Effective field theories and low-energy probes of new physics

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University of Zurich[UZH]

PAUL SCHERRER INSTITUT

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Lecture 1 (Thu, 08:30 – 10:00): EFT basics

- motivation
- EFT methods
- examples: Fermi theory / LEFT, SMEFT, χPT, HEFT

Lecture 2 (Fri, 16:30 – 18:00): applications

- low-energy searches and hadronic effects
- CP violation and the neutron EDM
- lepton-flavor violation

Goals of the lecture

- understand **basic concepts** of EFTs
- get familiar with a few **important examples**
- understand use of EFTs in the **search for new physics**
- learn about **interplay of different EFTs** in phenomenological examples
- learn about **theory challenges** in low-energy searches

Lecture 2: applications

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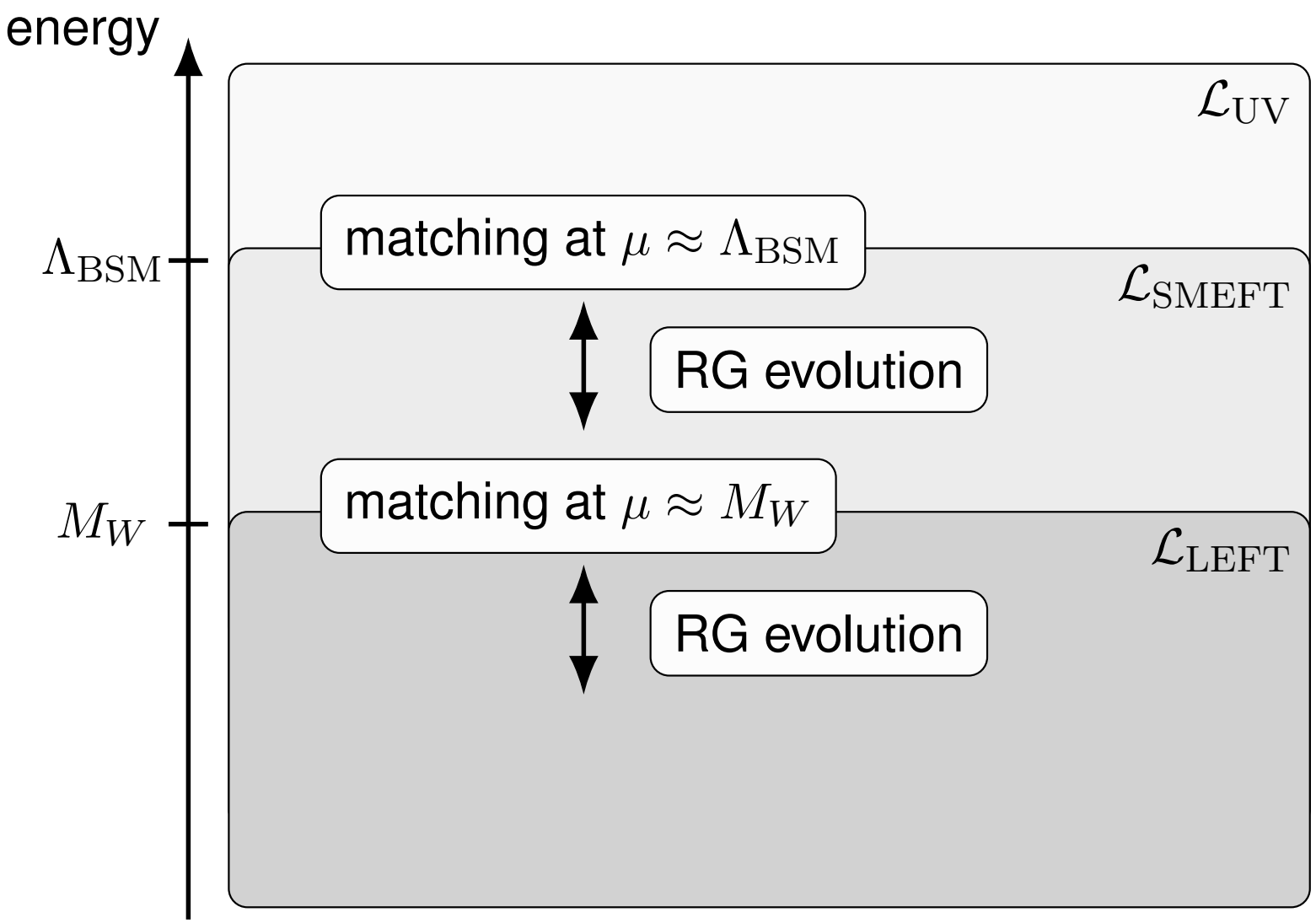
energy
$\Lambda_{\rm BSM}$
M_W
$\mathcal{L}_{\rm UV}$
matching at $\mu \approx \Lambda_{\rm BSM}$
$\mathcal{L}_{\rm SMEFT}$
RG evolution
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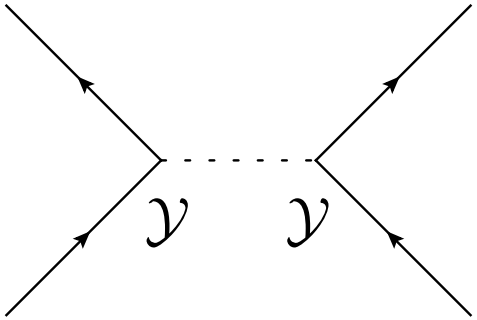
Integrating out weak-scale SM particles

consider Higgs-exchange diagram:



$$[\mathcal{Y}_\psi]_{rs} = \frac{1}{v_T}\,[M_\psi]_{rs}\,[1 + c_{H,\mathrm{kin}}] - \frac{v^2}{\sqrt{2}} C^*_{\psi H\,sr}$$

$\mathcal{Y}^2$ has terms of order $(m/v)^2$, mv/Λ^2, v^4/Λ^4

$\Rightarrow$ diagram $\mathcal{Y}^2/m_h^2$ is of same order as dimension-7 or 8 contributions in LEFT or dimension-8 in SMEFT

Integrating out weak-scale SM particles

- for SMEFT $\Rightarrow$ LEFT matching: rewrite terms

$$\ldots \frac{1}{\Lambda^n} = \underbrace{\ldots \frac{1}{v^n}}_{\text{LEFT counting}} \times \underbrace{\frac{v^n}{\Lambda^n}}_{\text{SMEFT counting}}$$

- tree-level matching simple: fix Higgs field to vev and compute **$\mathcal{W}/\mathcal{Z}$-exchange diagrams**

One-loop matching

→ Dekens, Stoffer, JHEP **10** (2019) 197

- simplify matching calculation:

$$\text{tree}_{\text{LEFT}} + \text{loop}_{\text{LEFT}}\Big|_{\mu_W} = \text{tree}_{\text{SMEFT}} + \text{loop}_{\text{SMEFT}}\Big|_{\mu_W}$$

- **expand loops** in all low scales

$$\text{tree}_{\text{LEFT}} + \cancel{\text{loop}^{\text{exp}}_{\text{LEFT}}}\Big|_{\mu_W} = \text{tree}_{\text{SMEFT}} + \text{loop}^{\text{exp}}_{\text{SMEFT}}\Big|_{\mu_W}$$

Lecture 2: applications

Strong interaction at low energies

- running QCD coupling:

$$\alpha_s(\bar{\mu}) = \frac{\alpha_s(\bar{\mu}_0)}{1 + \frac{\alpha_s(\bar{\mu}_0)}{4\pi}\beta_0 \log\left(\frac{\bar{\mu}^2}{\bar{\mu}_0^2}\right)}$$

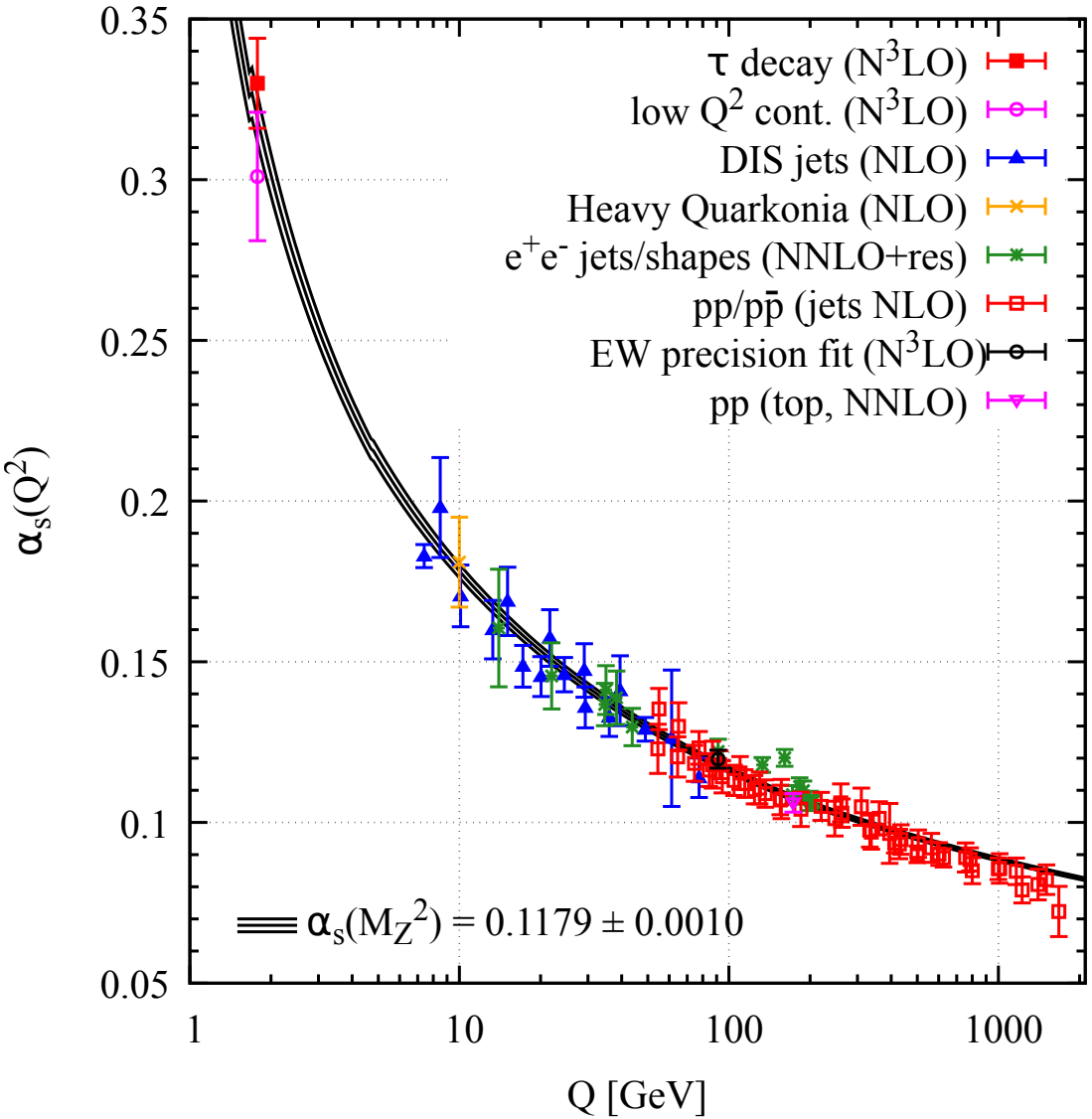
  at high energies, $\bar{\mu} \to \infty$: **asymptotic freedom**

