A Class on Spin Dynamics in Accelerators and the Physics and Technology of Polarized Beams

USPAS Spin Class 2020

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Melville, Long Island, New York

Held in the US Particle Accelerator School Summer Session: June 15–26, 2020
Polarization at RHIC and at the Electron Ion Collider
Spin Simulation Techniques

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The Electron-Ion Collider

“The announcement by DOE of CD-0 and site selection for the EIC occurred on January 9, 2020: DOE selected BNL to host the EIC. The facility is estimated to cost between $1.6B and $2.6B.”

eRHIC Design Concept

- Based on RHIC ion complex:
  - Polarized protons from OPPIS
  - Ions, polarized 3He and d, from EBIS
  - Booster and AGS injectors
  - Acceleration/storage in RHIC Yellow

- Adding an electron complex, in RHIC tunnel
  - Polarized electron source
  - 400 MeV linac
  - Rapid-cycling synchrotron
  - 5 to 18 GeV storage ring

- Large acceptance detectors
  - At IP6 and IP8
Table 1.3: Initial luminosity parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>hadron</th>
<th>electron</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center of mass energy [GeV]</td>
<td>104.9</td>
<td></td>
</tr>
<tr>
<td>Energy [GeV]</td>
<td>275</td>
<td>10</td>
</tr>
<tr>
<td>Number of bunches</td>
<td>290</td>
<td></td>
</tr>
<tr>
<td>Particles per bunch [10^{10}]</td>
<td>10.2</td>
<td>22.0</td>
</tr>
<tr>
<td>Beam current [A]</td>
<td>0.37</td>
<td>0.8</td>
</tr>
<tr>
<td>Horizontal emittance [nm]</td>
<td>17.9</td>
<td>20.0</td>
</tr>
<tr>
<td>Vertical emittance [nm]</td>
<td>8.5</td>
<td>4.2</td>
</tr>
<tr>
<td>Hor. β-function at IP $\beta'_x$ [cm]</td>
<td>90</td>
<td>81</td>
</tr>
<tr>
<td>Vert. β-function at IP$\beta'_y$ [cm]</td>
<td>5.9</td>
<td>12.1</td>
</tr>
<tr>
<td>Horizontal/Vertical fractional betatron tunes</td>
<td>0.3/0.31</td>
<td>0.09/0.12</td>
</tr>
<tr>
<td>Horizontal divergence $d\sigma_x^H / ds$ [mrad]</td>
<td>0.141</td>
<td>0.157</td>
</tr>
<tr>
<td>Vertical divergence $d\sigma_y^V / ds$ [mrad]</td>
<td>0.380</td>
<td>0.186</td>
</tr>
<tr>
<td>Horizontal beam-beam parameter $\xi_x$</td>
<td>0.0079</td>
<td>0.1</td>
</tr>
<tr>
<td>Vertical beam-beam parameter $\xi_y$</td>
<td>0.0029</td>
<td>0.085</td>
</tr>
<tr>
<td>IBS growth time longitudinal/horizontal [hours]</td>
<td>8/18</td>
<td>-</td>
</tr>
<tr>
<td>Synchrotron radiation power [MW]</td>
<td>-</td>
<td>2.95</td>
</tr>
<tr>
<td>Bunch length [cm]</td>
<td>9.9</td>
<td>1.9</td>
</tr>
<tr>
<td>Hourglass and crab reduction factor</td>
<td>0.74</td>
<td></td>
</tr>
<tr>
<td>Luminosity [10^{33} cm^{-2} sec^{-1}]</td>
<td>1.05</td>
<td></td>
</tr>
</tbody>
</table>

