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Dynamic modelling of 2G-HTS systems

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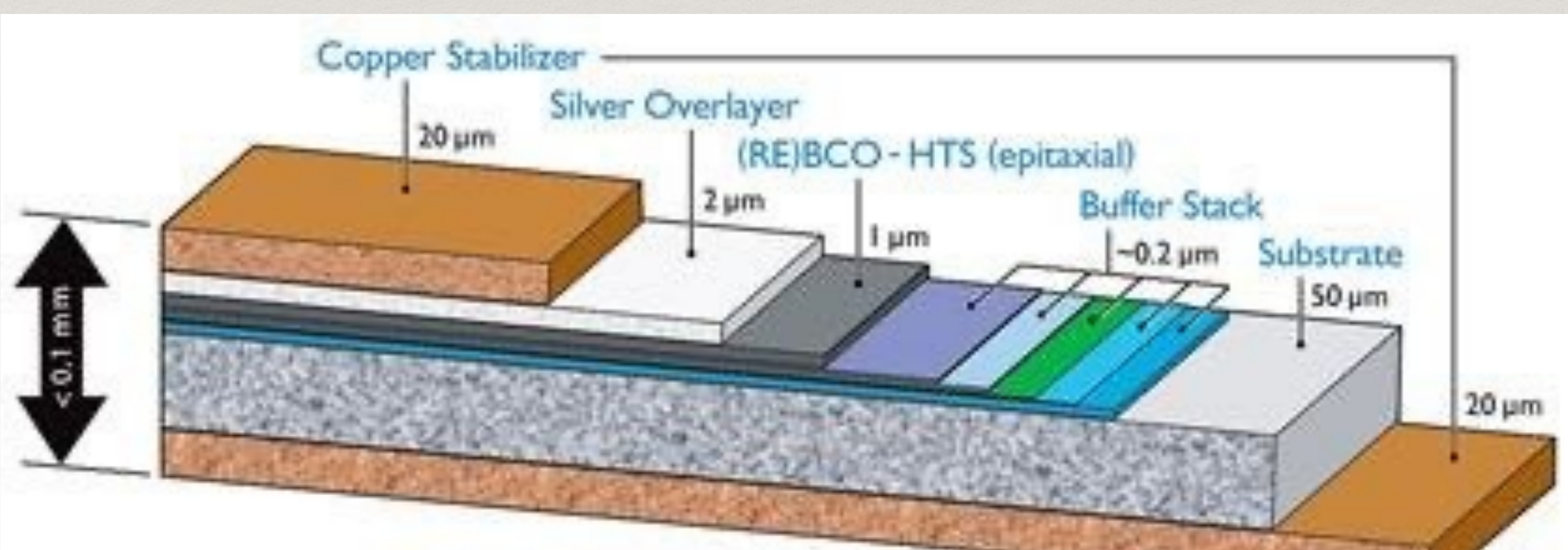
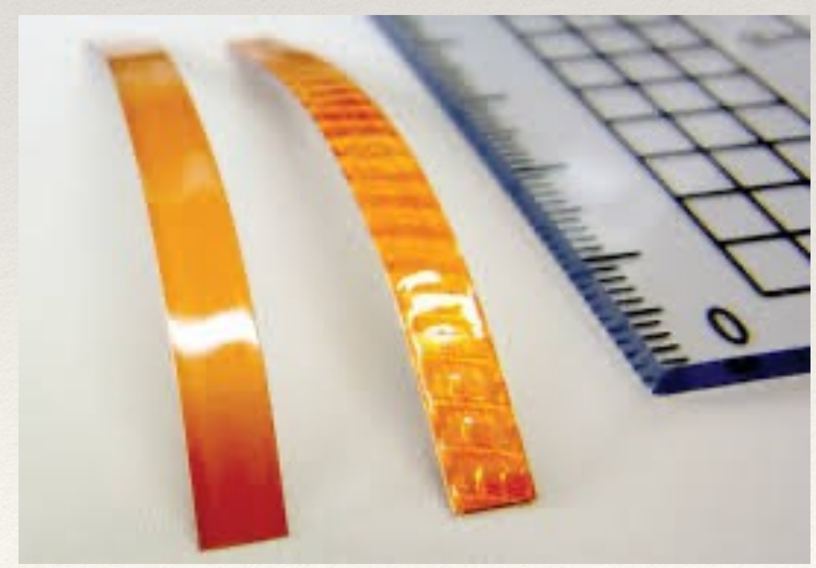
Content

REBCO or GdBCO tapes

- Second Generation of High Temperature Superconductors (2G HTS)
- Different technologies:
    - With or without stabilizer (Cu)
    - With, without and partial insulation
    - Tape width [mm] (critical current I_c [A], SF - 77 K): 2 (50), 3 (75), 4 (100), 6 (120), 12 (300)
    - Critical temperature T_c: ~92 K



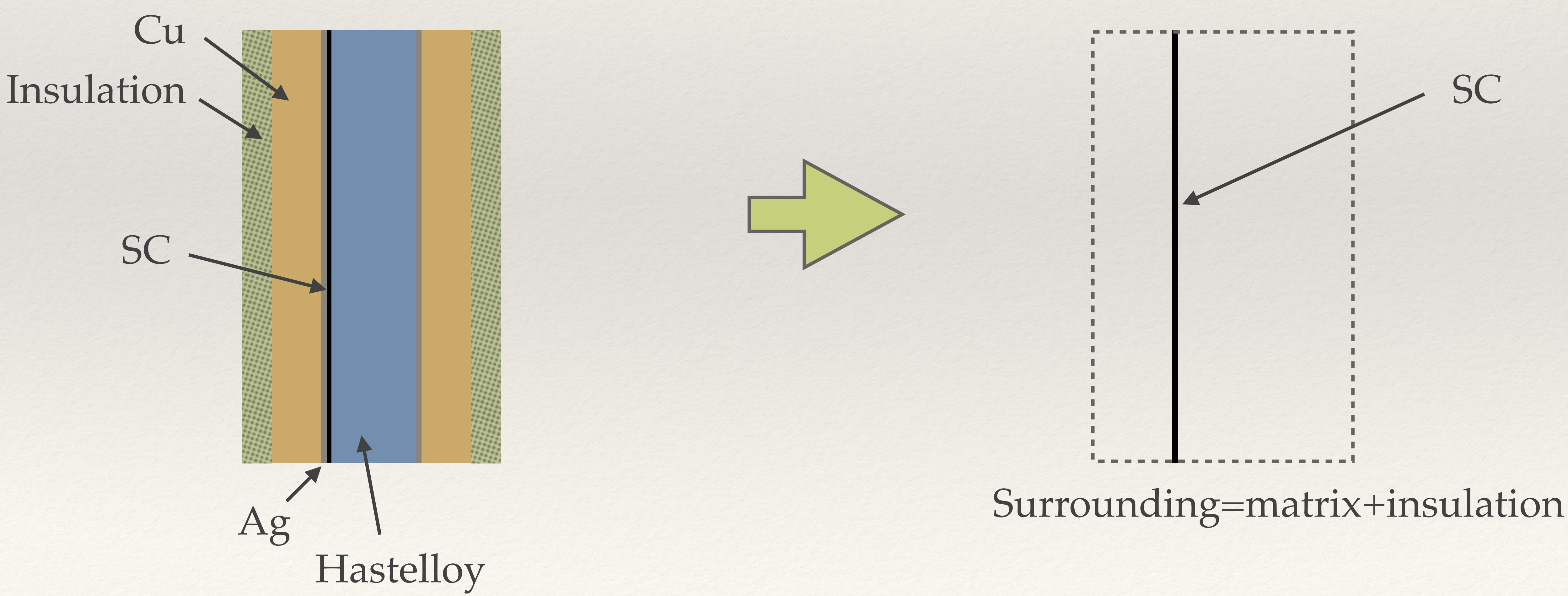
Superpower, Inc.: SCS4050



https://www.nanowerk.com/news2/newsid=32602.php

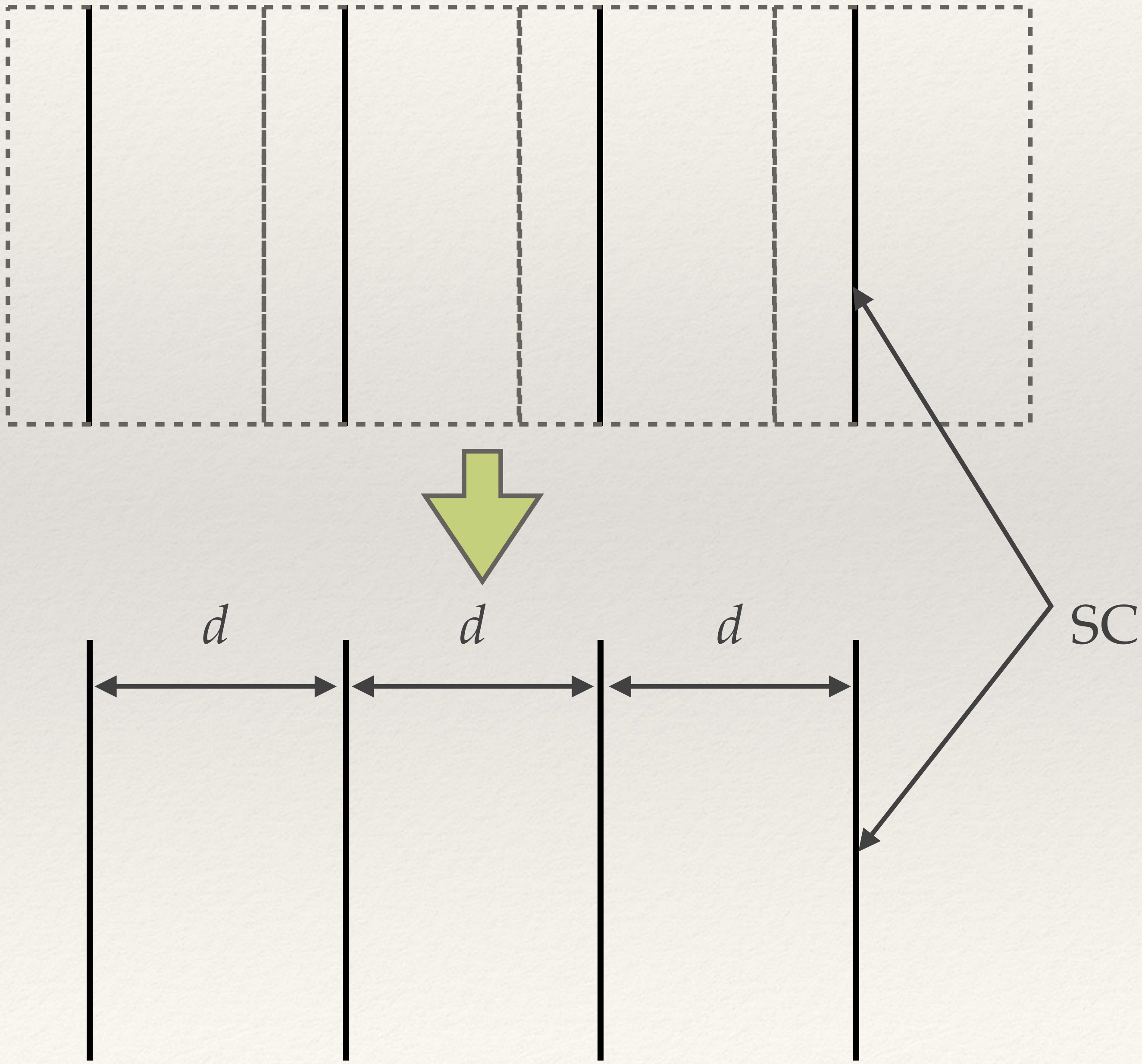
Model of a single tape

- The matrix refers to all the materials that are not superconductor except the insulation: Cu+Ag+Hastelloy+buffer
- Induced current at around I_c. Therefore, the matrix is not considered since its resistance is nearly always larger than the resistance of the superconductor layer ($R_m >> R_{sc}$)



Model of a stack

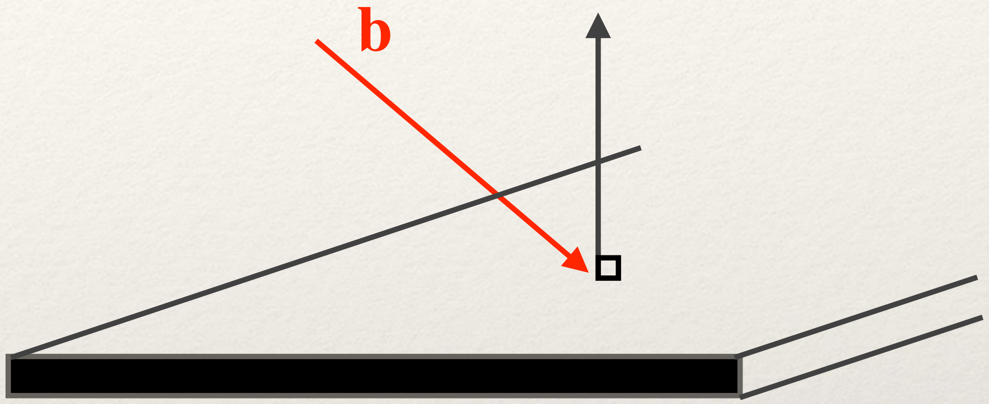
- Having the model of a single tape, the model of a stack is just a repeat over space
- Anything that is not the superconductor becomes part of the surrounding