  at low energies: α_s diverges for $\bar{\mu} \to \Lambda_{\mathrm{QCD}}$,

  non-perturbative regime

- below $\sim 2 \ldots 3$ GeV: perturbative QCD not applicable

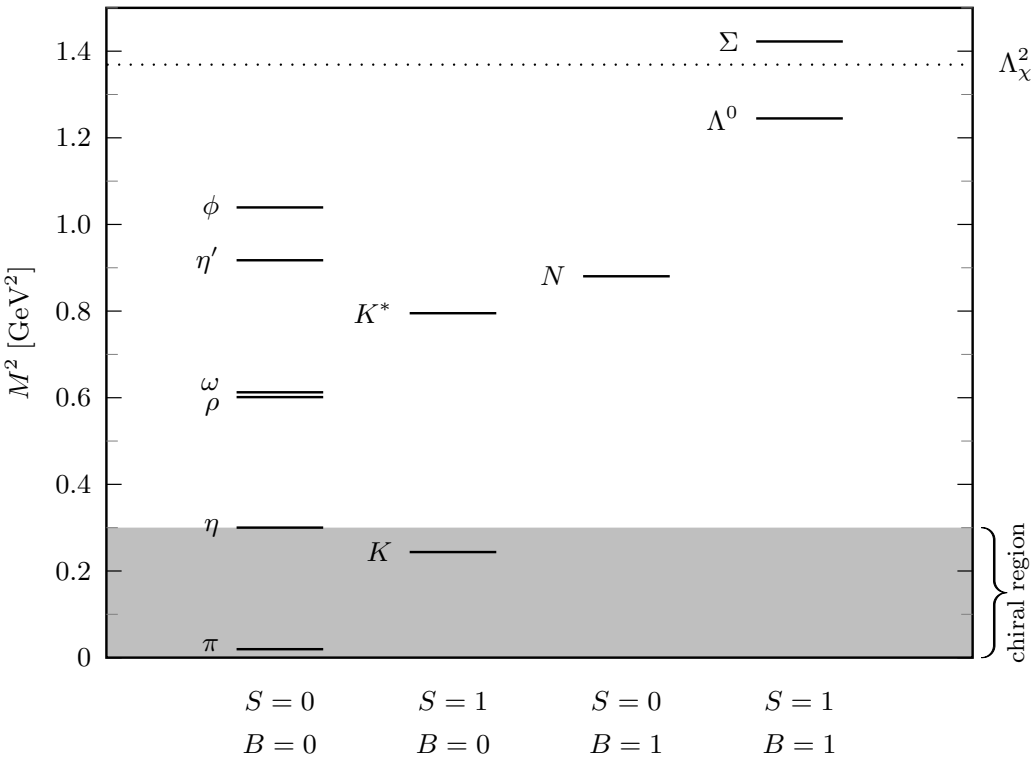
Running of α_s



→ P.A. Zyla et al. (PDG), Prog. Theor. Exp. Phys. (2020) 083C01

QCD at low energies

- observed degrees of freedom are color-neutral **hadrons**, not quarks and gluons
- low-energy spectrum contains an **octet of pseudoscalars**: $\pi^{\pm}$, π^0, $K^{\pm}$, K^0, $\bar{K}^0$, η



Chiral symmetry

- in the **chiral limit** $m_{u,d,s} \to 0$, the three-flavor QCD Lagrangian has a $U(3)_L \times U(3)_R$ symmetry
- $U(1)_A$ broken by quantum anomaly
  $\Rightarrow SU(3)_L \times SU(3)_R \times U(1)_V$ **chiral symmetry**
- not observed in spectrum $\Rightarrow$ **spontaneous symmetry breaking** to $SU(3)_V \times U(1)_V$

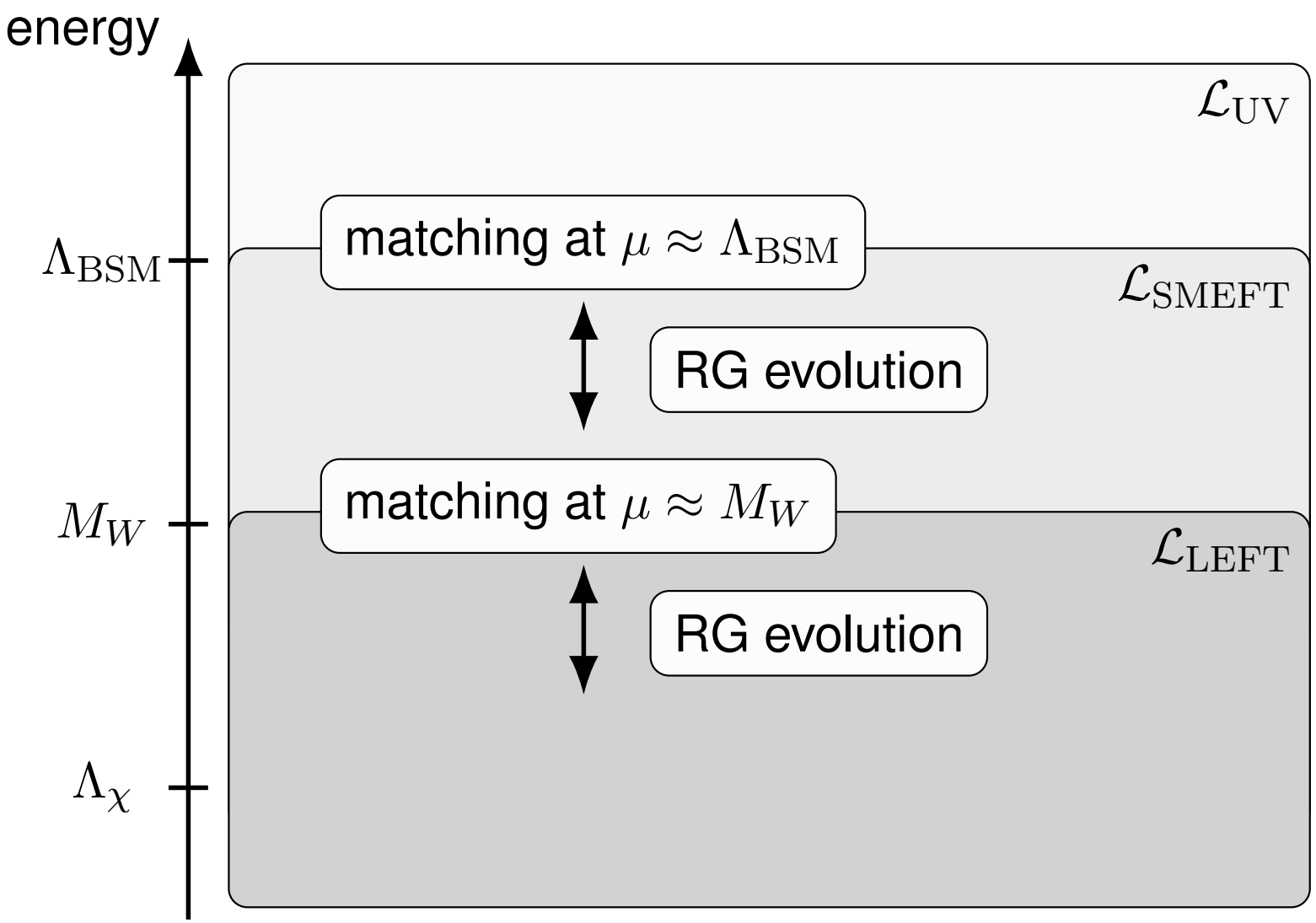
Chiral symmetry

- **Goldstone's theorem**: massless spin-0 particle for each broken generator of a continuous symmetry
- **CCWZ construction**: Goldstone bosons transform non-linearly under chiral group $(L, R) \in SU(3)_L \times SU(3)_R$:

  $\rightarrow$ Callan, Coleman, Wess, Zumino (1969)

$$U(x) = \exp\left(i\frac{\pi(x)}{F_0}\right) \xmapsto{\chi} RU(x)L^\dagger$$

$$\pi(x) = \sum_{a=1}^{8} \pi^a \lambda^a = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}$$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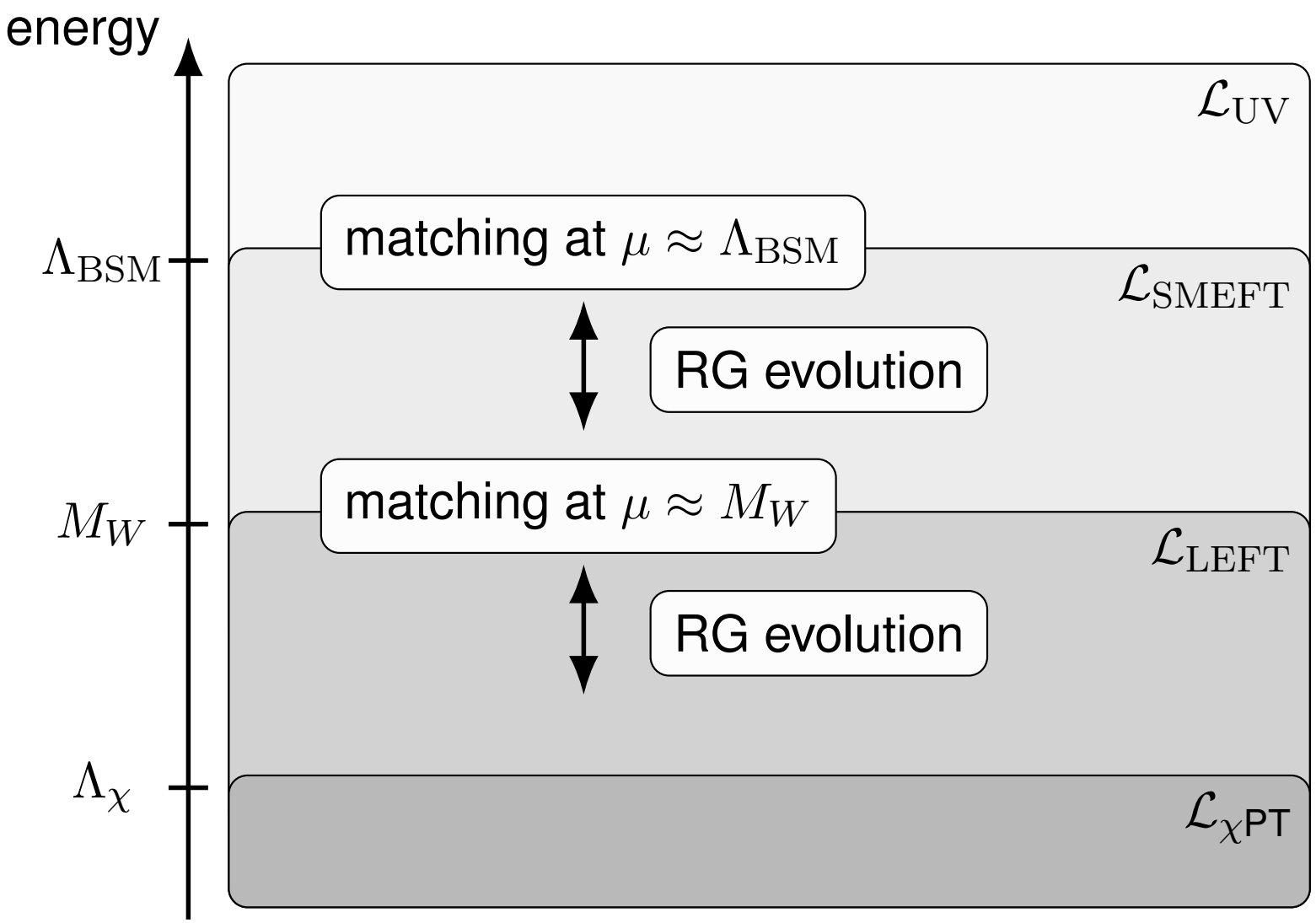
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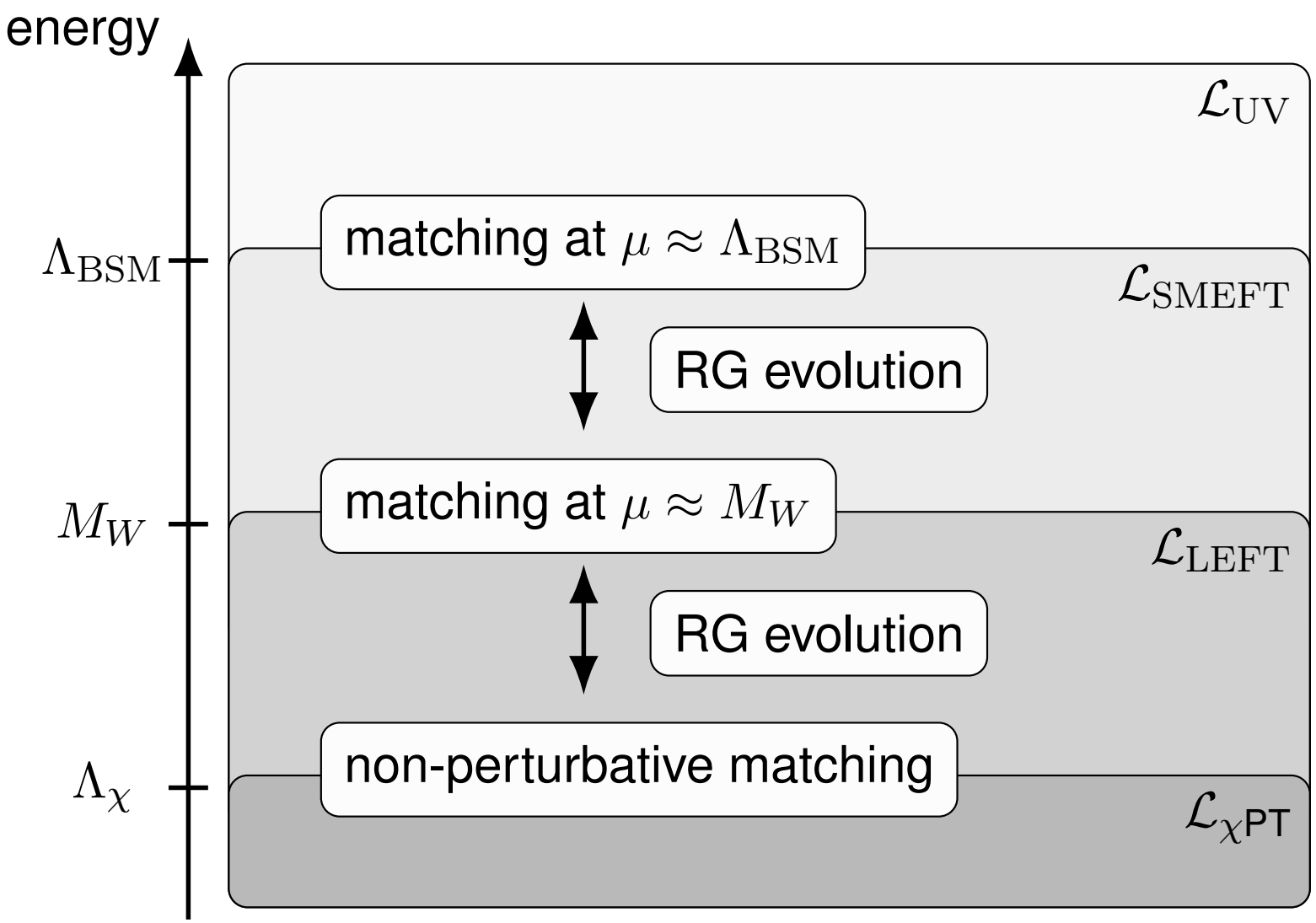
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RG evolution
non-perturbative matching
$\mathcal{L}_{\chi\rm PT}$