Table 1.1: Maximum luminosity parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>hadron</th>
<th>electron</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center-of-Mass Energy [GeV]</td>
<td>104.9</td>
<td></td>
</tr>
<tr>
<td>Energy [GeV]</td>
<td>275</td>
<td>10</td>
</tr>
<tr>
<td>Number of Bunches</td>
<td>1320</td>
<td></td>
</tr>
<tr>
<td>Particles per Bunch [10^{10}]</td>
<td>6.0</td>
<td>15.1</td>
</tr>
<tr>
<td>Beam Current [A]</td>
<td>1.0</td>
<td>2.5</td>
</tr>
<tr>
<td>Horizontal Emittance [nm]</td>
<td>9.2</td>
<td>20.0</td>
</tr>
<tr>
<td>Vertical Emittance [nm]</td>
<td>1.3</td>
<td>1.0</td>
</tr>
<tr>
<td>Hor. β-function at IP $\beta'_x$ [cm]</td>
<td>90</td>
<td>42</td>
</tr>
<tr>
<td>Vert. β-function at IP$\beta'_y$ [cm]</td>
<td>4.0</td>
<td>5.0</td>
</tr>
<tr>
<td>Hor./Vert. Fractional Betatron Tunes</td>
<td>0.305/0.31</td>
<td>0.08/0.06</td>
</tr>
<tr>
<td>Horizontal Divergence $d\sigma_x^H / ds$ [mrad]</td>
<td>0.101</td>
<td>0.219</td>
</tr>
<tr>
<td>Vertical Divergence $d\sigma_y^V / ds$ [mrad]</td>
<td>0.179</td>
<td>0.143</td>
</tr>
<tr>
<td>Horizontal Beam-Beam Parameter $\xi_x$</td>
<td>0.013</td>
<td>0.064</td>
</tr>
<tr>
<td>Vertical Beam-Beam Parameter $\xi_y$</td>
<td>0.007</td>
<td>0.1</td>
</tr>
<tr>
<td>IBS Growth Time longitudinal/horizontal [hr]</td>
<td>2.19/2.06</td>
<td>-</td>
</tr>
<tr>
<td>Synchrotron Radiation Power [MW]</td>
<td>-</td>
<td>9.18</td>
</tr>
<tr>
<td>Bunch Length [cm]</td>
<td>5</td>
<td>1.9</td>
</tr>
<tr>
<td>Hourglass and Crab Reduction Factor [16]</td>
<td>0.87</td>
<td></td>
</tr>
<tr>
<td>Luminosity [10^{34} cm^{-2} sec^{-1}]</td>
<td>1.05</td>
<td></td>
</tr>
</tbody>
</table>
1 Hadron polarization at eRHIC

Polarized proton beams at the RHIC complex
• AGS

◊ An on-line model of the AGS uses the 3-D OPERA field maps of the two helical snakes

◊ Full acceleration cycle simulations are part of polarization transmission and improvement studies (thousands of particles, 150000 turns, 30 minutes/cycle on NERSC CPU cluster)

Particle excursion over full AGS cycle.

◊ Optimization of AGS parameter settings routinely based on simulations.

[Y. Dutheil, BNL C-AD, PhD dissertation, April 2015].

Evolution of the polarization along AGS acceleration cycle, 10^3 particles, 6-D bunch.

Loss at \( G\gamma \sim 5 \) (a pair of weak vertical intrinsic resonances) and at \( G\gamma = 36 + Q_v \) in the present hypotheses.

• RHIC (1): Acceleration cycle to 100 GeV

◊ Goal: comparison of polarization transport with two different optics:
  - a new, Run 15 optics
  - earlier Run 13 optics

◊ Isolated snake resonance: \(< S_y >= 1 - 8\frac{|e|^2}{\chi^2} \sin^2 \frac{\pi \lambda}{N_s} (1 - \frac{|e|^2}{\chi^2} \sin^2 \frac{\pi \lambda}{N_s})\)

8 particles on 10 rms invariants are plotted. Envelopes (blue) are from theory.

Normalized invariants over 500,000 turns:

\(x \times \sqrt{\frac{p}{p_0}} \) and \(y \times \sqrt{\frac{p}{p_0}}\) over 2 million turns:
RHIC (2): use snake field maps → orbit defect tilts \( \tilde{\mathbf{n}} \)

\[
\begin{aligned}
\begin{cases}
 x = x_0 + \frac{B_0}{k^2 B \rho} \left[ \cos(k s + \alpha) - \cos(\alpha) \right] + \left[ x_0' + \frac{B_0}{k B \rho} \sin(\alpha) \right] s
 y = y_0 + \frac{B_0}{k^2 B \rho} \left[ \sin(k s + \alpha) - \sin(\alpha) \right] + \left[ y_0' - \frac{B_0}{k B \rho} \cos(\alpha) \right] s
\end{cases}
\end{aligned}
\]

\[
\begin{aligned}
\begin{cases}
 x = x_0 + \frac{B_0}{k^2 B \rho} \left[ \cos(k s) - 1 \right] + x_0' s
 y = y_0 + \frac{B_0}{k^2 B \rho} \sin(k s) + y_0' s
\end{cases}
\end{aligned}
\]

\( x_0, \ , x_0', \ y_0, y_0' \) the initial coordinates (taken wrt snake axis),
\( \alpha \) the angle of the field to the vertical at entrance,
\( k = \text{pitch} = \frac{2\pi}{L} = 2.61800 \), length \( L = 2.4 \text{ m} \).