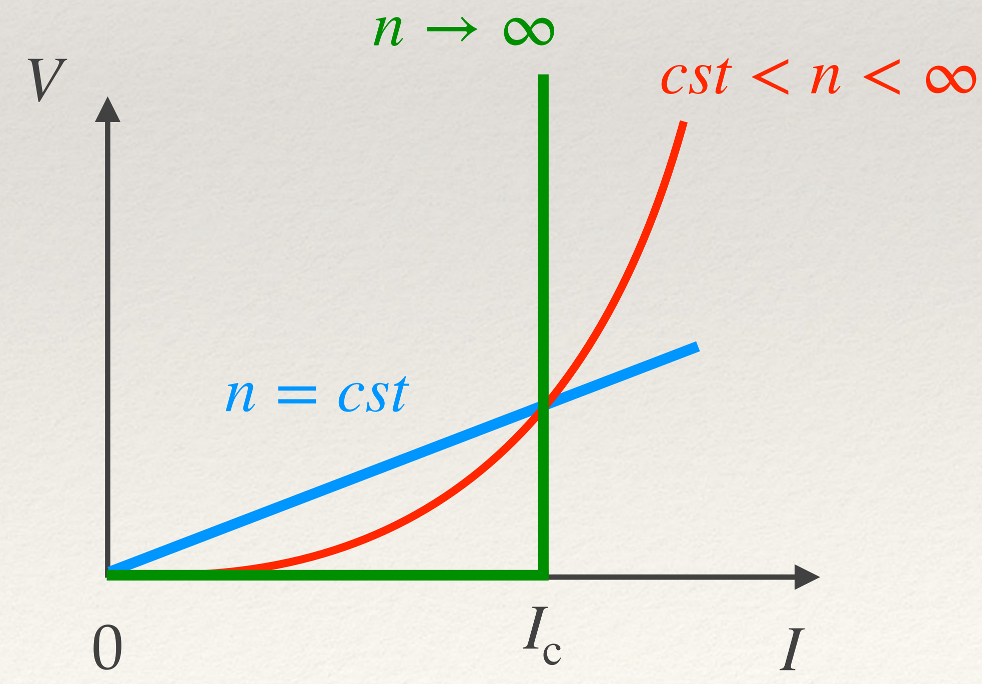
“Properties” of the superconductor

- Permeability of vacuum: $\mu_0 = 4\pi 10^{-7}$ H/m
- Resistivity depending on temperature, magnetic flux density and current density: $\rho\left(T, \mathbf{b}, |\mathbf{j}|\right)$

Include its orientation



- Engineering properties depending on temperature and magnetic flux density:
  - Critical current I_c: “measure of the capacity to transmit current”
  - Index value n: “Measure of the rate of transition from the superconducting state to the normal resistive state”. For an “ideal” superconductor: $n \to \infty$



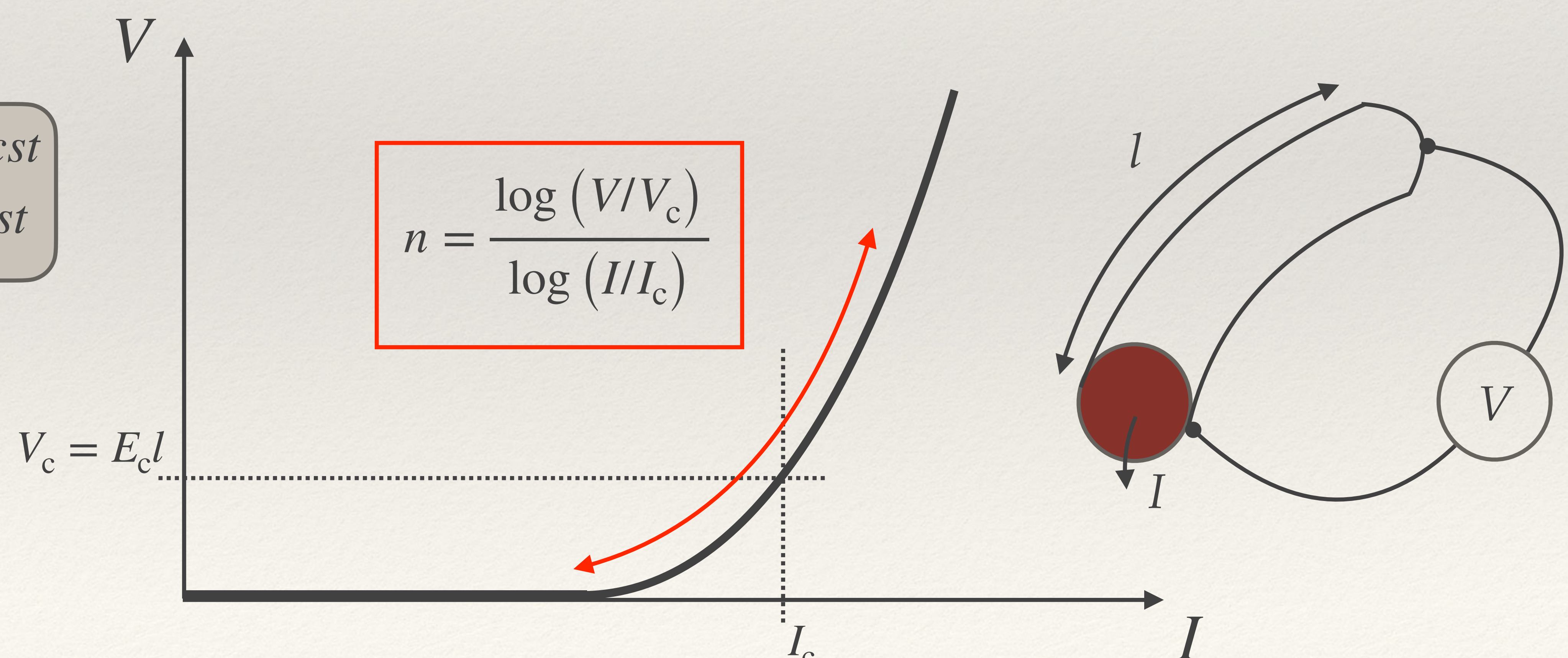
Power law: Critical current I_c and index n

- Power model of the $V - I$ characteristics of a superconductor around its critical current I_c:

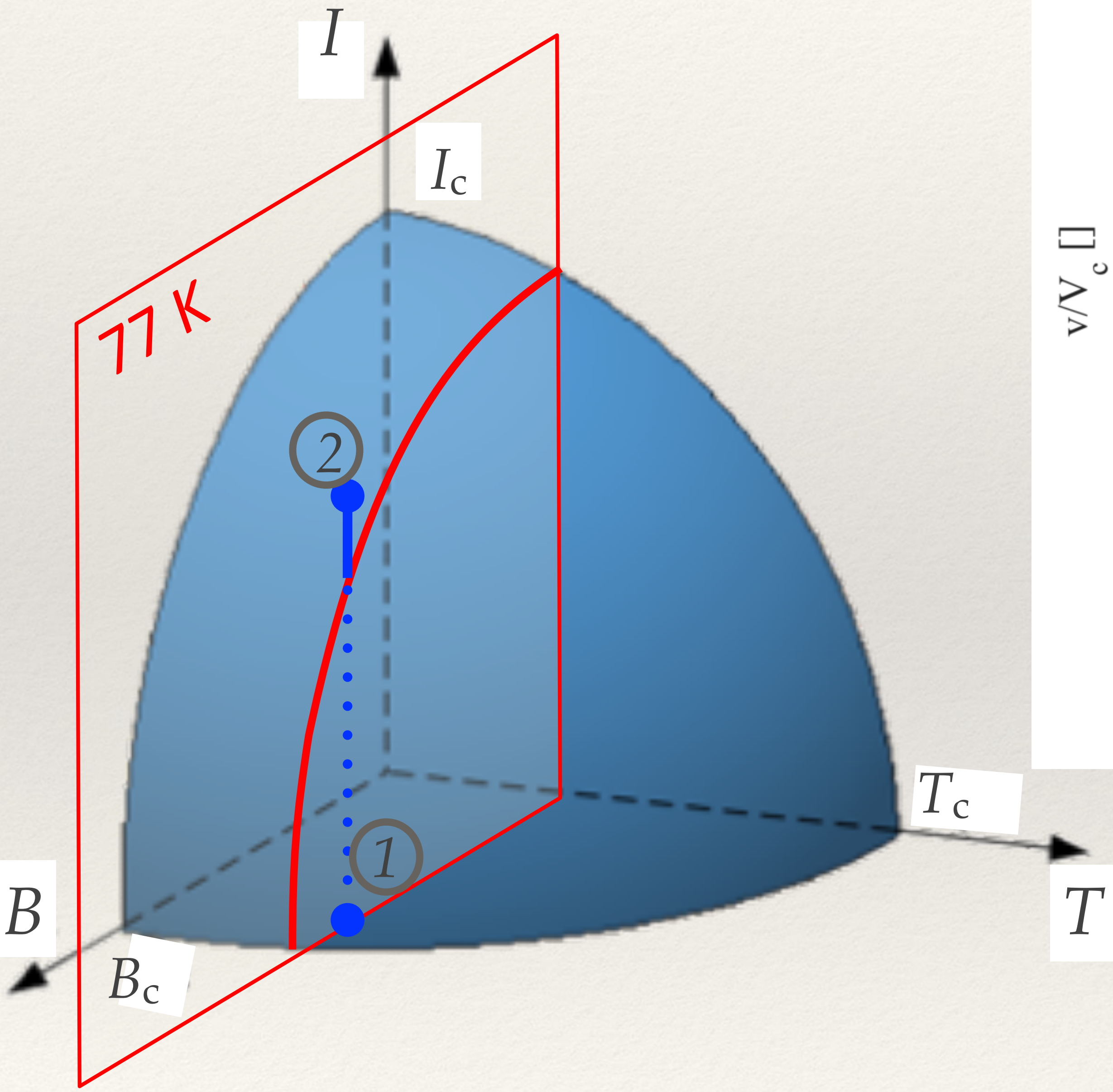
$$V = V_c \left(\frac{I}{I_c}\right)^n \quad \Rightarrow \quad E = \underbrace{\left(\frac{E_c}{J_c^n} J^{n-1}\right)}_{\rho_{sc}} J$$

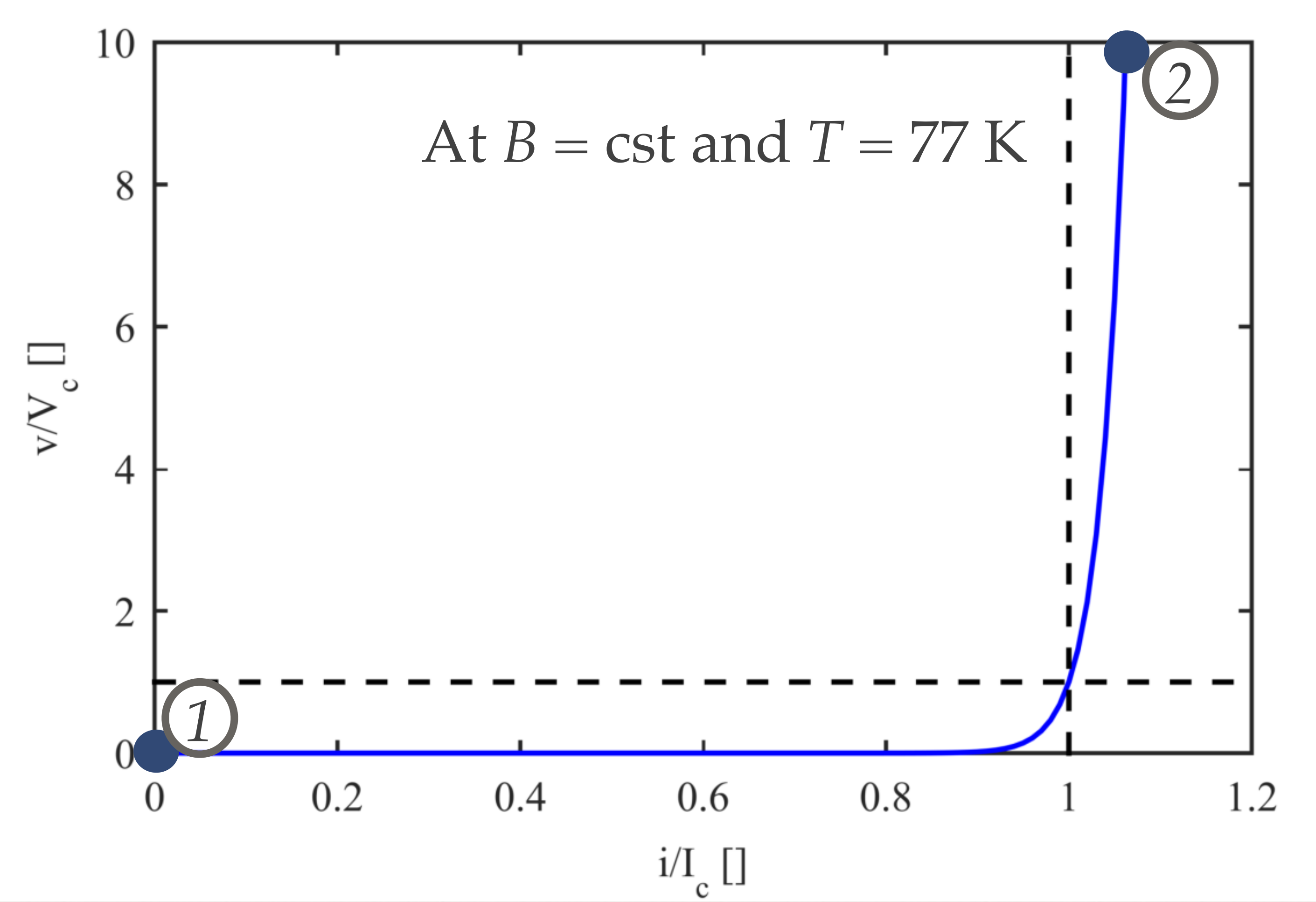
with $V_c = E_c l$, l is the length between the voltage taps and E_c is the electrical field assumed constant across the thickness of the tape with values ranging from 0.1 to 10 μV/cm