Chiral perturbation theory (χPT)

$\rightarrow$ Weinberg (1968), Gasser, Leutwyler (1984/1985)

- degrees of freedom: lightest hadrons as Goldstone bosons
- symmetries: $SU(3)_L \times SU(3)_R$ (flavor) chiral symmetry, spontaneously broken to $SU(3)_V$, Lorentz symmetry, ...
- power counting: expansion in $p/\Lambda_\chi \ll 1$, $M_{\rm GB}/\Lambda_\chi \ll 1$

Chiral perturbation theory (χPT)

- **leading-order chiral Lagrangian**:

$$\mathcal{L}_{p^2} = \frac{F_0^2}{4}\langle (D_\mu U)(D^\mu U)^\dagger \rangle + \frac{F_0^2 B_0}{2}\langle M_q U^\dagger + U M_q^\dagger \rangle$$

- two **"low-energy constants"** (Wilson coefficients of χPT):
  - F_0: pion decay constant
  - B_0: scalar singlet condensate, $B_0 = -\frac{1}{3F_0^2}\langle 0|\bar{q}q|0\rangle_0$
- derivative expansion, power counting no longer in mass dimensions, intertwined with loop expansion: $\Lambda_\chi = 4\pi F_0$
- describes, e.g., $\pi\pi \to \pi\pi$, $\pi^+ \to \mu^+\nu_\mu$, ...
- originally **QCD as UV theory**, but inclusion of photons and leptons and higher-dimension LEFT operators possible

Lecture 2: applications

Scalar sector of the SM

- usual linear representation:

$$\mathcal{L}_{\text{scalar}} = (\partial_\mu H)^\dagger (\partial^\mu H) - \lambda \left(H^\dagger H - \frac{1}{2} v^2 \right)^2$$

$$H(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_2(x) + i\phi_1(x) \\ h(x) + v - i\phi_3(x) \end{pmatrix}$$

- **spontaneous symmetry breaking** $SO(4) \to SO(3)$, or equivalently $SU(2)_L \times SU(2)_R \to SU(2)_V$, symmetry-breaking pattern identical to χPT
- make a field redefinition: $\mathcal{L}_{\text{scalar}}$ can be written in **non-linear form**:

$$\mathcal{L}_{\text{scalar}} = \frac{1}{2}\partial_\mu \hat{h} \partial^\mu \hat{h} - \frac{1}{2} M_H^2 \hat{h}^2 + \frac{v^2}{4} \mathcal{F}(\hat{h}/v) \langle (\partial_\mu U)(\partial^\mu U)^\dagger \rangle - V(\hat{h}/v)$$

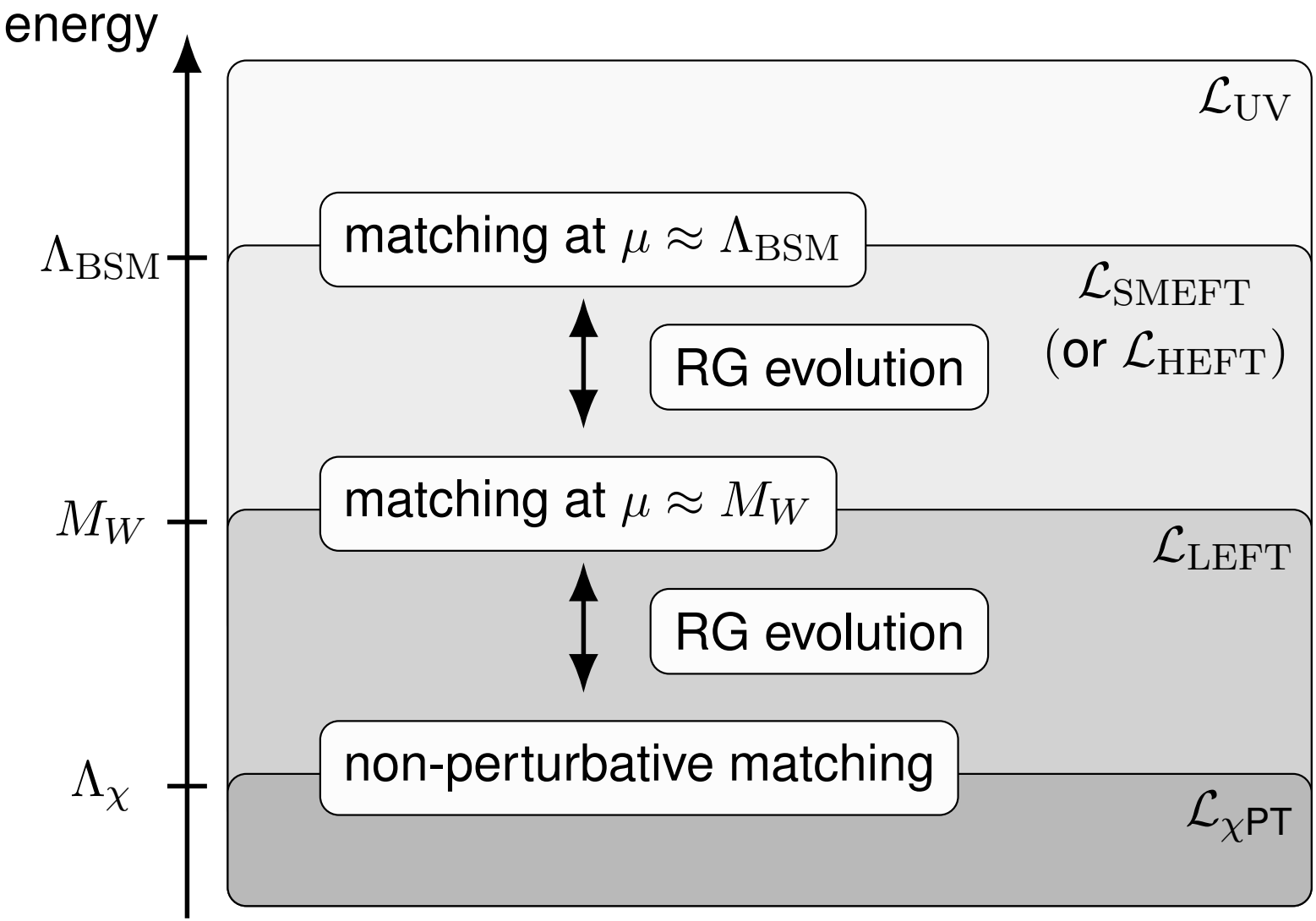
Scalar sector of the SM: extend to an EFT

- include higher-order operators: essentially χPT with additional scalar singlet $\hat{h}$ (Higgs boson)
- all sectors of the SM can be included in the theory
- EFT for physics beyond the SM, but **more general than SMEFT**: Higgs not restricted to be part of EW doublet

HEFT / EWχL

- degrees of freedom: all SM particles, but Higgs generic scalar singlet
- symmetries: $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge invariance, spontaneously broken to $SU(3)_c \times U(1)_{\mathrm{em}}$; Lorentz invariance;
- power counting: nonlinear realization leads to chiral loop expansion

A tower of EFTs:



Two questions

discuss with your neighbor for 2 minutes:

Q1

In the LEFT, we encountered the dipole operator $\mathcal{O}_{e\gamma} = \bar{e}_L \sigma^{\mu\nu} e_R F_{\mu\nu}$. Why are there no dimension-5 dipole operators in the SMEFT?

Q2

What is the main difference between the (bosonic sector of) HEFT / EWχL and χPT?

Lecture 2: applications

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Direct searches for new physics

- mainly LHC searches: try to directly produce new particles, search for bump in spectrum
- so far no evidence of new particles: **SM works** surprisingly well!
- **absence of a discovery** in contrast to widely shared expectations/hopes—crisis for particle physics?
- measurements are pushing the scale of new physics to **higher energies** (barring very weak coupling)
  $\Rightarrow$ EFT methods can be applied

Direct searches for new physics

Indirect searches for new physics

- what if LHC searches will not find new particles?
- **indirect searches**: do not produce new particles directly
- try to find their indirect quantum effects at **lower energies**
- LHC searches are becoming indirect “low-energy” searches, too!
- single indirect signal: **presence** of new physics
- identification of its **nature** will require a multitude of signals

Indirect searches for new physics

Indirect searches for new physics

What are good observables for indirect searches?

