



The Cabibbo-Angle Anomaly (and $b \rightarrow s\ell^+\ell^-$)

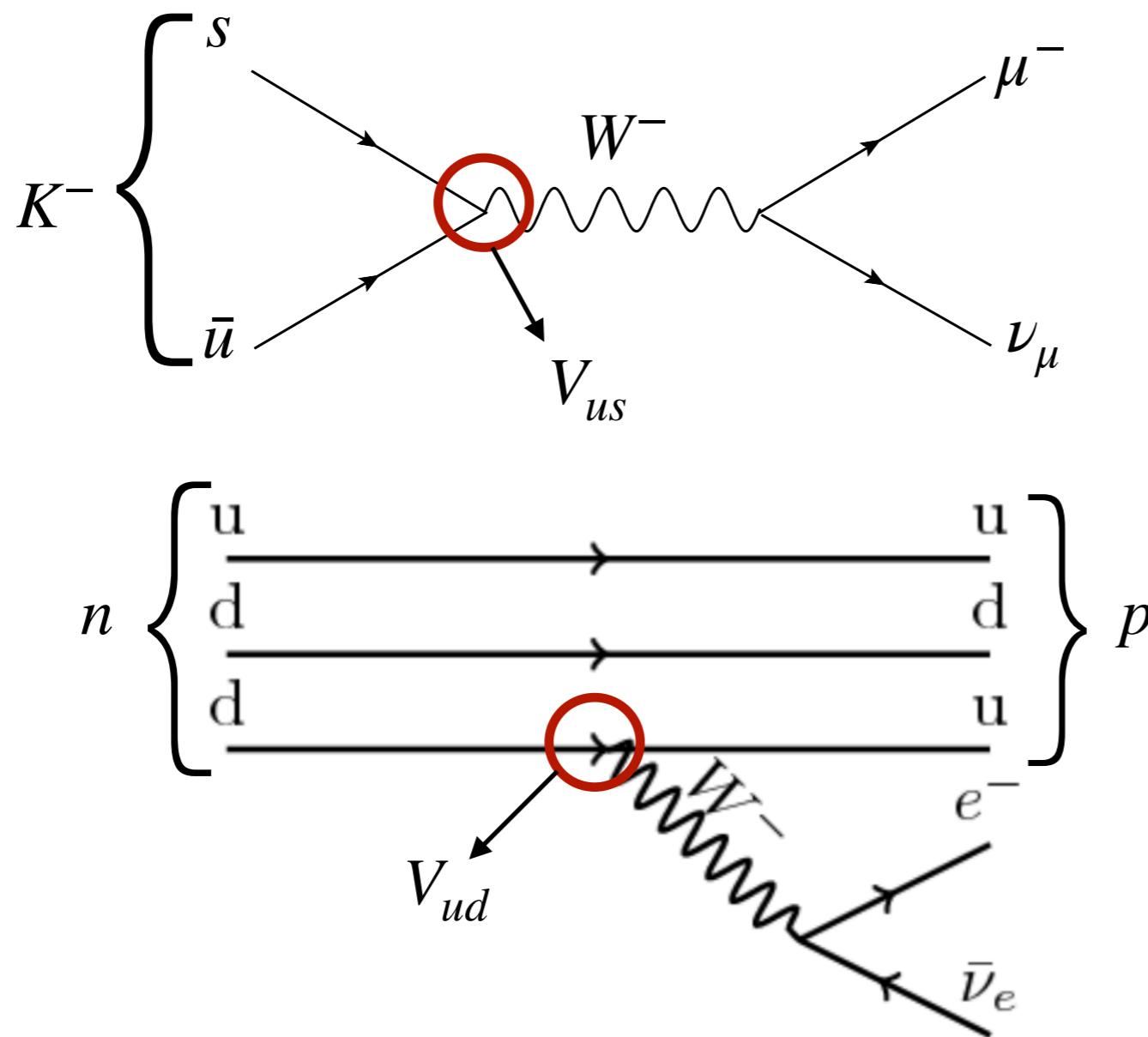
Claudio Andrea Manzari

based on: [1912.08823](#)

preliminary work of B.Capdevila, A.Crivellin, C.A.Manzari, M.Montull

The CKM matrix

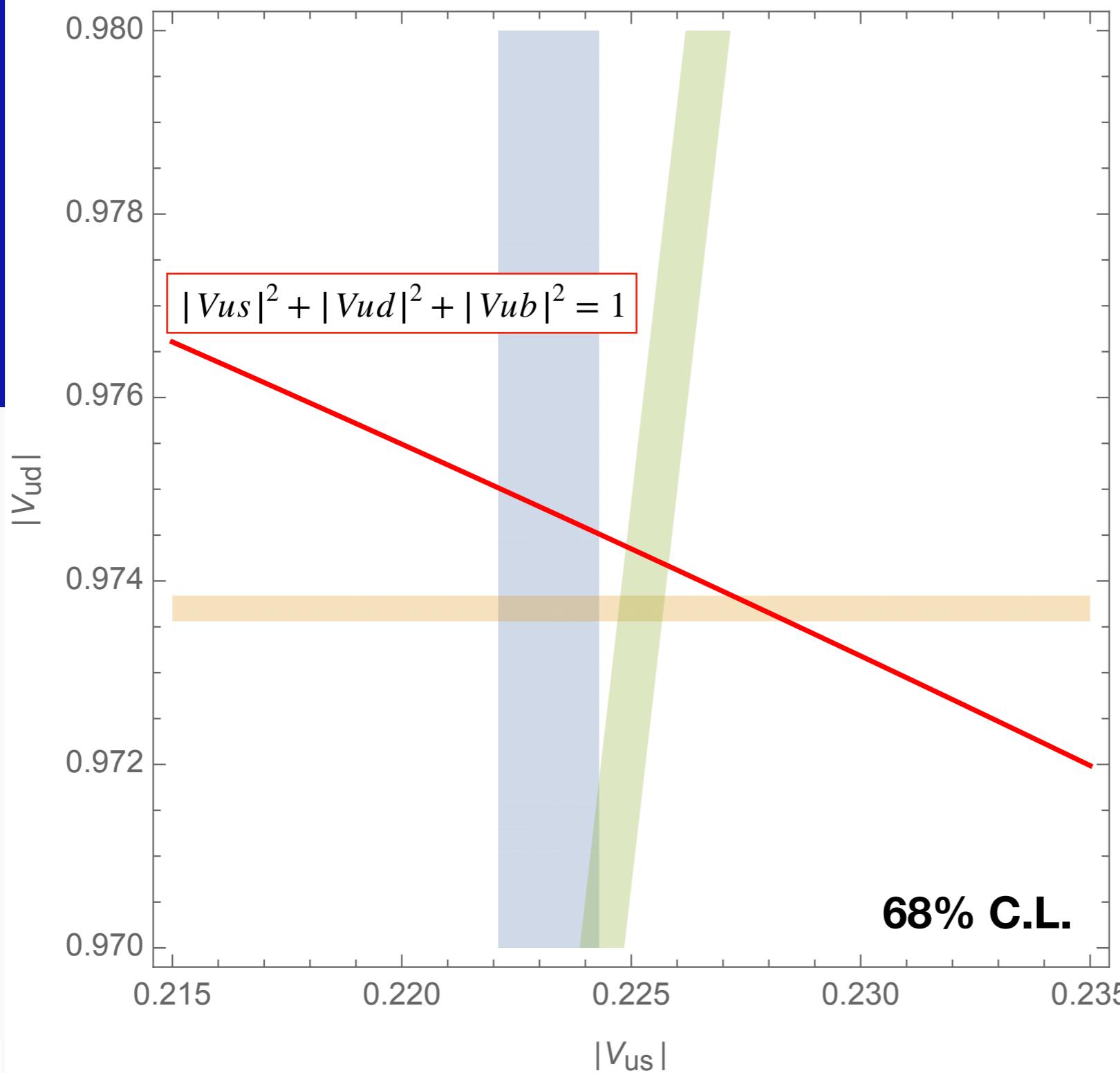
The unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix parametrize the misalignment between the up- and down-quark Yukawa couplings in the physical basis with diagonal mass matrices.



$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

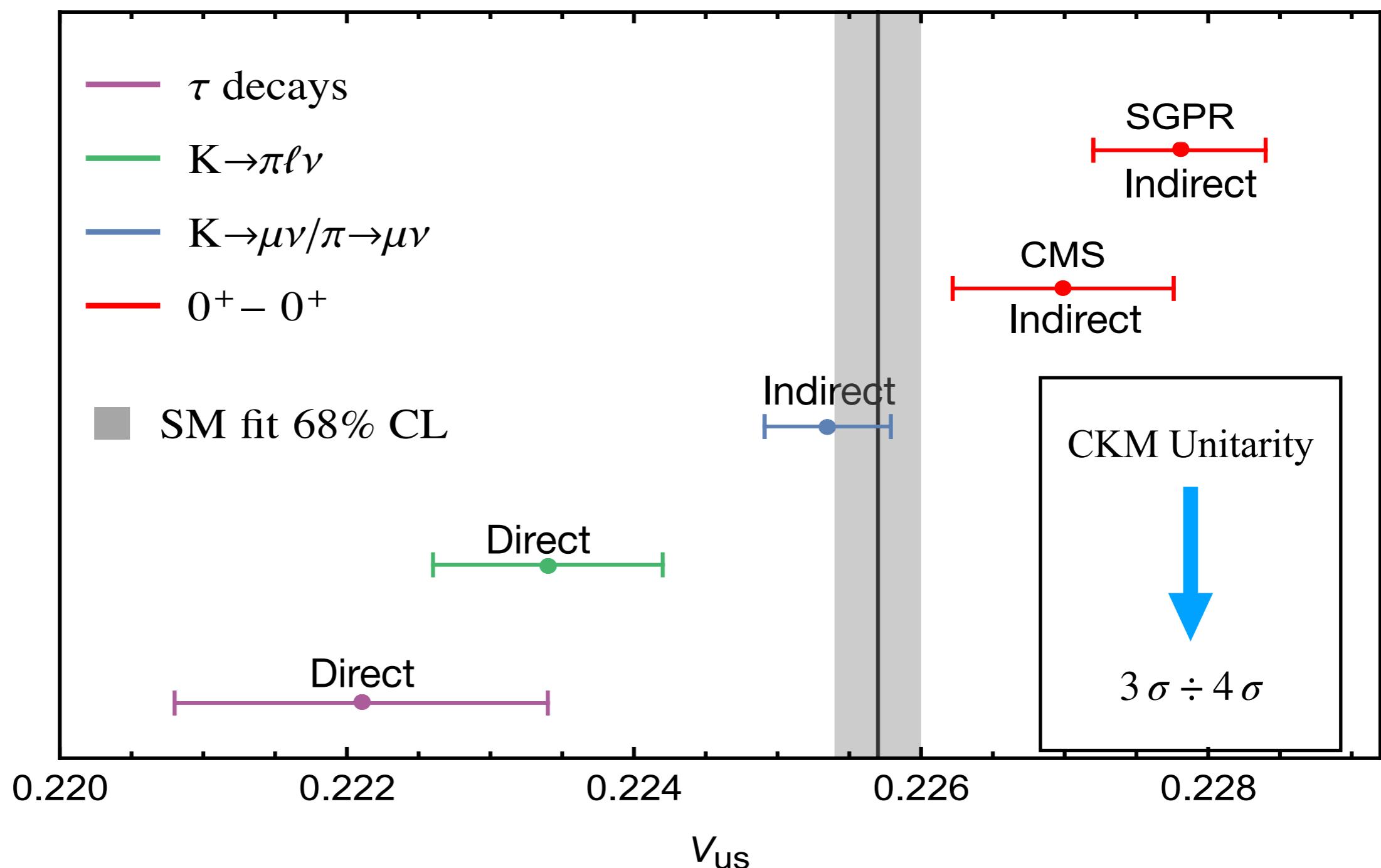
The Anomaly



- $K \rightarrow \pi \ell \nu + f_+(0)$:
 $|V_{us}| = 0.2232(11)$
- $\frac{K \rightarrow \mu \nu}{\pi \rightarrow \mu \nu} + \frac{f_{K^\pm}}{f_{\pi^\pm}}$
 $\left| \frac{V_{us}}{V_{ud}} \right| = 0.2313(5)$
- $0^+ - 0^+ + \text{corrections}$
 $|V_{ud}|_{CMS} = 0.97389(18)$
 $|V_{ud}|_{SGPR} = 0.97370(14)$