$T_{op} \simeq cst$
$B_a = cst$



Critical surface and *V-I* characteristics

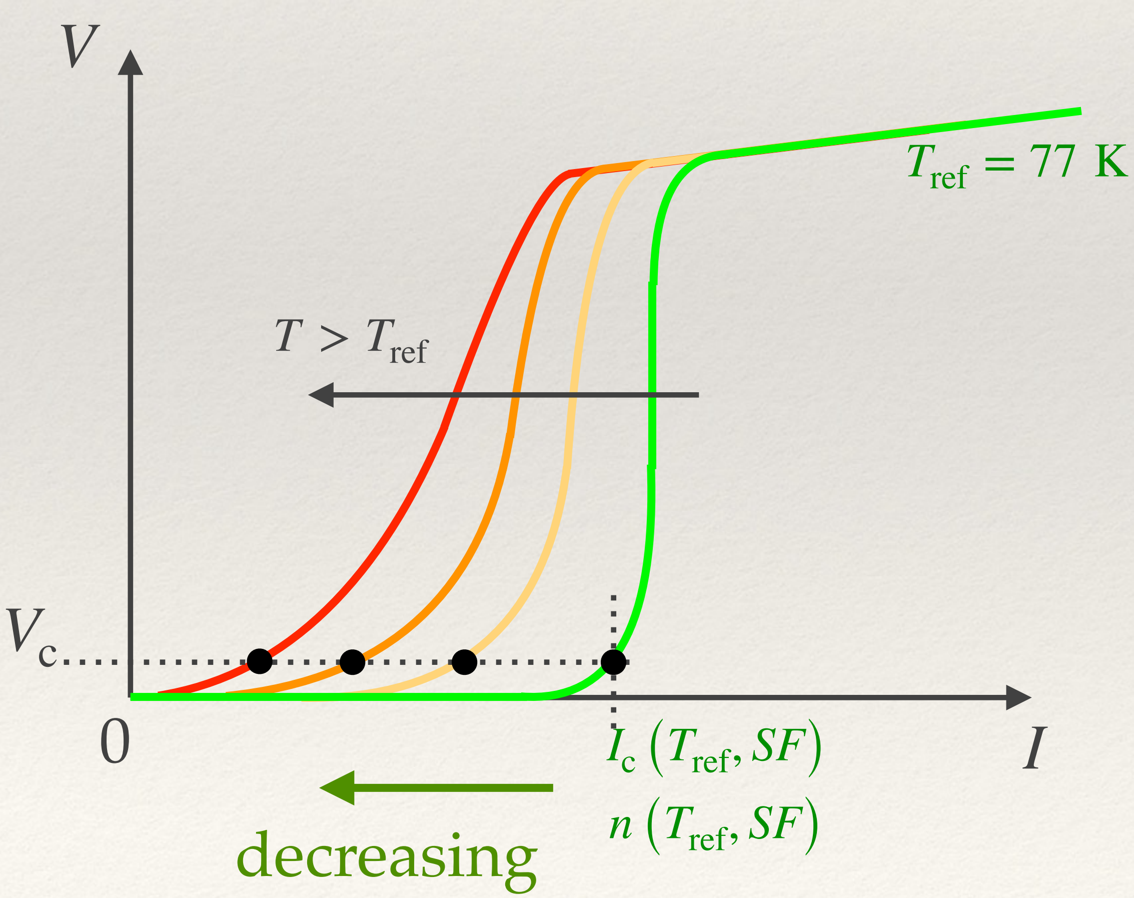


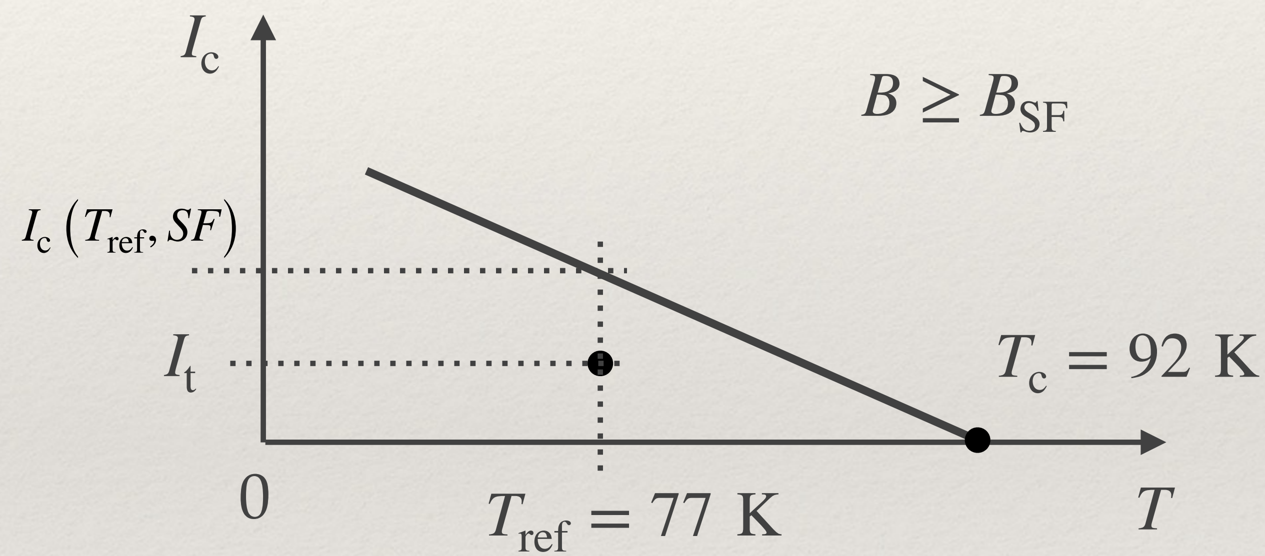


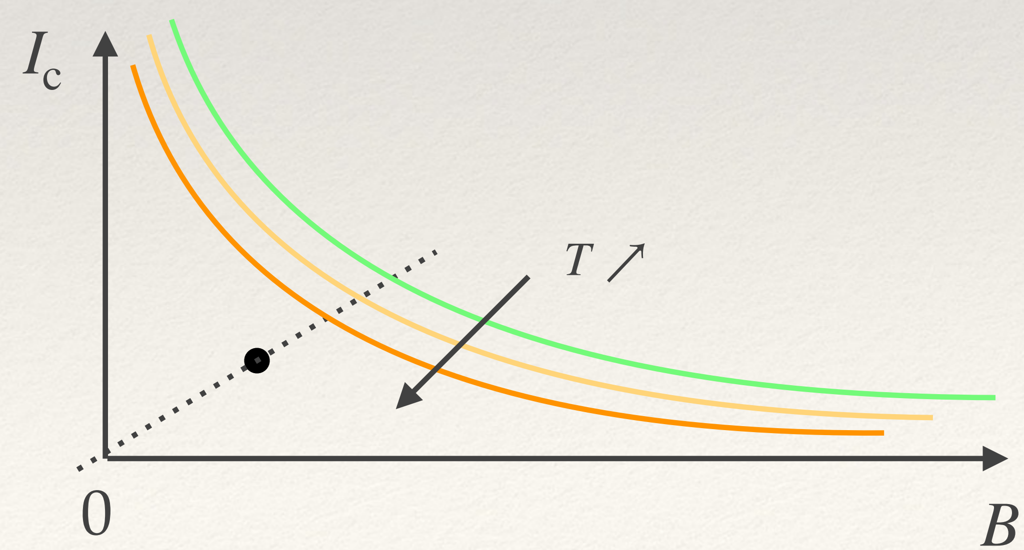
Modified from: B.B. Jensen, et al, US-China Education Review A, vol. 3, no. 3, 141-152, 2013.

Evolution of I_c and index n

- The characteristic V-I depends on temperature T and magnetic flux density B
- At current magnitude passed I_c, the matrix acts as a shunt at a near constant resistance value
- The index value follows the same degradation as the temperature increase and/or the magnetic field. However, in most of the models, it is assumed constant







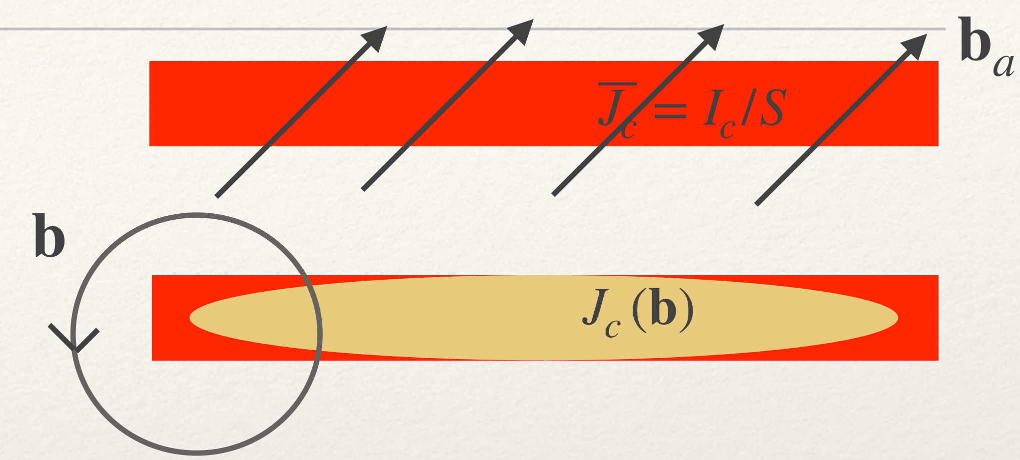
From I_c to J_c

- Known values from DC measurements: I_c ; derived value: J_c
- Two approaches:
  - Uniform distribution of current (most of instances, background field $\mathbf{b}_a$ >> self-field):

$$I_c = \int_{\Omega_s c} \mathbf{j}_c(\mathbf{b}) \cdot \mathrm{d}\mathbf{S} \simeq J_c\left(\mathbf{b}_a\right) \times S = I_c\left(\mathbf{b}_a\right)$$

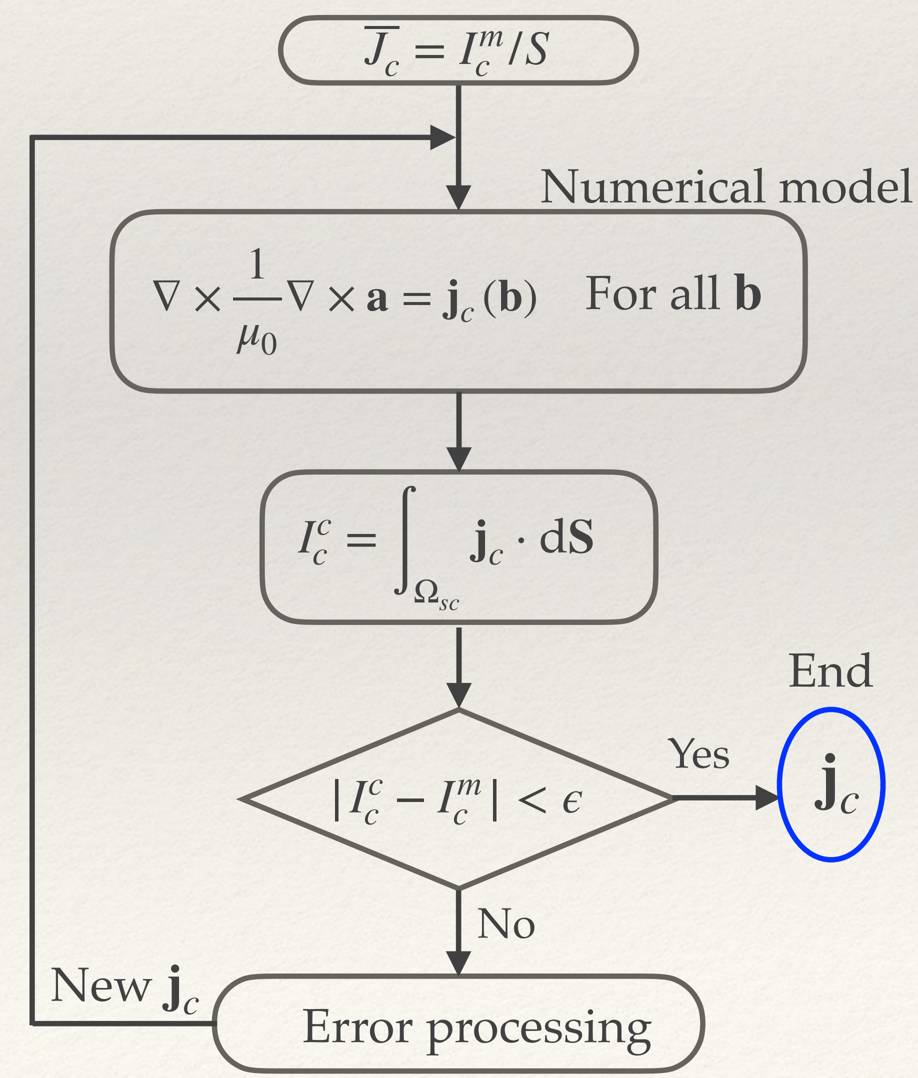
  - Expected distribution (more accurate for low field):

$$I_c(\mathbf{b}) = \int_{\Omega_{sc}} \mathbf{j}_c(\mathbf{b}) \cdot \mathrm{d}\mathbf{S} \neq J_c\left(\mathbf{b}_a\right) \times S$$



How to [Zermeño, 2017] [HTSModelling, 12]: numerical model to determine the actual current density distribution - iteration on the computed critical current I_c^c obtained from integral of the local critical current density J_c until convergence to the actual critical current I_c^m.

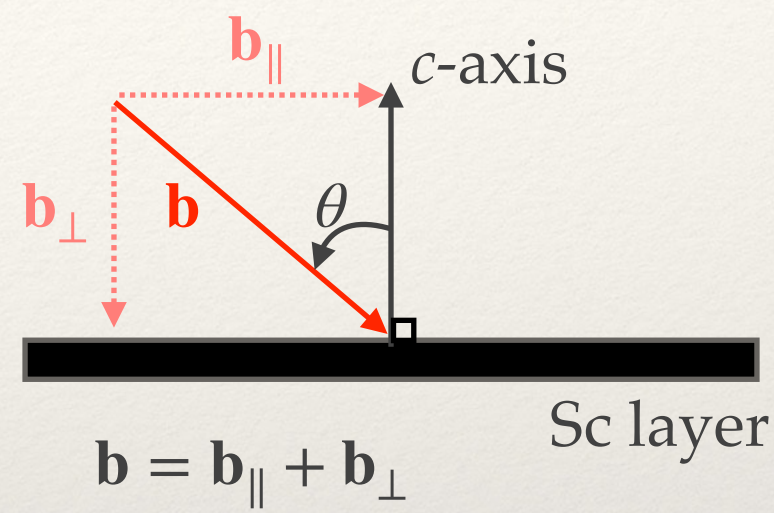
N.B.: The estimated current density J_c can be obtained from the modified Kim's relation solving the inverse problem by finding the parameters k, α and B_0 for instance.



