Effective field theories and low-energy probes of new physics

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Lecture 1 (Thu, 08:30 – 10:00): EFT basics

- motivation
- EFT methods
- examples: Fermi theory / LEFT, SMEFT, χPT, HEFT

Lecture 2 (Fri, 16:30 – 18:00): applications

- low-energy searches and hadronic effects
- CP violation and the neutron EDM
- lepton-flavor violation

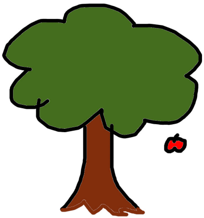
Goals of the lecture

- understand **basic concepts** of EFTs
- get familiar with a few **important examples**
- understand use of EFTs in the **search for new physics**
- learn about **interplay of different EFTs** in phenomenological examples
- learn about **theory challenges** in low-energy searches

Lecture 1: EFT basics

Lecture 1: EFT basics

Effective theories:



- speed of apple hitting ground:

$$mgh = \frac{mv^2}{2} \quad \Rightarrow \quad v = \sqrt{2gh}$$

- but: gravitational potential not linear:

$$\phi(r) = -\frac{GM}{r}$$
$$v = \sqrt{2gh}\left[1 - 2\frac{h}{R} + \mathcal{O}\left(\frac{h^2}{R^2}\right)\right]$$

- $h/R \approx 10^{-6}$ for a typical apple tree
- Newtonian potential only first approximation to GR

Effective theories:

- use **hierarchy of scales**
- expand in **small** (dimensionless) **parameter**
- **improve** results **systematically** to desired accuracy
- **limited range of validity**, but efficient description of the physics of interest
- roughly speaking a Taylor expansion, but there is a little more to it...

Effective **field** theories:

- apply concept to QFTs
- EFT: generic QFT **without** restriction of **renormalizability**

Why is it useful? Why should we care?

- calculations become **easier**
- calculations become **more accurate**

- **every theory** is an effective theory
- parametrize ignorance systematically and generically (model independence): typically small number of assumptions needed

Application to particle physics

- **Standard Model** (SM) works beautifully!
  - QED sector: tested at the 10^{-9} level
  - Higgs sector: tested at the $\mathcal{O}(10\%)$ level
- LHC: no direct evidence of new particles
- a few **anomalies**: muon $g-2$, B physics, . . .
- SM leaves **open questions**: dark matter, baryon asymmetry, neutrino masses, . . .
- SM is a **first approximation** in a systematic EFT for new physics

A little bit of slang

- IR: infrared, low energies, long distances, “soft”
- UV: ultraviolet, high energies, short distances, “hard”

- top-down perspective: underlying (more fundamental) UV theory is known, can derive EFT directly from it
- bottom-up perspective: construct EFT without explicitly referring to the underlying theory (too complicated or unknown)

Lecture 1: EFT basics

Lecture 1: EFT basics

A scalar toy example

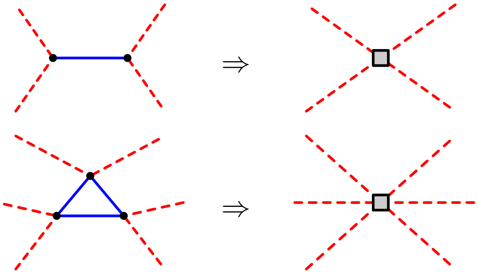
- one light particle, one heavy particle

$$\mathcal{L}_{\mathrm{UV}} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{m^2}{2}\phi^2 - \frac{M^2}{2}\Phi^2$$
$$- \frac{\lambda_0}{4!}\phi^4 - \frac{\lambda_1}{2}M\phi^2\Phi - \frac{\lambda_2}{4}\phi^2\Phi^2 - \frac{\lambda_3}{3!}M\Phi^3 - \frac{\lambda_4}{4!}\Phi^4$$
$$+ \mathcal{L}_{\mathrm{ct}} \,.$$

- assume $m \ll M$

At low energies

- consider $2 \to 2$ or $3 \to 3$ scattering of light particles:

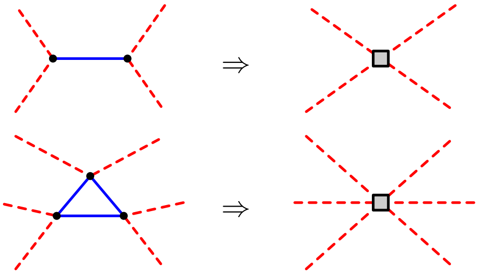


- at tree level: expand propagator for $(p_1 + p_2)^2 = s \ll M^2$:

$$\frac{i}{s - M^2} = \frac{-i}{M^2}\left(1 + \frac{s}{M^2} + \mathcal{O}\left(\frac{s^2}{M^4}\right)\right)$$

At low energies

- consider $2 \to 2$ or $3 \to 3$ scattering of light particles:



- low-energy effect of UV physics can be described in terms of (additional) **local interactions**: ϕ^4, ϕ^6, $\ldots$

