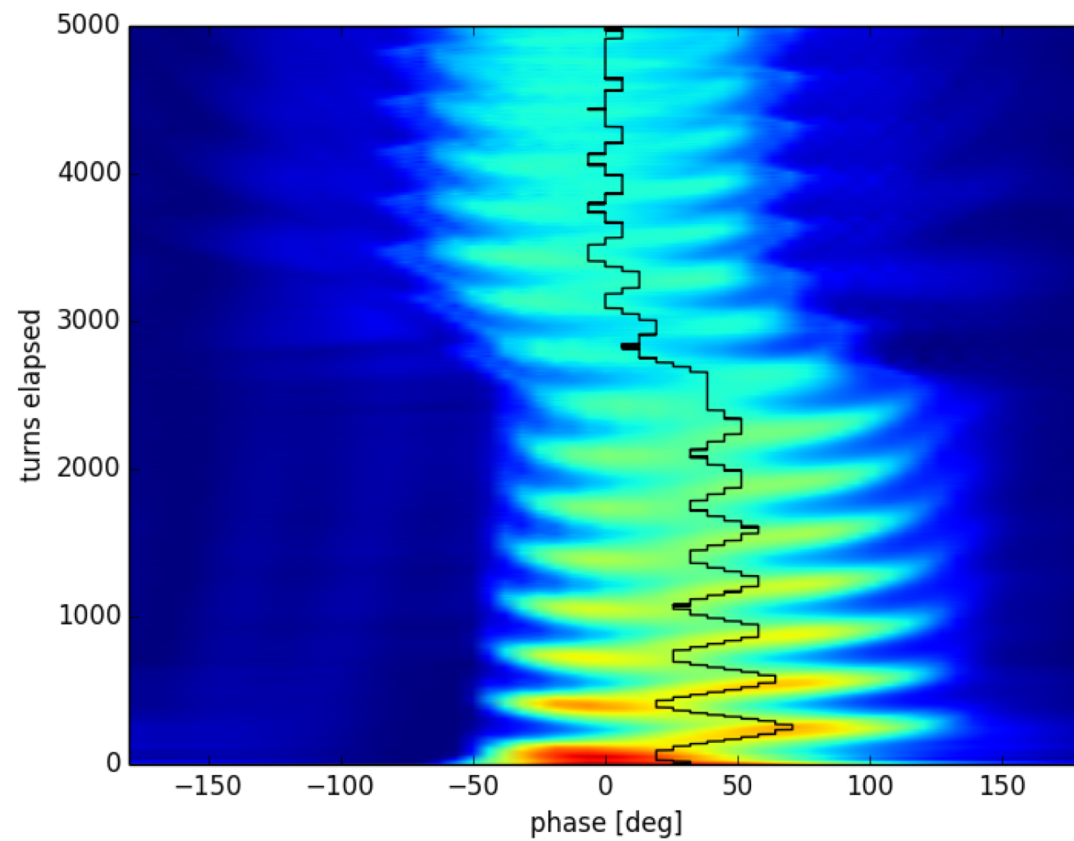
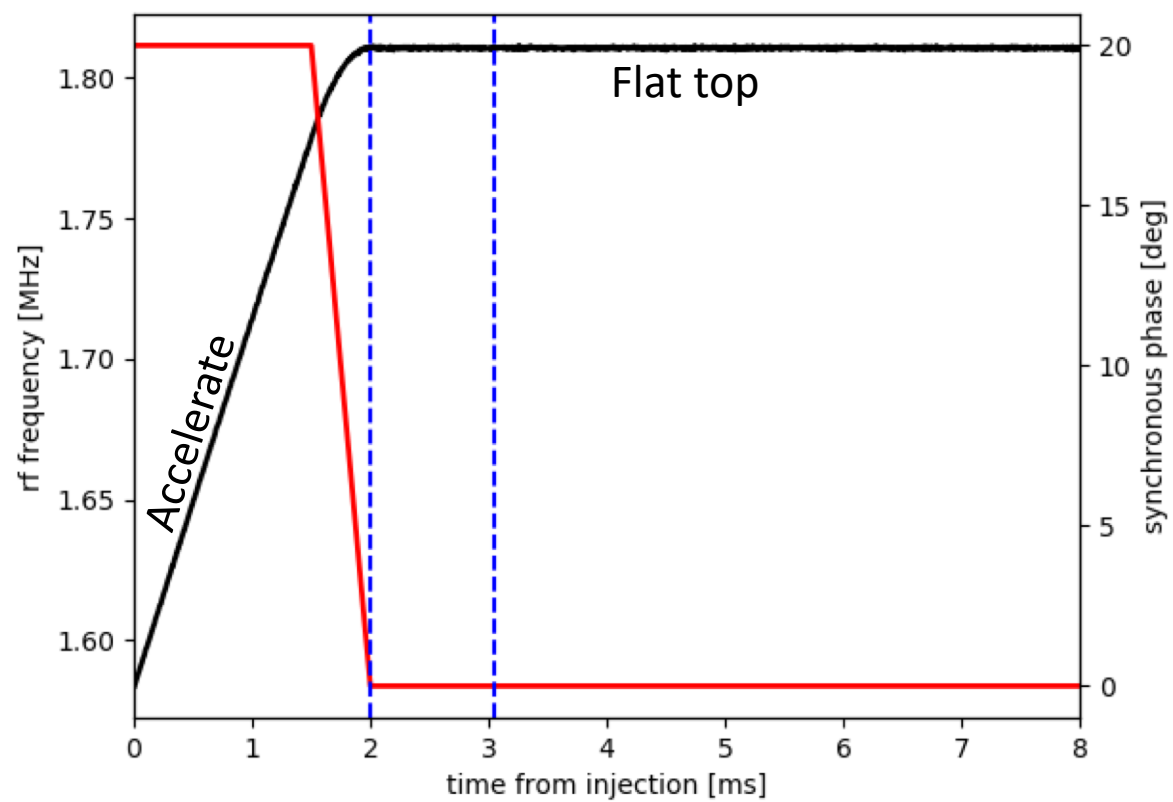
$$T = \oint \left( 2h\omega_0\eta \left[ H_0 - \frac{\omega_0 eV}{2\pi\beta^2 E} [\cos \phi - \cos \phi_s + (\phi - \phi_s) \sin \phi_s] \right] \right)^{-1/2} d\phi,$$