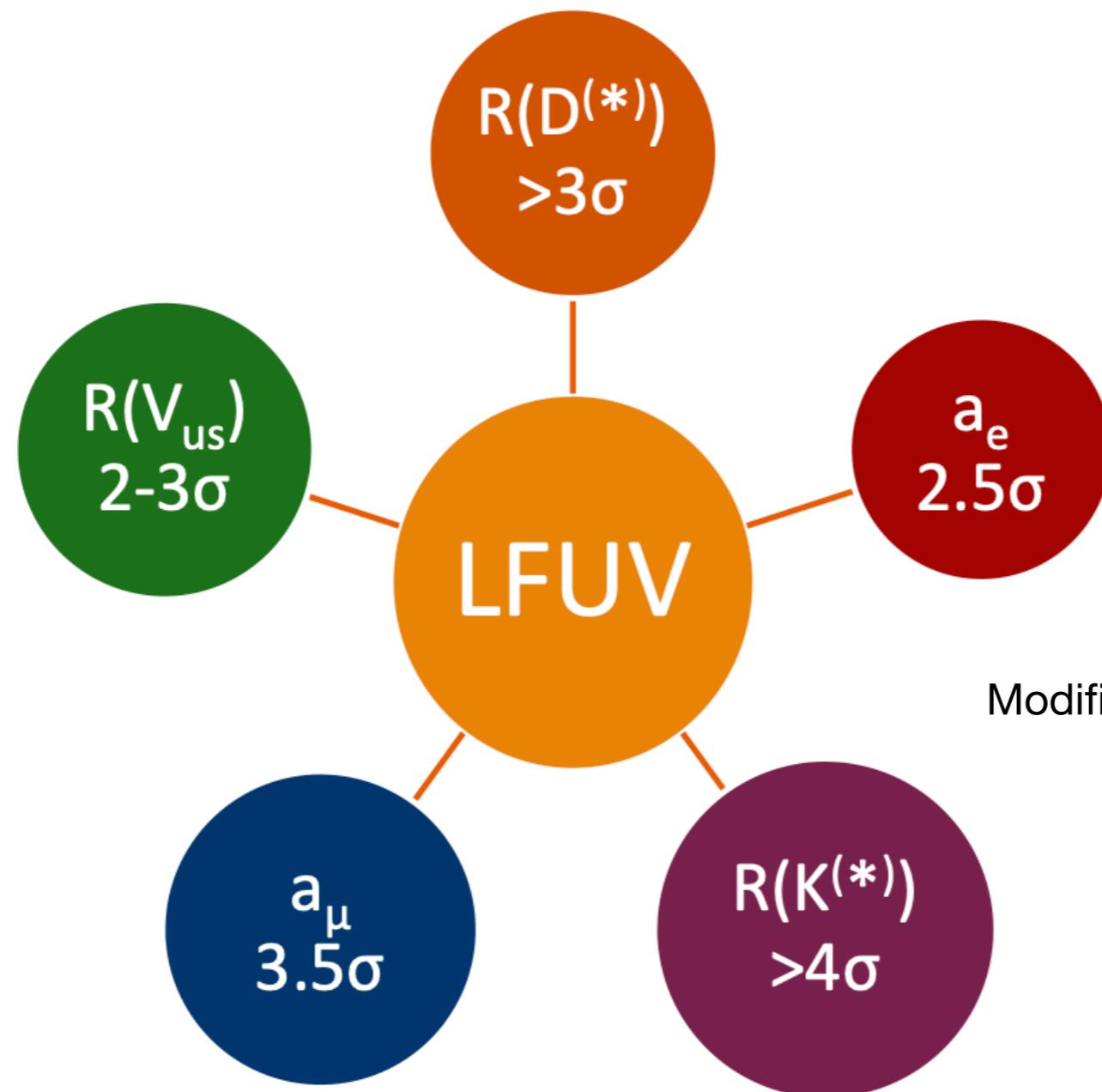
The Anomaly

There is a tension between the different determinations of V_{us}



LFUV

$R(V_{us}) = \frac{V_{us}^{K_{\mu 2}}}{V_{us}^{\beta}}$ as a test of LFUV complements to an already interesting picture



$$R(V_{us}) \Big|_{\text{SM}} = 1$$

$$R(V_{us}) \simeq 1 + \underbrace{\left(\frac{V_{ud}^{\mathcal{L}}}{V_{us}^{\mathcal{L}}} \right)}_{\sim 20} \epsilon_{\mu\mu}$$

Modification of the coupling of the W with muons

Modified Neutrino Couplings

Minimal impact: we modify only the couplings of W and Z with neutrinos

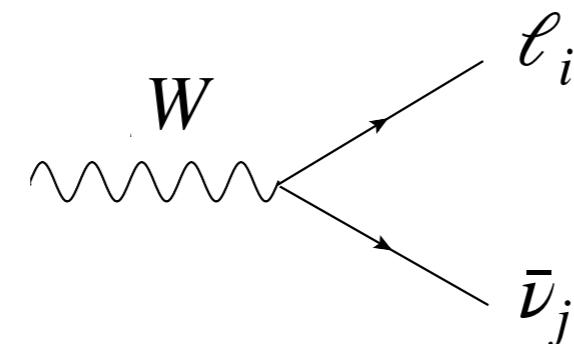


- EW observables
- Low energy observables (K, π, τ, W decays)

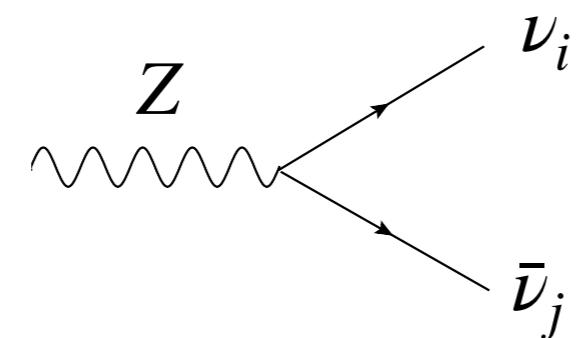
There is 1 Operator which modifies only neutrino couplings :

dim = 6

$$\bar{L}_i \gamma_\mu \tau^I L_j H^\dagger i D_I^\mu H \quad \text{with} \quad \tau^I = (1, -\sigma_1, -\sigma_2, -\sigma_3)$$



$$\frac{-ig_2}{\sqrt{2}} \Rightarrow \frac{-ig_2}{\sqrt{2}} \left(\delta_{ij} + \frac{1}{2} \epsilon_{ij} \right)$$



$$\frac{-ig_2}{2c_W} \Rightarrow \frac{-ig_2}{2c_W} \left(\delta_{ij} + \epsilon_{ij} \right)$$

LFV Parameters

Non-diagonal elements of ϵ_{ij} lead to charged lepton flavour violation

$$\text{Br}[\mu \rightarrow e\gamma] \rightarrow |\epsilon_{e\mu}| \leq 10^{-5}$$

$$\text{Br}[\tau \rightarrow \mu\gamma] \rightarrow |\epsilon_{\tau\mu}| \leq 10^{-2}$$

$$\text{Br}[\tau \rightarrow e\gamma] \rightarrow |\epsilon_{\tau e}| \leq 10^{-2}$$

In flavour conserving processes do not interfere with the SM contributions, and enter only quadratically, therefore they are further suppressed.

Neglected in what follows

Parameters and Observables

NP Parameters :

$$\epsilon_{ee}, \quad \epsilon_{\mu\mu}, \quad \epsilon_{\tau\tau}$$

EW Parameters

$$G_F, \quad \alpha, \quad M_Z$$

$$G_F^{\text{exp}} = G_F^{\mathcal{L}} \left(1 + \frac{1}{2} \epsilon_{ee} + \frac{1}{2} \epsilon_{\mu\mu} \right)$$

V_{us} Observables

Not affected

$$|V_{us}^{K_{\mu^3}}| \approx |V_{us}^{\mathcal{L}}| \left(1 - \frac{1}{2} \epsilon_{ee} \right)$$

$$|V_{us}^{\beta}| \approx \sqrt{1 - |V_{ud}^{\mathcal{L}}|^2} \left(1 - \frac{1}{2} \epsilon_{\mu\mu} \right)^2$$

$$|V_{us}^{\tau \rightarrow K\nu}| \approx |V_{us}^{\mathcal{L}}| \left(1 - \frac{1}{2} \epsilon_{ee} - \frac{1}{2} \epsilon_{\mu\mu} + \frac{1}{2} \epsilon_{\tau\tau} \right)$$

$$|V_{us}^{K/\pi}|$$

$$|V_{us}^{\tau \rightarrow K/\pi}|$$

$$|V_{us}^{\tau \rightarrow X\nu}|$$

Parameters and Observables

Low Energy Observables

These measurements together with the EW precision tests constraint the size of our parameters

$$\frac{\pi \rightarrow \mu\nu}{\pi \rightarrow e\nu} \sim \frac{\pi \rightarrow \mu\nu}{\pi \rightarrow e\nu} \Bigg|_{\text{SM}} (1 + \frac{1}{2}\epsilon_{\mu\mu} - \frac{1}{2}\epsilon_{ee})$$

$$\left\{ \begin{array}{ll} \frac{K \rightarrow \mu\nu}{K \rightarrow e\nu} & \frac{\tau \rightarrow \mu\nu\nu}{\tau \rightarrow e\nu\nu} \\ \frac{K \rightarrow \pi\mu\nu}{K \rightarrow \pi e\nu} & \frac{W \rightarrow \mu\nu}{W \rightarrow e\nu} \end{array} \right.$$

$$\frac{\tau \rightarrow e\nu\nu}{\mu \rightarrow e\nu\nu} \sim \frac{\tau \rightarrow e\nu\nu}{\mu \rightarrow e\nu\nu} \Bigg|_{\text{SM}} (1 + \frac{1}{2}\epsilon_{\tau\tau} - \frac{1}{2}\epsilon_{\mu\mu})$$