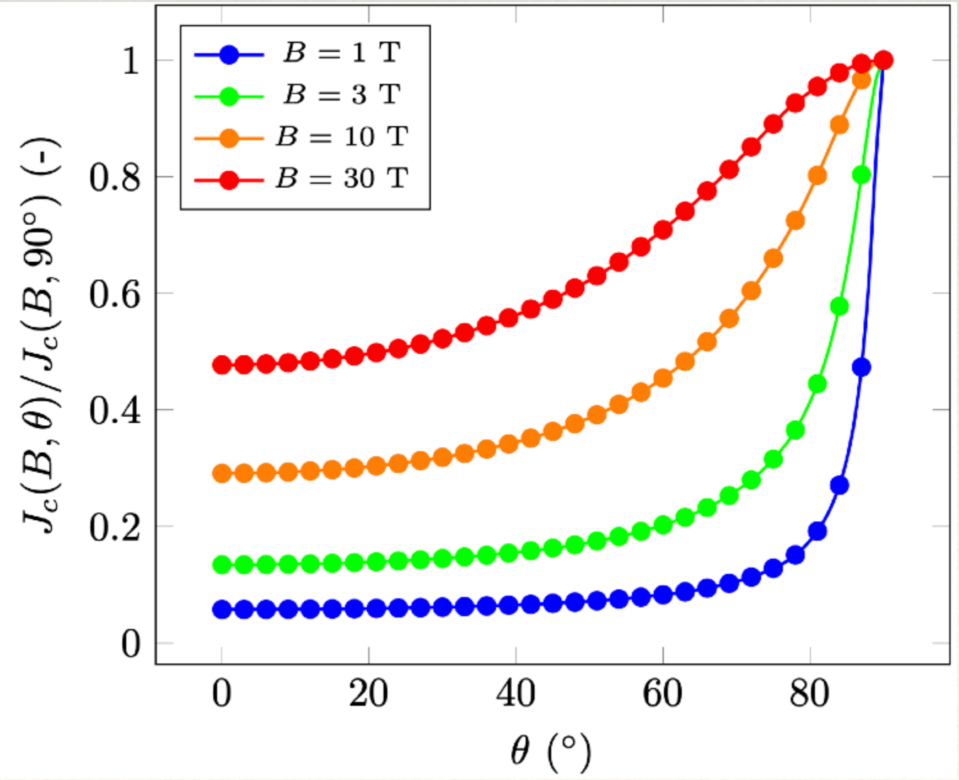
Critical current density J_c and n index

- Critical current density J_c:
  - Magnetic field dependence: dependence upon intensity and direction of the incidence of the magnetic flux density ("orientation") through the **modified Kim's relation** [Kim, 1962]:

$$J_c(\mathbf{b}) = \frac{J_{c0}}{\left(1 + \frac{\sqrt{k^2 b_{\parallel}^2 + b_{\perp}^2}}{B_0}\right)^{\alpha}}$$

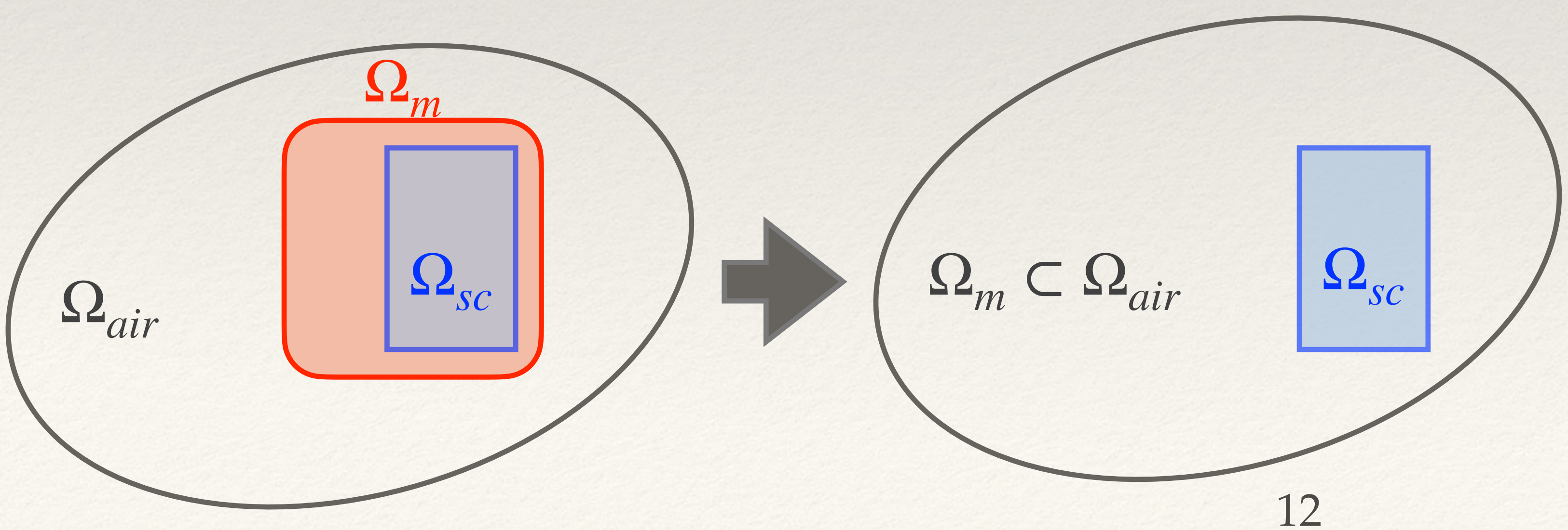


  - Temperature dependence (typically neglected):
    - $J_{c0} = \left(\frac{T_c - T}{T_c - T_{ref}}\right) J_{c00}$, J_{c00} is a constant (self-field at $T_{\text{ref}} = 77$ K)
- Index n:
  - As stated previously, in most of the cases, the index is assumed constant.
  - To our knowledge, no generic model of the dependence of the n index either on temperature or magnetic flux density on a wide range is available [Sass, 2015][Lee, 2015]

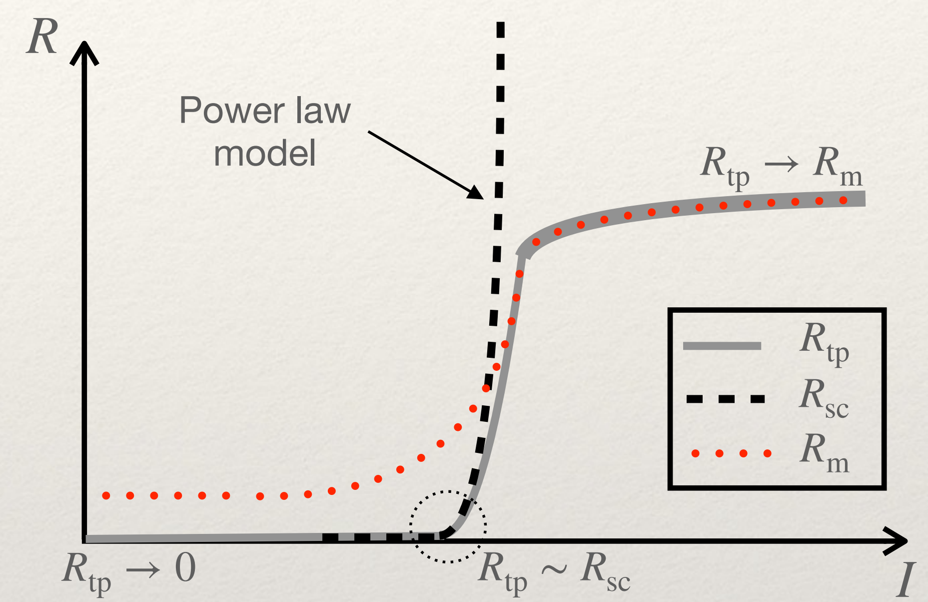


Power law: Range of validity

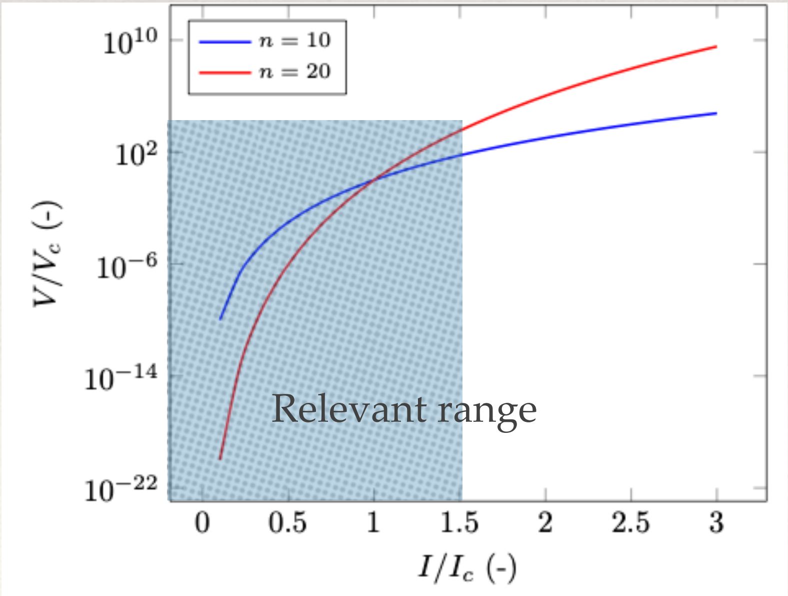
- Power law: $\mathbf{e} = \frac{E_c}{J_c^n} j^{n-1}\mathbf{j}$, with $E_c = 1\ \mu\text{V}/\text{cm}$
- Nonlinear resistivity: $\rho = \frac{E_c}{J_c^n} J^{n-1}$, with J_c the critical current density and index n
- **Validity** around J_c.
- Models assuming $R_m >> R_{sc}$ so the current induced in the matrix/stabilizer (m) is completely negligible => matrix/stabilizer lumped into the surrounding medium (depending on model: fictitious resistivity, $\rho_{\Omega_{air}} = 1\ \Omega\text{m}$ [Berrospe-Juarez, 2018])



Combined: SC and stabilizer*



SC alone*



*: conceptual figures

Maxwell's equations and constitutive relations

- Maxwell's equations:
  - Maxwell-Ampére (MA): $\nabla \times \mathbf{h} = \mathbf{j}$
  - Maxwell-Faraday (MF): $\nabla \times \mathbf{e} = -\partial_t \mathbf{b}$
- Constitutive laws (the nonlinearity is built inside the resistivity of the superconductor):
  - $\mathbf{b} = \mu_0 \mathbf{h}$
  - $\mathbf{e} = \rho \left(T, \mathbf{b}, |\mathbf{j}| \right) \mathbf{j}$

Potentials in electromagnetism

- Electric potential: $\nabla \times \mathbf{e} = 0 \Rightarrow \mathbf{e} = -\nabla V$
- Without current: $\nabla \times \mathbf{h} = 0 \Rightarrow \mathbf{h} = -\nabla \phi$
- Magnetic vector potential: $\nabla \cdot \mathbf{b} = 0 \Rightarrow \mathbf{b} = \nabla \times \mathbf{a}$
- Current vector potential: $\nabla \cdot \mathbf{j} = 0 \Rightarrow \mathbf{j} = \nabla \times \mathbf{T}$
- Couling magnetic and electric model: $\mathbf{e} = -\partial_t \mathbf{a} - \nabla V$

$$\nabla \times \mathbf{e} + \partial_t \mathbf{b} = 0 \Rightarrow \nabla \times \left(\mathbf{e} + \partial_t \mathbf{a}\right) = 0 \Rightarrow \mathbf{e} + \partial_t \mathbf{a} = -\nabla V$$

Two formulations of the Maxwell equations

- To model 2G HTS, various formulations of the Maxwell equations have been considered in 2D: *H*, *H-A*, *T-A*, *H-*ϕ, *A-V*
- Depending on the dimension (2D, 3D) and the problem at hand (presence of ferromagnetic material for instance => choice of appropriate formulation)
- We are going to present:
    - The "classic" *H* formulation, the variable is the magnetic field
    - The "new" *T-A* formulation, the variables are the vector current potential *T* and the vector magnetic potential *A*

“Classic” H formulation

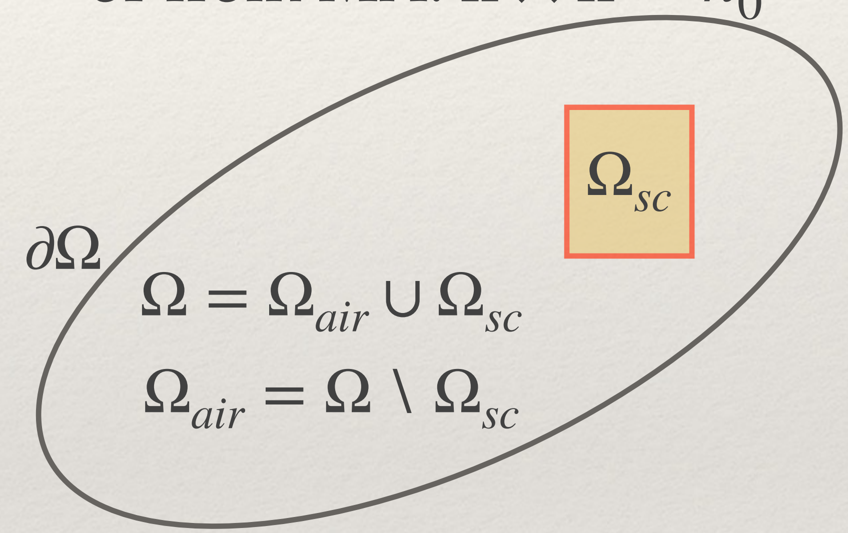
- H formulation (“classic”, used to cross-check models):

$$\nabla \times \big(\rho\,(\mathbf{b})\,\underbrace{\nabla \times \mathbf{h}}_{=\mathbf{j}}\big) + \mu_0 \partial_t \mathbf{h} = 0$$

- Magnetic Gauss law checked by setting initial condition: $\mathbf{h}\,(t=0) = \overrightarrow{cst}$: $\nabla \cdot \mathbf{b} = 0$ (since $\nabla \cdot \nabla \times \mathbf{e} = 0$)