Lecture 1: EFT basics

Three EFT core principles

1. determine the relevant **degrees of freedom**
   $\Rightarrow$ building blocks of Lagrangian
2. determine the **symmetries** of the problem
   $\Rightarrow$ constrain interactions
3. establish a **power counting**
   $\Rightarrow$ turn approximation into systematic expansion: what is a small parameter? requirement to obtain finite complexity

Three EFT core principles: toy example

$$\mathcal{L}_{\mathrm{UV}} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{m^2}{2}\phi^2 - \frac{M^2}{2}\Phi^2 - \frac{\lambda_0}{4!}\phi^4 - \frac{\lambda_1}{2}M\phi^2\Phi - \frac{\lambda_2}{4}\phi^2\Phi^2 - \frac{\lambda_3}{3!}M\Phi^3 - \frac{\lambda_4}{4!}\Phi^4 + \mathcal{L}_{\mathrm{ct}}$$

we consider scattering processes with $\frac{p}{M} \ll 1$:

1. **degrees of freedom**:
   only light particle ϕ
   (heavy particle Φ never produced as real particle)
2. **symmetries**:
   $\phi \mapsto -\phi$: only even powers of ϕ in Lagrangian
3. **power counting**

$$\delta = \frac{p}{\Lambda} \ll 1, \quad \Lambda: \text{ UV scale, } \Lambda \sim M$$

Lecture 1: EFT basics

Constructing an EFT

constructing an EFT starts by writing down the Lagrangian

- effects of heavy physics encoded in additional **local interactions**
- figure out the form of the Lagrangian: write down **all possible** interaction terms
- make sure all operators respect the symmetries
- every operator comes with an (a priori) arbitrary coefficient, which is a free parameter of the EFT
  $\Rightarrow$ **Wilson coefficients**
- power counting assigns an order to each operator $\Rightarrow$ only a **finite number of terms** at each order

Removing redundancies in the operator basis

- momentum conservation: **total derivatives** (usually) not of interest (there are notable exceptions, though)

$$\mathcal{O}_{\text{tot.der.}} = \partial_\mu (\mathcal{O}')^\mu$$

$\Rightarrow$ vertex rule:  $\propto (p_1 + p_2 + \ldots + p_n)_\mu = 0$

- operators containing the **classical equations of motion** (EOM) can be removed by a **field redefinition**

$$E[\phi] := \frac{\delta S}{\delta \phi}, \quad \phi = \phi' + \epsilon F[\phi']$$

$$\Rightarrow \quad \mathcal{L}[\phi] = \mathcal{L}[\phi'] + \epsilon F[\phi'] \frac{\delta S[\phi']}{\delta \phi'} + \mathcal{O}(\epsilon^2)$$

Removing redundancies in the operator basis

- momentum conservation: **total derivatives** (usually) not of interest (there are notable exceptions, though)

$$\mathcal{O}_{\text{tot.der.}} = \partial_\mu (\mathcal{O}')^\mu$$

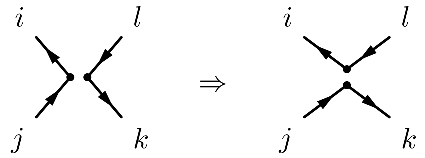
$\Rightarrow$ vertex rule:  $\propto (p_1 + p_2 + \ldots + p_n)_\mu = 0$

- operators containing the **classical equations of motion** (EOM) can be removed by a **field redefinition**
- field redefinitions (even non-linear ones) do not change the S-matrix

Removing redundancies in the operator basis

- further redundancies, e.g., Fierz relations for Dirac matrices Γ in $D = 4$:

$$\Gamma^a_{ij}\Gamma^b_{kl} = \sum_{c,d} \frac{1}{16} \mathrm{Tr}[\Gamma^a \Gamma_c \Gamma^b \Gamma_d] \, \Gamma^d_{il} \Gamma^c_{kj}$$



Constructing an EFT: toy example

$$\mathcal{L}_{\mathrm{UV}} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{m^2}{2}\phi^2 - \frac{M^2}{2}\Phi^2 - \frac{\lambda_0}{4!}\phi^4 - \frac{\lambda_1}{2}M\phi^2\Phi - \frac{\lambda_2}{4}\phi^2\Phi^2 - \frac{\lambda_3}{3!}M\Phi^3 - \frac{\lambda_4}{4!}\Phi^4 + \mathcal{L}_{\mathrm{ct}}$$

- interaction terms: a priori **all possible monomials** made of ϕ and its derivatives

$$\mathcal{L}_{\mathrm{EFT}} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m_\phi^2}{2}\phi^2 - \frac{\lambda^4}{4!}\phi^4 + \cdots$$

- $\phi \mapsto -\phi$ symmetry: only **even number of fields** ϕ
- Lorentz invariance: only **even number of derivatives**