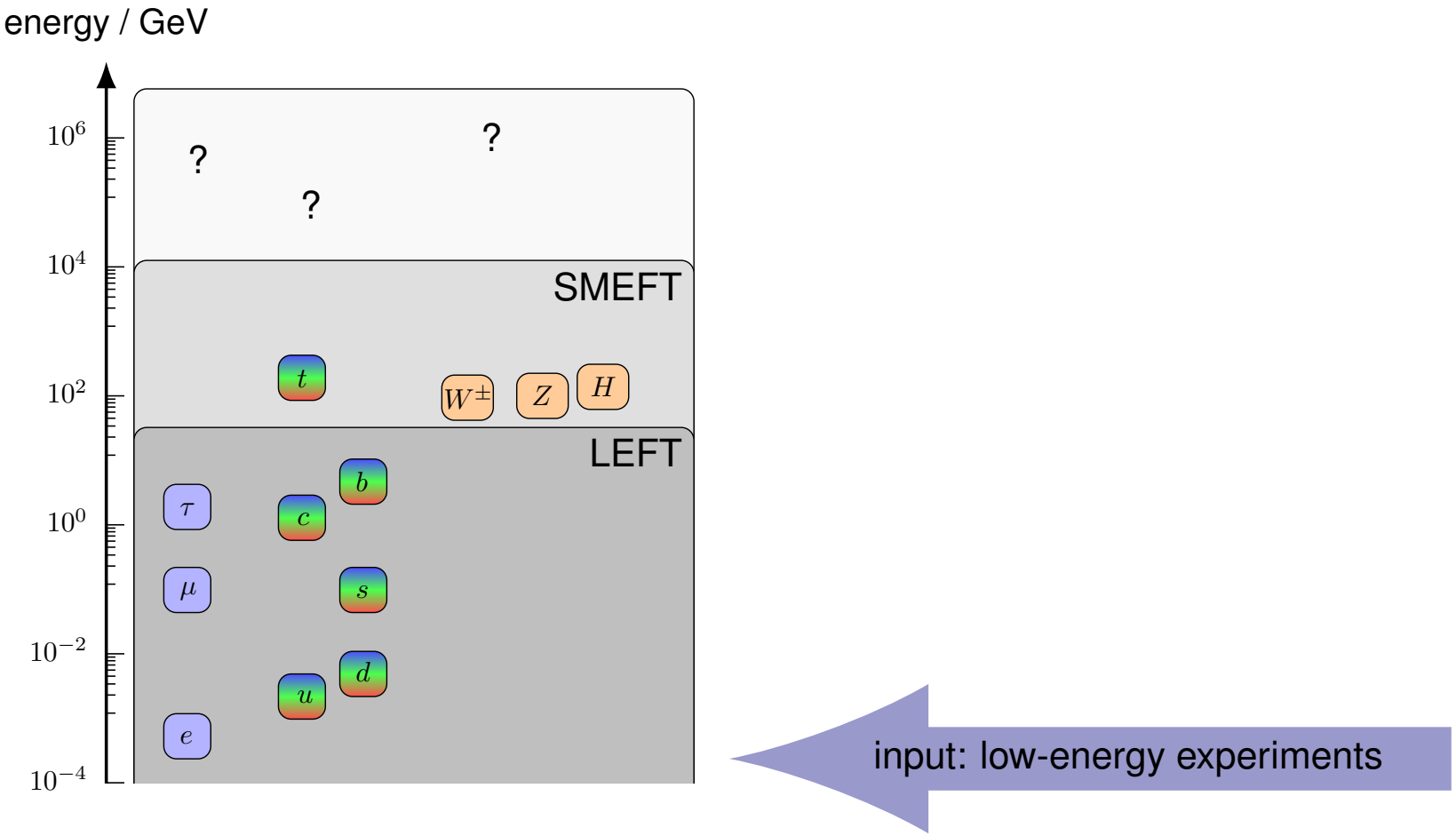
1. if there is a (significant) **SM contribution**: need to control SM uncertainties, ideally to similar level as experimental precision
2. easier: SM contribution **strongly suppressed** compared to potential new physics (loop suppression, CKM suppression, GIM, etc.)
3. even better: **process forbidden** (or completely negligible) in the SM

Lecture 2: applications

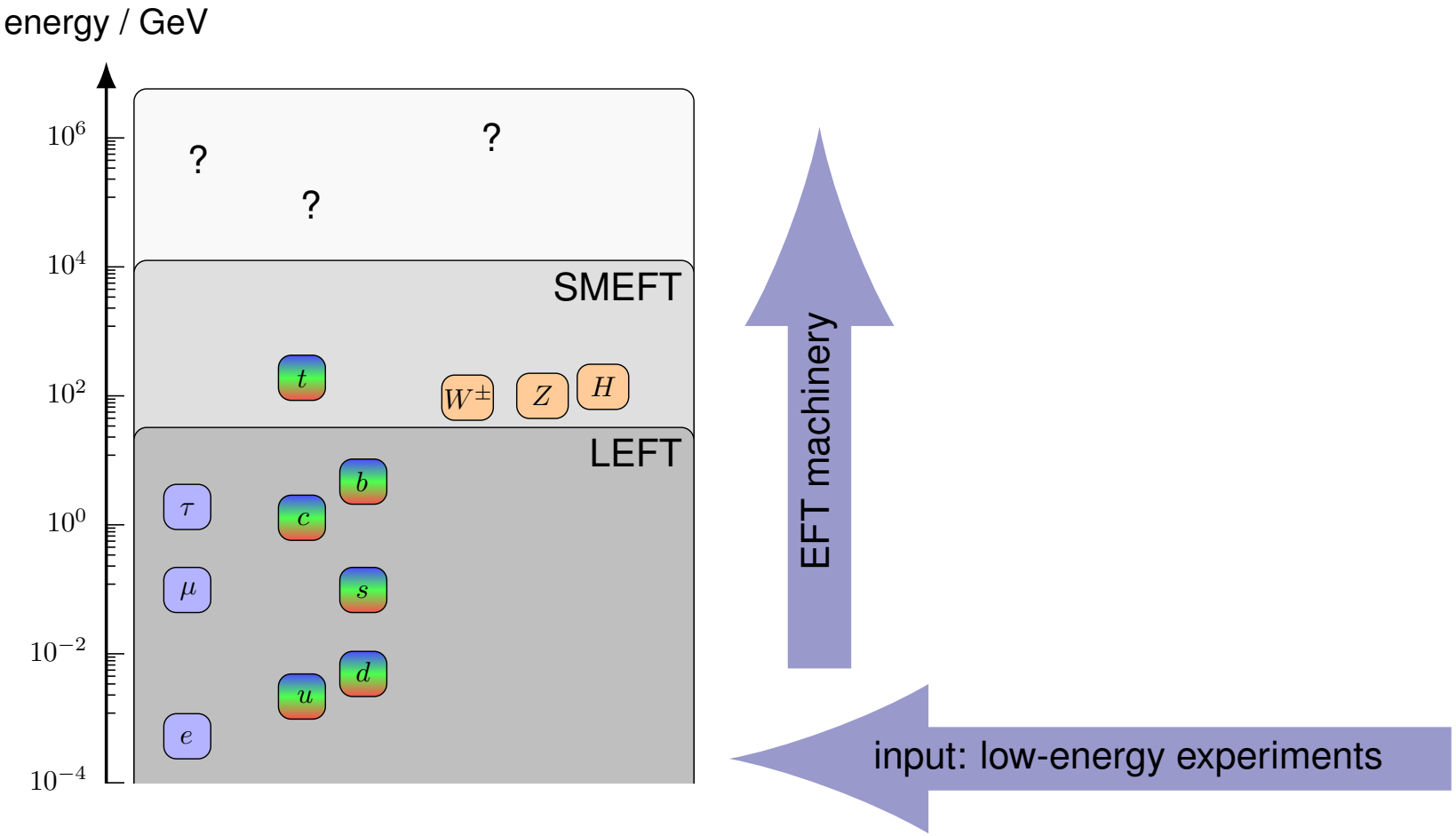
Where should we best search for new physics?

- deficiencies of SM provide some guidance
- a priori: try to search **wherever possible**, considering criteria for promising observables
- EFTs provide a systematic and model-independent framework for **connecting widely different energy regimes**
- (motivated) UV models can give a guidance where interesting effects might occur
  $\Rightarrow$ much stronger correlations between observables than in EFTs, but model dependent
- first take **existing bounds** systematically into account: match UV models to EFTs

Low-energy precision searches



Low-energy precision searches



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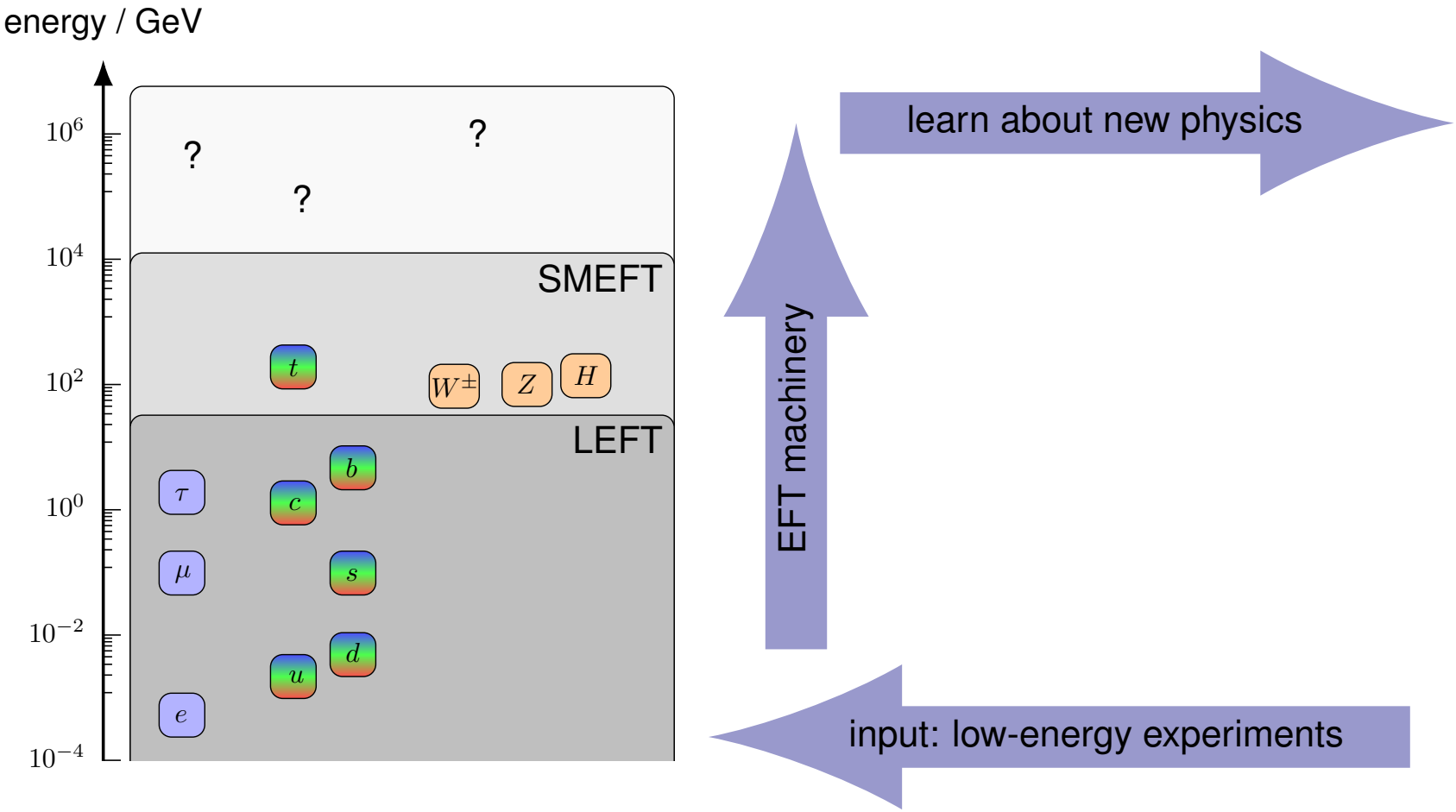

Lecture 2: applications

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Magnetic moments

- relation of spin and magnetic moment of a lepton:

$$\vec{\mu}_\ell = g_\ell \frac{e}{2m_\ell}\vec{s}$$

  g_ℓ: Landé factor, gyromagnetic ratio

- Dirac's prediction: $g_e = 2$
- anomalous magnetic moment: $a_\ell = (g_\ell - 2)/2$
- helped to establish QED and QFT as the framework for elementary particle physics

Dipole moments in QFT

- consider vertex function

$$= (-ie\mathsf{q}_e)\,\bar{u}(p')\Gamma^\mu(p,p')u(p)\,, \quad k = p' - p$$

- Lorentz invariance gives **form-factor decomposition**

$$\Gamma^\mu(p,p') = \gamma^\mu F_E(k^2) + i\frac{\sigma^{\mu\nu}k_\nu}{2m_\ell}F_M(k^2) + \frac{\sigma^{\mu\nu}k_\nu}{2m_\ell}\gamma_5 F_D(k^2) + \frac{k^2\gamma^\mu - k^\mu \not{k}}{m_\ell^2}\gamma_5 F_A(k^2)$$

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anomalous magnetic moment: $a_\ell = \frac{1}{2}(g_\ell - 2) = F_M(0)$

Dipole moments in QFT

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electric dipole moment: $d_\ell = -\frac{e\mathsf{q}_e}{2m_\ell}F_D(0)$

Electron vs. muon magnetic moments

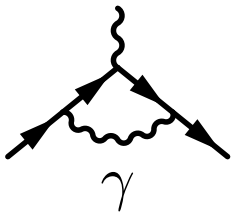
- influence of heavier virtual particles of mass M scales in many cases as ("naive scaling")

$$\frac{\Delta a_\ell}{a_\ell} \propto \frac{m_\ell^2}{M^2}$$

- $(m_\mu/m_e)^2 \approx 4 \times 10^4 \Rightarrow$ muon is much more sensitive to **new physics**, but also to **EW and hadronic contributions**
- a_τ experimentally not yet known precisely enough