- Observed low synchrotron tune may be the result of multi-harmonic components.
- Proposal: Include voltage and phase of these additional harmonics as free parameters in the tomographic reconstruction.

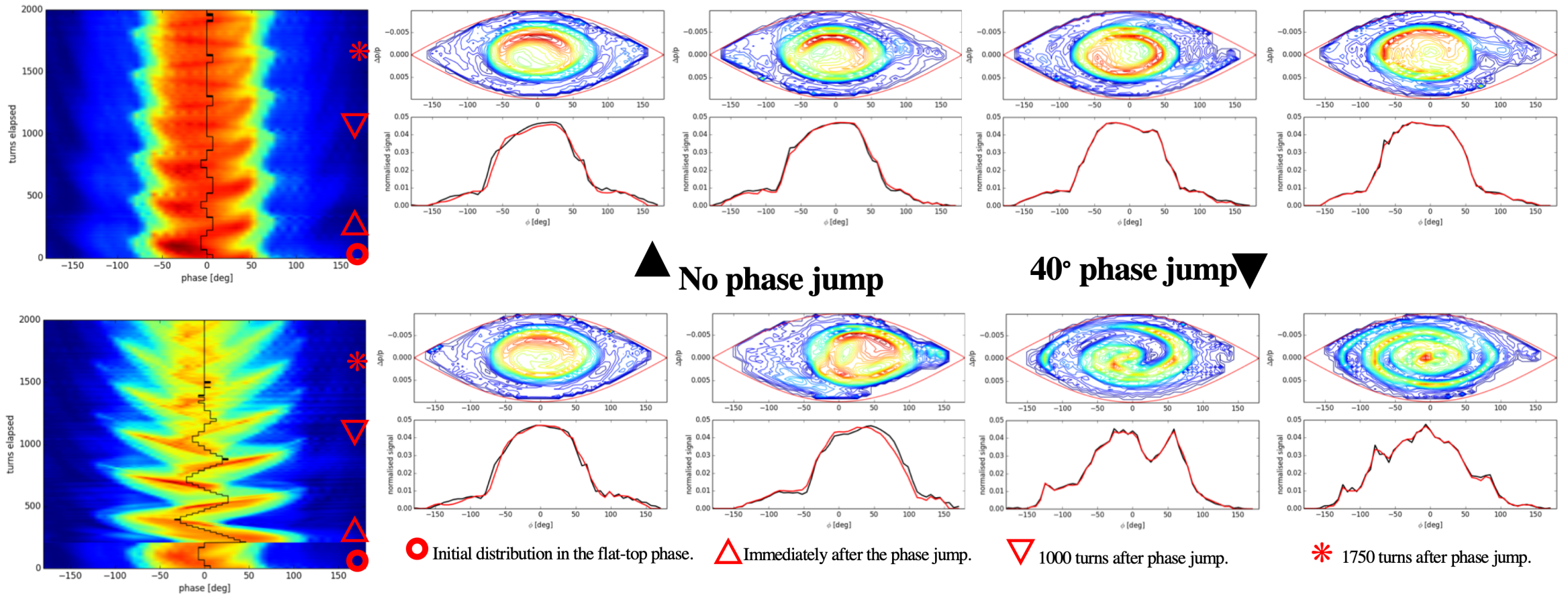
# KURNS beam survival (Y.Ishi, FFA'19)