Constructing an EFT: toy example

$$\mathcal{L}_{\mathrm{UV}} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{m^2}{2}\phi^2 - \frac{M^2}{2}\Phi^2$$
$$- \frac{\lambda_0}{4!}\phi^4 - \frac{\lambda_1}{2}M\phi^2\Phi - \frac{\lambda_2}{4}\phi^2\Phi^2 - \frac{\lambda_3}{3!}M\Phi^3 - \frac{\lambda_4}{4!}\Phi^4 + \mathcal{L}_{\mathrm{ct}}$$

- interaction terms: a priori **all possible monomials** made of ϕ and its derivatives

$$\mathcal{L}_{\mathrm{EFT}} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m_\phi^2}{2}\phi^2 - \frac{\lambda^4}{4!}\phi^4 + \sum_i \frac{C_i^{(d)}}{\Lambda^{d-4}}\mathcal{O}_i^{(d)}$$

- $\phi \mapsto -\phi$ symmetry: only **even number of fields ϕ**
- Lorentz invariance: only **even number of derivatives**

Constructing an EFT: toy example

$$\mathcal{L}_{\mathrm{UV}} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{m^2}{2}\phi^2 - \frac{M^2}{2}\Phi^2 - \frac{\lambda_0}{4!}\phi^4 - \frac{\lambda_1}{2}M\phi^2\Phi - \frac{\lambda_2}{4}\phi^2\Phi^2 - \frac{\lambda_3}{3!}M\Phi^3 - \frac{\lambda_4}{4!}\Phi^4 + \mathcal{L}_{\mathrm{ct}}$$

- redundancies: remove **total derivatives**, e.g.

$$\partial_\mu(\phi^3\partial^\mu\phi)\,,\ \Box(\phi^4)\,,\ \ldots$$

- **EOM**: $(\Box + m_\phi^2 + \frac{\lambda}{3!}\phi^2)\phi = 0 \Rightarrow$ replace terms with $\Box\phi$
- EFT Lagrangian:

$$\mathcal{L}_{\mathrm{EFT}} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m_\phi^2}{2}\phi^2 - \frac{\lambda^4}{4!}\phi^4 + \frac{C^{(6)}}{\Lambda^2}\frac{\phi^6}{6!} + \mathcal{O}(\Lambda^{-4})$$

Λ: UV scale, bookkeeping device

Power counting

- assume $\mathcal{A}$ to be dimensionless amplitude: insert effective operator with mass dimensions d:

$$\mathcal{A} \propto \left(\frac{p}{\Lambda}\right)^{d-4} = \delta^{d-4}$$

- **power-counting formula**: sum over inserted operators

$$\mathcal{A} \propto \delta^n, \; n = \sum_i (d_i - 4)$$

- remember: starting point is the assumption $m \ll \Lambda$, $p \ll \Lambda$

Lecture 1: EFT basics

Renormalization?

- EFT involves infinite tower of higher-dimension operators
- *hold on! EFT is **not renormalizable**!!! how can we make sense of that?*

Two questions

discuss with your neighbor for 1 minute:

Q1

How can the EFT have any predictability if it involves an **infinite number of parameters**?

How is it possible to work with a non-renormalizable theory?

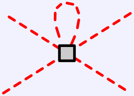
Two questions

discuss with your neighbor for 2 minutes:

Q2

In loops, we integrate over all loop momenta ℓ, even $\ell > \Lambda$.

Regulating UV divergence with a cutoff:



$$= i\frac{C^{(6)}}{2\Lambda^2}\int_{\Lambda_c}\frac{d^4\ell}{(2\pi)^4}\frac{i}{\ell^2 - m_\phi^2} \approx i\frac{C^{(6)}}{32\pi^2}\frac{\Lambda_c^2}{\Lambda^2}$$

This is $\mathcal{O}(\delta^0)$ instead of $\mathcal{O}(\delta^2)$, **violating power counting**!

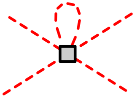
Do EFTs break down at the loop level?

Dimensional regularization

- use a **good regulator**, which respects power counting, symmetries (e.g., chiral symmetry), gauge invariance
  $\Rightarrow$ **dimensional regularization**, space-time dimension $D = 4 - 2\varepsilon$
- introduction of **renormalization scale μ** to keep couplings dimensionless
- cancel $\frac{1}{\varepsilon^n}$ loop divergences by local counterterms
  $\mu^\varepsilon \frac{1}{\varepsilon} = \frac{1}{\varepsilon} + \log(\mu) + \mathcal{O}(\varepsilon)$
  $\Rightarrow$ only **logarithmic μ-dependence**

Dimensional regularization

- **UV scale Λ**: only appears in insertion of higher-dimension operators as $\frac{1}{\Lambda^n}$
- only possible dimensional parameters in the numerator are **IR scales**
- power counting works at the loop level:



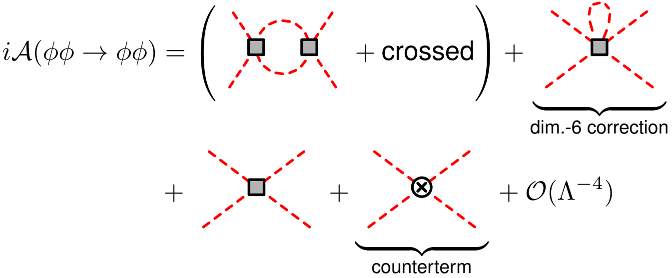
$$= i\mu^{2\varepsilon}\frac{C^{(6)}}{2\Lambda^2}\int\frac{d^D\ell}{(2\pi)^D}\frac{i}{\ell^2-m_\phi^2}$$
$$= -i\frac{C^{(6)}}{32\pi^2}\frac{m_\phi^2}{\Lambda^2}\left[\frac{1}{\varepsilon}+\log\left(\frac{\bar{\mu}^2}{m_\phi^2}\right)+1+\mathcal{O}(\varepsilon)\right]$$

- **no quadratic divergences** in the EFT: no hard cutoff

Renormalization: toy example

$$\mathcal{L}_{\mathrm{EFT}} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m_\phi^2}{2}\phi^2 - \frac{\lambda^4}{4!}\phi^4 + \frac{C^{(6)}}{\Lambda^2}\frac{\phi^6}{6!} + \mathcal{O}(\Lambda^{-4})$$

- appropriate Green's functions determine counterterms
- e.g., **renormalization of quartic coupling**:
  split $\lambda^{\mathrm{bare}} = (1+\delta Z_\lambda)\lambda^{\mathrm{ren}}\,\mu^{2\varepsilon}$, fix δZ_λ from



Renormalization-group equations (RGEs)

- physics is **independent of** artificial scale μ
- logarithmic μ-dependence from loops cancelled by μ-dependence of renormalized couplings $\Rightarrow$ RGEs

$$\frac{d}{d\log\bar{\mu}}\lambda = \beta[\lambda, m_\phi, \{C_j\}]\,,$$
$$\frac{d}{d\log\bar{\mu}}C_i = \beta_i[\lambda, m_\phi, \{C_j\}]$$

- solution of RGEs: **resummation of logarithms**

$$\lambda(\bar{\mu}) = \frac{\lambda(\bar{\mu}_0)}{1 - \frac{3\lambda(\bar{\mu}_0)}{32\pi^2}\log\left(\frac{\bar{\mu}^2}{\bar{\mu}_0^2}\right)} + \mathcal{O}(\delta^2)$$

- dependence of parameters on μ: **running/mixing**

Lecture 1: EFT basics

Matching to the UV theory

- **goal of the matching**: express renormalized EFT parameters in terms of renormalized UV-theory parameters
- choose matching scale $\mu = \mu_{\text{match}}$ and require equality of **S-matrix elements** of light particles in EFT and UV theory:

$$\mathcal{A}_{\text{EFT}} = \mathcal{A}_{\text{UV}}$$

- more practical: **off-shell matching** of one-light-particle-irreducible (1LPI) Green's functions (requires subsequent EFT basis change)
- can be done diagrammatically, or directly in path integral: **"integrating out"** heavy particles

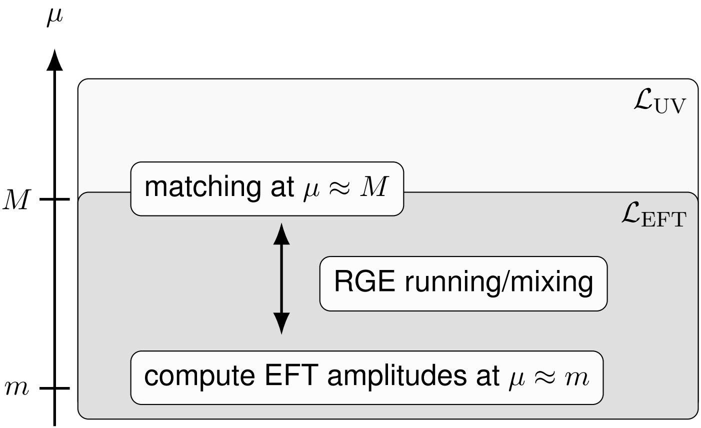
Matching to the UV theory

- UV theory might be **non-perturbative**: matching requires non-perturbative calculation (lattice simulation)
- UV theory might be unknown...

Matching at loop level

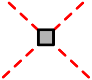
- EFT reproduces **IR physics** of the UV theory, including the entire cut structure
- matching determines Wilson coefficients: local EFT terms, polynomial in IR scales
  $\Rightarrow$ **expand** both sides of matching equation in IR scales **before integration**
- expanded loop integrals only contain UV scales $\Rightarrow$ **major simplification**
  $\Rightarrow$ Wilson coefficients absorb entire UV physics
- EFT loops only contain IR scales

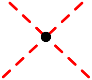
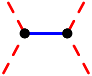
Sketch of the EFT strategy