$\mathbf{h}\,|_{\partial\Omega} = 0$

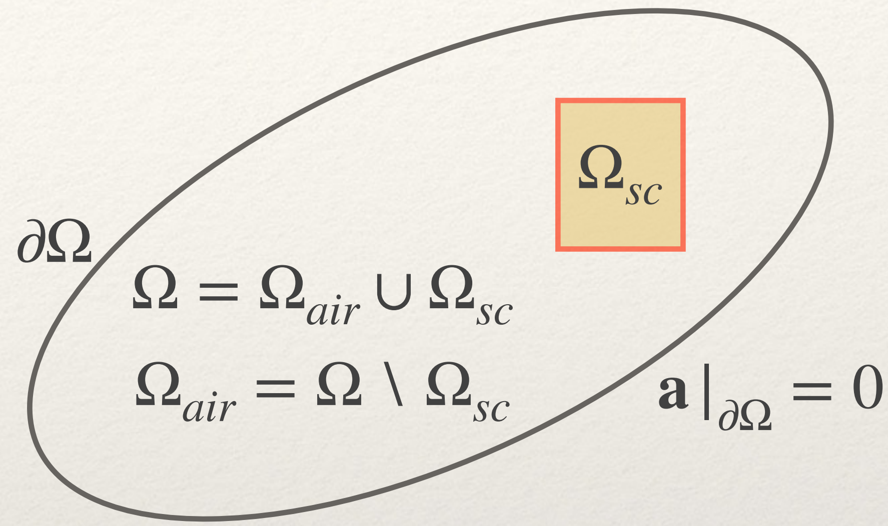
or from MA: $\mathbf{h} \times \mathbf{n} = h_0$



Latest *T-A* Formulations of Maxwell's equations

- *A*-formulation:

$$\nabla \times \frac{1}{\mu_0} \nabla \times \mathbf{a} = \mathbf{j}$$



- *T*, *A* - formulation (latest)[Zhang, 2017],[Berrospe-Juarez, 2019][HTSModelling, 21]:

T -formulation

$$\nabla \times \left[\rho(\mathbf{b}) \nabla \times \mathbf{T}\right] = -\partial_t \mathbf{b}$$

A - formulation

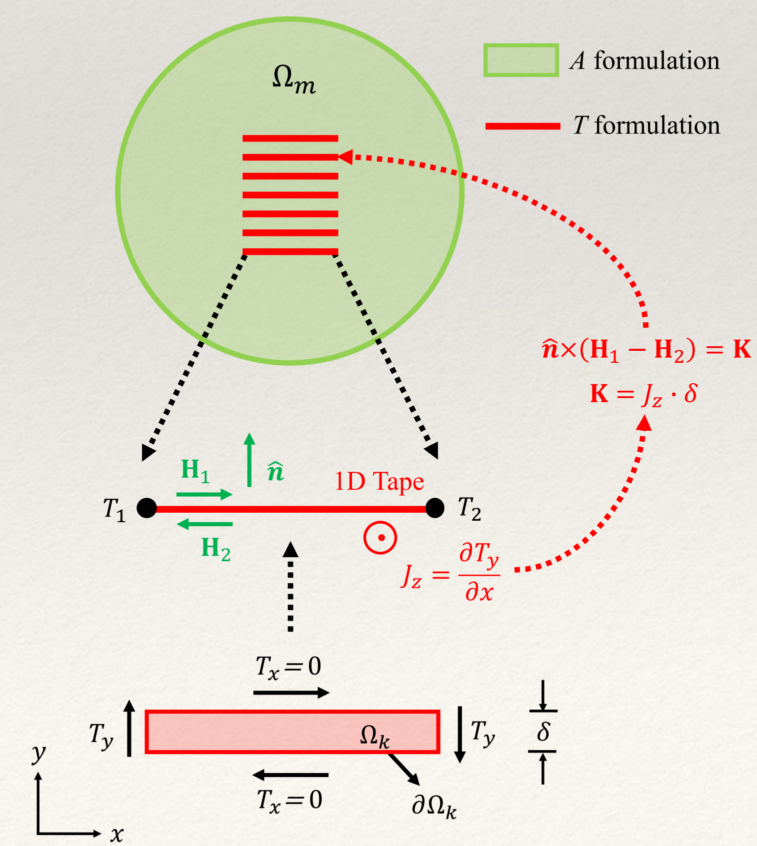
$$\nabla \times \frac{1}{\mu_0} \nabla \times \mathbf{a} = 0$$

The tapes are modeled as 1D lines

Thickness of SC layer

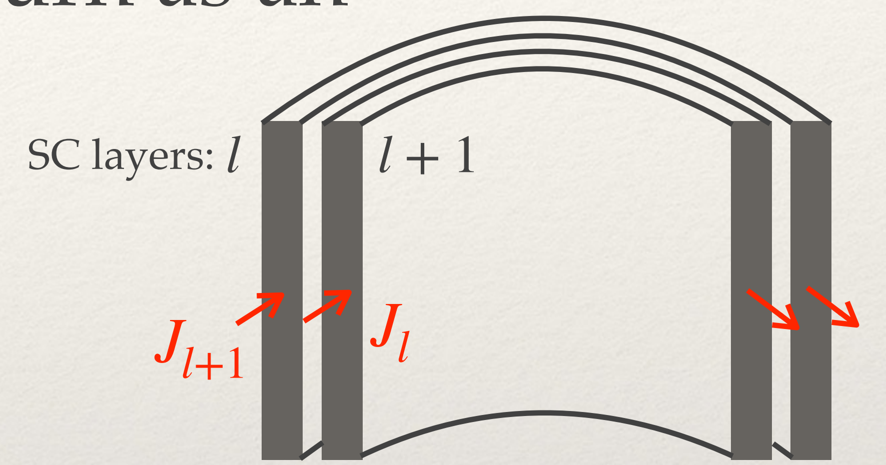
$$\mathbf{n} \times \left(\mathbf{h}_1 - \mathbf{h}_2\right) = \delta \underbrace{\nabla \times \mathbf{T}}_{\mathbf{j}}$$

Surface current **K**



Impressed transport current

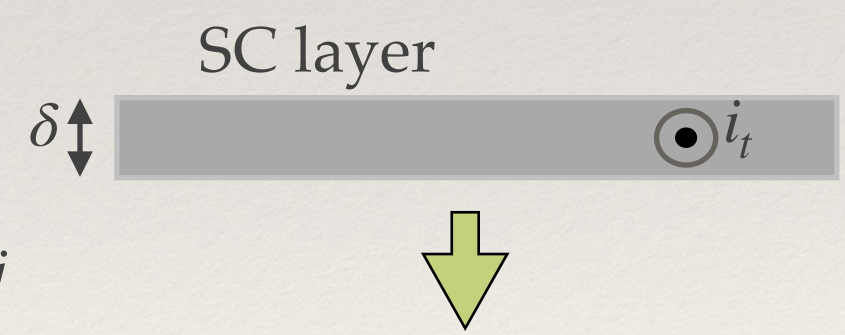
- Transport current i_t to be impressed turn by turn as an additional constraint to the model.



$J_l \neq J_{l+1}$ But $i_t = i_l = i_{l+1}$

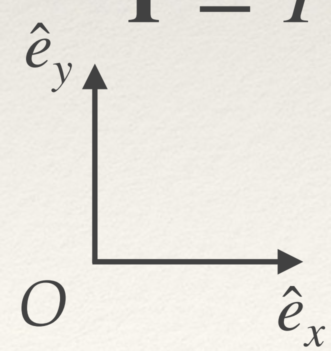
- *H* - formulation, at turn *l*:

$$i_t = i_l = \int_{\Omega_{sc_l}} \mathbf{j}_l \cdot d\mathbf{S} \qquad \mathbf{j}_l = \nabla \times \mathbf{h}$$



- *T*, *A* - formulation

$$\mathbf{T} = T(x,t)\,\hat{e}_j$$



$$\frac{i_t}{\delta} = T_l - \underbrace{T_r}_{=0}$$

$$i_t = \int_{\Omega_{sc}} \mathbf{j} \cdot d\mathbf{S} = \oint_{\partial\Omega_{sc}} \mathbf{T} \cdot d\boldsymbol{\gamma}$$

Estimation of losses

- The instantaneous losses over the tape *i* are given by:

$$Q_{sc,i} = \iint_{\Omega_{sc,i}} \mathbf{E} \cdot \mathbf{J} \, \mathrm{d}\Omega \quad \text{or} \quad Q_{\mathrm{sc,i}} = \delta \int_{\partial\Omega_{sc,i}} \mathbf{E} \cdot \mathbf{j} \, \mathrm{d}l$$

H formulation

T-*A* formulation

- The total average losses are computed as:

$$\tilde{Q}_{\mathrm{tot}} = \frac{1}{t_{\mathrm{p}}} \int_{t_{\mathrm{p}}} \sum_{i=1}^{N_{\mathrm{tp}}} Q_{sc,i} \, \mathrm{d}t$$

Analysis tools

- **Definition of a reference model**: all the tapes in the stack are simulated or full model

- Total losses - relative error defined as: $e_{qr} = \left| \dfrac{Q_r - Q_k}{Q_r} \right|$

where Q_r and Q_k are the total losses of the reference model and of the iteration k of the model under evaluation

- Current density - coefficient of determination or R^2 (indicates the likeliness of the distribution across the width of the tapes between models):

$$R_k^2 = 1 - \frac{\sum_{l=1}^{N_s} \left(J_l^r - J_l^{fk}\right)^2}{\sum_{l=1}^{N_s} \left(J_l^r - \overline{J^r}\right)^2}$$

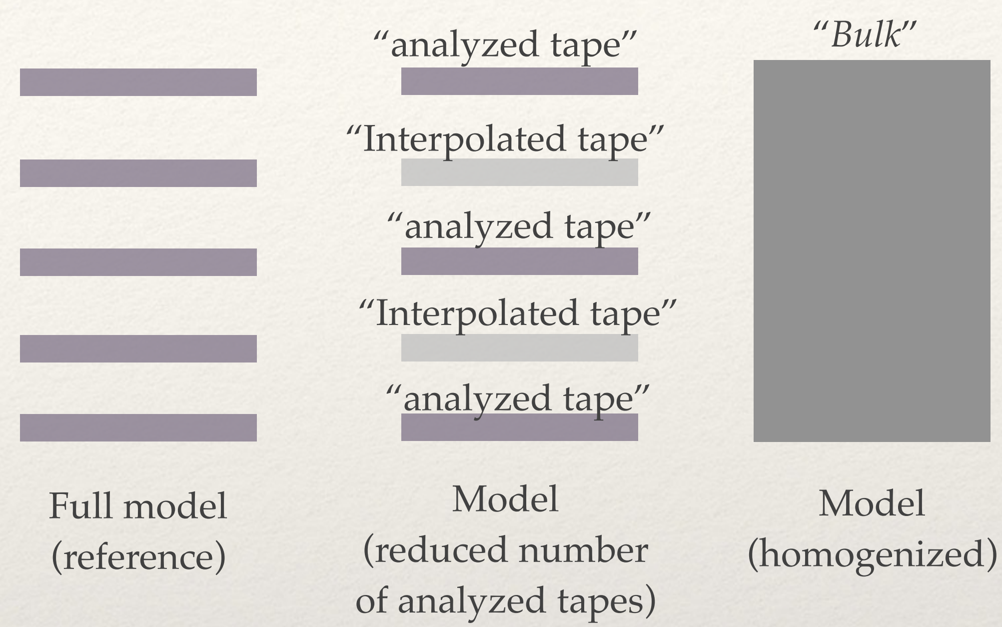
where,

J_l^r is a vector of current density containing the current density distribution of all the tapes for all the computed times obtained from the reference model.

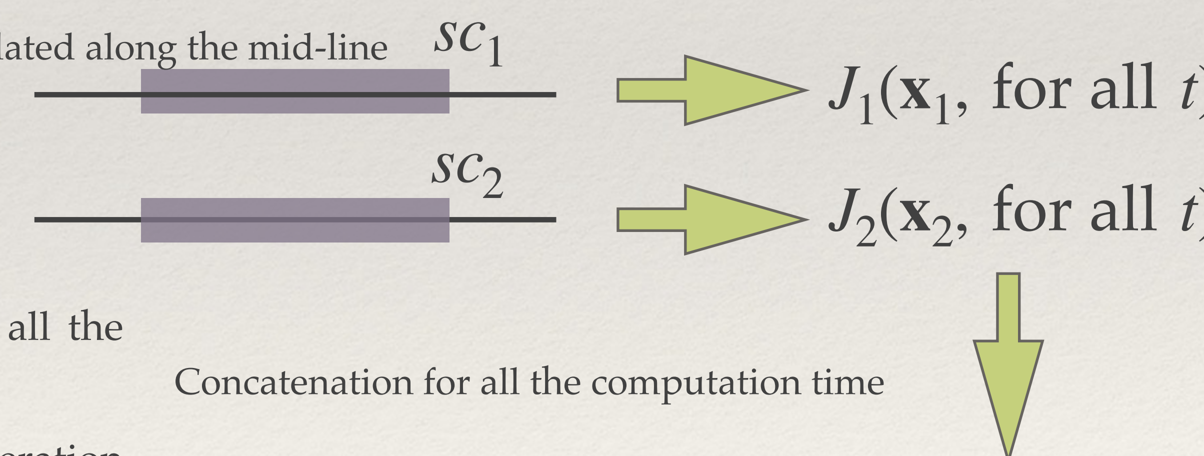
J_l^{fk} is a vector of current density for all the tapes obtained from interpolation at iteration k.

$\overline{J^r}$ is the average of the current density of the tapes at all times from the reference model.

