Zurich PhD Student Seminar 2019

NNLO corrections in massive QED

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Based on 1811.06461, 1909.10244

why NNLO & massive?

phenomenological reasons

- Low-energy precision experiments: MEG, Mu3e, MUSE, MUonE
- SM background has to be known precisely
- NNLO with $m_e \neq 0$ for $\mu \to e\nu\nu$, $\mu p \to \mu p$, $ep \to ep$, $\mu e \to \mu e$
- e.g. $\mu - e$ scattering at 10ppm: NNLO $\alpha^2 \log^2 m_e \gg 10^{-5}$

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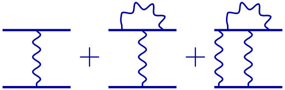
- e.g. $\mu - e$ scattering at 10ppm: NNLO $\alpha^2 \log^2 m_e \gg 10^{-5}$

technical reasons

- simplifies real corrections
- initial state logarithms explicit

NNLO basics

$$\sigma = \int \mathrm{d}\Phi_n |\mathcal{A}|^2 = \int \mathrm{d}\Phi_2 \Bigg|$$



$$+ \ldots \Bigg|^2$$

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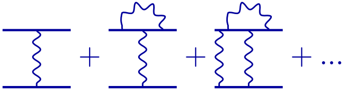
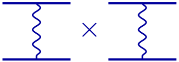
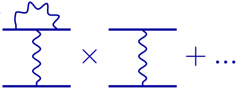


LO: $\int \mathrm{d}\Phi_2 \big(\ldots \big) \sim \alpha^2$



NLO: $\int \mathrm{d}\Phi_2 \big(\ldots + \ldots \big) \sim \alpha^3$



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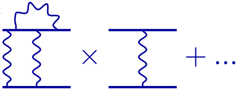

NNLO basics

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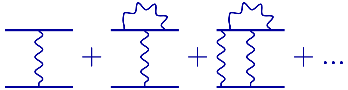
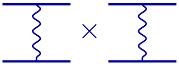


LO: $\int \mathrm{d}\Phi_2 \left(\quad \times \quad \right) \sim \alpha^2$



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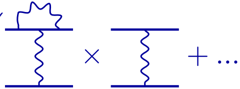

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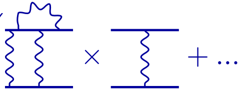
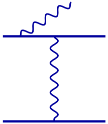


Divergences in loops

- UV divergences
  $\rightarrow$ renormalization
- IR divergences?

NNLO basics: IR divergences

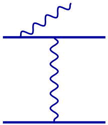
$$\int \mathrm{d}\Phi_\gamma$$



$$\sim \quad \int_0 \mathrm{d}E_\gamma \int_0 \mathrm{d}\theta \, \frac{1}{E_\gamma(1-\beta_e\cos\theta)}$$

NNLO basics: IR divergences

$$\int \mathrm{d}\Phi_\gamma$$



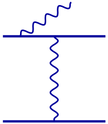
$$\sim \quad \int_0 \mathrm{d}E_\gamma \int_0 \mathrm{d}\theta \, \frac{1}{E_\gamma(1-\beta_e\cos\theta)}$$

Two types of IR singularities

- soft: $E_\gamma \to 0$
- collinear: $\theta \to 0$ (if $\beta_e = 1 \leftrightarrow m_e = 0$)

NNLO basics: IR divergences

$$\int \mathrm{d}\Phi_\gamma$$



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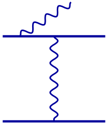
KLN theorem

- IR divergences from final state real emissions cancel the ones from the loops

NNLO basics: IR divergences

$\int \mathrm{d}\Phi_\gamma$



$$\sim \int_0^{E_{\text{res}}} \mathrm{d}E_\gamma \int_0^{\theta_{\text{res}}} \mathrm{d}\theta \, \frac{1}{E_\gamma(1-\beta_e\cos\theta)}$$

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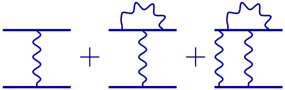
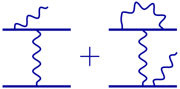
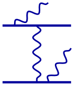


$$+ \ldots \Bigg|^2$$

$$+ \int \mathrm{d}\Phi_3 \Bigg|$$


$$+ \ldots \Bigg|^2$$

$$+ \int \mathrm{d}\Phi_4 \Bigg|$$


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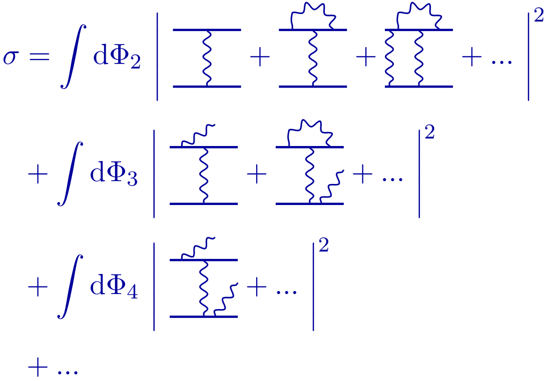
NNLO basics



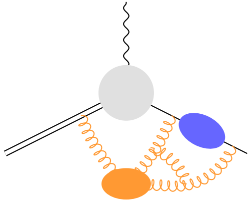
conflict

- phase space harder if massless
- loop integrals harder if massive

$\Rightarrow$ 'massification' & FKS^2

‘massification’

simple process ($\mu \to e\nu\nu$)