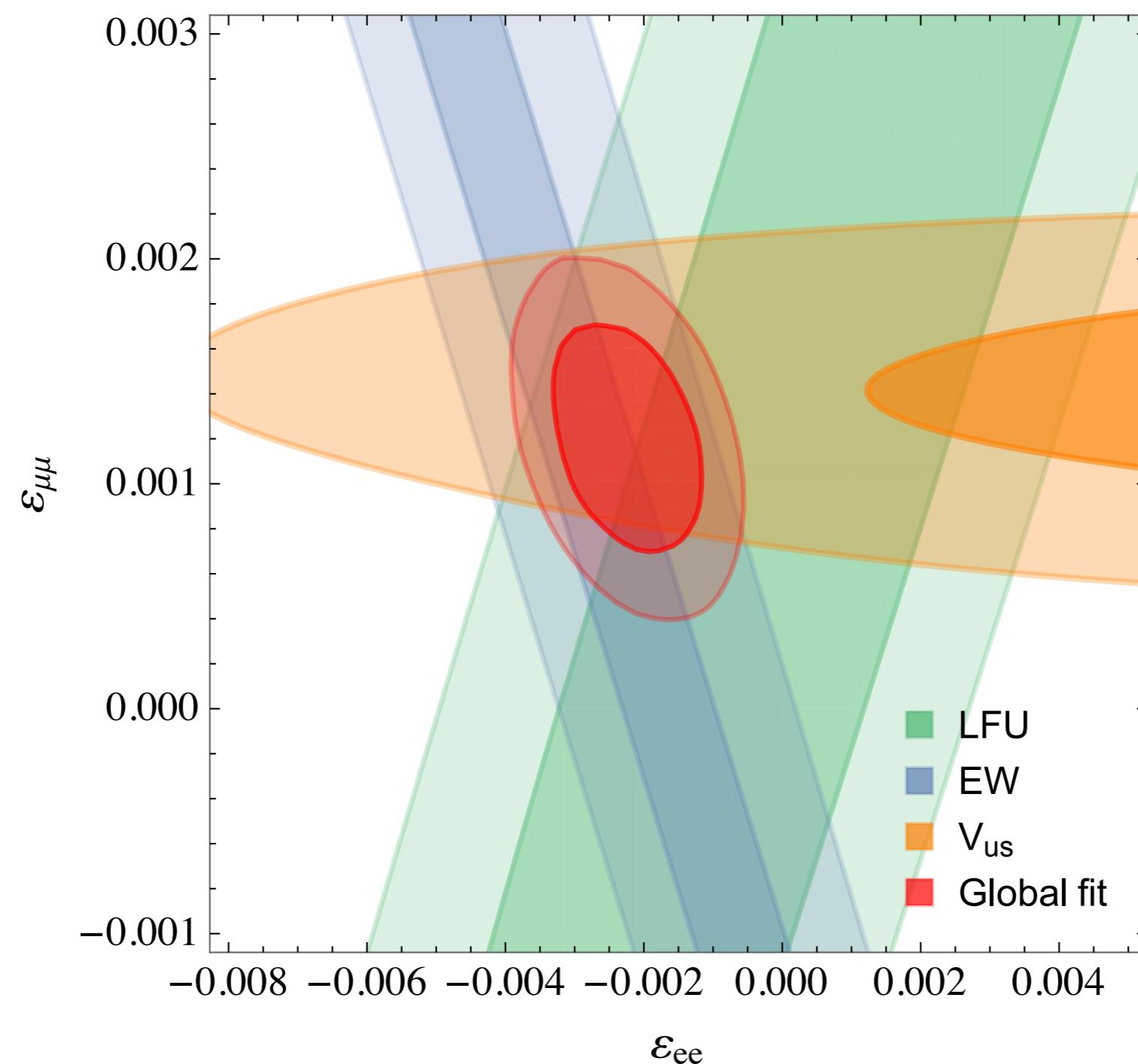
$$\left\{ \begin{array}{ll} \frac{\tau \rightarrow \pi\nu}{\pi \rightarrow \mu\nu} & \frac{\tau \rightarrow K\nu}{K \rightarrow \mu\nu} \\ \frac{W \rightarrow \tau\nu}{W \rightarrow \mu\nu} & \end{array} \right.$$

$$\frac{\tau \rightarrow \mu\nu\nu}{\mu \rightarrow e\nu\nu} \sim \frac{\tau \rightarrow \mu\nu\nu}{\mu \rightarrow e\nu\nu} \Bigg|_{\text{SM}} (1 + \frac{1}{2}\epsilon_{\tau\tau} - \frac{1}{2}\epsilon_{ee})$$

$$\left\{ \frac{W \rightarrow \tau\nu}{W \rightarrow e\nu} \right.$$

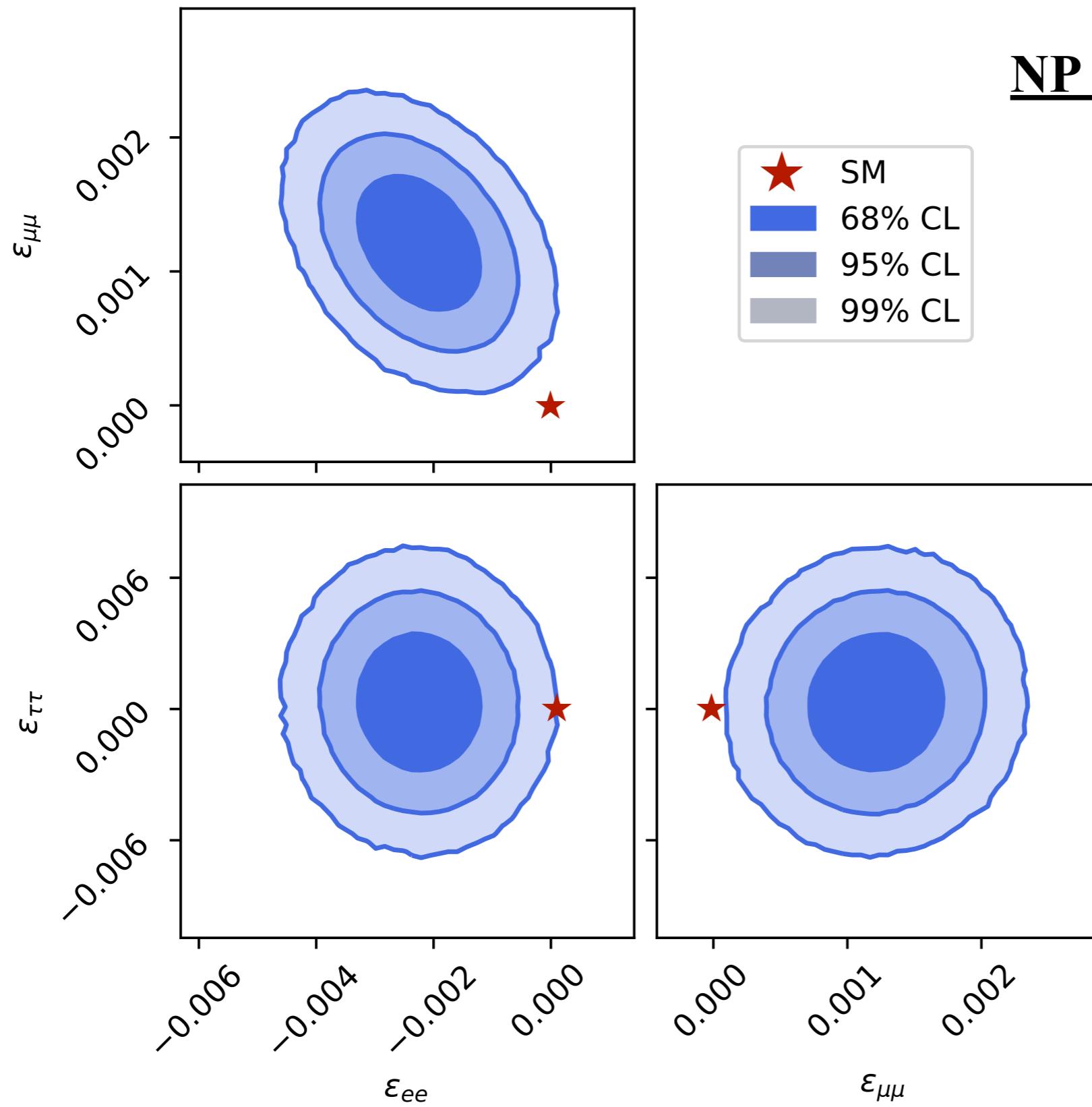
A global fit to all the data is necessary!

Contributions to the Fit



Contributions to the global fit from each class of observables. 1σ and 2σ regions are shown in the ϵ_{ee} vs $\epsilon_{\mu\mu}$ plane, marginalising over $\epsilon_{\tau\tau}$.

Global Fit

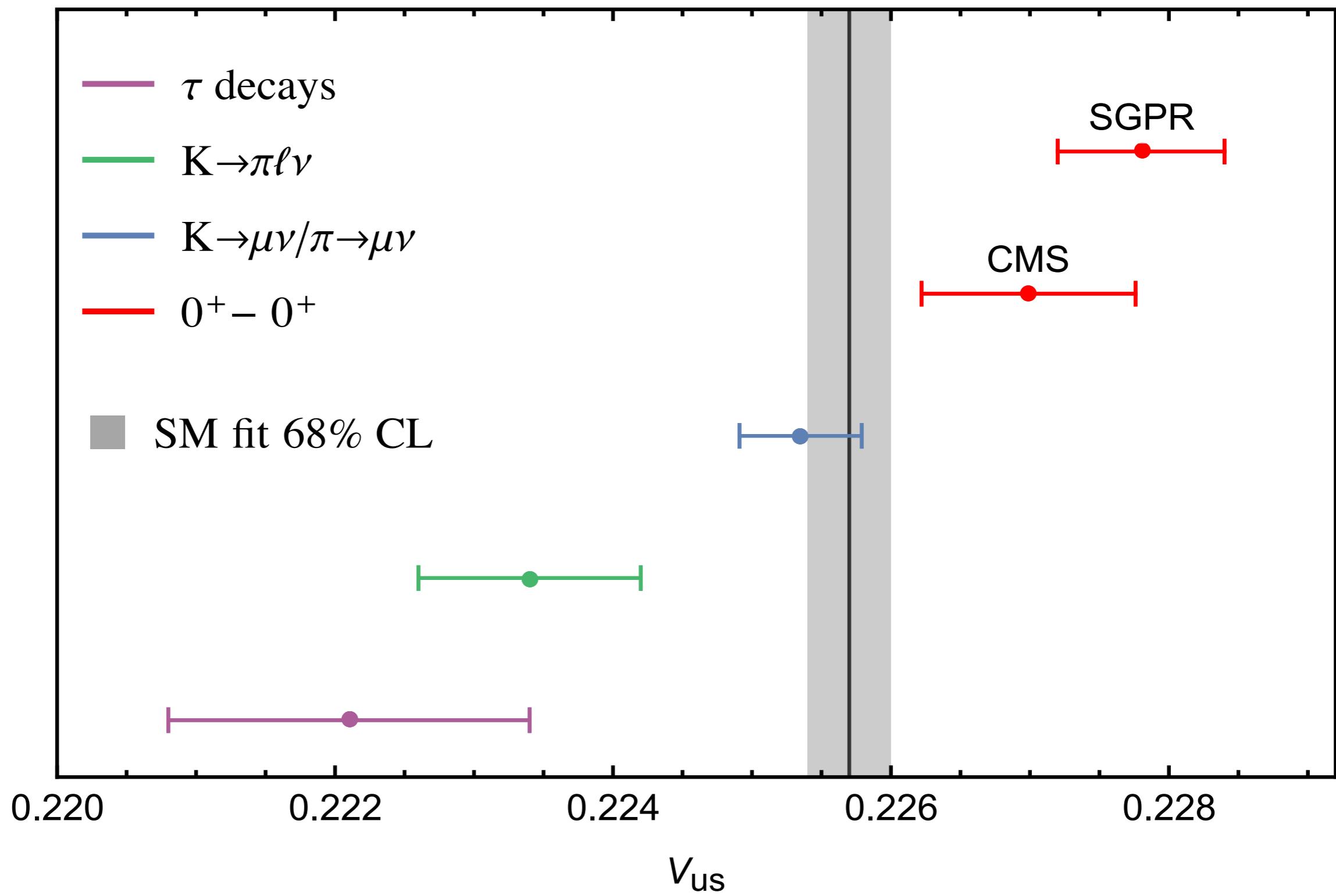


NP preferred at more than
99% C.L.

