

Zurich PhD Student Seminar 2019

NNLO corrections in massive QED

Tim Engel

Paul Scherrer Institut / Universität Zürich

10TH OCTOBER 2019

Based on 1811.06461, 1909.10244

phenomenological reasons

- Low-energy precision experiments: MEG, Mu3e, MUSE, MUonE
- SM background has to be known precisely
- NNLO with $m_e \neq 0$ for $\mu \rightarrow e\nu\nu$, $\mu p \rightarrow \mu p$, $ep \rightarrow ep$, $\mu e \rightarrow \mu e$
- e.g. $\mu - e$ scattering at 10ppm: NNLO $\alpha^2 \log^2 m_e \gg 10^{-5}$

phenomenological reasons

- Low-energy precision experiments: MEG, Mu3e, MUSE, MUonE
- SM background has to be known precisely
- NNLO with $m_e \neq 0$ for $\mu \rightarrow e\nu\nu$, $\mu p \rightarrow \mu p$, $ep \rightarrow ep$, $\mu e \rightarrow \mu e$
- e.g. $\mu - e$ scattering at 10ppm: NNLO $\alpha^2 \log^2 m_e \gg 10^{-5}$

technical reasons

- simplifies real corrections
- initial state logarithms explicit

$$\sigma = \int d\Phi_n |\mathcal{A}|^2 = \int d\Phi_2 \left| \begin{array}{c} \text{---} \\ | \quad \text{---} \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ | \quad \text{---} \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ | \quad \text{---} \\ \text{---} \end{array} + \dots \right|^2$$

The diagrammatic equation shows the cross-section σ as an integral over phase space $d\Phi_n$ of the squared amplitude $|\mathcal{A}|^2$. This is equal to an integral over the two-particle phase space $d\Phi_2$ of the squared sum of diagrams. The diagrams shown are:

- A tree-level diagram with two external lines and a wavy internal line.
- A one-loop diagram with a wavy internal line and a gluon loop (represented by a curly line) on top.
- A two-loop diagram with a wavy internal line, a gluon loop on top, and a ghost loop (represented by a dashed line) on the right.

$$\int d\Phi_\gamma \text{ [diagram]} \sim \int_0^1 dE_\gamma \int_0^1 d\theta \frac{1}{E_\gamma(1 - \beta_e \cos \theta)}$$

The diagram shows a vertical wavy line (photon) between two horizontal lines (fermions). A second wavy line (photon) is emitted from the upper fermion line, forming a loop with the vertical wavy line.

$$\int d\Phi_\gamma \text{ [diagram of a fermion line with a photon emission] } \sim \int_0^1 dE_\gamma \int_0^{\pi} d\theta \frac{1}{E_\gamma(1 - \beta_e \cos \theta)}$$

Two types of IR singularities

- soft: $E_\gamma \rightarrow 0$
- collinear: $\theta \rightarrow 0$ (if $\beta_e = 1 \leftrightarrow m_e = 0$)

$$\int d\Phi_\gamma \text{ [diagram of a fermion line with a photon emission] } \sim \int_0^1 dE_\gamma \int_0^1 d\theta \frac{1}{E_\gamma(1 - \beta_e \cos \theta)}$$

Two types of IR singularities

- soft: $E_\gamma \rightarrow 0$
- collinear: $\theta \rightarrow 0$ (if $\beta_e = 1 \leftrightarrow m_e = 0$)

KLN theorem

- IR divergences from final state real emissions cancel the ones from the loops

$$\int d\Phi_\gamma \text{ [diagram of a fermion line with a wavy photon line branching off] } \sim \int_0^{E_{\text{res}}} dE_\gamma \int_0^{\theta_{\text{res}}} d\theta \frac{1}{E_\gamma(1 - \beta_e \cos \theta)}$$

Two types of IR singularities

- soft: $E_\gamma \rightarrow 0$
- collinear: $\theta \rightarrow 0$ (if $\beta_e = 1 \leftrightarrow m_e = 0$)

KLN theorem

- IR divergences from final state real emissions cancel the ones from the loops

$$\begin{aligned}
 \sigma &= \int d\Phi_2 \left| \begin{array}{c} \text{tree} \\ \text{tree} \end{array} + \begin{array}{c} \text{tree} \\ \text{loop} \end{array} + \begin{array}{c} \text{tree} \\ \text{loop} \\ \text{loop} \end{array} + \dots \right|^2 \\
 &+ \int d\Phi_3 \left| \begin{array}{c} \text{loop} \\ \text{tree} \end{array} + \begin{array}{c} \text{loop} \\ \text{loop} \end{array} + \dots \right|^2 \\
 &+ \int d\Phi_4 \left| \begin{array}{c} \text{loop} \\ \text{loop} \\ \text{loop} \end{array} + \dots \right|^2 \\
 &+ \dots
 \end{aligned}$$

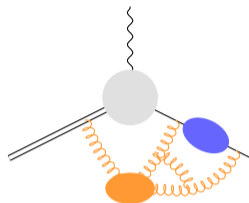
$$\begin{aligned}
 \sigma &= \int d\Phi_2 \left| \begin{array}{c} \text{tree} \\ + \text{1-loop} \\ + \text{2-loop} \\ + \dots \end{array} \right|^2 \\
 &+ \int d\Phi_3 \left| \begin{array}{c} \text{1-loop} \\ + \text{2-loop} \\ + \dots \end{array} \right|^2 \\
 &+ \int d\Phi_4 \left| \begin{array}{c} \text{2-loop} \\ + \dots \end{array} \right|^2 \\
 &+ \dots
 \end{aligned}$$

conflict

- phase space harder if massless
 - loop integrals harder if massive
- ⇒ 'massification' & FKS²

simple process ($\mu \rightarrow e\nu\nu$)