- $\mathcal{A}_\mu(m) = \mathcal{S} \times Z \times \mathcal{A}_\mu(0) + \mathcal{O}(m \log m)$
- $Z \supset \log(m)$:
  process indep. jet fct.
- $\mathcal{S} \supset \log(m)$:
  process dep. soft fct. (easy)

[Becher, Melnikov 07; TE, Gnendiger, Signer, Ulrich 18]

'massification'

simple process ($\mu \to e\nu\nu$)

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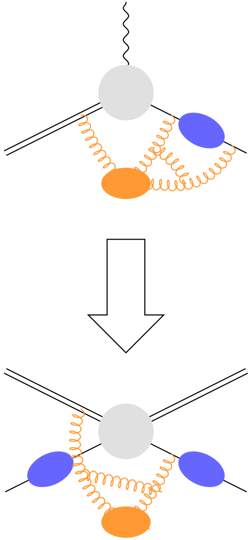

complex process ($\mu e \to \mu e$)

- $\mathcal{A}_{\mu e}(m) = \mathcal{S}' \times Z \times Z \times \mathcal{A}_{\mu e}(0) + \mathcal{O}(m \log m)$

FKS^2 subtraction scheme

idea

$$\underbrace{\int \mathrm{d}\Phi_\gamma \Big(\mathcal{M}(E_\gamma) - \mathcal{M}_{\text{CT}}\Big)}_{\text{complicated \& finite} \,\to\, \text{numerically}} \qquad + \qquad \underbrace{\int \mathrm{d}\Phi_\gamma \mathcal{M}_{\text{CT}}}_{\text{divergent \& easy} \,\to\, \text{analytically}}$$

FKS^2 subtraction scheme

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FKS (NLO)

$$\sigma^{(1)} = \sigma^{(1)}_n(\xi_c) + \sigma^{(1)}_{n+1}(\xi_c)$$

$$\sigma^{(1)}_n(\xi_c) = \int \mathrm{d}\Phi^{d=4}_n \left(\mathcal{M}^{(1)}_n + \hat{\mathcal{E}}(\xi_c)\,\mathcal{M}^{(0)}_n\right)$$

$$\sigma^{(1)}_{n+1}(\xi_c) = \int \mathrm{d}\Phi^{d=4}_{n+1} \left(\frac{1}{\xi_1}\right)_c \left(\xi_1\,\mathcal{M}^{(0)f}_{n+1}\right)$$

[Frixione, Kunszt, Signer 95]

FKS^2 (NNLO)

$$\sigma^{(2)} = \sigma_n^{(2)}(\xi_c) + \sigma_{n+1}^{(2)}(\xi_c) + \sigma_{n+2}^{(2)}(\xi_c)$$

$$\sigma_n^{(2)}(\xi_c) = \int \mathrm{d}\Phi_n^{d=4} \left(\mathcal{M}_n^{(2)} + \hat{\mathcal{E}}(\xi_c)\,\mathcal{M}_n^{(1)} + \frac{1}{2!}\mathcal{M}_n^{(0)}\hat{\mathcal{E}}(\xi_c)^2 \right)$$

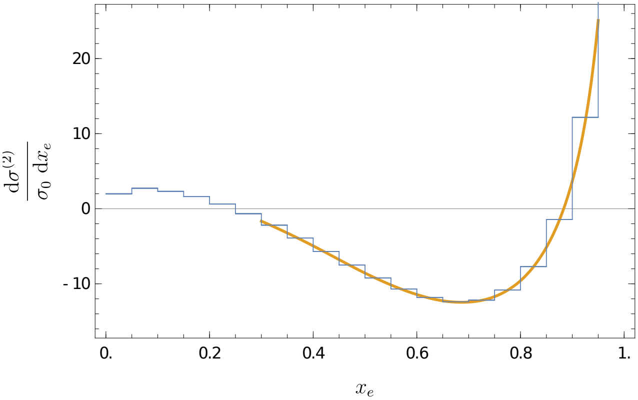
$$\sigma_{n+1}^{(2)}(\xi_c) = \int \mathrm{d}\Phi_{n+1}^{d=4} \left(\frac{1}{\xi}\right)_c \left(\xi\,\mathcal{M}_{n+1}^{(1)f}(\xi_c)\right)$$

$$\sigma_{n+2}^{(2)}(\xi_c) = \int \mathrm{d}\Phi_{n+2}^{d=4} \left(\frac{1}{\xi_1}\right)_c \left(\frac{1}{\xi_2}\right)_c \left(\xi_1\xi_2\,\mathcal{M}_{n+2}^{(0)f}\right)$$

[TE, Signer, Ulrich 19]

results: muon decay

Electron energy spectrum ($x_e \sim E_e$):



[Arbuzov, Czarnecki, Gaponenko; Arbuzov, Melnikov 02, Anastasiou, Melnikov, Petriello 05]

conclusion

- NNLO corrections in massive QED phenomenologically relevant
- ‘massification’ & FKS^2 applied to muon decay (proof of concept)
- work in progress: $\mu e \to \mu e$, $\mu p \to \mu p$ and $ep \to ep$

questions?