Matching λ on shell in the toy example

- match on-shell amplitude:

$$i\mathcal{A}_{\mathrm{EFT}}(\phi\phi \to \phi\phi) =$$

$$= -i\lambda$$

$$i\mathcal{A}_{\mathrm{UV}}(\phi\phi \to \phi\phi) =$$

$$+ \Bigg($$

$$+ \text{crossed} \Bigg)$$

$$= -i\lambda_0 + (-i\lambda_1 M)\left(\frac{i}{s-M^2} + \frac{i}{t-M^2} + \frac{i}{u-M^2}\right)(-i\lambda_1 M)$$

- expand propagators:

$$\frac{i}{s-M^2} = \frac{-i}{M^2}\left(1 + \frac{s}{M^2} + \dots\right)$$

Matching λ on shell in the toy example

- UV amplitude becomes

$$i\mathcal{A}_{\mathrm{UV}}(\phi\phi \to \phi\phi) = -i\lambda_0 + i\lambda_1^2 \left(3 + \frac{s+t+u}{M^2}\right) + \dots$$
$$= -i\lambda_0 + i\lambda_1^2 \left(3 + \frac{4m^2}{M^2}\right) + \dots$$

- obtain EFT parameter in terms of UV parameters:

$$\lambda = \lambda_0 - \lambda_1^2 \left(3 + \frac{4m^2}{M^2}\right) + \mathcal{O}(\delta^4) + \mathcal{O}(\text{1-loop})$$

Lecture 1: EFT basics

Renormalization-group equations (RGEs)

- consider again RGE solution for coupling:

$$\lambda(\bar{\mu}) = \frac{\lambda(\bar{\mu}_0)}{1 - \frac{3\lambda(\bar{\mu}_0)}{32\pi^2}\log\left(\frac{\bar{\mu}^2}{\bar{\mu}_0^2}\right)}$$

- re-expand around $\lambda(\bar{\mu}_0) = 0$: **fixed-order result**

$$\lambda(\bar{\mu}) = \lambda(\bar{\mu}_0)\left[1 + \frac{3\lambda(\bar{\mu}_0)}{32\pi^2}\log\left(\frac{\bar{\mu}^2}{\bar{\mu}_0^2}\right) + \ldots\right]$$

$\Rightarrow$ good approximation if $\left|\frac{\lambda(\bar{\mu}_0)}{16\pi^2}\log\left(\frac{\bar{\mu}^2}{\bar{\mu}_0^2}\right)\right| \ll 1$

- if $\mu \ll \mu_0$: **log becomes large**!

Resumming large logs

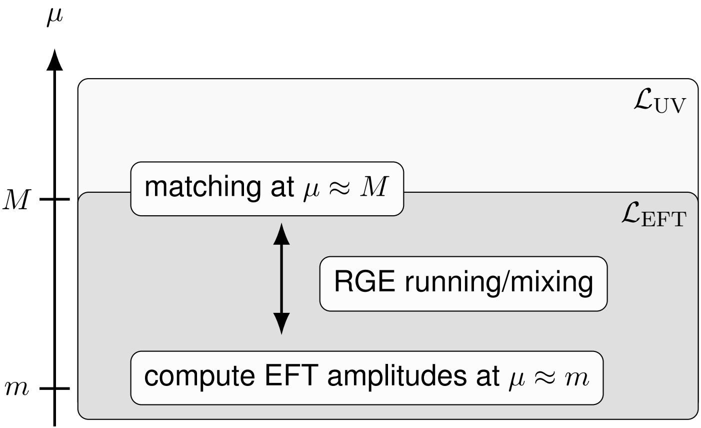
- fixed-order perturbation theory:

$$
\begin{aligned}
C(\mu) = C(\mu_0)\Big\{ 1 &+ \lambda \left[a_{1,1} \log\left(\frac{\mu^2}{\mu_0^2}\right) + a_{1,0} \right] \\
&+ \lambda^2 \left[a_{2,2} \log^2\left(\frac{\mu^2}{\mu_0^2}\right) + a_{2,1} \log\left(\frac{\mu^2}{\mu_0^2}\right) + a_{2,0} \right] \\
&+ \lambda^3 \left[a_{3,3} \log^3\left(\frac{\mu^2}{\mu_0^2}\right) + a_{3,2} \log^2\left(\frac{\mu^2}{\mu_0^2}\right) + a_{3,1} \log\left(\frac{\mu^2}{\mu_0^2}\right) + a_{3,0} \right] + \ldots \Big\}
\end{aligned}
$$

- RG-improved perturbation theory

$$
\begin{aligned}
C(\mu) = C(\mu_0)\Big\{ & \left[1 + a_{1,1}\lambda \log\left(\frac{\mu^2}{\mu_0^2}\right) + a_{2,2}\lambda^2 \log^2\left(\frac{\mu^2}{\mu_0^2}\right) + a_{3,3}\lambda^3 \log^3\left(\frac{\mu^2}{\mu_0^2}\right) + \ldots \right] \\
&+ \lambda \left[a_{1,0} + a_{2,1}\lambda \log\left(\frac{\mu^2}{\mu_0^2}\right) + a_{3,1}\lambda^2 \log^2\left(\frac{\mu^2}{\mu_0^2}\right) + \ldots \right] \\
&+ \lambda^2 \left[a_{2,0} + a_{3,1}\lambda \log\left(\frac{\mu^2}{\mu_0^2}\right) + \ldots \right] \\
&+ \lambda^3 \left[a_{3,0} + \ldots \right] + \ldots \Big\}
\end{aligned}
$$