SM contributions to $g-2$

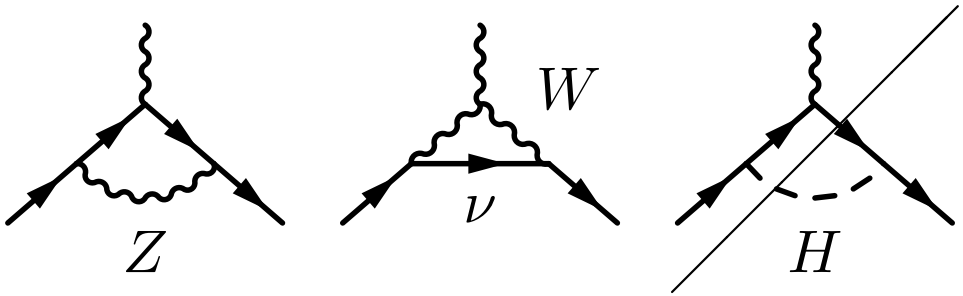
- QED contribution starts at 1 loop (Schwinger term)



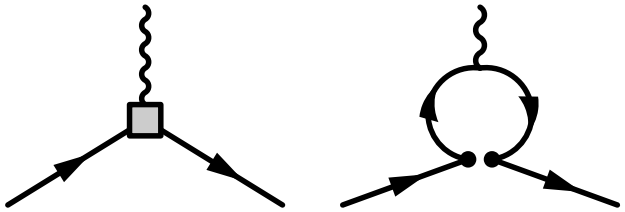
$$a_\ell = \frac{\alpha}{2\pi}$$

- SM electroweak contribution via the LEFT up to dim.6:

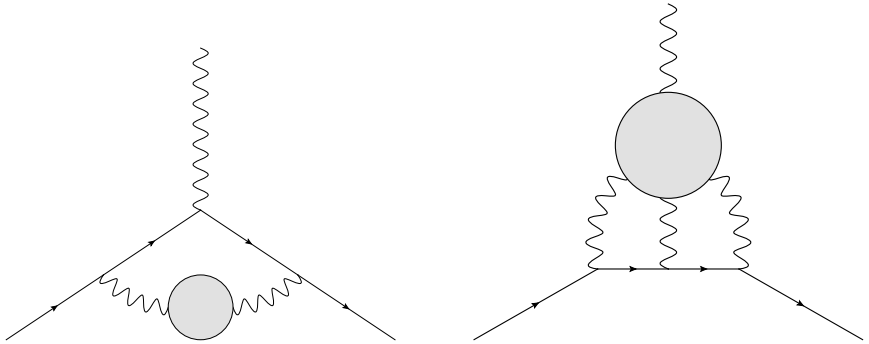
SM:



LEFT:



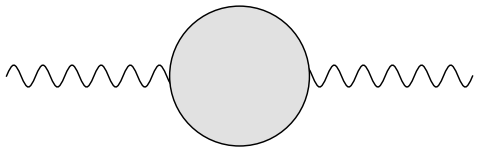
Hadronic contributions in the SM



- strong interaction at low energies: **non-perturbative effects**
- either simulate QCD on the **lattice** or use **dispersion relations**

Hadronic vacuum polarization (HVP)

photon HVP function:



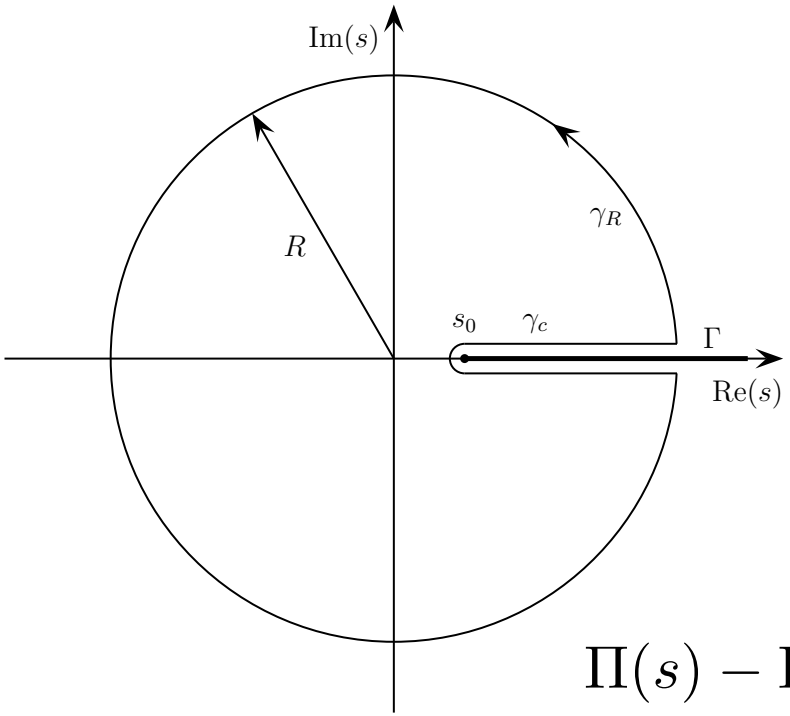
$$= i(q^2 g_{\mu\nu} - q_\mu q_\nu)\Pi(q^2)$$

unitarity of the S-matrix implies the optical theorem:

$$\mathrm{Im}\Pi(s) = \frac{s}{e(s)^2}\sigma(e^+e^- \to \mathrm{hadrons})$$

Dispersion relation

causality implies **analyticity**:



Cauchy integral formula:

$$\Pi(s) = \frac{1}{2\pi i} \oint_\gamma \frac{\Pi(s')}{s'-s} ds'$$

deform integration path:

$$\Pi(s) - \Pi(0) = \frac{s}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\mathrm{Im}\Pi(s')}{(s'-s-i\epsilon)s'} ds'$$

HVP contribution to $(g-2)_\mu$

$$a_\mu^{\mathrm{HVP}} = \frac{m_\mu^2}{12\pi^3}\int_{s_{\mathrm{thr}}}^{\infty} ds\,\frac{\hat{K}(s)}{s}\,\sigma(e^+e^- \to \mathrm{hadrons})$$

- basic principles: unitarity and analyticity
- direct **relation to data**: total hadronic cross section $\sigma(e^+e^- \to \mathrm{hadrons}) \Rightarrow$ **R ratio**
- dedicated e^+e^- program (BaBar, Belle, BESIII, CMD3, KLOE, SND)

Discuss with your neighbor for 2 minutes:

Q1

How does the naive scaling of heavy-particle contributions to $g-2$ arise?

$$\frac{\Delta a_\ell}{a_\ell} \propto \frac{m_\ell^2}{M^2}$$

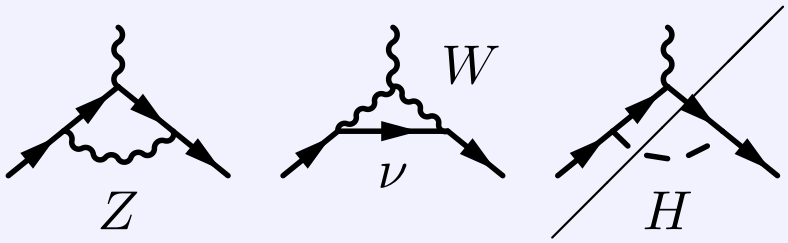
In which cases do we get a different scaling?

Q2

Which SM diagrams correspond to which LEFT diagrams?

Why does the Higgs-exchange diagram not contribute at dimension 6 in the LEFT?

SM:



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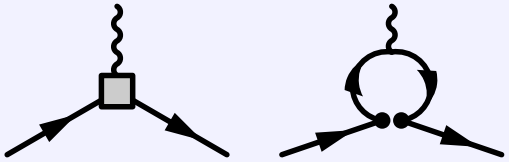
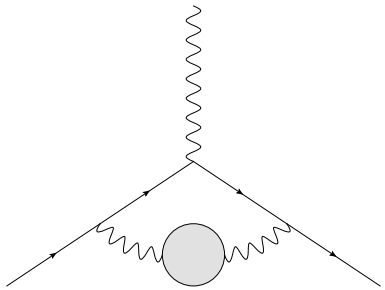


SM (according to white paper) vs. experiment

→ T. Aoyama *et al.*, Phys. Rept. **887** (2020) 1-166

	$10^{11} \times a_\mu$	$10^{11} \times \Delta a_\mu$
QED total	$116\,584\,718.931$	0.104
EW	153.6	1.0
HVP	$6\,845$	40
HLbL	92	18
SM total	$116\,591\,810$	43
experiment (E821+E989)	$116\,592\,061$	41
difference exp−theory	251	59

Hadronic vacuum polarization



- final white-paper number: data-driven evaluation

$$a_\mu^{\text{LO HVP, pheno}} = 6\,931(40) \times 10^{-11}$$

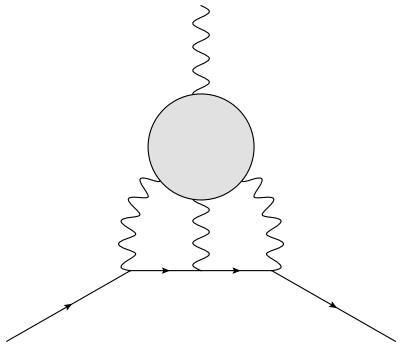
- previous average of published lattice-QCD results

$$a_\mu^{\text{LO HVP, lattice average}} = 7\,116(184) \times 10^{-11}$$

- most precise lattice-QCD result → S. Borsanyi *et al.*, Nature (2021)

$$a_\mu^{\text{LO HVP, lattice}} = 7\,075(55) \times 10^{-11}$$

Hadronic light-by-light scattering



- dispersion relations + hadronic models (LO, without charm)

$$a_\mu^{\text{HLbL, pheno}} = 89(19) \times 10^{-11}$$

- lattice-QCD results

$a_\mu^{\text{HLbL, lattice}} = 79(35) \times 10^{-11} \rightarrow$ T. Blum *et al.*, PRL **124** (2020) 132002

$a_\mu^{\text{HLbL, lattice}} = 106.8(15.9) \times 10^{-11} \rightarrow$ E.-H. Chao *et al.*, EPJC **81** (2021) 651

Muon anomalous magnetic moment $(g-2)_\mu$

recent and future experimental progress:

- FNAL will improve precision further: **factor of 4 wrt E821**
- theory: clarify **tensions** and **reduce uncertainty** in hadronic contributions!
- new physics interpretation?

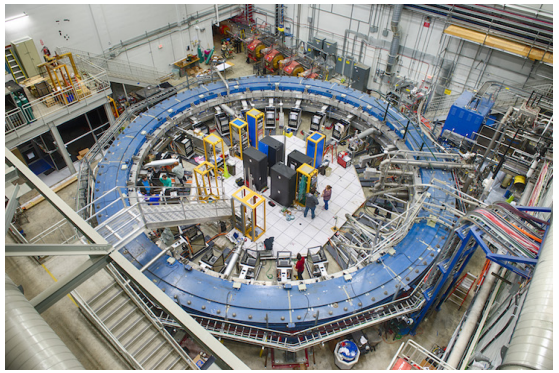
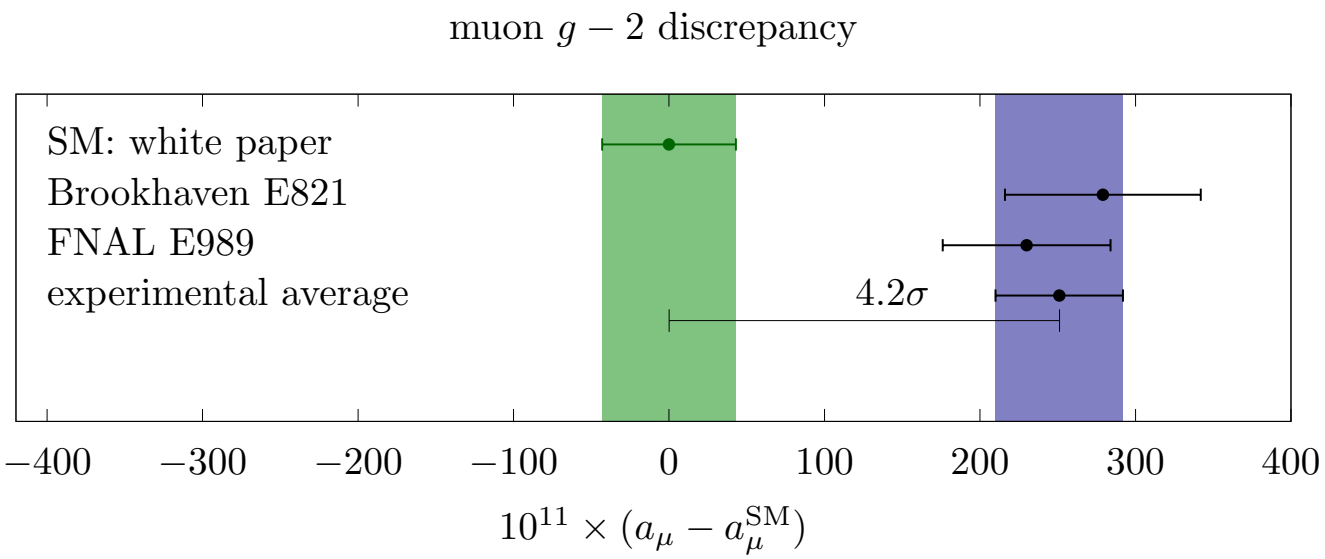