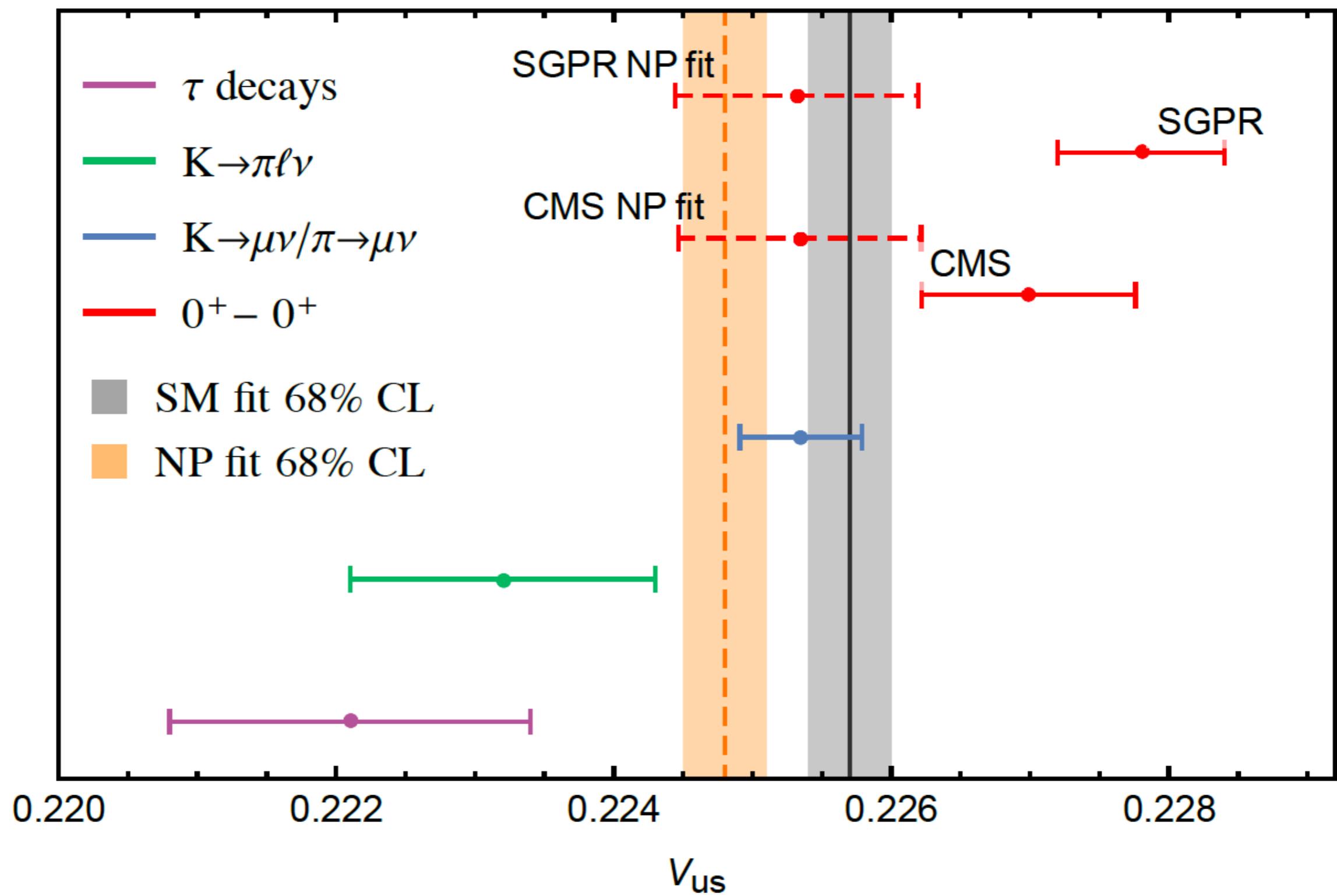
$IC_{SM} \approx 73$

$IC_{NP} \approx 63$

Global Fit

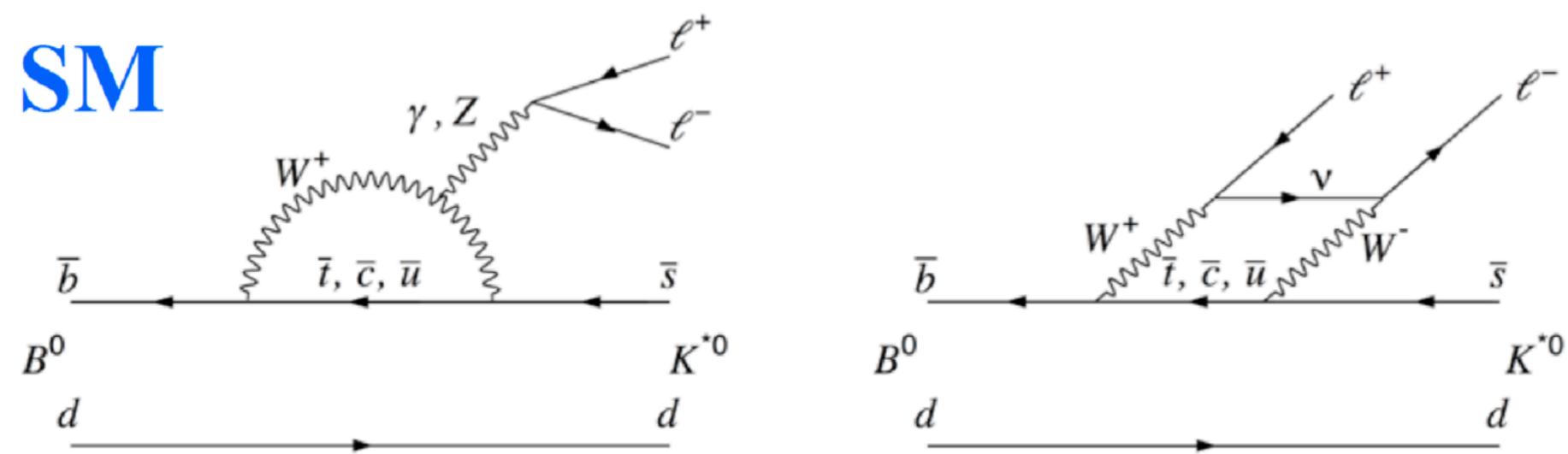


Global Fit



Combining CAA with $b \rightarrow s\ell^+\ell^-$

Many observables related to the flavour-changing neutral-current transition $b \rightarrow s\ell^+\ell^-$ exhibit deviations from SM expectations.



Due to their suppression in the SM, they have a high sensitivity to potential NP contributions.

To perform a global fit to all the data we work within the model-independent approach of the effective Hamiltonian:

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i C_i O_i$$

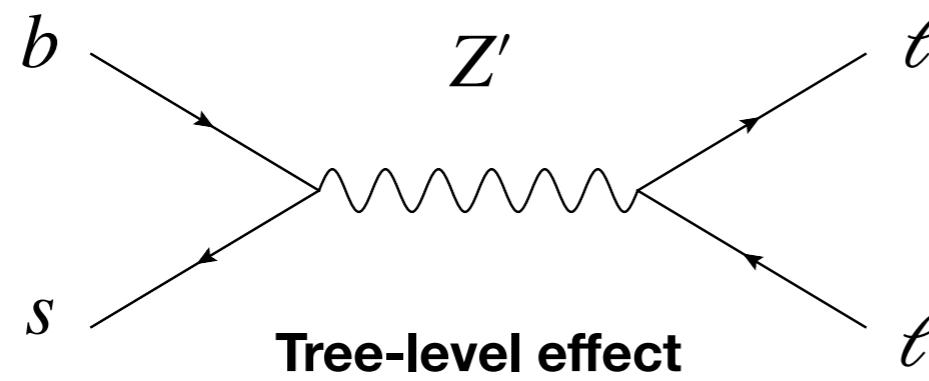
$$O_9 = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma_\mu \ell)$$

$$O_{10} = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma_\mu \gamma_5 \ell)$$

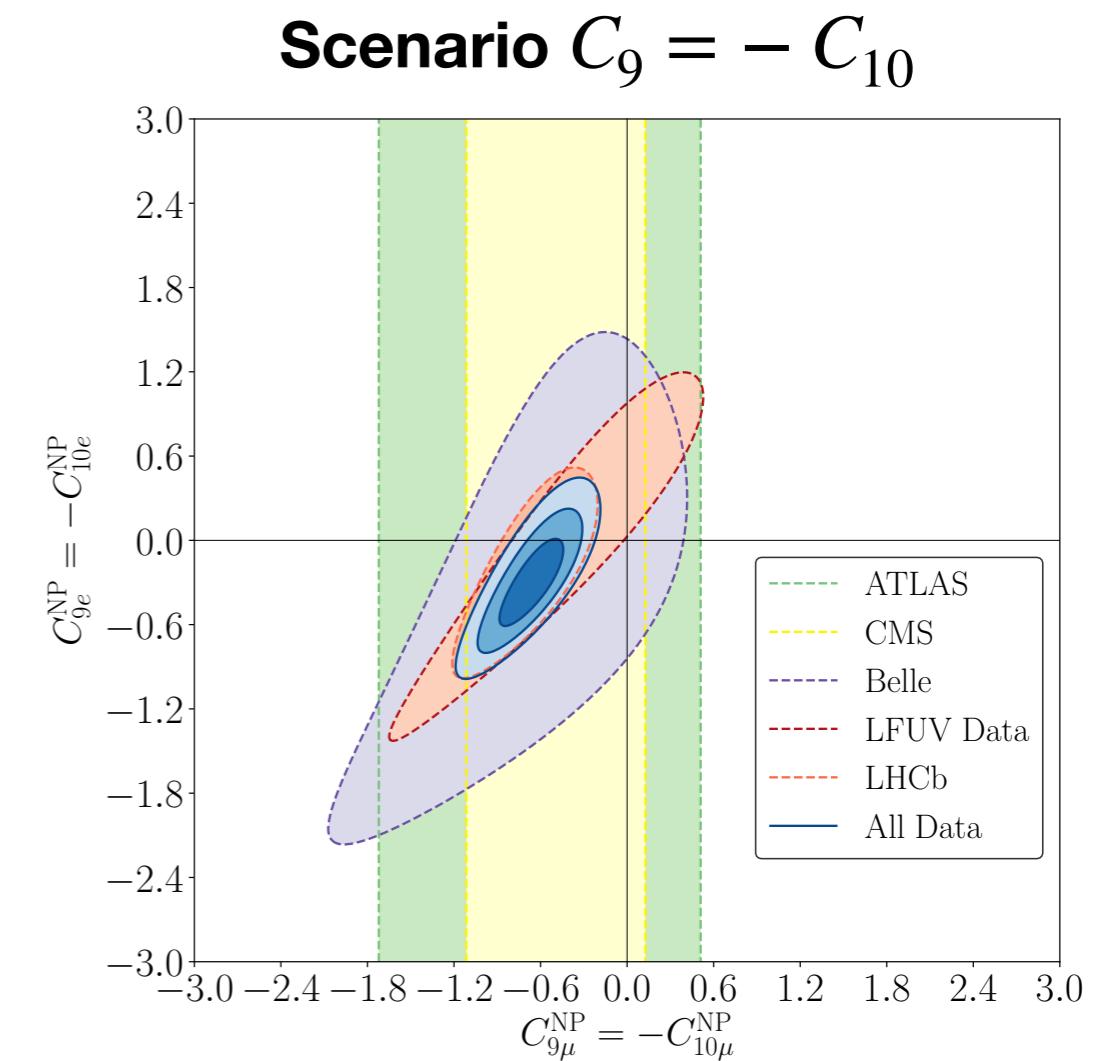
Z' and W'

Z' models provide a promising solution to $b \rightarrow s\ell\ell$, but we need:

1. flavour violating couplings to quarks;
2. non-universal couplings to leptons;