- Can we reduce the effect of resonance crossing by reducing the longitudinal emittance?

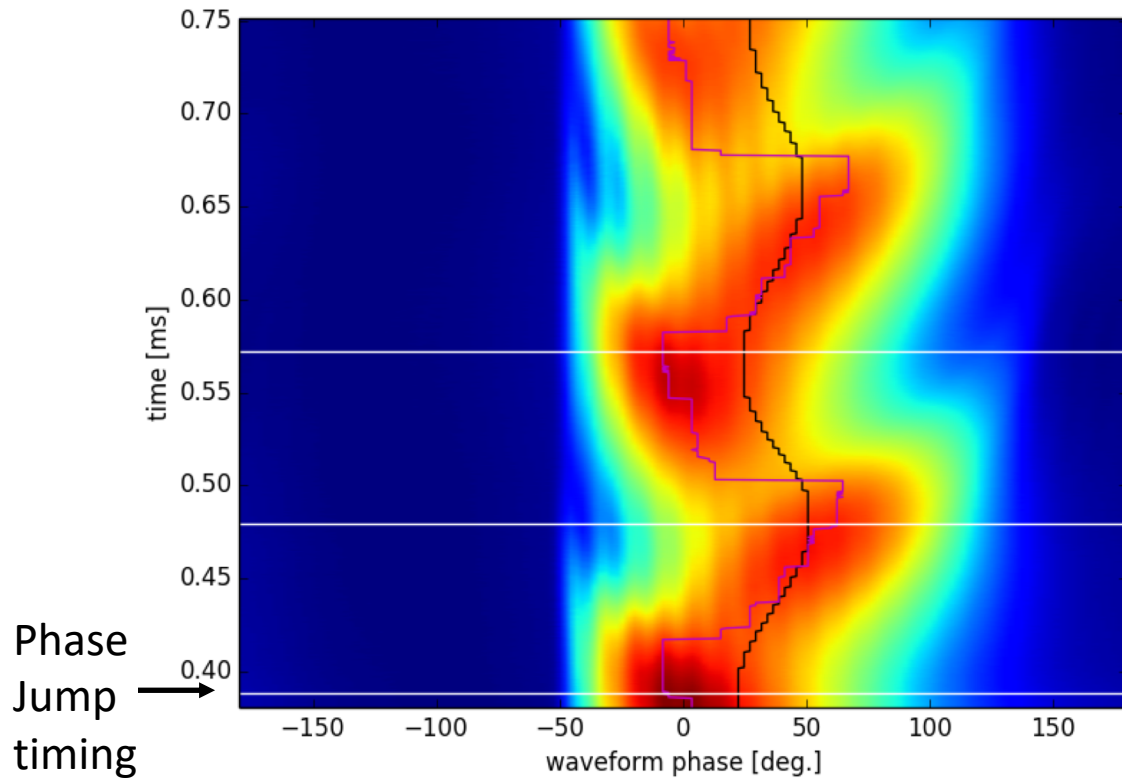


# Phase jump during flat top

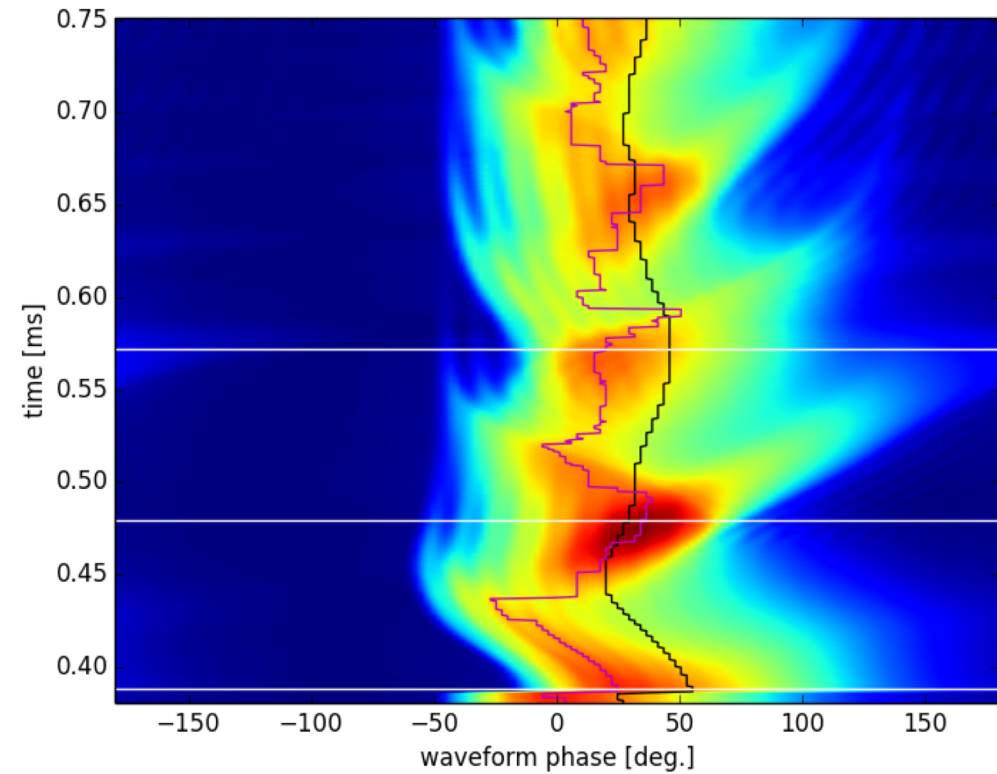