EFT achieves RG improvement



RG improvement

- **matching**: choose $\mu_{\rm match} \approx M$ (scale of UV thresholds), determine Wilson coefficients $C_i^{(d)}(\mu_{\rm match})$ in terms of UV parameters

  $\Rightarrow$ **no large logs in matching**, $\log(\mu_{\rm match}/M)$ is small
- **running/mixing**: solve RGEs, achieve resummation of large logs, determine $C_i^{(d)}(\mu)$ at $\mu \approx m$ (IR scale)
- **amplitudes in EFT**: calculate low-energy process with $\mu \approx m$

  $\Rightarrow$ **no large logs in amplitudes**, $\log(\mu/m)$ is small

RG-improved perturbation theory

- in general **superior procedure** to a fixed-order calculation in the full theory!
- RG resummation sometimes **very large effects** (e.g., QCD effects in B physics)

Discuss with your neighbor for 3 minutes:

Consider QED with photons and electrons:

$$\mathcal{L}_{\mathrm{UV}} = \mathcal{L}_{\mathrm{QED}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\not{D} - m_e)\psi$$

At very low energies, $E \ll m_e$, photon-photon scattering $\gamma\gamma \to \gamma\gamma$ can be described by an EFT. Discuss the following aspects of this EFT:

- degrees of freedom, symmetries, power counting
- form of the EFT Lagrangian
- renormalization
- matching

Lecture 1: EFT basics

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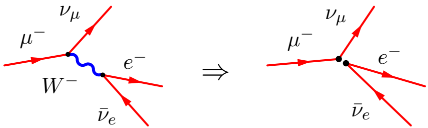
Fermi theory

- Enrico Fermi (1933/34): **four-fermion interaction** to describe β decay; historically one of the first EFTs
- also describes **muon decay**



- in the SM:

$$\mathcal{L}_{\mathrm{EW}}^{\mathrm{cc}} = -\frac{g_2}{\sqrt{2}} W_\mu^+ j_W^\mu + \mathrm{h.c.}$$

$$j_W^\mu = \sum_{\ell=e,\mu,\tau} \bar{\nu}_\ell \gamma^\mu P_L \ell + \text{quark currents}\,,$$

Fermi theory

- expand **W propagator** $D^W_{\mu\nu}(p) = \frac{-i}{p^2 - M_W^2}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{M_W^2}\right)$

  for $p^2 \ll M_W^2$:

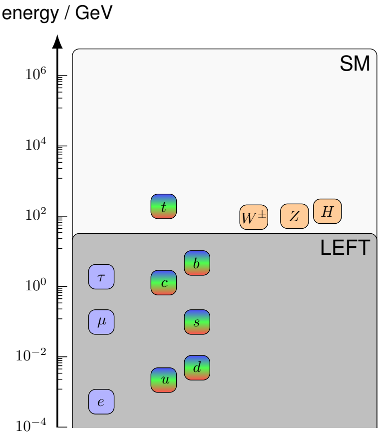
$$\frac{1}{p^2 - M_W^2} = -\frac{1}{M_W^2}\left(1 + \frac{p^2}{M_W^2} + \frac{(p^2)^2}{M_W^4} + \dots\right)$$

- **tree-level matching**:

$$\begin{aligned}\mathcal{L}_{\text{Fermi}} &= \frac{C}{\Lambda^2}(\bar\nu_\mu \gamma^\mu P_L \mu)(\bar e \gamma_\mu P_L \nu_e) + \text{h.c.} \\ &= -\frac{4G_F}{\sqrt{2}}(\bar\nu_\mu \gamma^\mu P_L \mu)(\bar e \gamma_\mu P_L \nu_e) + \text{h.c.}\end{aligned}$$

- **Fermi constant** $G_F = \frac{\sqrt{2}g_2^2}{8M_W^2} = \frac{1}{\sqrt{2}v^2}$ related to Higgs vev, $v \approx 246\,$GeV

Low-energy EFT below the weak scale (LEFT)



- degrees of freedom: SM particles w/o { t, H, $W^\pm$, Z }
- symmetries: $SU(3)_c \times U(1)_{\text{em}}$ gauge invariance; Lorentz invariance, accidental SM symmetries
- power counting: expansion in $m/v \ll 1$, $p/v \ll 1$

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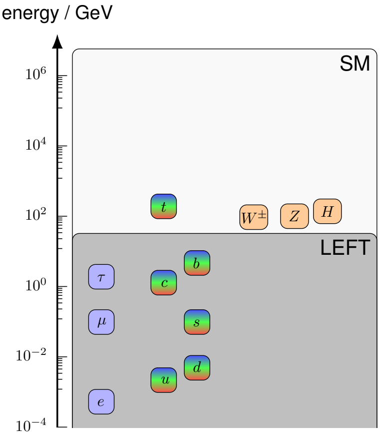