Tree-level effect

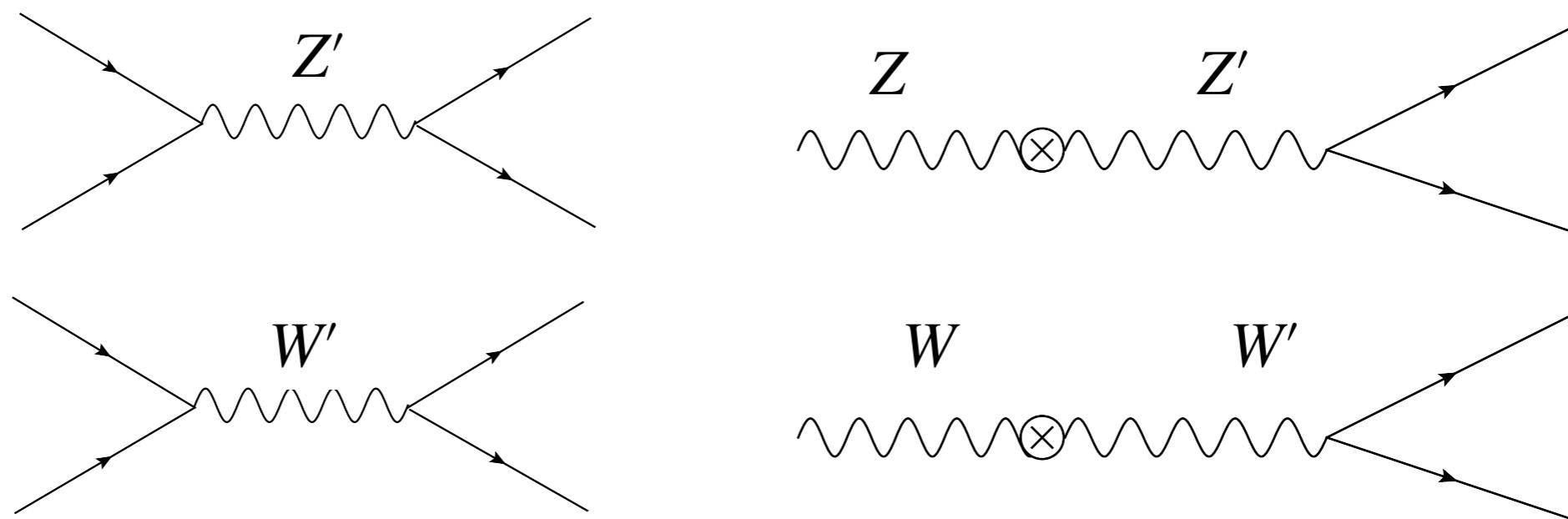


Is there a correlation between the Cabibbo Angle Anomaly and $b \rightarrow s\ell\ell$?

Prompted by the Z' , we can attempt to solve **the CAA anomaly with a W'**

The Vector Triplet Model

A new heavy Vector Triplet coupling to left-handed fermions provides an interesting solution



Deeper impact: **modified W and Z couplings & direct effects from W' and Z'**



- EW observables
- Low energy observables (K, π, τ, W decays)
- $b \rightarrow s\ell\ell$

The Vector Triplet Model

$$\mathcal{L}_X^{\text{int}} = -g_{ji}^\ell X_a^\mu \bar{\ell}_j \gamma_\mu \frac{\sigma^a}{2} \ell_i - g_{ji}^q X_a^\mu \bar{q}_j \gamma_\mu \frac{\sigma^a}{2} q_i - \left(i g_X^{D\phi} X_a^\mu \phi^\dagger \frac{\sigma^a}{2} D_\mu \phi + \text{h.c.} \right)$$

NP Parameters :

$$g_{ee}^\ell, \quad g_{\mu\mu}^\ell, \quad g_{\tau\tau}^\ell, \quad g^q, \quad g_X^{D\phi}$$

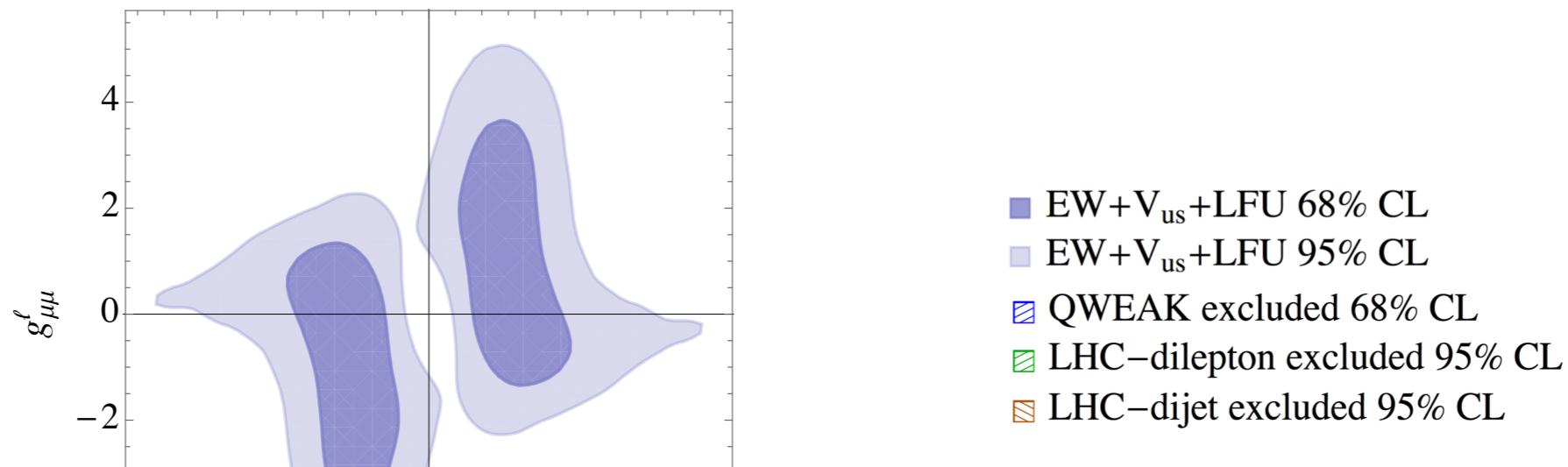
EW Parameters :

$$G_F, \quad \alpha, \quad M_Z$$

$$M_Z^{\text{exp}} = M_Z^{\mathcal{L}} \sqrt{\left(1 - \frac{|g_X^{D\phi}|^2 v^2}{4M_X^2} \right)}$$

$$G_F^{\text{exp}} = G_F^{\mathcal{L}} \left(1 + \frac{g_X^{D\phi} (g_{11}^\ell + g_{22}^\ell) v^2}{2M_X^2} \right) + \frac{g_{11}^\ell g_{22}^\ell}{4\sqrt{2}M_X^2}$$

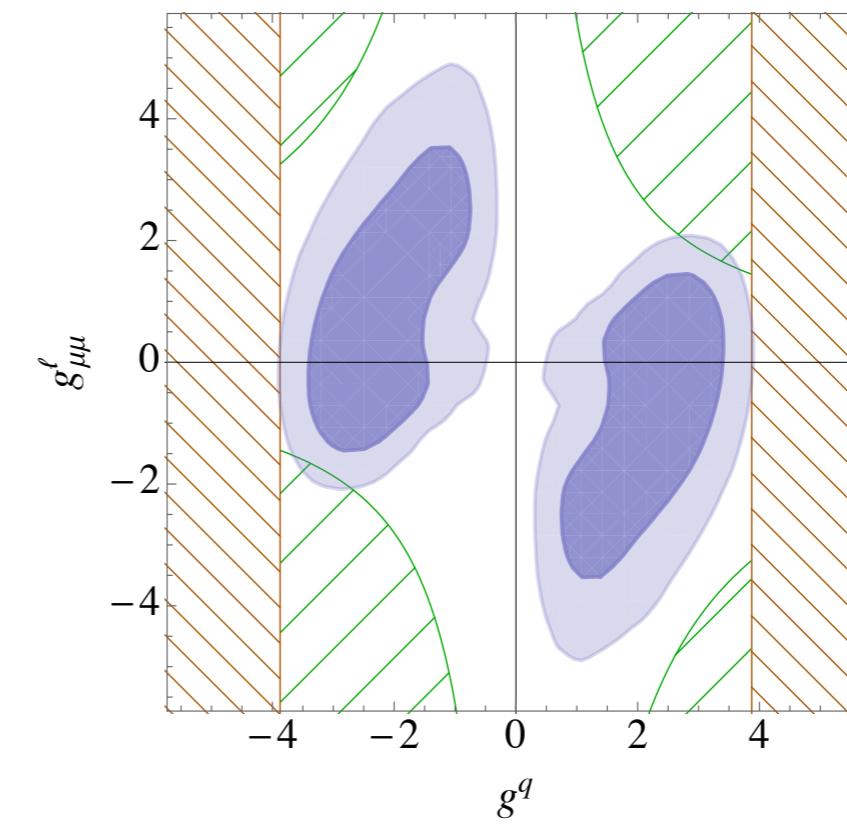
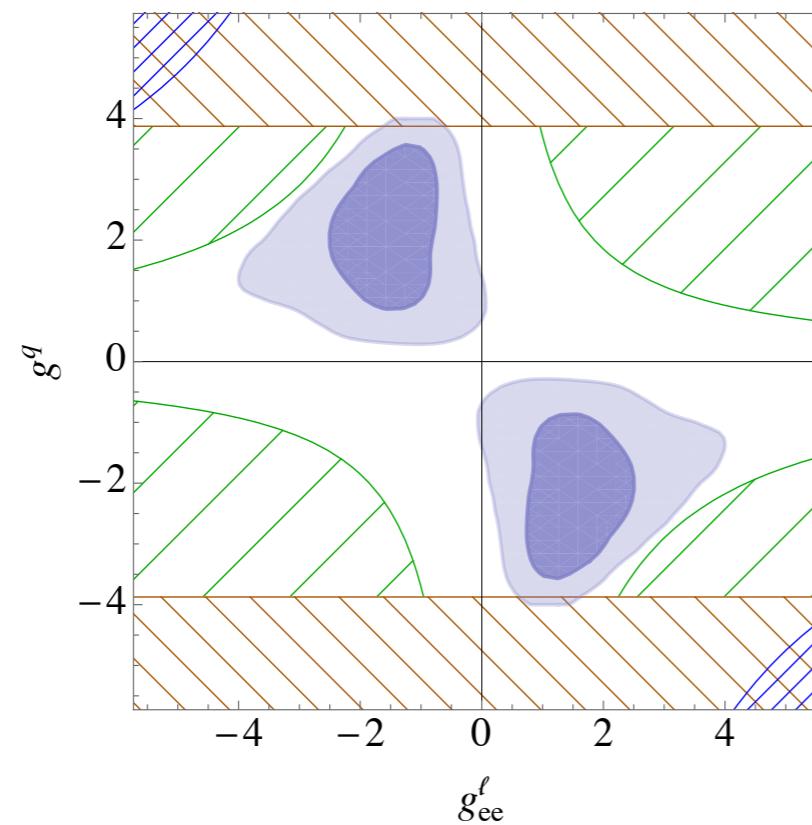
EW+LFU+ V_{us}