- Magnetic flux density - "visual" and error on center field (SCIF)



J interpolated along the mid-line



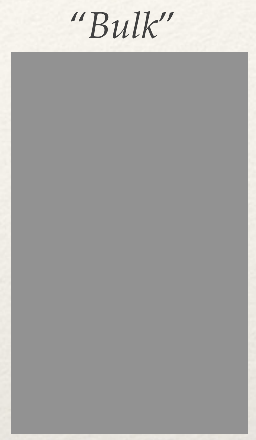
Concatenation for all the computation time

$$J = \left(J_1, J_2, \ldots, J_m\right)$$

Number of time steps: n_t
Number of tapes: m
Number of elements in the tapes: n_e
$N_s = \dim(J) = n_t \times n_e \times m$

Techniques to lower the computation burden

- Objectives:
  - **The reduction of the computational load** (reduction of the size of the problem)
  - **A lesser computation time**, targeting "real-time" simulations (full computation time comparable to the characteristic time of the response of the system to changes)
- Techniques:
  - **Homogenization:** transform a set of tapes into a bulk with an "equivalent" current density
  - **Multi-scaling:** solve a global problem and use appropriate boundary conditions to solve a detailed local problem ("and the way around")
  - **Reduced models**: model of a subset of all the tapes (referred here to as analyzed tapes) and then interpolation techniques to propagate the information to the remaining tapes



Model
(homogenized)

Model
(reduced number
of analyzed tapes)

Techniques: Homogenization

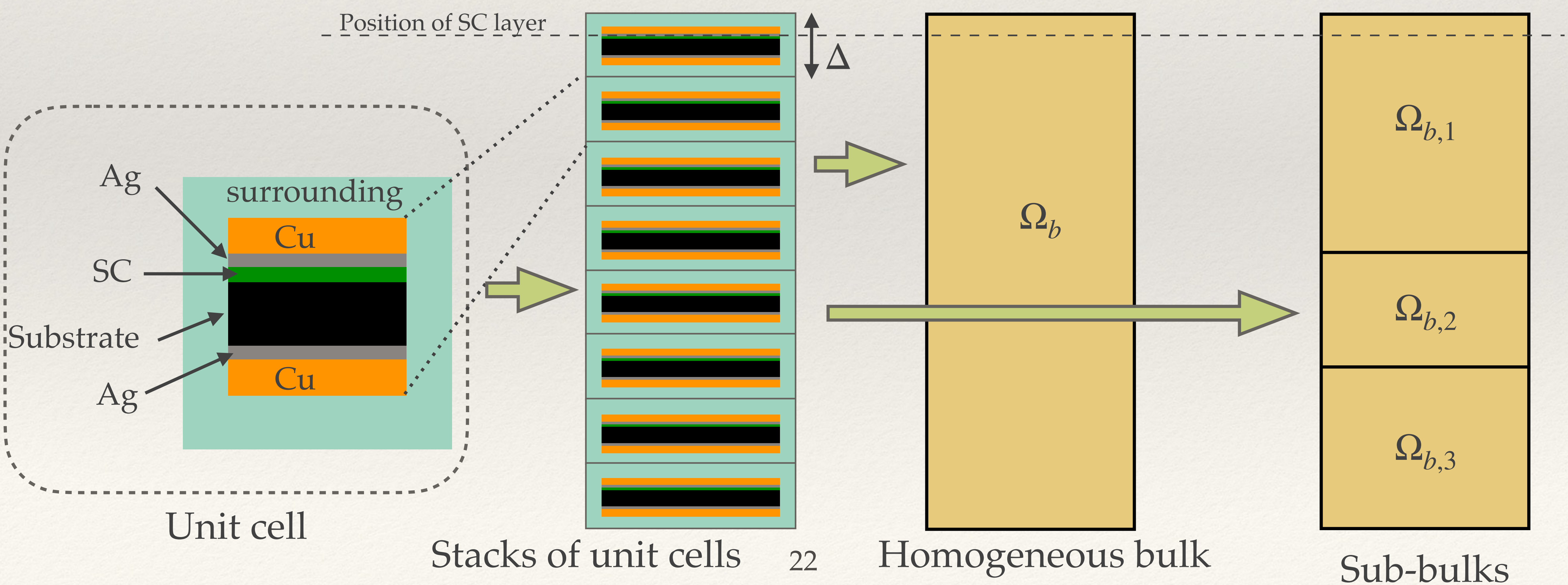
- Conditions to apply homogenization:
  - Evenly distributed tapes
  - Identical tapes -> a bulk
  - Constant permeability
- The stack of tapes is transformed into an homogeneous bulk, with anisotropic properties
- Additional constraint for impressed transport current: $N_l i_t = \int_{\Omega_{b,l}} \mathbf{j}_l \cdot \, d\mathbf{S}$
- Clear advantage: a reduced number of Degrees Of Freedom (DOF)

δ: thickness of SC layer

Δ: thickness of unit cell

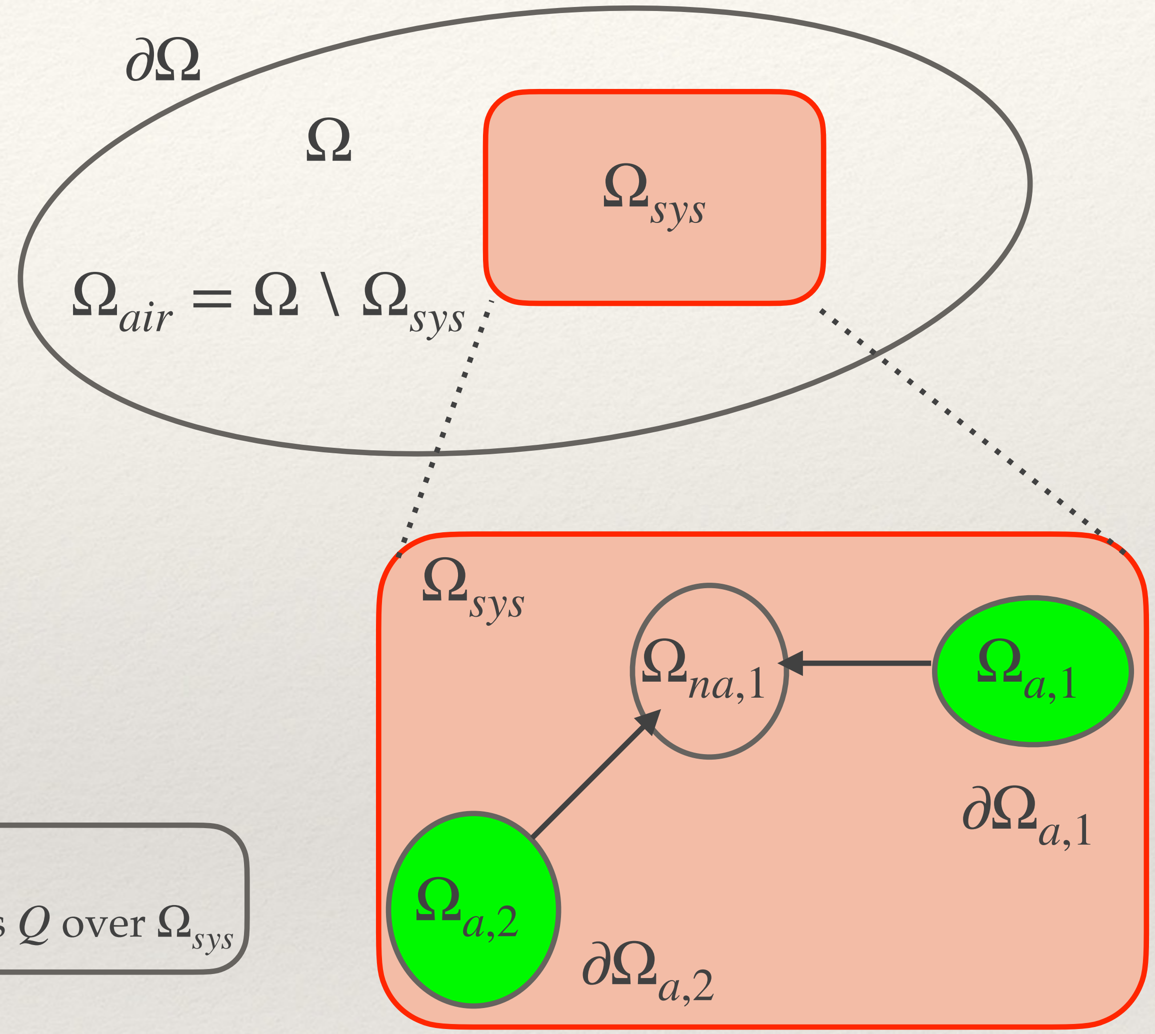
N_l: Number of unit cells

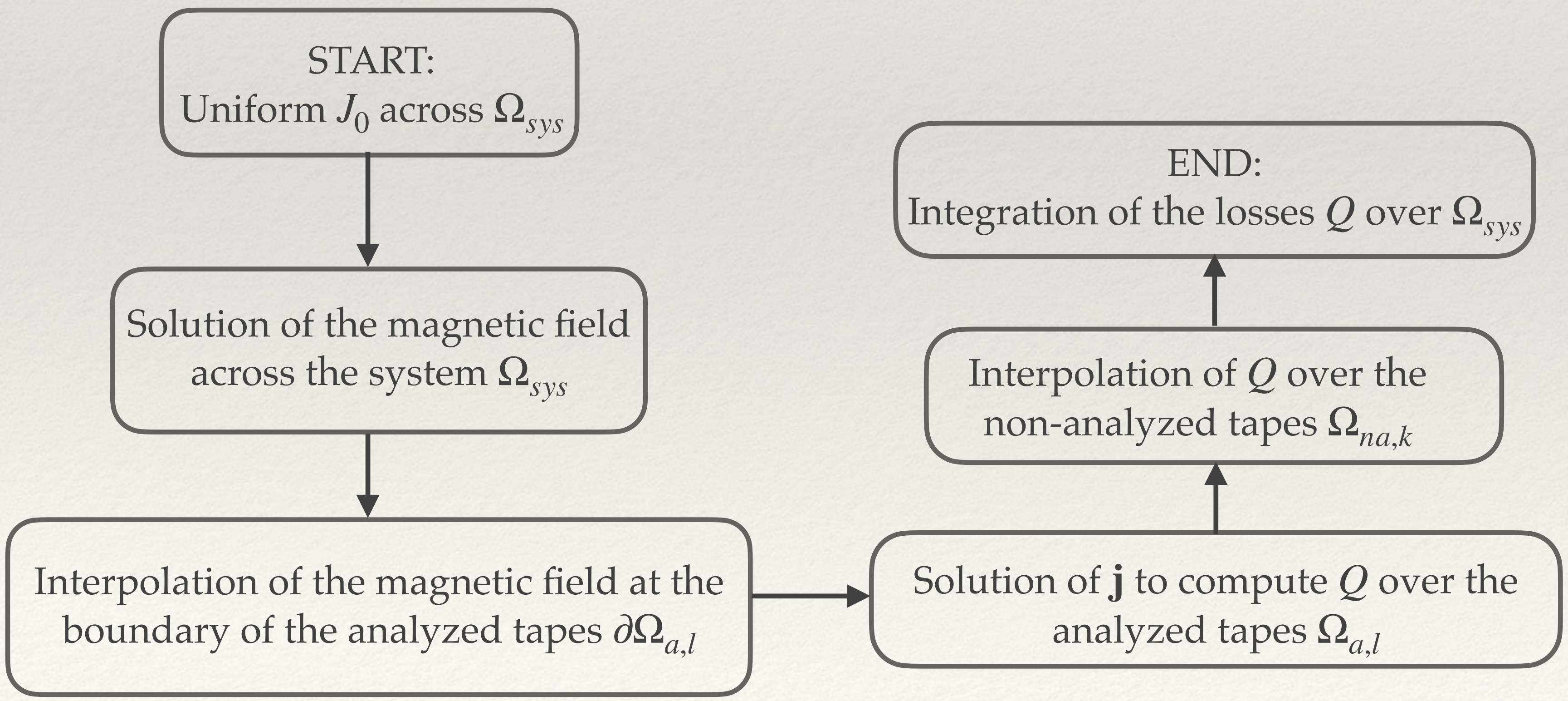
$$J_{c,b_l} = \frac{\delta}{\Delta} \times J_c$$



Techniques: multi-scaling

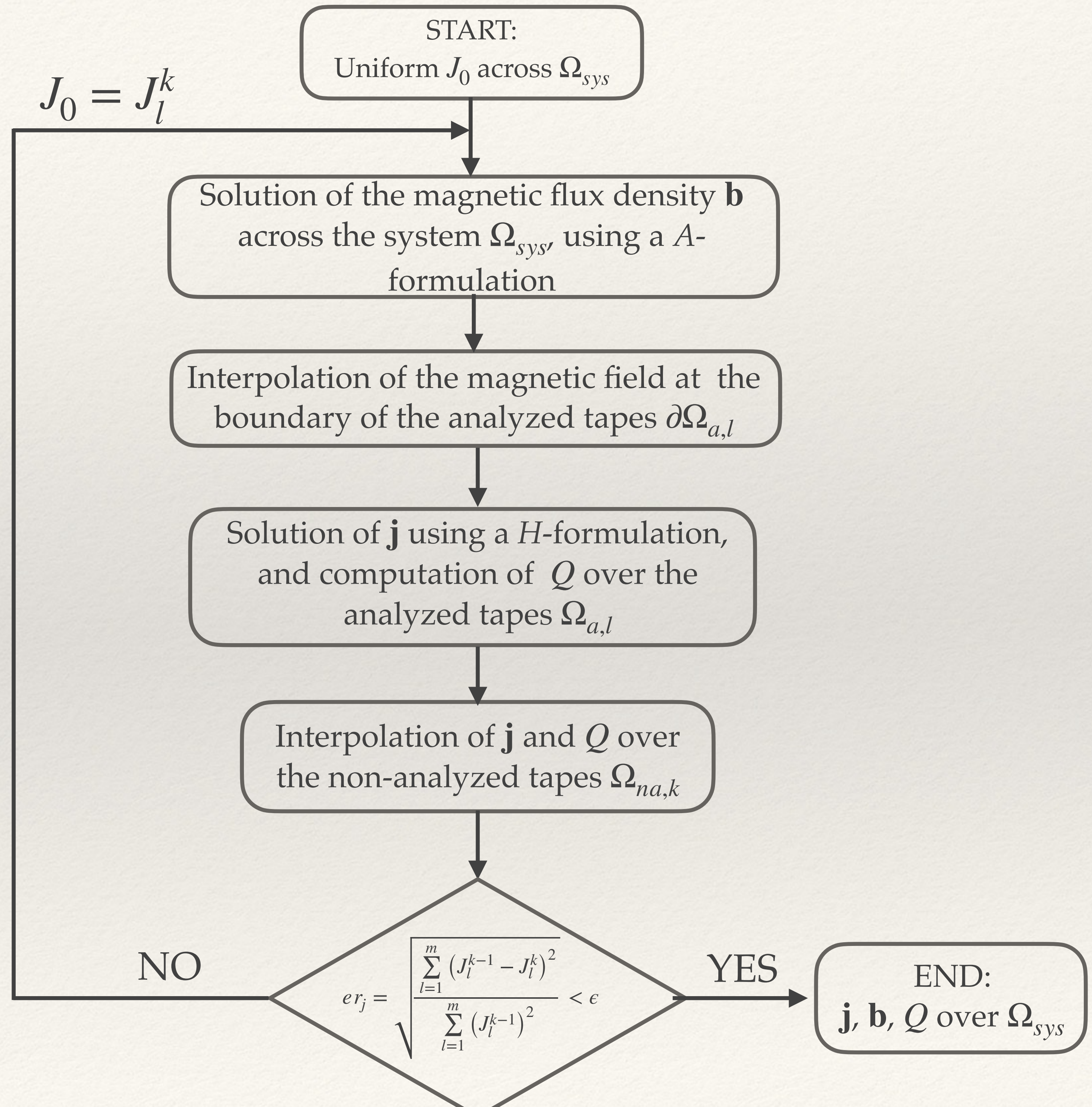
- The multi-scale approach does not require any pre-requisites.
- The model is split into a global model (the whole system, Ω) and a set of local detailed models $\Omega_{a,l}$
- Not all the tapes are modeled, only a fraction for computation efficiency (analyzed tapes)
- An interpolation technique is used to propagate the information to the remaining, non-analyzed tapes $\Omega_{na,k}$