From a field map model (i.e., field map of a 10.4 meter, 4-module RHIC snake):

- Non-circular, \((x,y)\) projection, theory and numerical, 23, 100, 250, 300 GeV.
- Spin components along orbits.
• RHIC (3): Spin flipper

◊ Sweep $Q_{\text{osc}}$ through $Q_s = \frac{1}{2}^+ :$

a matter of a $10^4 \sim 5 \times 10^5$ turn tracking around RHIC

◊ AC-dipole + DC-rotator assembly excites a single, isolated resonance, at $Q_s = Q_{\text{osc}}$.

![ACD bump 1 and 2](image)

Spin flip efficiency vs. crossing speed:

Very good accord obtained between measurements and numerical simulations

![Graph showing spin flip efficiency vs. crossing speed](image)

Image at $1 - Q_s$ is not excited:

![Graph showing frequency mismatch](image)

Multiple crossing:

$$\delta\nu_s = \frac{1 + G\gamma}{\pi} \Delta D' \frac{\Delta p}{p}$$
Towards eRHIC

- **Goal proton polarization at eRHIC:** 70% up to 275 GeV.
  For the record: today, using 2 snakes: close to 60% at best, at 255 GeV.

- **Add polarized 3He beams:** polarization to 70%, also requiring 6 snakes.

- **Add polarized Deuteron beams, 70%**.

---

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Moderate luminosity without cooling</th>
<th>High luminosity with cooling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam energy [GeV]</td>
<td>41 100 275</td>
<td>41 100 275</td>
</tr>
<tr>
<td>Bunch intensity [10^{11}]</td>
<td>0.6 1.1 1.1</td>
<td>0.11 0.6 0.6</td>
</tr>
<tr>
<td>RMS Horizontal normalized emittance [mm mrad]</td>
<td>3.3 4.5 4.1</td>
<td>2.0 3.6 2.7</td>
</tr>
<tr>
<td>RMS Vertical normalized emittance [mm mrad]</td>
<td>2.5 2.5 2.5</td>
<td>0.11 0.19 0.38</td>
</tr>
<tr>
<td>Longitudinal bunch area [eV sec]</td>
<td>0.5 0.6 0.8</td>
<td>0.2 0.4 0.4</td>
</tr>
<tr>
<td>RF frequency [MHz]</td>
<td>225 225 563</td>
<td>563 563 563</td>
</tr>
<tr>
<td>RF voltage [MV]</td>
<td>8 11 24</td>
<td>8 16 18</td>
</tr>
<tr>
<td>RMS momentum spread [10^{-4}]</td>
<td>15 8.7 6.6</td>
<td>10.3 9.1 4.6</td>
</tr>
<tr>
<td>RMS bunch length [cm]</td>
<td>13 11 7</td>
<td>7.5 7 5</td>
</tr>
<tr>
<td>Longitudinal emittance IBS growth time [hours]</td>
<td>10.3 8.0 10.1</td>
<td>3.2 2.4 2.2</td>
</tr>
<tr>
<td>Horizontal emittance IBS growth time [hours]</td>
<td>8.0 13.7 9.2</td>
<td>2.0 2.0 2.1</td>
</tr>
</tbody>
</table>

(without coupling)
• Booster: add $^3$He instrumentation

◊ AC dipole & tune kicker at booster E3 straight:

◊ $f_{\text{osc}} \approx 250 \text{ kHz}$

◊ This year, 2020:
- Will establish crossing $G \gamma = 0 + Q_y$ ($Q_y \approx 4.2$) with polarized protons.
- Spin dynamics conditions similar to $^3$He crossing $12 - Q_y$ for eRHIC.