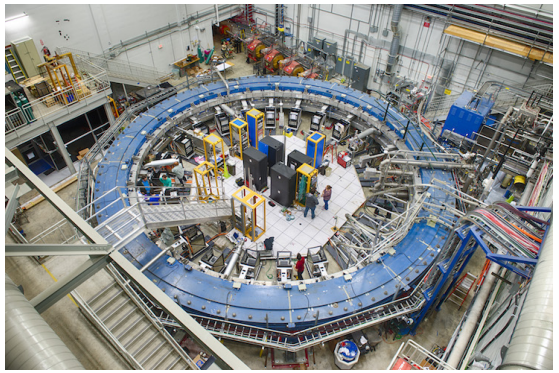
Photo: Glukicov (License: CC-BY-SA-4.0)



Muon anomalous magnetic moment $(g-2)_\mu$

recent and future experimental progress:

- FNAL will improve precision further: **factor of 4 wrt E821**
- theory: clarify **tensions** and **reduce uncertainty** in hadronic contributions!
- new physics interpretation?



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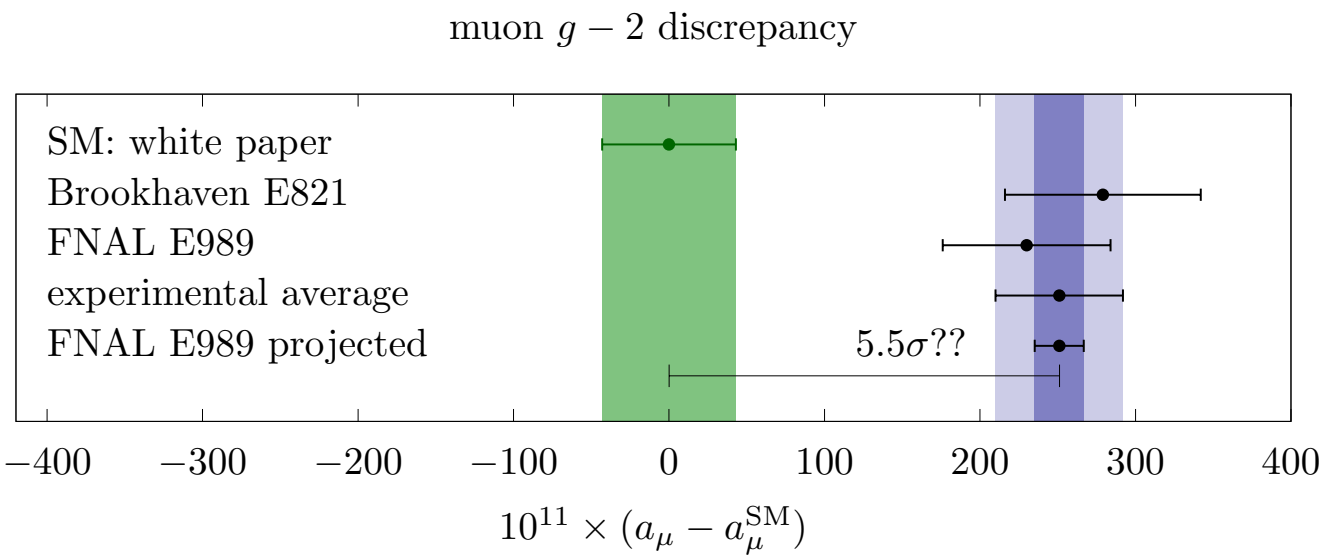

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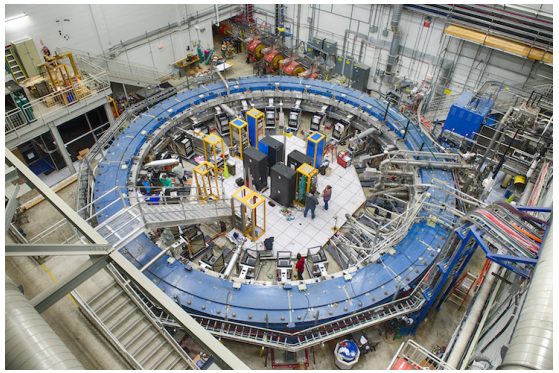

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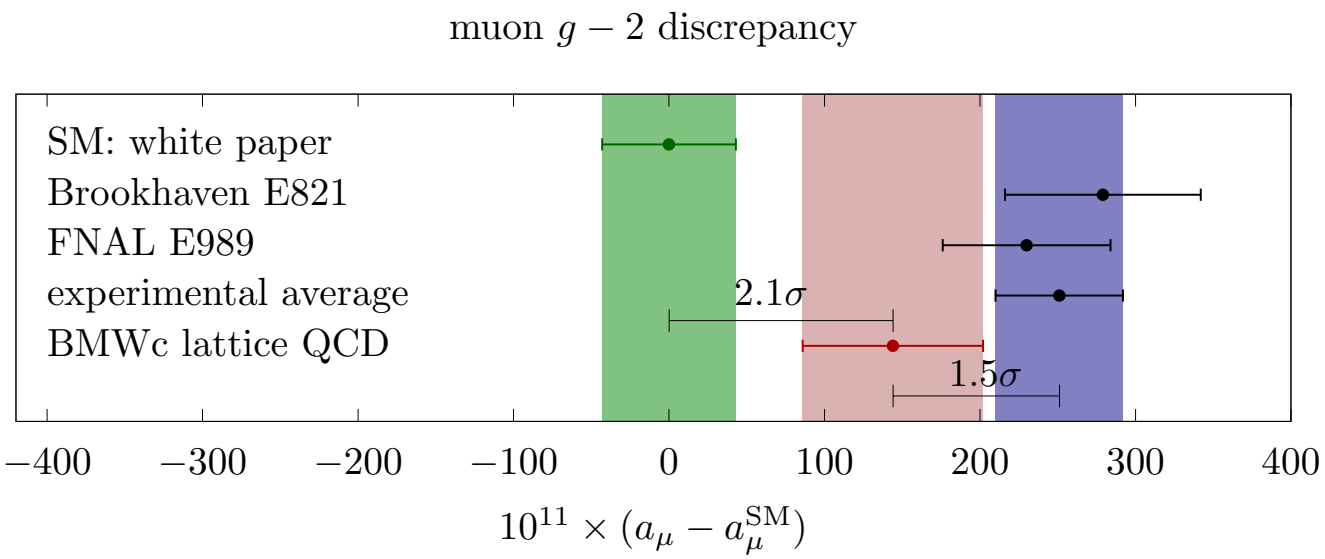


Lecture 2: applications

CP violation: a case for new physics

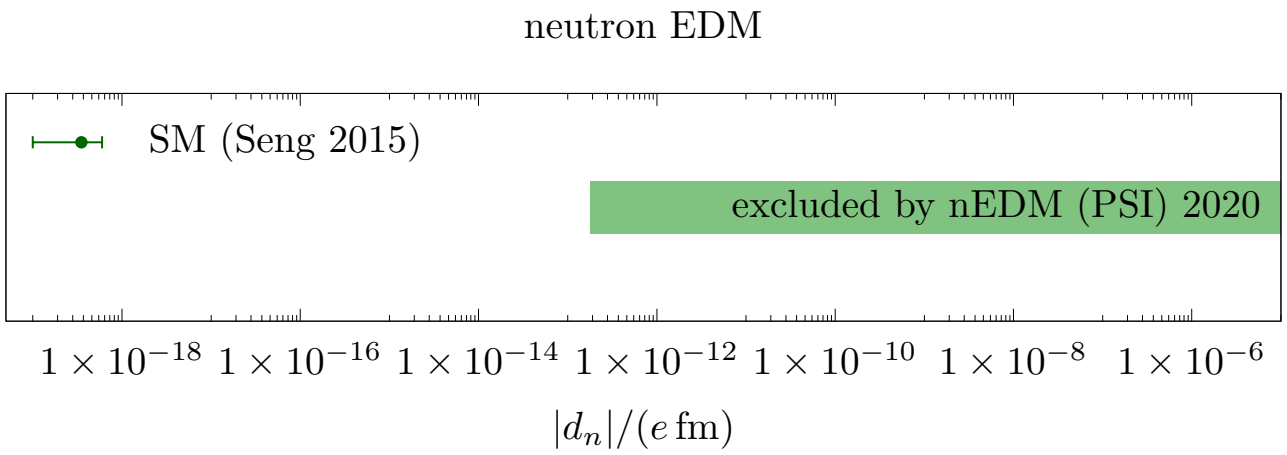
- **baryon asymmetry** in the universe requires more CP violation than Standard Model (SM) can provide
- **electric dipole moments** (EDMs) are sensitive probes of CP violation
- **SM (CKM) contribution** many orders of magnitude below current limits
- non-observation leads to strong **constraints on CP-violating sources**
- observation would be a clear **signal of physics beyond the SM** or QCD θ-term

Experimental progress in near future

neutron EDM:

- SM prediction tiny
- current limit: $|d_n| < 1.8 \times 10^{-13} e\,\mathrm{fm}$

  $\rightarrow$ nEDM Collaboration, PRL **124** (2020) 081803
- n2EDM experiment at PSI will improve sensitivity by **two orders** of magnitude



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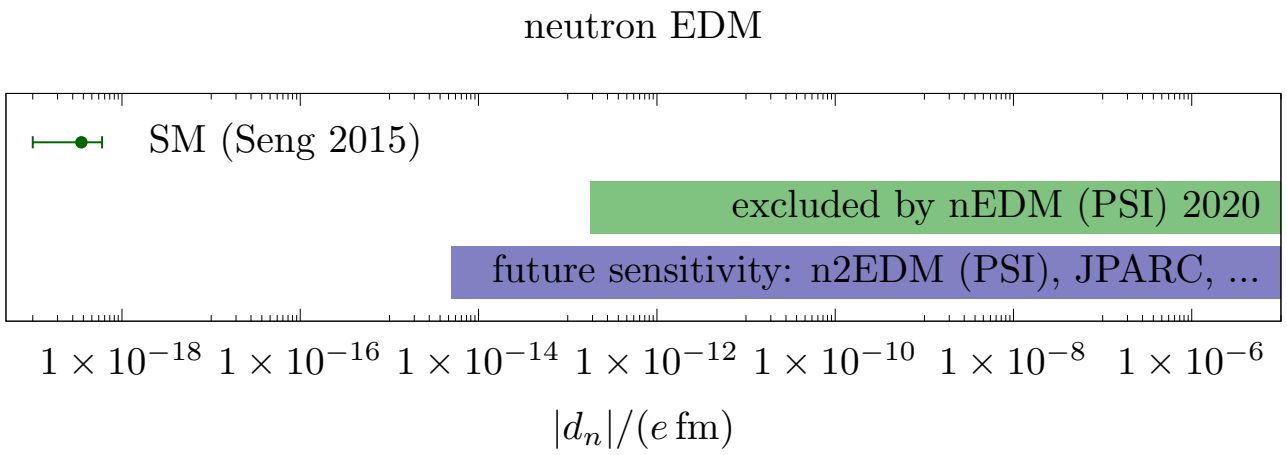


Theory challenges

- non-observation: how to turn experimental bounds into **best generic constraints** on new physics?
- observation: how to **disentangle different possible sources** of CP violation?