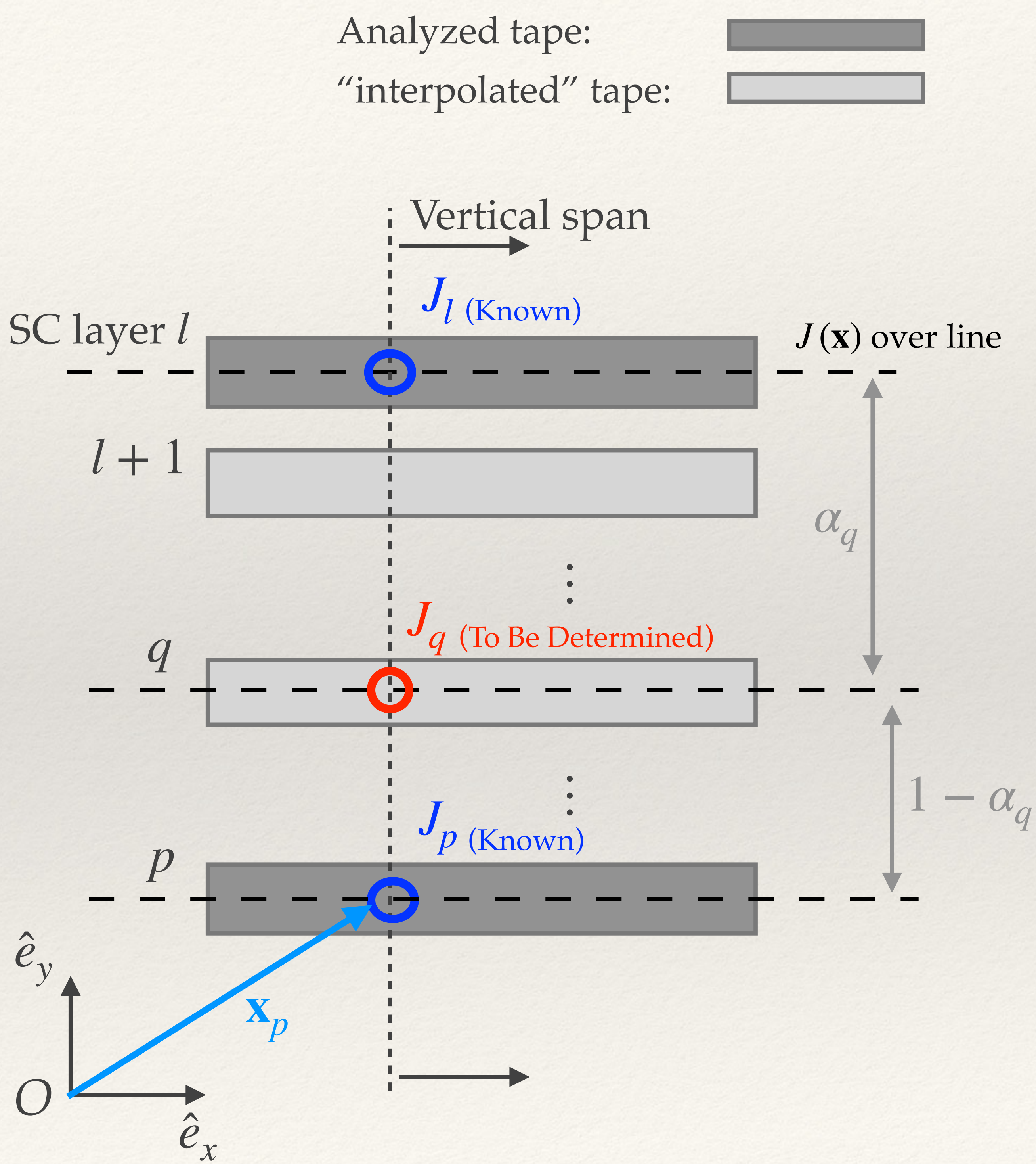
Techniques: iterative multi-scaling

- Modification of the classic multi-scale approach to include a feedback loop to the J distribution
- The gain over the global losses Q is miscellaneous
- However, large increase in the accuracy of: **b**, **j**, and local Q



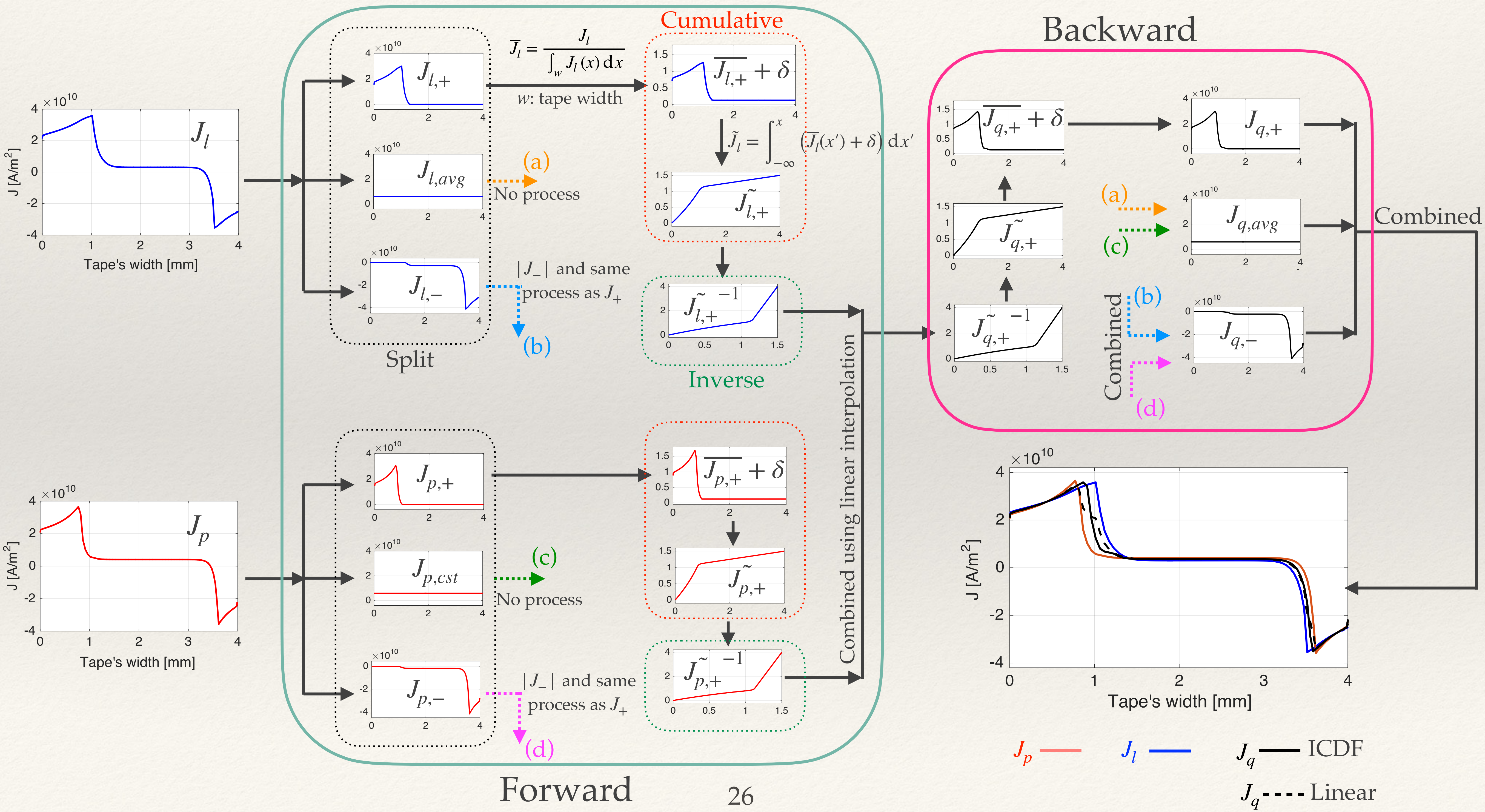
Techniques: Interpolations

- Interpolation techniques: solving the current density J in some analyzed tapes and interpolate J for the remaining tapes
- Two approaches:
  - "Classic" linear interpolation: $J_q = (1-\alpha_q)J_l\left(\mathbf{x}_l\right) + \alpha_q J_p\left(\mathbf{x}_p\right)$, with $\alpha_q = \dfrac{|\mathbf{x}_q - \mathbf{x}_l|}{|\mathbf{x}_p - \mathbf{x}_l|}$, $\mathbf{x} = x\hat{e}_x + y\hat{e}_y$
  - Inverse Cumulative Density Function (ICDF) method

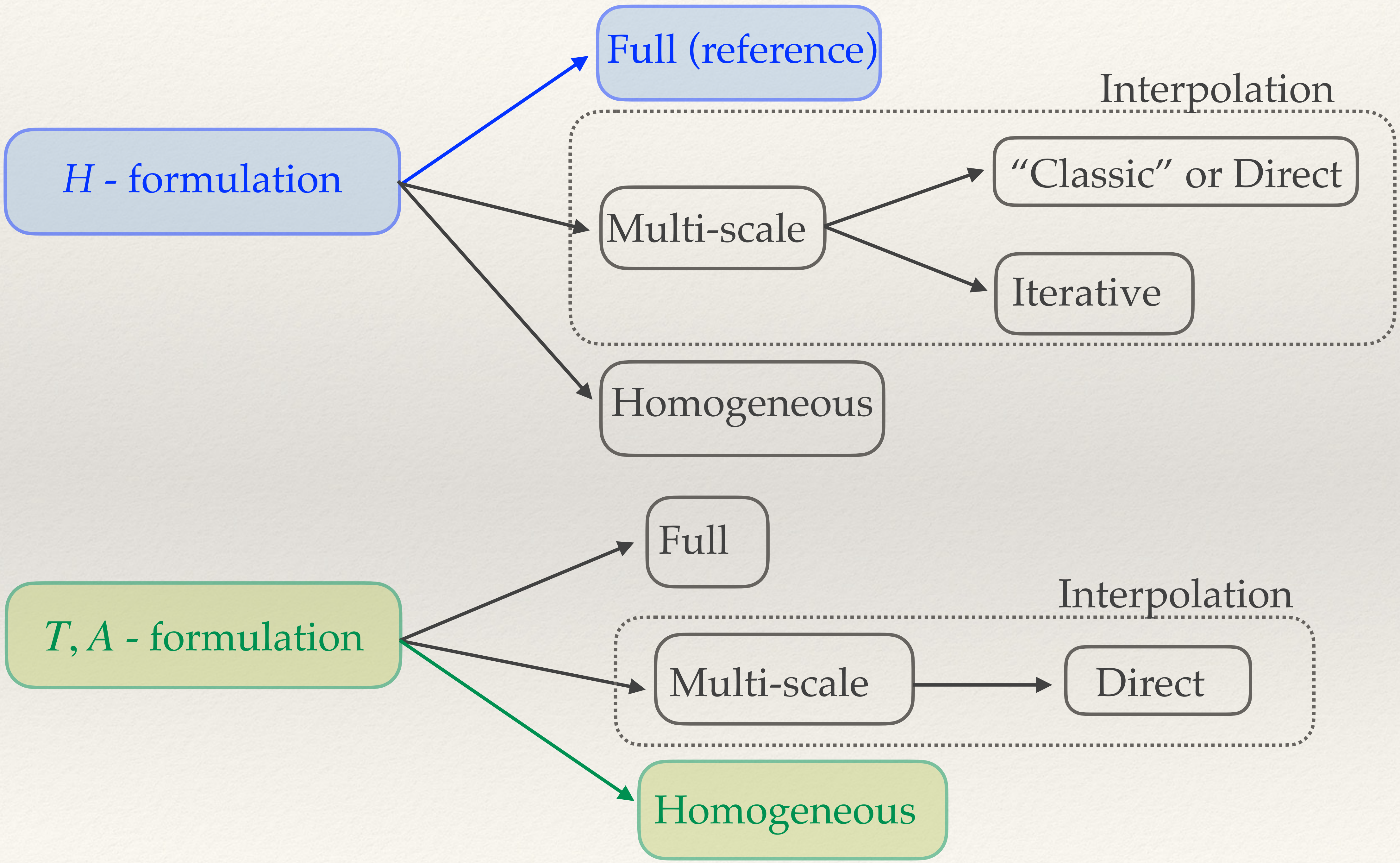


Techniques: ICDF method

- Inverse Cumulative Density Function (ICDF) method [Bonneel, 2011], [Berrospe-Juarez, SUST-2018]