- $^3\text{He}$ in AGS
- AGS with $^3\text{He}$: $G = -4.18$. AGS partial snakes are 2.3 times stronger than for protons (14% and 25%).
- This is ok for overcoming 1.5 times stronger resonances.
- Issue here: snakes induce $\vec{n}_0$ tilt resulting in harmful 81 resonance series $G_\gamma = \text{int.} \pm Q_x$, stronger with $^3\text{He}$.
- A possibility: stronger cold snake/wider spin tune gap to put both $Q_x, Q_y$ in the gap.
- Proof-of-principle simulation:
  - a 2000-$^3\text{He}$ bunch in $15\pi \mu\text{m}$ emittance:

<table>
<thead>
<tr>
<th>B max (T)</th>
<th>Warm snake</th>
<th>Cold snake</th>
</tr>
</thead>
<tbody>
<tr>
<td>B nominal (T)</td>
<td>proton: $^3\text{He}^2+$</td>
<td>$^3\text{He}$</td>
</tr>
<tr>
<td>snake angle (deg.)</td>
<td>$\approx 11$</td>
<td>$\approx 21$</td>
</tr>
<tr>
<td>&quot;$\gamma$ e snake&quot;</td>
<td>$^3\text{He}^2+$</td>
<td>$^3\text{He}$</td>
</tr>
<tr>
<td>$\approx 26$</td>
<td>$\approx 44$</td>
<td></td>
</tr>
<tr>
<td>$6$</td>
<td>$12$</td>
<td></td>
</tr>
<tr>
<td>$15$</td>
<td>$25$</td>
<td></td>
</tr>
</tbody>
</table>

- With $^3\text{He}$, snakes are $|G_{^3\text{He}}|/G_p = 2.3$ times stronger than for protons:

At identical normalized emittance, strengths of intrinsic resonances satisfy

$$\left| \frac{\epsilon_{^3\text{He}}}{\epsilon_p} \right| = \sqrt{\frac{G_{^3\text{He}}}{G_p}} \approx 1.5$$

- $Q_x and Q_y$ in spin gap never happens:

$\Rightarrow G_\gamma = \text{int.} \pm Q_{x,y}$ never happens:
• RHIC with 6 Snakes

◊ Polarization goal at store, $42 \sim 275$ GeV : 70%

◊ RHIC Blue and Yellow rings today reach 60% polarization in 255 GeV proton runs, with 2 snakes per ring.

◊ Stronger $^3$He resonance strength $(\times 1.5)$ require more snakes.

◊ Foreseen scheme:
  - 6 snakes ensure $N_{\text{snakes}} > 5 |\epsilon_{\text{int}}|_{\text{max}} \approx 4$,
  - $2\pi/6$ distance ensures energy-independent $Q_s$,
  - snake axes at $\phi_k = \pm 45^\circ$ yield $Q_s = \frac{1}{\pi} \sum_{k=1}^{6} (-1)^k \phi_k = 3/2$
  - build 4 additional snakes from existing, like-helicity, rotator modules

A glimpse on snake resonance crossing simulations, 411-$Q_y$ and 393+$Q_y$
Electron polarization in eRHIC

Studies in eRHIC include SR induced spin diffusion:

STOCHASTIC ENERGY LOSS BY SR ➞ SPIN DIFFUSION

- Sokholov-Ternov self-polarization - time scale 30 minutes at 18 GeV in eRHIC eSR
- Spin diffusion causes depolarization ➔ ring design has to work out time constant as large as possible.
- The result is an asymptotic equilibrium polarization - goal at the EIC: 70%

Ref.: eRHIC pCDR, BNL C-AD (2018), to be published.
• Polarization lifetime in eRHIC storage ring

• Typically:

(i) track particles and spins, including Monte Carlo SR,

(ii) produce polarization landscape, i.e., $P_{eq}$ versus ring rigidity setting


![Graph showing polarization decay with time]

The diffusion time constant $\tau_D$ is obtained from linear regression $P/P_0 = \exp(-t/\tau_D) \approx 1 - t/\tau_D$.

Given that $\tau_{eq} = (1/\tau_{ST} + 1/\tau_D)^{-1}$,

then, $P_{eq}$ stems from $P_{eq} = P_{ST} \times \tau_{eq}/\tau_{ST}$.

![Graph showing polarization decay with time]
• Exploration: an ergodic approach to polarization lifetime:

Stochastic spin motion observed at IP, single particle. A linear regression on $P/P_0 = \exp(-\frac{t}{\tau_D}) \approx 1 - \frac{t}{\tau_D}$ provides the diffusion time constant $\tau_D$.

An energy scan of the diffusion time constant provides the diffusion landscape.