- $\mathcal{A}_\mu(m) = \mathcal{S} \times \mathcal{Z} \times \mathcal{A}_\mu(0) + \mathcal{O}(m \log m)$
- $\mathcal{Z} \supset \log(m)$:
process indep. jet fct.
- $\mathcal{S} \supset \log(m)$:
process dep. soft fct. (easy)



[Becher, Melnikov 07; TE, Gnendiger, Signer, Ulrich 18]

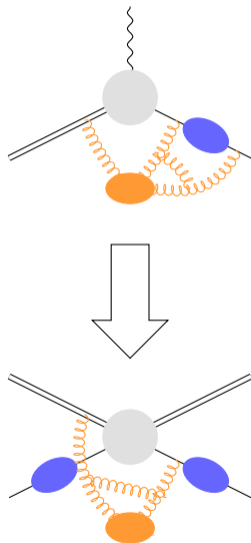
simple process ($\mu \rightarrow e\nu\nu$)

- $\mathcal{A}_\mu(m) = \mathcal{S} \times Z \times \mathcal{A}_\mu(0) + \mathcal{O}(m \log m)$
- $Z \supset \log(m)$:
process indep. jet fct.
- $\mathcal{S} \supset \log(m)$:
process dep. soft fct. (easy)

[Becher, Melnikov 07; TE, Gneidiger, Signer, Ulrich 18]

complex process ($\mu e \rightarrow \mu e$)

- $\mathcal{A}_{\mu e}(m) = \mathcal{S}' \times Z \times Z \times \mathcal{A}_{\mu e}(0) + \mathcal{O}(m \log m)$



idea

$$\underbrace{\int d\Phi_\gamma (\mathcal{M}(E_\gamma) - \mathcal{M}_{\text{CT}})}_{\text{complicated \& finite} \rightarrow \text{numerically}}
 \quad + \quad
 \underbrace{\int d\Phi_\gamma \mathcal{M}_{\text{CT}}}_{\text{divergent \& easy} \rightarrow \text{analytically}}$$

idea

$$\underbrace{\int d\Phi_\gamma (\mathcal{M}(E_\gamma) - \mathcal{M}_{CT})}_{\text{complicated \& finite} \rightarrow \text{numerically}} + \underbrace{\int d\Phi_\gamma \mathcal{M}_{CT}}_{\text{divergent \& easy} \rightarrow \text{analytically}}$$

FKS (NLO)

$$\begin{aligned}
 \sigma^{(1)} &= \sigma_n^{(1)}(\xi_c) + \sigma_{n+1}^{(1)}(\xi_c) \\
 \sigma_n^{(1)}(\xi_c) &= \int d\Phi_n^{d=4} \left(\mathcal{M}_n^{(1)} + \hat{\mathcal{E}}(\xi_c) \mathcal{M}_n^{(0)} \right) \\
 \sigma_{n+1}^{(1)}(\xi_c) &= \int d\Phi_{n+1}^{d=4} \left(\frac{1}{\xi_1} \right)_c (\xi_1 \mathcal{M}_{n+1}^{(0)f})
 \end{aligned}$$

FKS² (NNLO)

$$\sigma^{(2)} = \sigma_n^{(2)}(\xi_c) + \sigma_{n+1}^{(2)}(\xi_c) + \sigma_{n+2}^{(2)}(\xi_c)$$

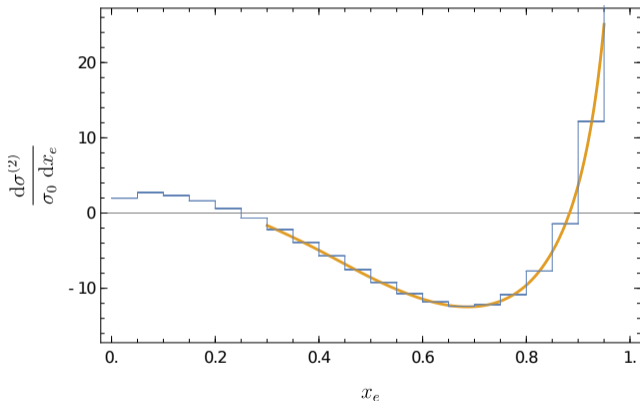
$$\sigma_n^{(2)}(\xi_c) = \int d\Phi_n^{d=4} \left(\mathcal{M}_n^{(2)} + \hat{\mathcal{E}}(\xi_c) \mathcal{M}_n^{(1)} + \frac{1}{2!} \mathcal{M}_n^{(0)} \hat{\mathcal{E}}(\xi_c)^2 \right)$$

$$\sigma_{n+1}^{(2)}(\xi_c) = \int d\Phi_{n+1}^{d=4} \left(\frac{1}{\xi} \right)_c \left(\xi \mathcal{M}_{n+1}^{(1)f}(\xi_c) \right)$$

$$\sigma_{n+2}^{(2)}(\xi_c) = \int d\Phi_{n+2}^{d=4} \left(\frac{1}{\xi_1} \right)_c \left(\frac{1}{\xi_2} \right)_c \left(\xi_1 \xi_2 \mathcal{M}_{n+2}^{(0)f} \right)$$

[TE, Signer, Ulrich 19]

Electron energy spectrum ($x_e \sim E_e$):



[Arbuzov, Czarnecki, Gaponenko; Arbuzov, Melnikov 02, Anastasiou, Melnikov, Petriello 05]

conclusion

- NNLO corrections in massive QED phenomenologically relevant
- 'massification' & FKS² applied to muon decay (proof of concept)
- work in progress: $\mu e \rightarrow \mu e$, $\mu p \rightarrow \mu p$ and $ep \rightarrow ep$

questions?