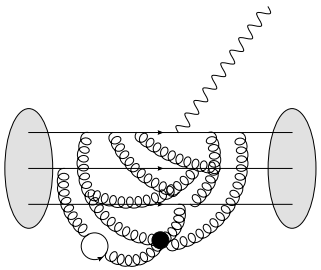
$\Rightarrow$ work with generic, **model-independent** EFT framework

$\Rightarrow$ **accuracy of theoretical description** needs to match experimental precision

$\Rightarrow$ **control uncertainties**, in particular non-perturbative aspects

Neutron EDM in LEFT

- calculate matrix element in LEFT at a renormalization scale of $\mu \sim 2 \ldots 3$ GeV
- contribution schematically given as

$d_N \sim$  $= \sum_i L_i \, \langle N | \mathcal{O}_i | N \gamma \rangle$

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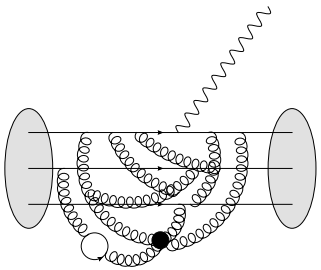
$d_N \sim$  $= \sum_i L_i \langle N|\mathcal{O}_i|N\gamma\rangle$

LEFT operator coefficients

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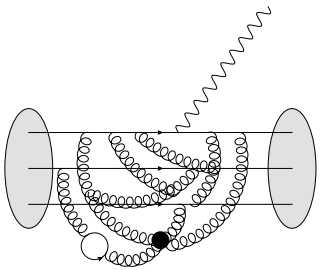
$d_N \sim$  $= \sum_i L_i \langle N|\mathcal{O}_i|N\gamma\rangle$

hadronic matrix element

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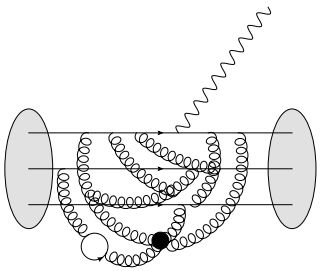
$d_N \sim$  $= \sum_i L_i \langle N|\mathcal{O}_i|N\gamma\rangle$

hadronic matrix element

- at present, **large uncertainties** on matrix elements dilute experimental sensitivity

Neutron EDM in LEFT

- hadronic EDMs (nEDM) complicated: **QCD is non-perturbative**
- any P-odd, CP-odd flavor-conserving operator **contributes non-perturbatively** to nEDM:
  - QCD θ-term
  - dimension-five quark (C)EDM operators
  - dimension-six three-gluon operator
  - dimension-six P/CP-odd four-fermion operators

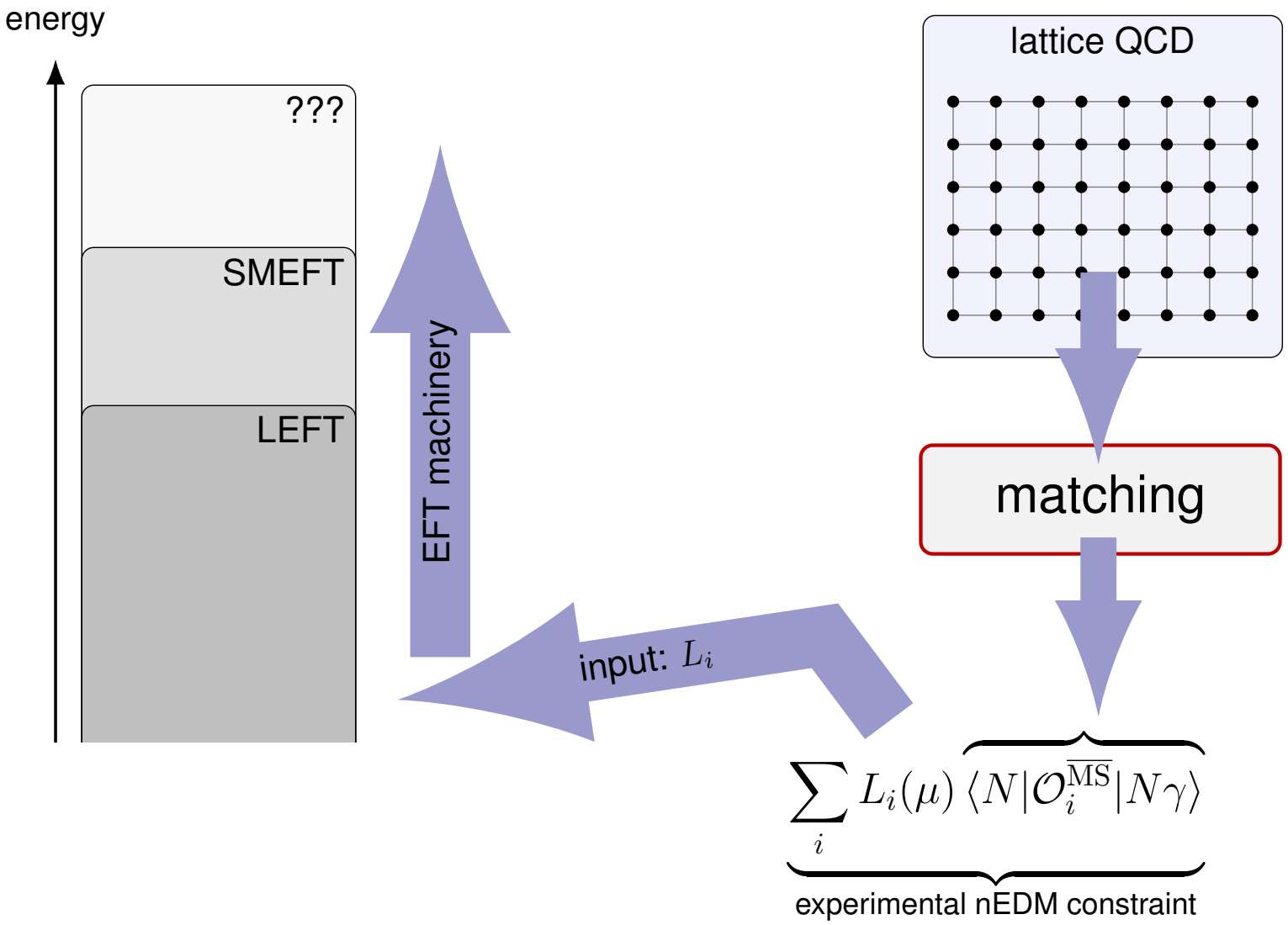
Neutron EDM in LEFT

$$
\begin{aligned}
d_N = &-(1.5 \pm 0.7) \times 10^{-3}\, \bar{\theta}\, e\ \mathrm{fm} \\
&-(0.20 \pm 0.01) d_u + (0.78 \pm 0.03) d_d + (0.0027 \pm 0.016) d_s \\
&-(0.55 \pm 0.28) e\, \tilde{d}_u - (1.1 \pm 0.55) e\, \tilde{d}_d + (??) e\, \tilde{d}_s \\
&+(50 \pm 40)\mathrm{MeV}\, e\, \tilde{d}_G + (??)\ \text{four-quark}
\end{aligned}
$$

→ Alarcon et al., arXiv:2203.08103

- ideally use **lattice QCD** to compute matrix elements
- problem with lattice and LEFT: $d_N \sim \sum_i L_i(\mu) \langle N | \mathcal{O}_i^{\overline{\mathrm{MS}}} | N\gamma \rangle$
  $\overline{\mathrm{MS}}$ cannot be implemented on the lattice!
- requires a **matching calculation**

Neutron EDM in LEFT



Matching to lattice QCD

- $\overline{\mathrm{MS}}$: subtraction of $1/\varepsilon$ poles in dimensional regularization
- **define renormalized operators** in a scheme amenable to lattice computations
- compute their matrix elements in lattice QCD
- calculate relation between $\overline{\mathrm{MS}}$ and lattice scheme **in perturbation theory** (at $\mu \sim 2 \ldots 3$ GeV)
- use this matching to derive matrix elements of $\overline{\mathrm{MS}}$ operators

Lecture 2: applications

Lepton-flavor violation: $\mu \to e\gamma$

- forbidden in the SM
- **impressive constraint** from MEG (PSI)
  $\to$ MEG Collaboration, EPJ **C76** (2016) 8, 434

$$\mathrm{BR}(\mu \to e\gamma) < 4.2 \times 10^{-13}$$

- puts limits on beyond SM physics equivalent to **many hundreds of TeV**
- induced in the LEFT at tree level by dimension-five **dipole operators**

$$\mathcal{O}_{\substack{e\gamma \\ e\mu}} = \bar{e}_L \sigma^{\mu\nu} \mu_R F_{\mu\nu}\,, \quad \mathcal{O}_{\substack{e\gamma \\ \mu e}} = \bar{\mu}_L \sigma^{\mu\nu} e_R F_{\mu\nu}\,, \quad \mathrm{h.c.}$$

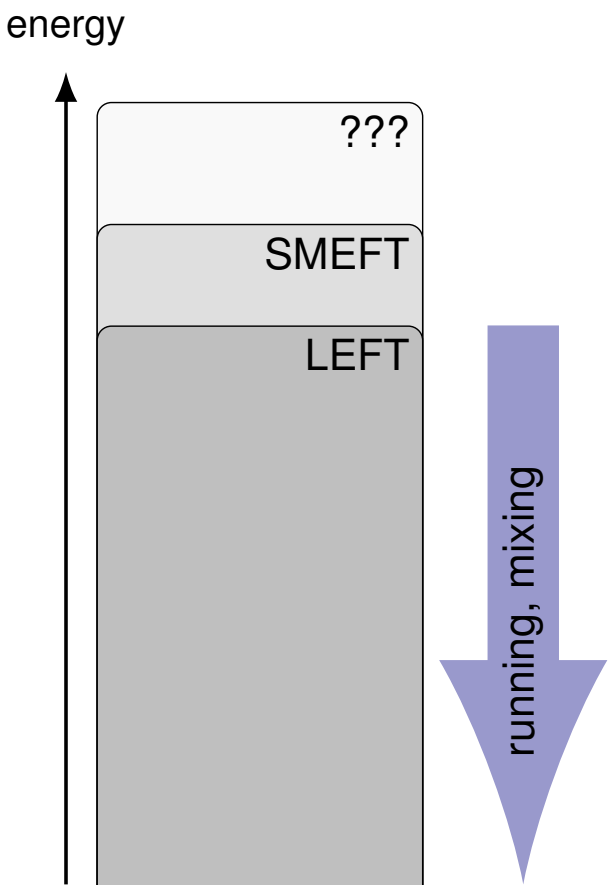
Lepton-flavor violation: $\mu \to e\gamma$

→ Dekens, Jenkins, Manohar, Stoffer, JHEP **01** (2019) 088

- **hadronic effects** can show up in purely leptonic process
- LF violation due to many operators, e.g.

$$\mathcal{O}_{eq}^{S,RR} = (\bar{e}_{Lp} e_{Rr})(\bar{q}_{Ls} q_{Rt})$$
$$\mathcal{O}_{eq}^{V,LL} = (\bar{e}_{Lp} \gamma^\mu e_{Lr})(\bar{q}_{Ls} \gamma_\mu q_{Lt})$$
$$\mathcal{O}_{eq}^{T,RR} = (\bar{e}_{Lp} \sigma^{\mu\nu} e_{Rr})(\bar{q}_{Ls} \sigma_{\mu\nu} q_{Rt})$$