$IC_{SM} \simeq 110.61$

$IC_{NP} \simeq 96.453$

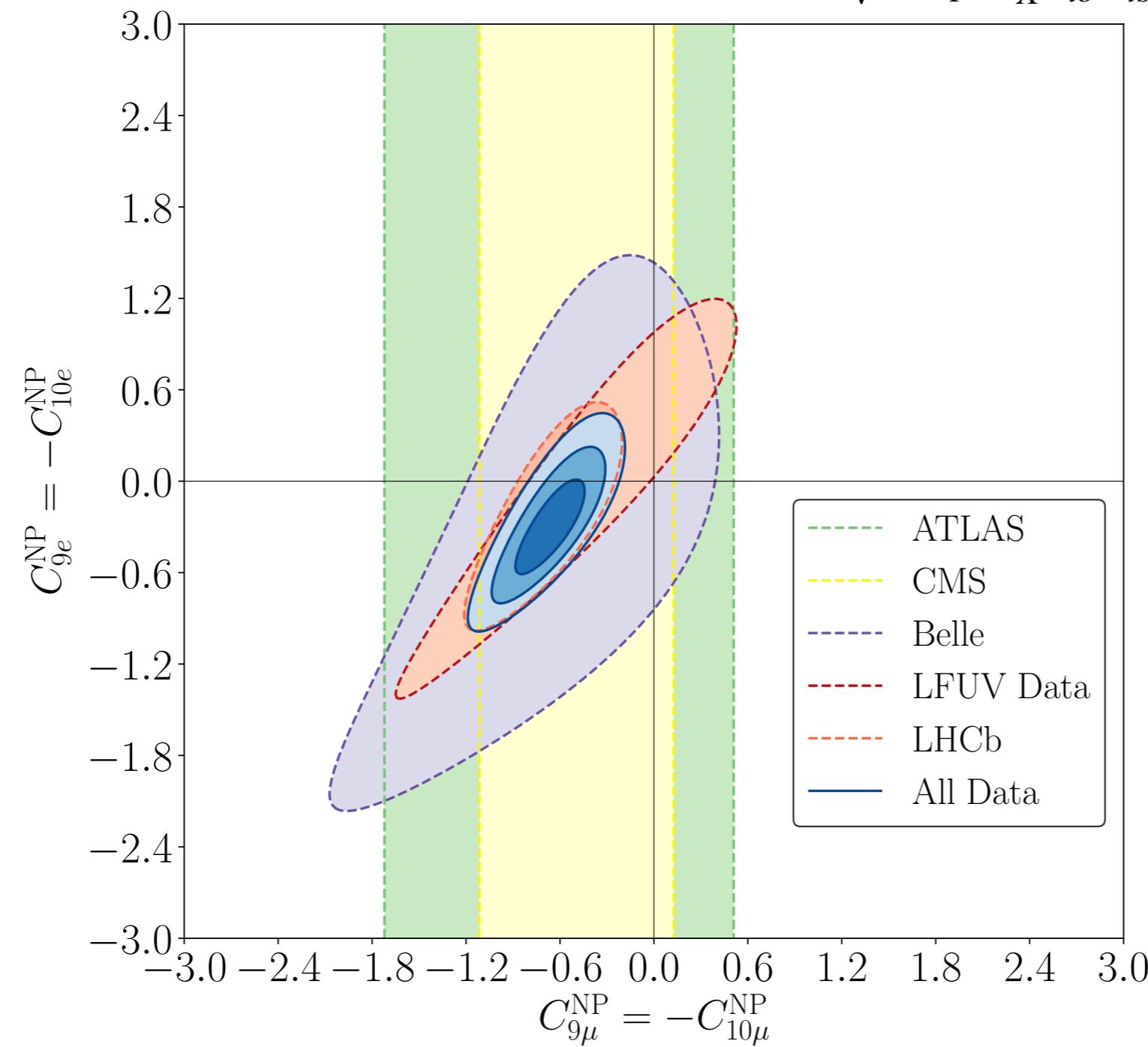
Preliminary



$b \rightarrow s\ell\ell$

Is there a connection between the Cabibbo Angle Anomaly and $b \rightarrow s\ell\ell$?

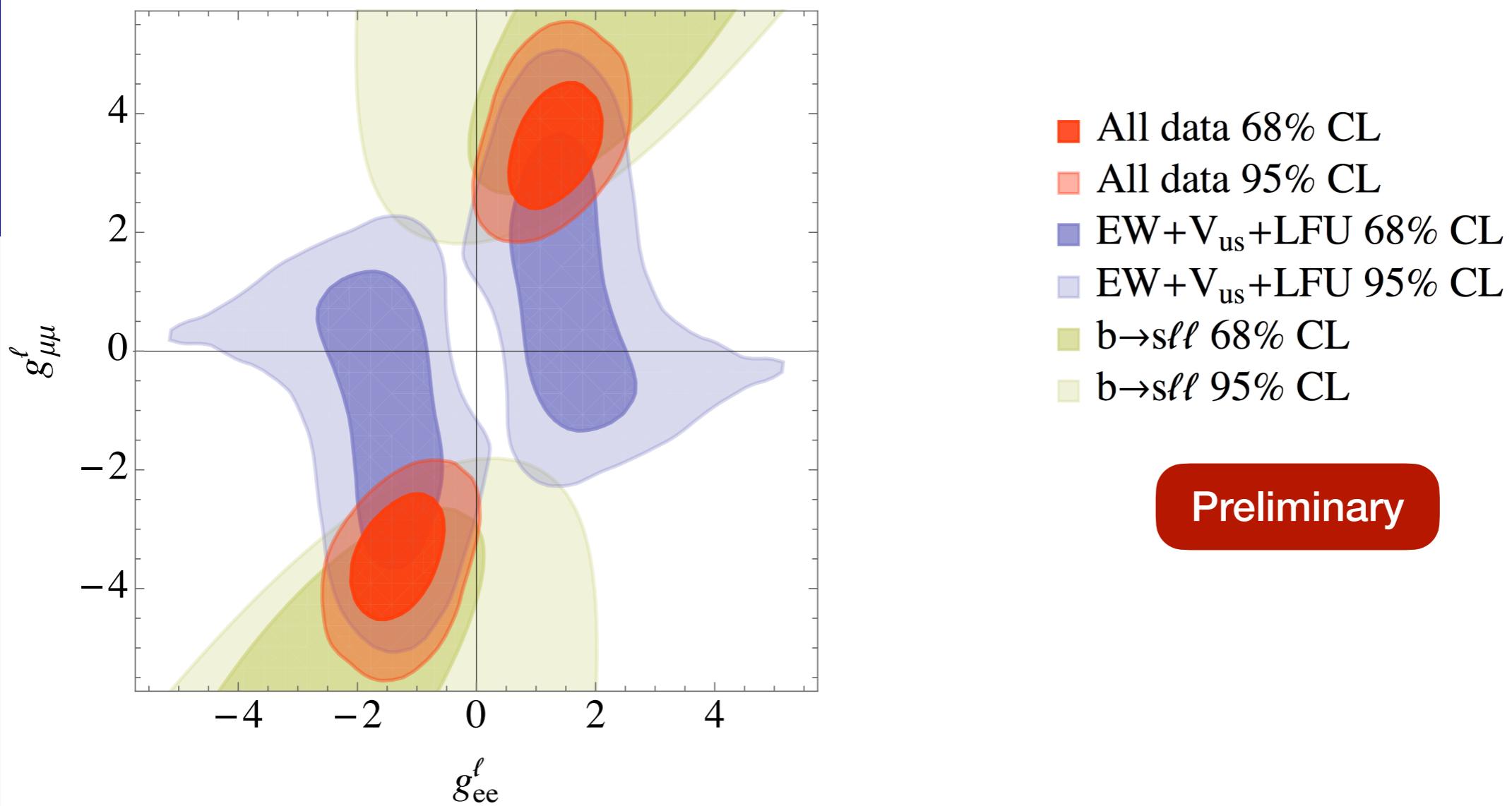
$$C_{9,j}^{NP} = -C_{10,j}^{NP} = -\frac{16\pi^2}{e^2} \frac{g_{23}^d g_{jj}^\ell}{16\sqrt{2} G_F M_X^2 V_{tb} V_{ts}^*}$$



Global Fit

$IC_{SM} \simeq 167.72$

$IC_{NP} \simeq 102.25$



Conclusions (I)

- There is a tension in the determination of V_{us} from different processes
- It can be seen as an evidence of LFUV completing an already interesting picture
- We tried to solve the tension modifying the couplings of neutrinos with gauge bosons
- The global fit to EW, LFU and V_{us} prefers LFUV NP at more than 99% C.L.

Conclusions (II)

- We tried to solve the anomaly with a different tree-level effects, and the Vector Triplet model turned to be a good candidate
- With this simplified model, we are able to explain $b \rightarrow s\ell\ell$ and CAA simultaneously

This results are of notable importance for research at PSI, since they emphasise the need for precise tests of LFU

Example

$$R_{\mu/e}^{\pi, \text{exp}} = 1.0010 \pm 0.0009$$

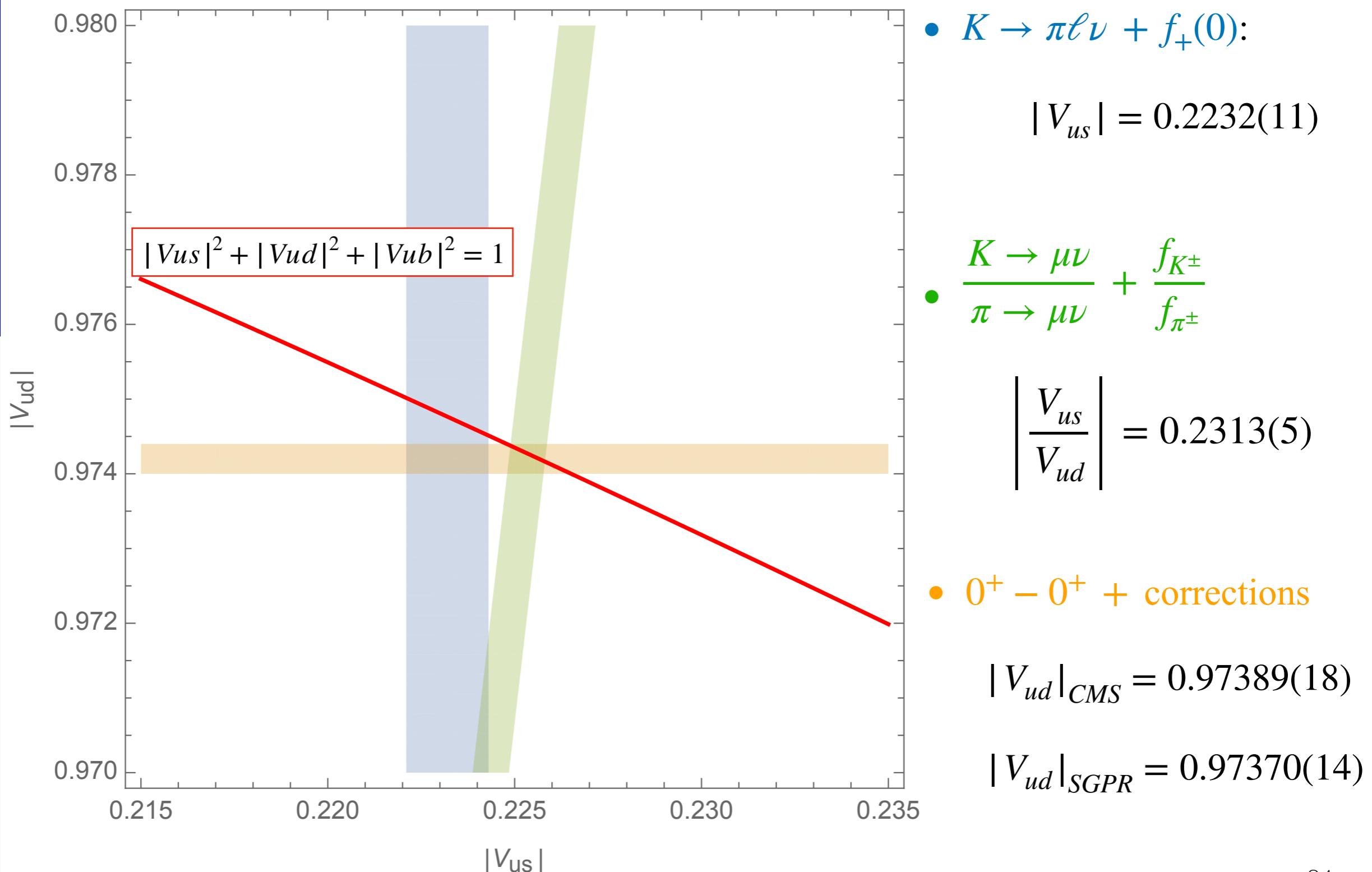
$$R_{\mu/e}^{\pi, \text{SM}} = 1 \rightarrow R_{\mu/e}^{\pi} = 1.00173 \pm 0.00043$$

PREDICTION WITH MODIFIED NEUTRINO COUPLINGS

Looking forward to see PEN results!!!