# Jump right after injection

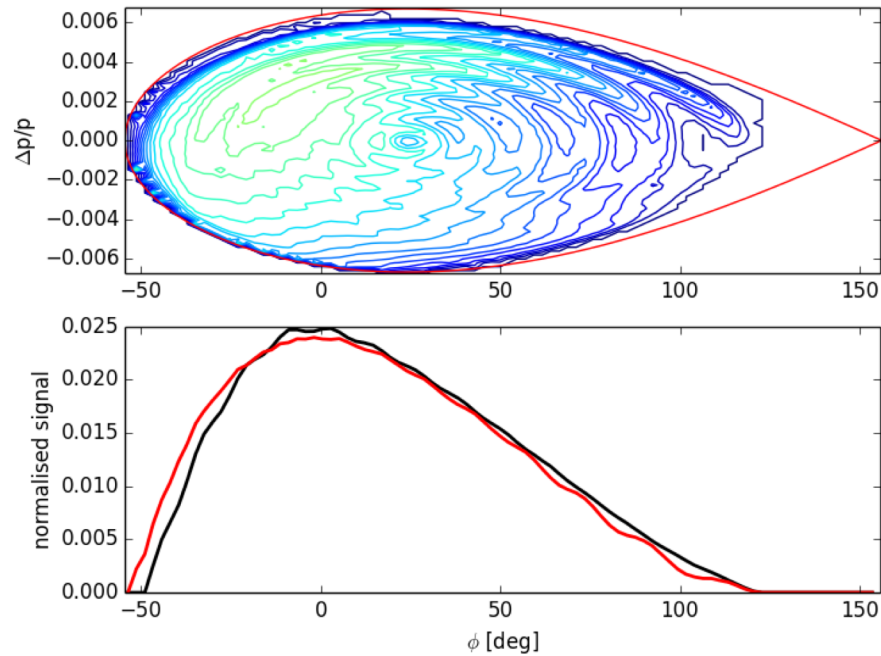


**No jump (Case 0)**

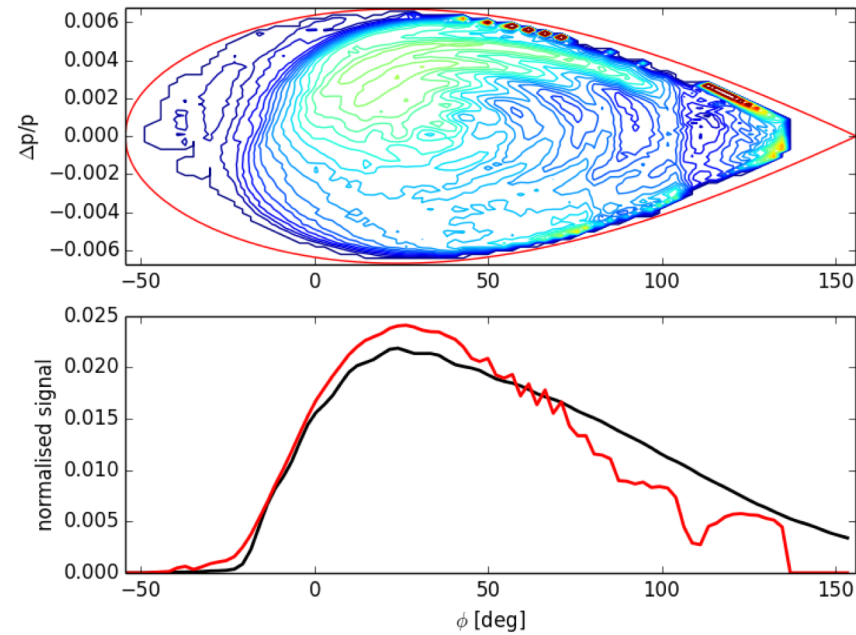


**+30 deg (Case 12)**

# Jump