Lepton-flavor violation: $\mu \to e\gamma$



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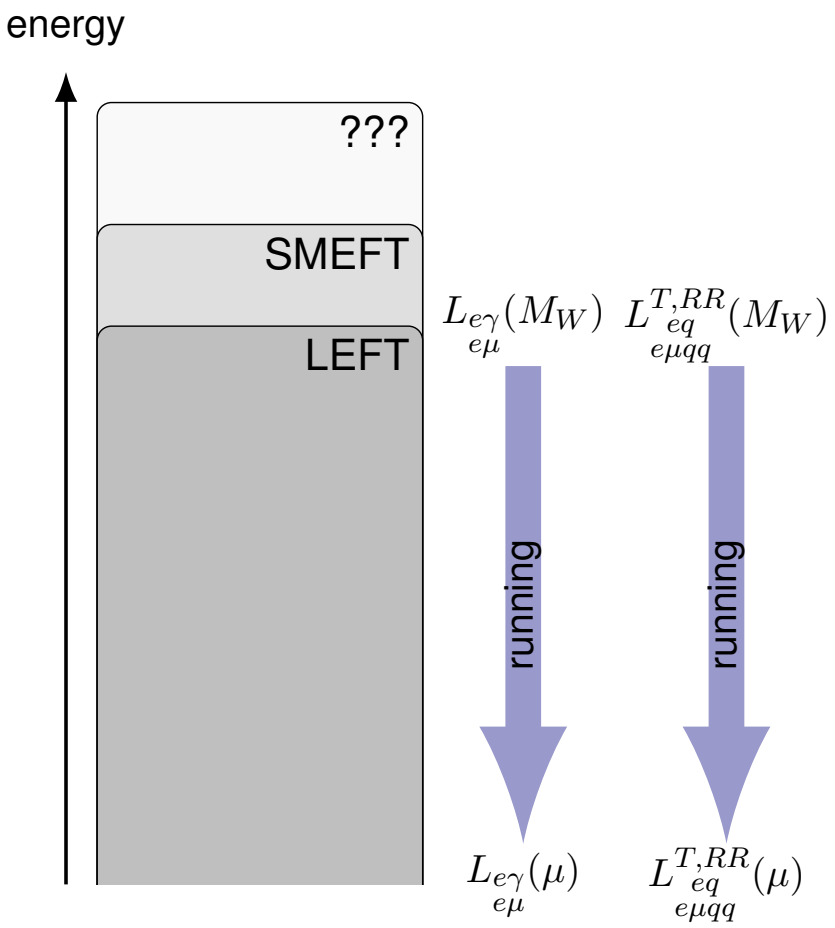

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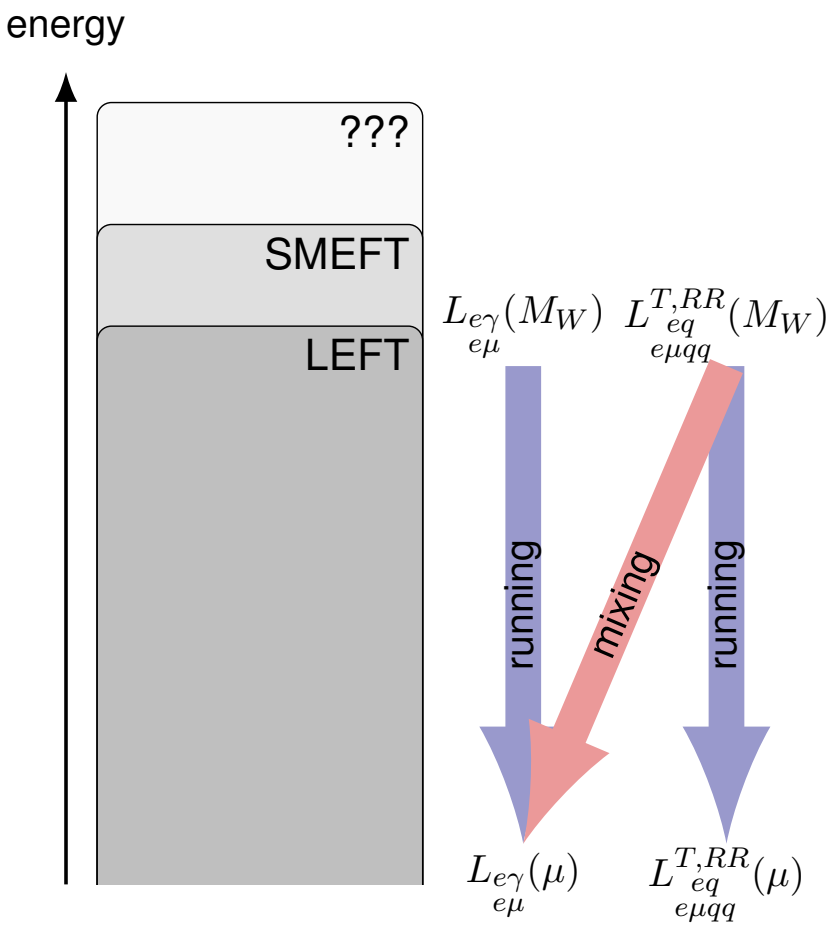
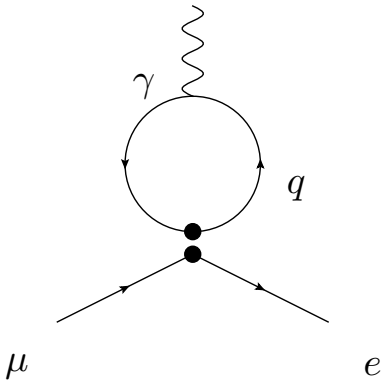
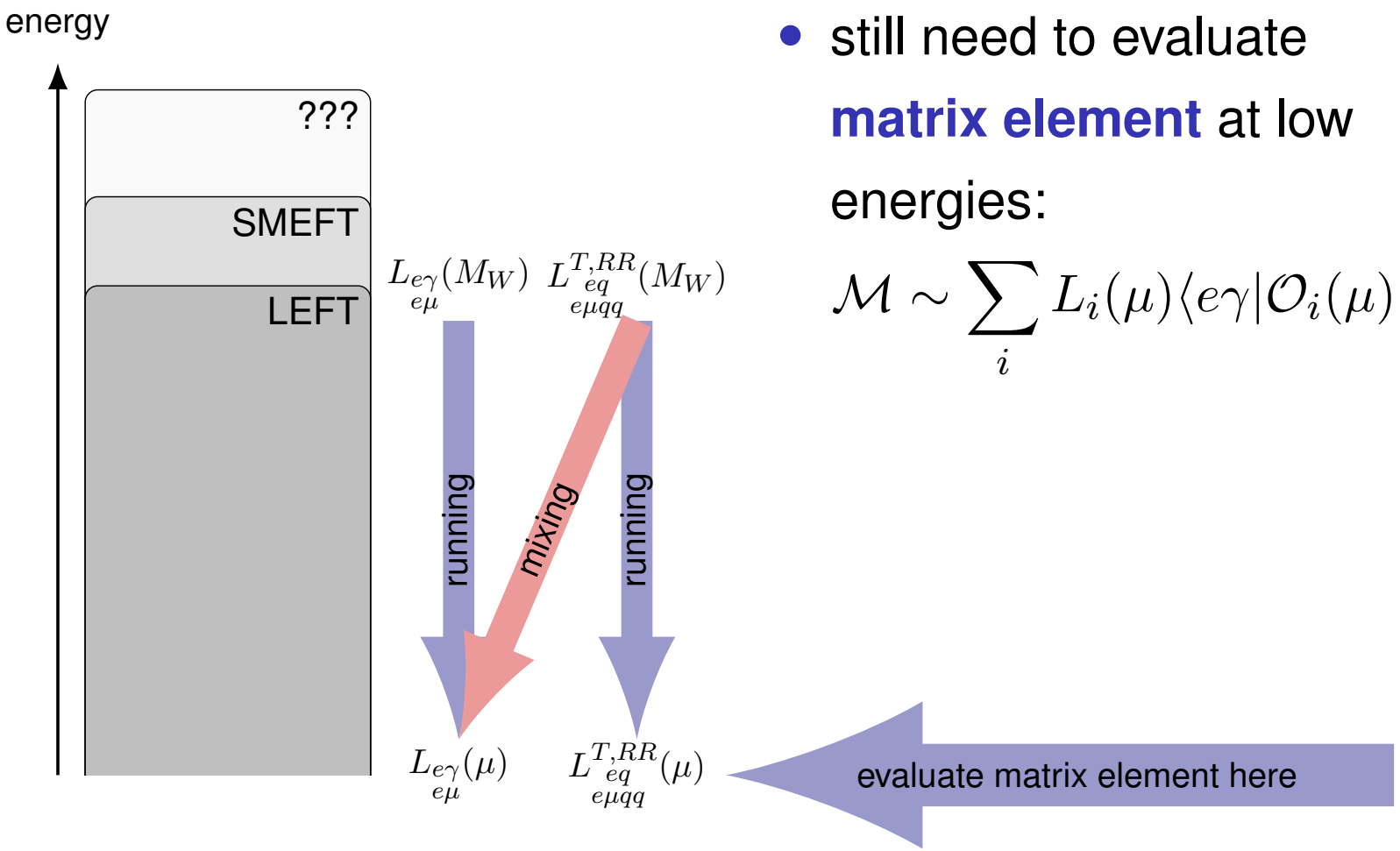


- **perturbative mixing** of semileptonic tensor operators into dipole



- **helicity flip** introduces suppression by light-quark mass

Lepton-flavor violation: $\mu \to e\gamma$

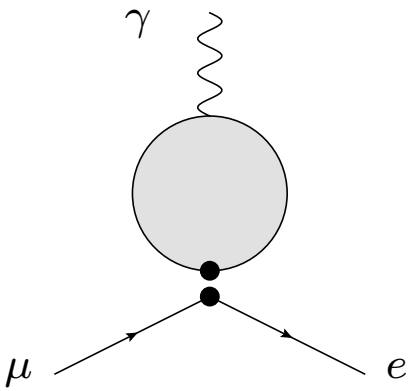


- still need to evaluate **matrix element** at low energies:

$$\mathcal{M} \sim \sum_i L_i(\mu) \langle e\gamma | \mathcal{O}_i(\mu) | \mu \rangle$$

Lepton-flavor violation: $\mu \to e\gamma$

- at lowest order in $\alpha_{\rm QED}$: $\langle\gamma(p,\epsilon)|S|0\rangle$ and $\langle\gamma(p,\epsilon)|V^\mu|0\rangle$ vanish due to Lorentz and gauge invariance
- **semileptonic tensor operators** contribute to $\mu \to e\gamma$:



- non-perturbative effects **not suppressed** by light quark masses

Matching to χPT

- matching of semileptonic operators to χPT is standard: external scalar, vector, and tensor sources

  $\rightarrow$ Gasser, Leutwyler (1984), Catà, Mateu (2007)

- at $\mathcal{O}(p^4)$:

$$\bar{q}_L \sigma^{\mu\nu} t_{\mu\nu} q_R \rightarrow \Lambda_1 \langle t_{\mu\nu} (U F_L^{\mu\nu} + F_R^{\mu\nu} U) \rangle + i\Lambda_2 \langle t^{\mu\nu} D_\mu U U^\dagger D_\nu U \rangle$$

- **no external Goldstone** bosons:

$$(\bar{\mu}_L \sigma^{\mu\nu} e_R)(\bar{q}_L \sigma_{\mu\nu} q_R) \rightarrow -2 Q_q e \, \Lambda_1 \, (\bar{\mu}_L \sigma^{\mu\nu} e_R) F_{\mu\nu} \, , \qquad q = u, d, s$$

- $\Lambda_{1,2}$: **low-energy constants** for χPT with tensor sources. NDA: $\Lambda_1 = c_T \frac{F_\pi}{4\pi}$ with $c_T = \mathcal{O}(1)$

Lepton-flavor violation: $\mu \to e\gamma$

- constraints on SMEFT operators at the weak scale through matching SMEFT $\Rightarrow$ LEFT
- two competing effects:
  - **perturbative RGE** mixing of tensor operators into dipoles when running from $\mu = M_W$ to $\mu = 2\,\text{GeV}$
  - **non-perturbative matching** effect proportional to c_T
- $\mu \to e\gamma$ gives **best limit** for strange-quark operator at the electroweak scale:

$$(c_T - 3.1) L^{T,RR}_{\substack{ed \\ e\mu ss}} < 2.8 \times 10^{-5}\,\text{TeV}^{-2}$$

Lecture 2: applications

Important EFT concepts

- EFT core principles
- Lagrangian construction
- renormalization and matching
- RG improvement

Discussed examples of EFTs

- LEFT, SMEFT, χPT, HEFT
- EFTs ideal framework to **connect different energy regimes** and combine different constraints

Low-energy searches

- low-energy precision searches: need to **control SM prediction**
- process ideally **strongly suppressed** or **forbidden** in SM
- non-perturbative effects at low energies:
  - often dominate SM uncertainty (e.g., $g-2$)
  - hadronic matrix elements of EFT operators required to extract information about EFT coefficients (e.g., nEDM)
  - showing up also as virtual effects in beyond SM contributions (e.g., $\mu \to e\gamma$)
- indirect searches may reach **scales much above** directly accessible energies

Summary and conclusions

Summary and conclusions