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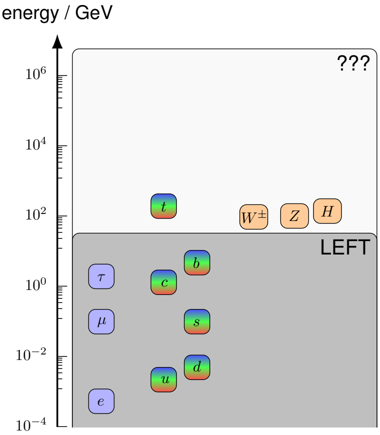

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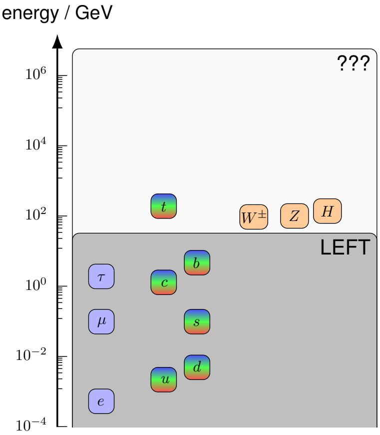

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LEFT

- QED + QCD plus tower of gauge-invariant effective operators:

$$\mathcal{L}_{\rm LEFT} = \mathcal{L}_{\rm QCD+QED} + \mathcal{L}_\nu + \sum_{d\geq 5} \sum_i L_i^{(d)} \mathcal{O}_i^{(d)}$$

$$\mathcal{L}_{\rm QCD+QED} = -\frac{1}{4} G^A_{\mu\nu} G^{A\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \theta_{\rm QCD} \frac{g^2}{32\pi^2} G^A_{\mu\nu} \widetilde{G}^{A\mu\nu}$$
$$+ \sum_{\psi = u,d,e} \overline{\psi} \left(i \not{D} - M_\psi P_L - M_\psi^\dagger P_R \right) \psi$$

$$\mathcal{L}_\nu = \bar{\nu}_L i \not{\partial} \nu_L - \frac{1}{2} \left(\nu_L^T C M_\nu \nu_L + \text{h.c.} \right)$$

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covariant derivative $D_\mu = \partial_\mu + igT^A G^A_\mu + ieQA_\mu$

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CP-violating QCD theta term

LEFT

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Majorana neutrino mass term, $\Delta L = \pm 2$

LEFT operators

$\rightarrow$ Jenkins, Manohar, Stoffer, JHEP **03** (2018) 016

- **dimension 5**: $\Delta B = \Delta L = 0$ **dipole** operators for $\psi = u, d, e$ and $\Delta L = \pm 2$ neutrino-dipole operators:

$$\mathcal{O}_{e\gamma} = \bar{e}_L \sigma^{\mu\nu} e_R F_{\mu\nu}$$

- **dimension 6**: CP-even and CP-odd **three-gluon** operators, as well as **four-fermion** operators

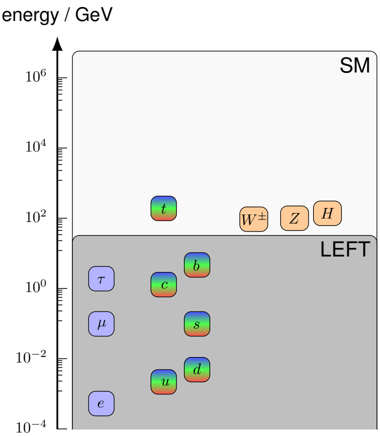
$$\mathcal{O}_G = f^{ABC} G_\mu^{A\nu} G_\nu^{B\lambda} G_\lambda^{C\mu}$$

$$\mathcal{O}_{4f} = (\bar{\psi}_p \Gamma \psi_r)(\bar{\psi}_s \Gamma \psi_t) \quad \Gamma : \text{Dirac/color structures}$$

- in total **5951 operators** at dimensions five and six, but this is mainly **flavor structure**

Lecture 1: EFT basics

Standard Model EFT (SMEFT)



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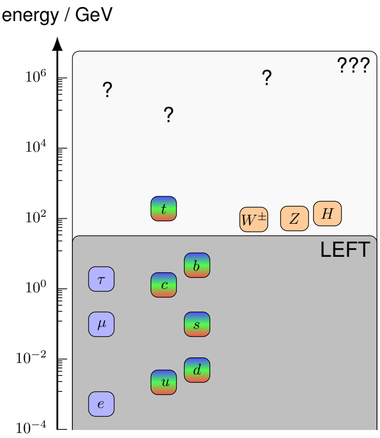

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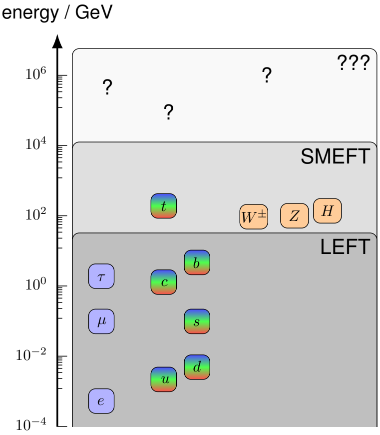