in phase space



**No jump**

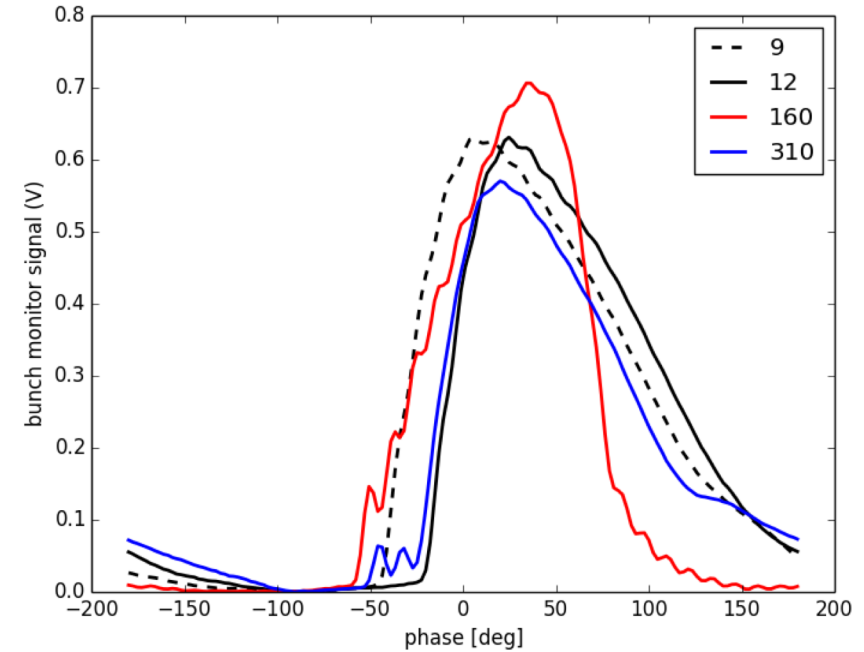
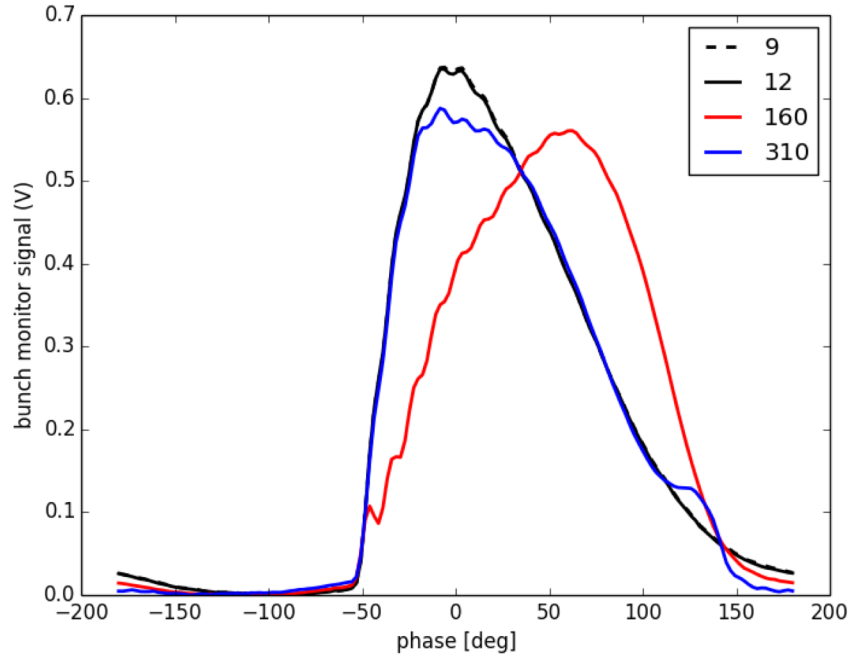


**+30 deg**

Tomography used to establish jump timing and amplitude.