Injector: Rapid cycling synchrotron
- Full Energy Injector Accelerates 400 MeV, 10 nC bunches from the linac, to full collision energy, 5-18 GeV.
- 100-200 ms acceleration ramp, 1 Hz repetition rate
- 95% spin transmission w/ rms orbit 1 mm, gradient error 0.1%
- RF system: normal-conducting 563 MHz cavities (located at IR10), total voltage 72 MV. Stainless steel vacuum chamber

- Spin-transparent lattice, principle:
  - Strong resonances occur at/near $Q_s = nPM \pm [Q_{\text{arcs}}]$
  - Take $P \times M = 6$ arcs $\times 32$ cells/arc $= 192$, and $Q_{\text{arcs}} > a \gamma_{\text{max}}$ (arc-cell $Q_y \approx 0.25$ or more)
- first strongest imperfection resonance at $Q_s = 0 + [Q_{\text{arcs}}] = 48$
- with $Q_y = 60.2$ here, first strong intrinsic resonance is at $Q_s = Q_y - 2P = 48.2$ closest to $0 \times PM + [Q_{\text{arcs}}]$,
  - both are to the right of the useful range:
  for $E: 400$ MeV $\rightarrow 20$ GeV, spin tune $Q_s : 0.9 \rightarrow 45.4$, 

Rms closed orbit: 1mm
Gradient errors: 0.1%
Wien filter spin rotator (2019)

350 keV electrons. Principle:

Spin Z-rotation, from longitudinal ($\parallel \hat{X}$) to transverse ($\parallel \hat{Y}$) over 150 cm:

Unequal E and B fringe field extents cause a shift of the orbit inside the Wien filter. The orbit is anyway zeroed at the exit, while maintaining $\theta_s = 30$ deg, by adjusting E and B amplitudes:
3 Spin simulation techniques

• A couple of slides about the code used in these simulations

https://sourceforge.net/projects/zgoubi/files/stats/map

◇ A stepwise ray-tracing technique: First version: 1972... ~ half a century ago!
◇ Spin installed in the late 1980s, for a partial snake project at Saturne, Saclay
◇ Code “repository-ed” in sourceforge in 2007
◇ 4000 downloads at this day (63 depuis la Confédération des Helvètes)
- **DOE FUNDED SBIR (Radiasoft, Boulder, CO, 2018-2020)**
  - This includes development of a graphical interface under SIREPO (Paul Moeller/Radiasoft)

Ref.: https://beta.SIREPO.com/#/accel
• NUMERICAL INTEGRATOR

◇ What needs be solved:

\[
\frac{d(m\tilde{v})}{dt} = q(\tilde{E} + \tilde{v} \times \tilde{B})
\]

\[
\frac{d\tilde{S}}{dt} = \frac{q}{m} \tilde{S} \times \tilde{\omega}
\]

\[
\tilde{\omega} = (1 + \gamma G)\tilde{B} + G(1 - \gamma)\tilde{B}_{//} + \gamma(G + \frac{1}{1 + \gamma})\tilde{E} \times \tilde{v}
\]

\[
G : \text{gyromagnetic factor, } \gamma : \text{Lorentz relativistic factor,}
\]
\[
c : \text{velocity of light, } q : \text{charge, } m : \text{mass.}
\]

◇ ODE solver: Taylor series in the step size \(\Delta s\):

\[
\tilde{a}(M_1) \approx \tilde{a}(M_0) + \frac{d\tilde{a}}{ds}(M_0) \Delta s + ... + \frac{d^n\tilde{a}}{ds^n}(M_0) \frac{\Delta s^n}{n!}
\]

(1)

- Solving particle motion: \(\tilde{a}\) stands for position \(\tilde{R}\) or velocity \(\tilde{v}\).
- Solving spin motion: \(\tilde{a}\) stands for the spin \(\tilde{S}\).
A note in passing: stepwise ray-tracing is as good at accelerating in a cyclotron ...

184 inch cyclotron at Berkeley, 100MeV/u ions (1940s)

Double-Dee acceleration to 10 MeV in 50 turns. Accuracy on time computation is critical:

\[
\cos\phi(W) = \cos\phi_0 + \pi \left[1 - \frac{\omega_{RF}}{\omega_{rev_0}} \left(1 + \frac{W}{2m_0c^2}\right)\right] \frac{W}{q\hat{V}}
\]

... as it is at tracking polarization in large colliders... we’ve seen that!

THANK YOU FOR YOUR ATTENTION