Backup

The Anomaly with NP



Modified Neutrino Couplings

Minimal impact: we modify only the couplings of W and Z with neutrinos



- EW observables
- Low energy observables (K, π, τ, W decays)

dim = 6

There is 1 Operator which modifies only neutrino couplings :

$$\bar{L}_i \gamma_\mu \tau^I L_j H^\dagger i D_I^\mu H \quad \text{with} \quad \tau^I = (1, -\sigma_1, -\sigma_2, -\sigma_3)$$

$$\frac{-ig_2}{\sqrt{2}} \bar{\ell}_i \gamma^\mu P_L \nu_j W_\mu \Rightarrow \frac{-ig_2}{\sqrt{2}} \bar{\ell}_i \gamma^\mu P_L \nu_j W_\mu \left(\delta_{ij} + \frac{1}{2} \varepsilon_{ij} \right)$$

$$\frac{-ig_2}{2c_W} \bar{\nu}_i \gamma^\mu P_L \nu_j Z_\mu \Rightarrow \frac{-ig_2}{2c_W} \bar{\nu}_i \gamma^\mu P_L \nu_j Z_\mu \left(\delta_{ij} + \varepsilon_{ij} \right)$$

Parameters of Fit I

Parameter	Prior	SM posterior
G_F [GeV $^{-2}$] [3]	$1.1663787(6) \times 10^{-5}$	★
α [3]	$7.2973525664(17) \times 10^{-3}$	★
$\Delta\alpha_{\text{had}}$ [3]	$276.1(11) \times 10^{-4}$	$275.4(10) \times 10^{-4}$
$\alpha_s(M_Z)$ [3]	$0.1181(11)$	★
m_Z [GeV] [7]	91.1875 ± 0.0021	91.1883 ± 0.0020
m_H [GeV] [9, 10]	125.16 ± 0.13	★
m_t [GeV] [11-13]	172.80 ± 0.40	172.96 ± 0.39

	Prior	NP-I posterior	NP-II posterior
$V_{us}^{\mathcal{L}}$	0.225 ± 0.010	0.2248 ± 0.0004	0.2248 ± 0.0004
ε_{ee}	0.00 ± 0.05	-0.0018 ± 0.0006	-0.0022 ± 0.0007
$\varepsilon_{\mu\mu}$	0.00 ± 0.05	0.0008 ± 0.0004	0.0012 ± 0.0003
$\varepsilon_{\tau\tau}$	0.00 ± 0.05	-0.0002 ± 0.0020	-0.0003 ± 0.0020

EW Observables of Fit I

Observable	Ref.	Measurement	SM Posterior	NP-I posterior	NP-II posterior	Pull I	Pull II
M_W [GeV]	[3]	80.379(12)	80.363(4)	80.371(6)	80.370(6)	0.67	0.59
Γ_W [GeV]	[3]	2.085(42)	2.089(1)	2.090(1)	2.090(1)	-0.02	-0.02
$\text{BR}(W \rightarrow \text{had})$	[3]	0.6741(27)	0.6749(1)	0.6749(2)	0.6749(1)	0	0
$\sin^2\theta_{\text{eff}}^{\text{lept}}(Q_{\text{FB}}^{\text{had}})$	[3]	0.2324(12)	0.2316(4)	0.2315(1)	0.2315(1)	-0.1	-0.1
$\sin^2\theta_{\text{eff(Tev)}}^{\text{lept}}$	[3]	0.23148(33)	0.2316(4)	0.2315(1)	0.2315(1)	0.17	0.17
$\sin^2\theta_{\text{eff(LHC)}}^{\text{lept}}$	[3]	0.23104(49)	0.2316(4)	0.2315(1)	0.2315(1)	-0.03	-0.03
P_τ^{pol}	[7]	0.1465(33)	0.1461(3)	0.1474(8)	0.1472(8)	-0.14	-0.09
A_ℓ	[7]	0.1513(21)	0.1461(3)	0.1474(8)	0.1472(8)	0.72	0.60
Γ_Z [GeV]	[7]	2.4952(23)	2.4947(6)	2.496(1)	2.496(1)	-0.11	-0.11
σ_h^0 [nb]	[7]	41.541(37)	41.485(6)	41.495(24)	41.493(24)	0.47	0.42
R_ℓ^0	[7]	20.767(35)	20.747(7)	20.749(7)	20.749(7)	0.06	0.06
$A_{\text{FB}}^{0,\ell}$	[7]	0.0171(10)	0.0160(7)	0.0163(2)	0.0163(2)	0.12	0.12
R_b^0	[7]	0.21629(66)	0.21582(1)	0.21582(1)	0.21582(1)	0	0
R_c^0	[7]	0.1721(30)	0.17219(2)	0.17220(2)	0.17220(2)	0	0
$A_{\text{FB}}^{0,b}$	[7]	0.0992(16)	0.1024(2)	0.1033(6)	0.1032(6)	-0.41	-0.36
$A_{\text{FB}}^{0,c}$	[7]	0.0707(35)	0.0731(2)	0.0738(4)	0.0738(4)	-0.20	-0.20
A_b	[7]	0.923(20)	0.93456(2)	0.9347(1)	0.9347(1)	-0.01	-0.01
A_c	[7]	0.670(27)	0.6675(1)	0.6680(4)	0.6680(3)	0	0

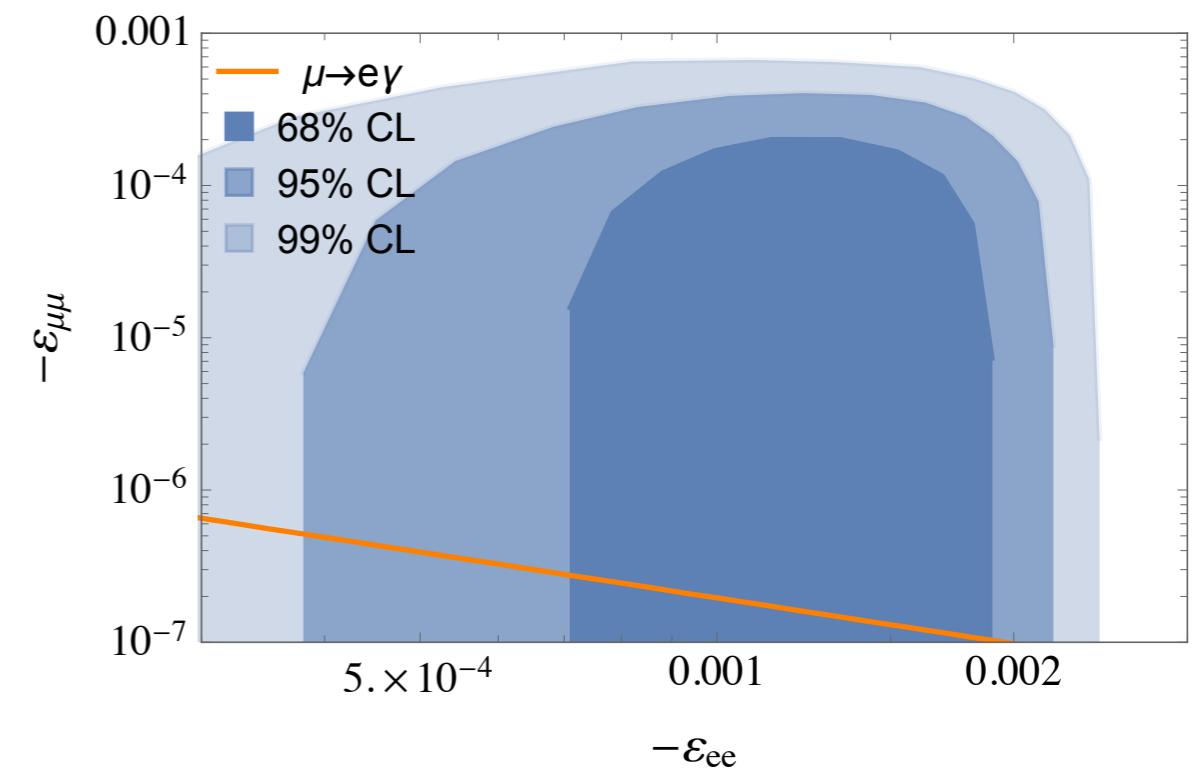
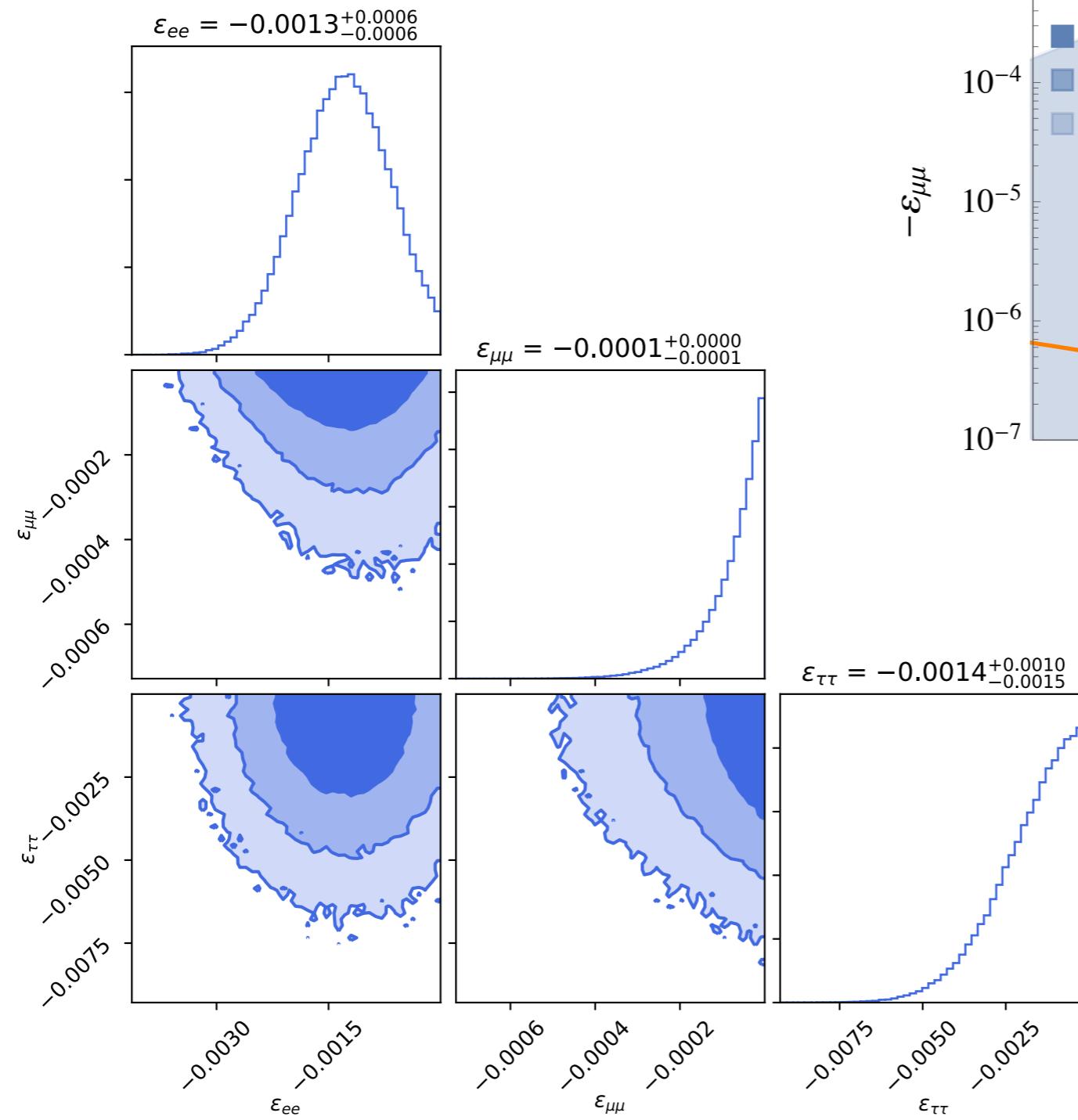
$$P(O_i) = \left| \frac{O_i^{\text{exp}} - O_i^{\text{SM}}}{\sqrt{(\sigma_i^{\text{exp}})^2 + (\sigma_i^{\text{SM}})^2}} \right| - \left| \frac{O_i^{\text{exp}} - O_i^{\text{NP}}}{\sqrt{(\sigma_i^{\text{exp}})^2 + (\sigma_i^{\text{NP}})^2}} \right|$$