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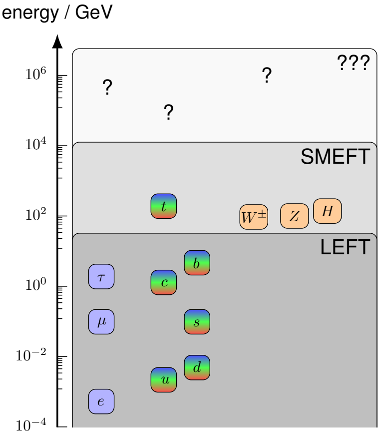

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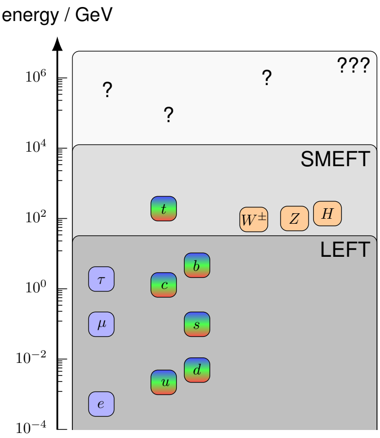

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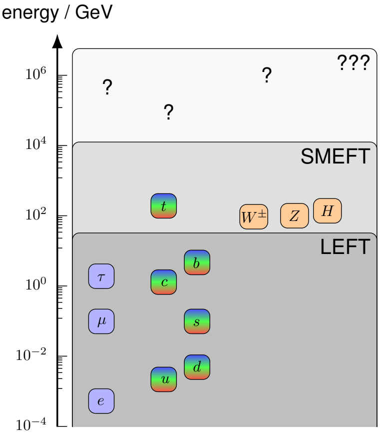

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SMEFT

- SM plus higher-dimension gauge-invariant effective operators $Q_i^{(d)}$:

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \sum_{d\geq 5} \sum_i C_i^{(d)} Q_i^{(d)}$$

- $\mathcal{L}_{\mathrm{SM}}$: "renormalizable" part, Higgs vev v is now IR scale!
- **power counting** of $d \geq 5$ Wilson coefficients: $C_i^{(d)} = \mathcal{O}(\Lambda^{4-d})$, UV scale Λ for bookkeeping
- dimension 5: Weinberg operator

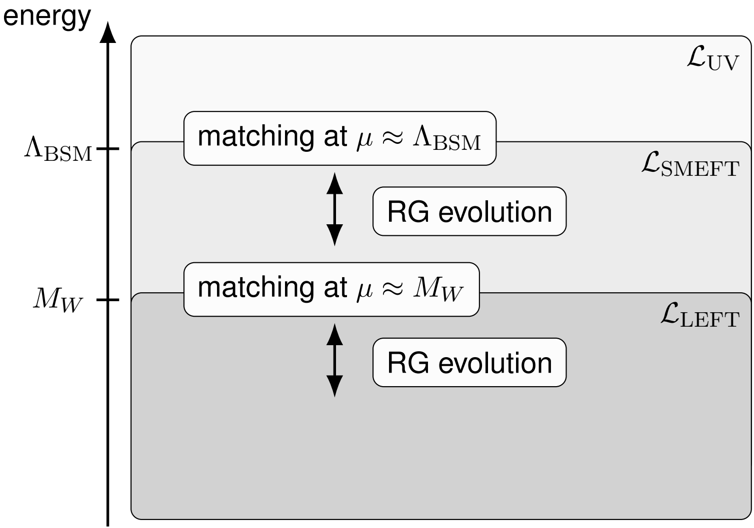
$$\mathcal{L}^{(5)} = C_{pr}^{(5)} \epsilon^{ij} \epsilon^{k\ell} (\bar{l}_{ip}^c l_{kr}) H_j H_\ell + \mathrm{h.c.}$$

- dimension 6: many operators...

SMEFT (in the broken phase)

- dimension-six modifications of fermion masses and Yukawa couplings $\Rightarrow$ no longer proportional
- modifications of gauge-boson mass terms

  $\rightarrow$ see also the Monday lecture by Martin Schmaltz
- weak charged and neutral currents modified as well, e.g. coupling of $\mathcal{W}^+$ to **right-handed current** $\bar{u}_R \gamma^\mu d_R$
- after rotation to mass eigenstates, modified weak currents lead to **non-unitary effective CKM** quark-mixing matrix


energy
Λ_{BSM}
M_W
$\mathcal{L}_{\mathrm{UV}}$
matching at $\mu \approx \Lambda_{\mathrm{BSM}}$
$\mathcal{L}_{\mathrm{SMEFT}}$
RG evolution
matching at $\mu \approx M_W$
$\mathcal{L}_{\mathrm{LEFT}}$
RG evolution

TBC on Friday...