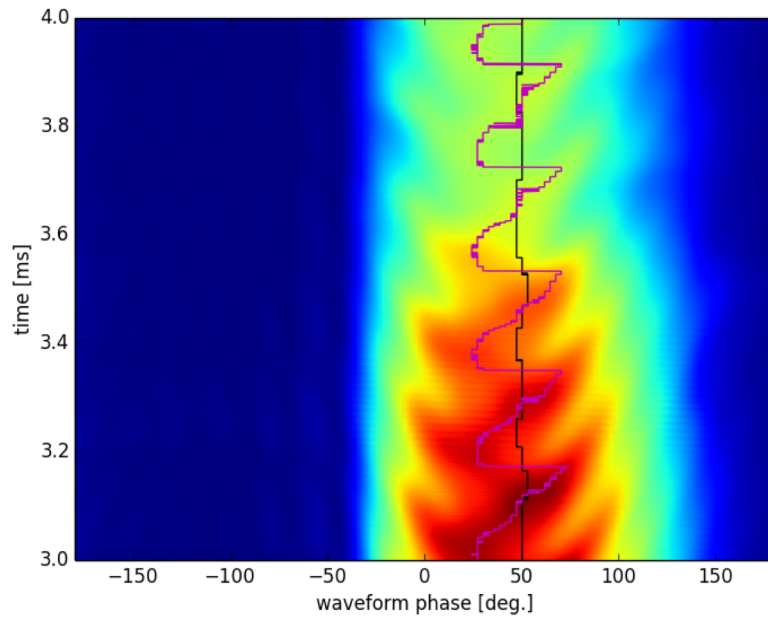
# Selected bunch profiles



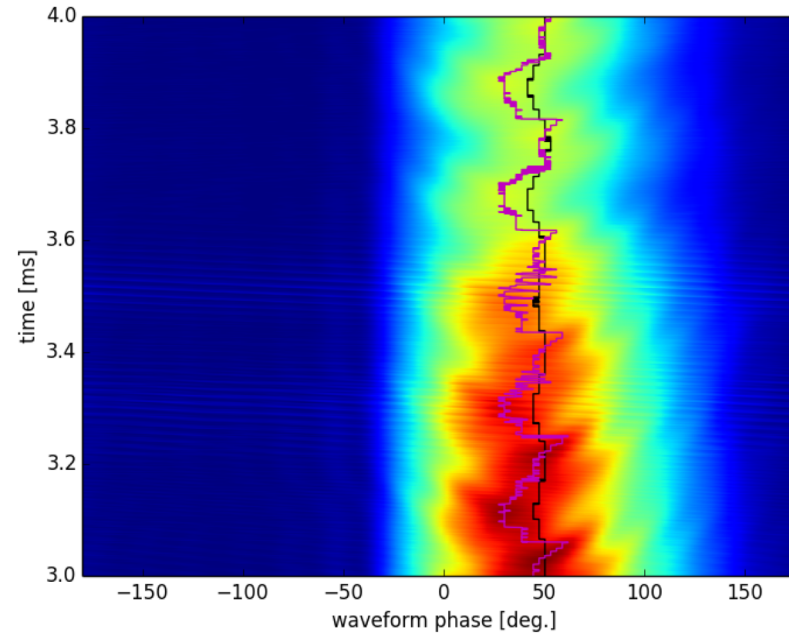
Turn	No jump	+30 deg
9	-5.9	-5.9
12	-8.2	24.7
160	62	34.1
310	-8.2	20