LFU & V_{us} Observables of Fit I

Observable	Ref.	Measurement	SM Posterior	NP-I posterior	NP-II posterior	Pull I	Pull II
$K \rightarrow \mu\nu$	[1, 14-16]	0.9978 ± 0.0020	1	1.00137 ± 0.00046	1.00173 ± 0.00043	-0.63	-0.82
$K \rightarrow e\nu$	[2, 3, 16-19]	1.0010 ± 0.0009	1	1.00137 ± 0.00046	1.00173 ± 0.00043	0.75	0.38
$\pi \rightarrow \mu\nu$	[3, 4]	1.0018 ± 0.0014	1	1.00137 ± 0.00046	1.00173 ± 0.00043	0.99	1.24
$\tau \rightarrow \mu\nu\bar{\nu}$	[1, 20, 21]	1.0010 ± 0.0025	1	1.00137 ± 0.00046	1.00173 ± 0.00043	0.25	0.11
$K \rightarrow \pi\mu\bar{\nu}$	[1, 5]	0.996 ± 0.010	1	1.00137 ± 0.00046	1.00173 ± 0.00043	-0.14	-0.17
$W \rightarrow \mu\bar{\nu}$	[6]	0.989 ± 0.012	1	1.00137 ± 0.00046	1.00173 ± 0.00043	-0.11	-0.14
$B \rightarrow D^{(*)}\mu\nu$	[3, 4]	1.0010 ± 0.0014	1	0.9997 ± 0.0010	0.9995 ± 0.0010	-0.04	-0.15
$B \rightarrow D^{(*)}e\nu$	[4]	0.9961 ± 0.0027	1	0.9997 ± 0.0010	0.9995 ± 0.0010	0.20	0.26
$\mu \rightarrow e\bar{\nu}\nu$	[4]	0.9860 ± 0.0070	1	0.9997 ± 0.0010	0.9995 ± 0.0010	0.06	0.09
$\tau \rightarrow \pi\nu$	[1, 5]	1.034 ± 0.013	1	0.9997 ± 0.0010	0.9995 ± 0.0010	-0.02	-0.03
$\pi \rightarrow \mu\nu$	[3, 4]	1.0029 ± 0.0014	1	1.0011 ± 0.0011	1.0013 ± 0.0011	1.06	1.17
$\tau \rightarrow \mu\nu\bar{\nu}$	[1, 5]	1.031 ± 0.013	1	1.0011 ± 0.0011	1.0013 ± 0.0011	0.10	0.11
$ V_{us}^{K\mu^3} $	[3, 22]	0.2234 ± 0.0008	$0.2257(3)$	0.22509 ± 0.00040	0.22516 ± 0.00040	0.81	0.74
$ V_{us}/V_{ud} ^{K/\pi}$	[22, 23]	0.2313 ± 0.0005	$0.2317(4)$	0.23078 ± 0.00044	0.23082 ± 0.00044	-0.16	-0.10
$ V_{us}^\tau _{\text{incl.}}$	[24, 25]	0.2195 ± 0.0019	$0.2257(3)$	0.22487 ± 0.00041	0.22491 ± 0.00041	0.48	0.45
$ V_{ud}^\beta _{\text{CMS}}$	[24, 25]	0.97389 ± 0.00018	$0.974185(79)$	0.97400 ± 0.00017	-	0.56	-
$ V_{ud}^\beta _{\text{SGPR}}$	[24, 26]	0.97370 ± 0.00014	$0.974185(79)$	-	0.97379 ± 0.00013	-	2.57

$$P(O_i) = \left| \frac{O_i^{\text{exp}} - O_i^{\text{SM}}}{\sqrt{(\sigma_i^{\text{exp}})^2 + (\sigma_i^{\text{SM}})^2}} \right| - \left| \frac{O_i^{\text{exp}} - O_i^{\text{NP}}}{\sqrt{(\sigma_i^{\text{exp}})^2 + (\sigma_i^{\text{NP}})^2}} \right|$$

Right-handed Neutrinos Fit



Lepton Radiative Decays Bounds

$$|\varepsilon_{e\mu}| \leq 1.4 \times 10^{-5}$$

$$|\varepsilon_{\mu\tau}| \leq 9.4 \times 10^{-3}$$

$$|\varepsilon_{e\tau}| \leq 1.1 \times 10^{-2}$$

$$|\varepsilon_{e\mu}| = \sqrt{|\varepsilon_{ee}| |\varepsilon_{\mu\mu}|}$$

Mixing for the Vector Triplet Model

After EWSB

$$M_0^2 = \begin{pmatrix} M_{Z^{(0)}}^2 & \frac{x}{c_W} \\ \frac{x^*}{c_W} & M_X^2 \end{pmatrix} \quad M_\pm^2 = \begin{pmatrix} M_{W^{(0)}}^2 & x \\ x^* & M_X^2 \end{pmatrix} \quad x = M_{W^{(0)}} \frac{g_X^{D\phi} v}{2}$$

$$\begin{pmatrix} W'_\pm \\ W_\pm \end{pmatrix} = \begin{pmatrix} X_\pm \cos\alpha_{WW'} - W_\pm^{(0)} \sin\alpha_{WW'} \\ X_\pm \sin\alpha_{WW'} + W_\pm^{(0)} \cos\alpha_{WW'} \end{pmatrix}$$

$$\sin\alpha_{WW'} \approx \frac{x}{M_X^2}$$

$$\sin\alpha_{ZZ'} \approx \frac{x}{M_X^2 x_W}$$

LHC bounds for the Vector Triplet Model

2-quarks-2-leptons

$$-\frac{4\pi}{(24 \text{TeV}^2)} \leq \frac{g_{11}^\ell g^q}{4M_{Z'}} \leq \frac{4\pi}{(37 \text{TeV}^2)}$$

$$-\frac{4\pi}{(20 \text{TeV}^2)} \leq \frac{g_{22}^\ell g^q}{4M_{Z'}} \leq \frac{4\pi}{(30 \text{TeV}^2)}$$

$$-10.5 \frac{M_{W'}^2}{(10 \text{TeV})^2} \leq g_{33}^\ell g^q \leq 0$$

2-jets

$$|g^q|^2 \leq 15 \frac{M_{Z'}^2}{(10 \text{TeV})^2}$$

APV & QWEAK bounds for the VT model

QWEAK

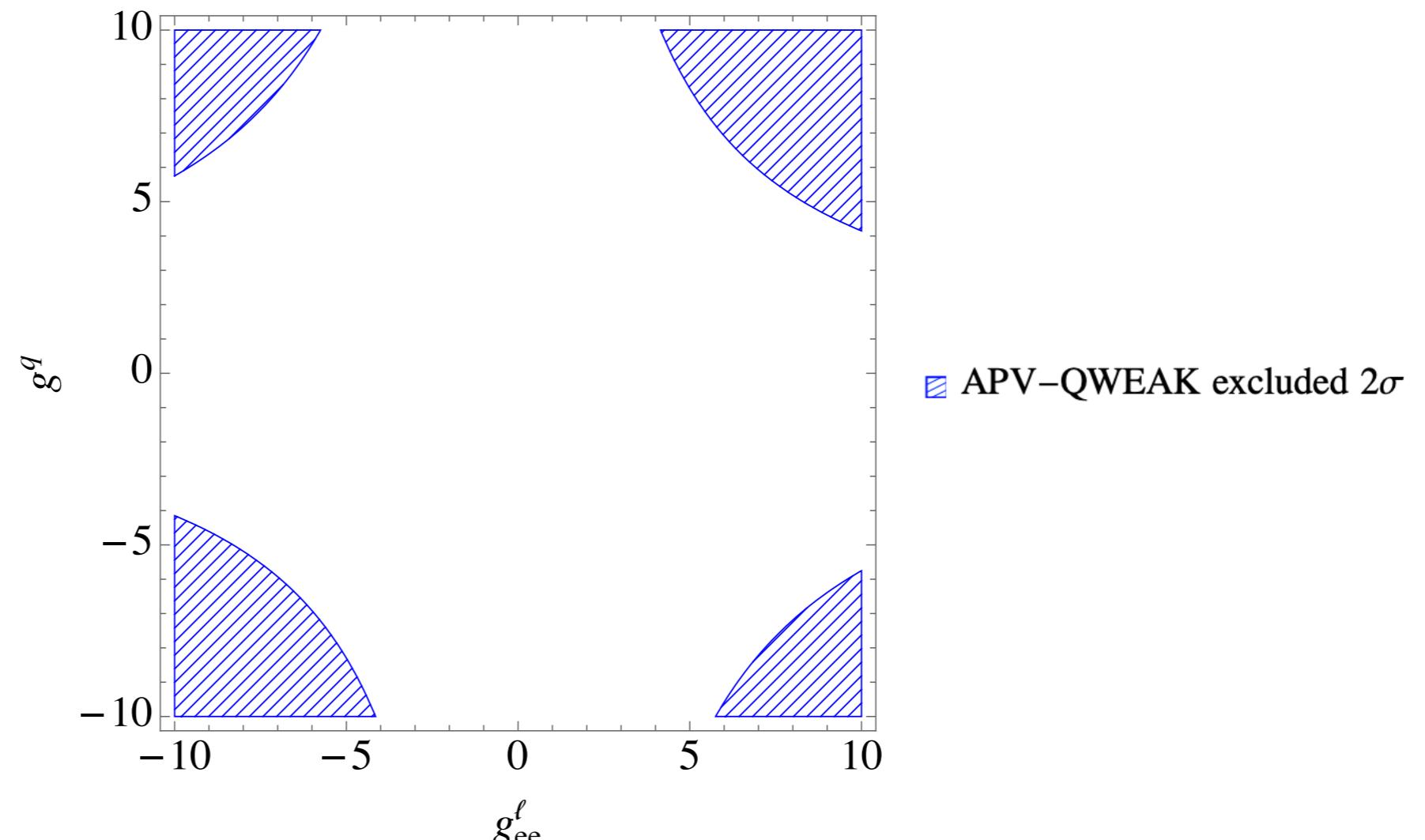
$$-2(2C_{1u} + C_{1d}) = 0.0719 \pm 0.0045$$

APV

$$-2(188C_{1u} + 211C_{1d}) = -72.62 \pm 0.43$$

$$C_{1d} = 0.3419 + \frac{\sqrt{2}}{G_F} \frac{g^q g_{11}^\ell}{16M_{Z'}^2}$$

$$C_{1u} = -0.1887 - \frac{\sqrt{2}}{G_F} \frac{g^q g_{11}^\ell}{16M_{Z'}^2}$$



Information Criterion

In a bayesian approach, the *Information Criterion* allows for a comparison between different models

$$IC = -2\log L + 4\sigma_{\log L}^2$$

average of the log-likelihood

variance of the log-likelihood

The second term takes into account the effective numbers of parameters in the model, allowing for a meaningful comparison of models with different number of parameters. Preferred models are expected to give smaller *IC* values