↓ +30.6deg

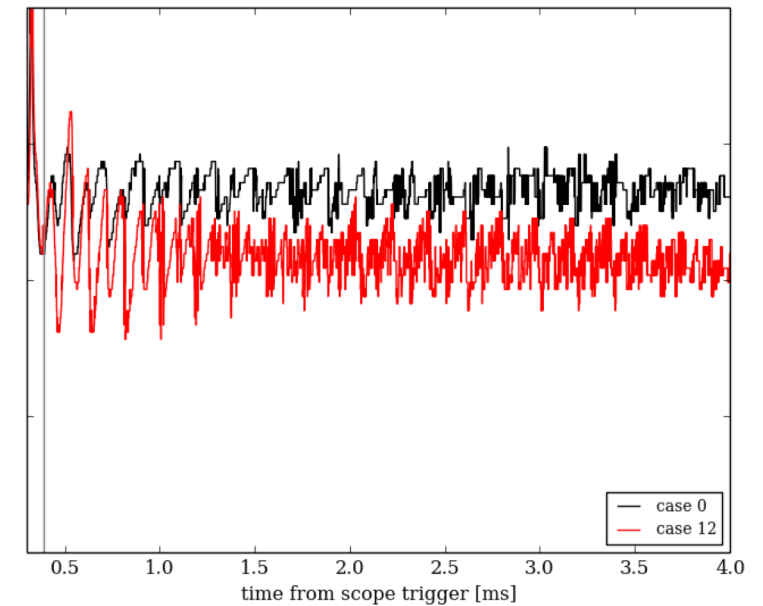
# Persistence of emittance reduction



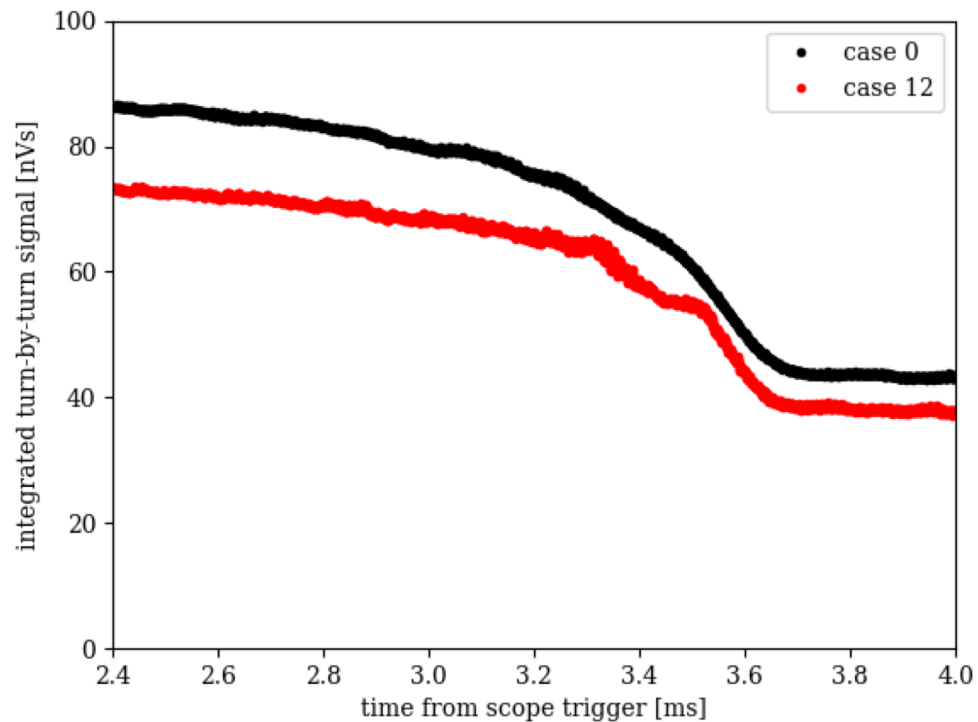
**No jump**



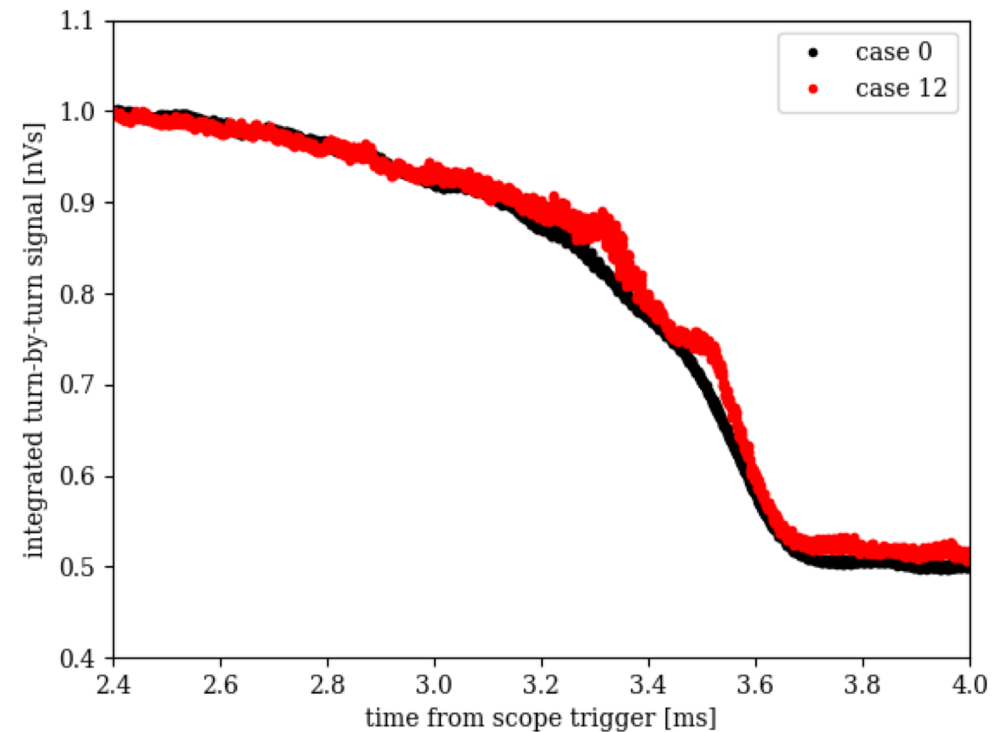
**+30 deg**



# Intensity during resonance crossing



Raw data



Normalised to value at 2.4ms

# Discussion/Future Work

- Longitudinal tomography can be applied to FFAs. Here the CERN tomography code was used with minimal modifications.
- Longitudinal parameters could be measured by including them as free parameters in the tomographic reconstruction.
- The longitudinal emittance can be reduced by applying a phase jump at the zero-crossing in the synchrotron oscillation before the bunch decoheres.