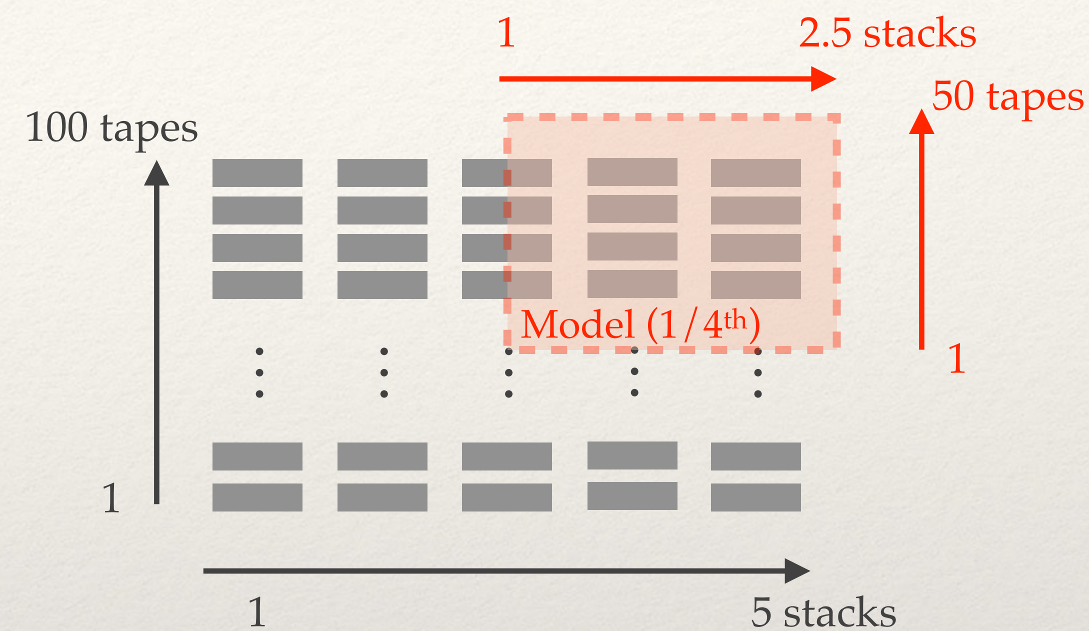


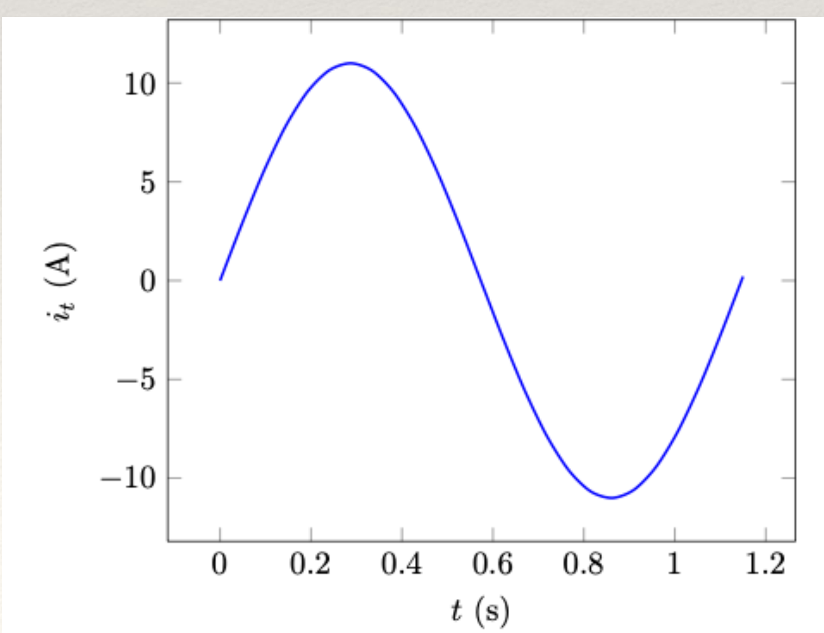
Summary of formulations and techniques used in the present case study



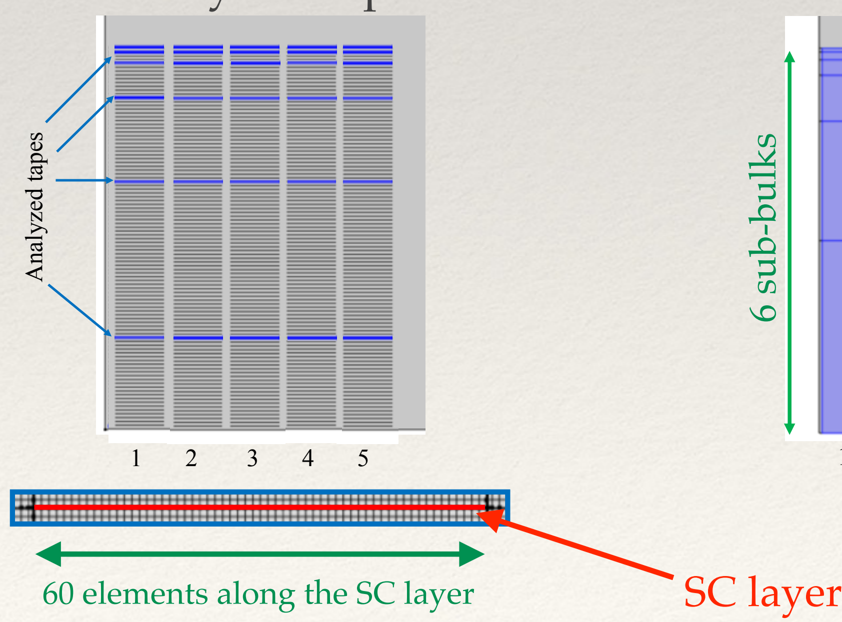
Case study: planar case

- Planar case: 10 stacks with 200 tapes per stack [Berrospe-Juarez, 2020]
- Cu-stabilized REBCO tapes (1μm thick, 4 mm wide SC) with $I_c\,(77\ \mathrm{K},\ \mathrm{SF}) = 112\ \mathrm{A}$
- Impressed sinusoidal transport current, $i_t = I_0 \sin(2\pi\nu t)$, with $I_0 = 11\mathrm{A}$ and $\nu = 50$ Hz (one full cycle)
- Power law and J_c dependent on magnetic flux density and its orientation with parameters: $n = 38$, $J_{c0} = 2.8 \times 10^{10}\,\mathrm{A/m^2}$, $B_0 = 0.04265\mathrm{T}$, $k = 0.29515$, $\alpha = 0.7$.

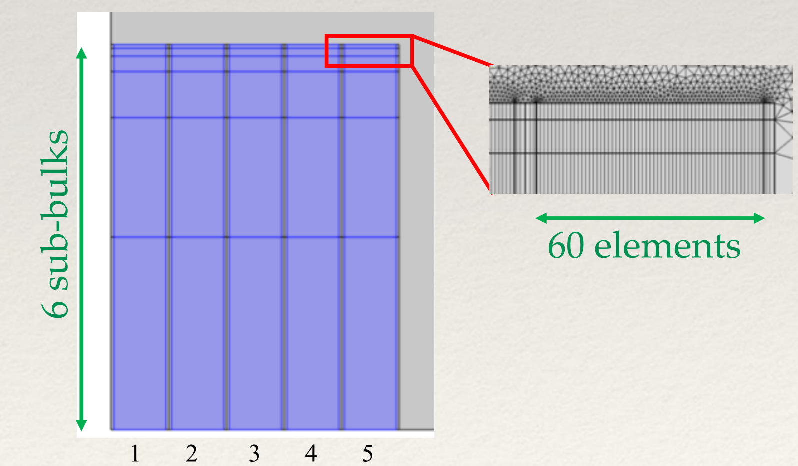




Multi-scale models
6 analyzed tapes

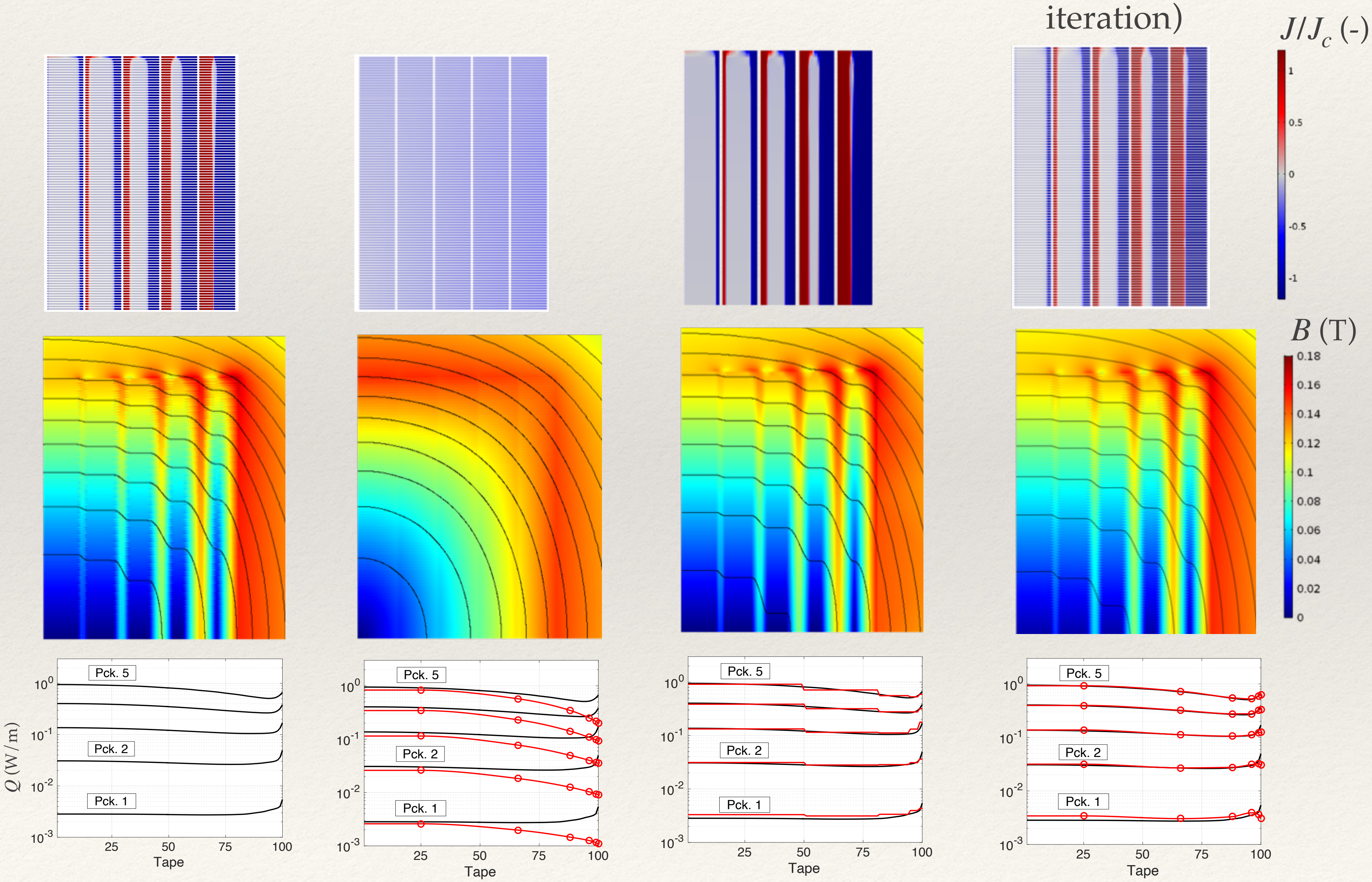


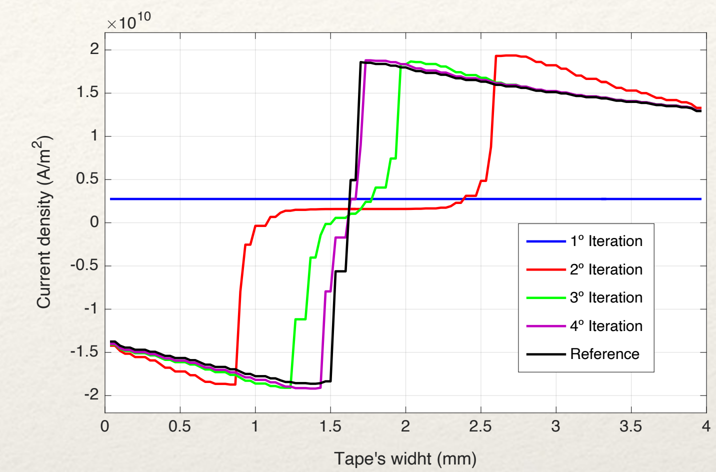
Homogeneous models

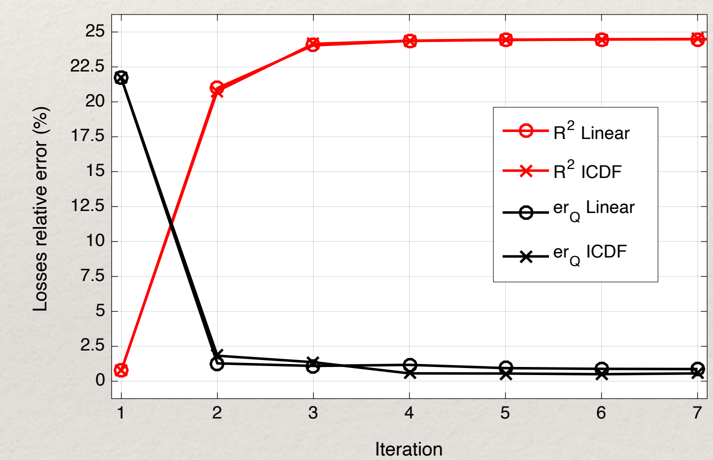


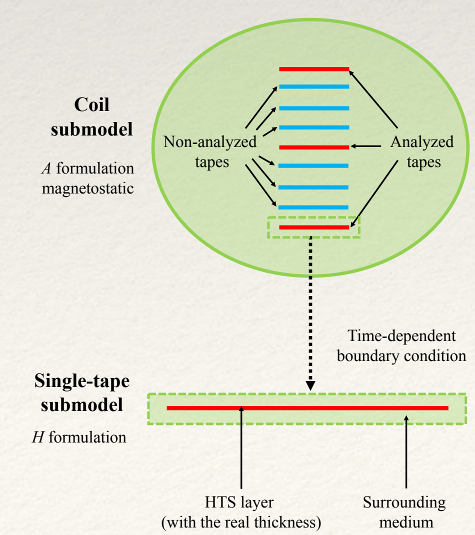
Results: *H*-formulation

H, full model | *H*, multi-scale model | *H*, homogeneous model | *H*, iterative multi-scale model (7th iteration)



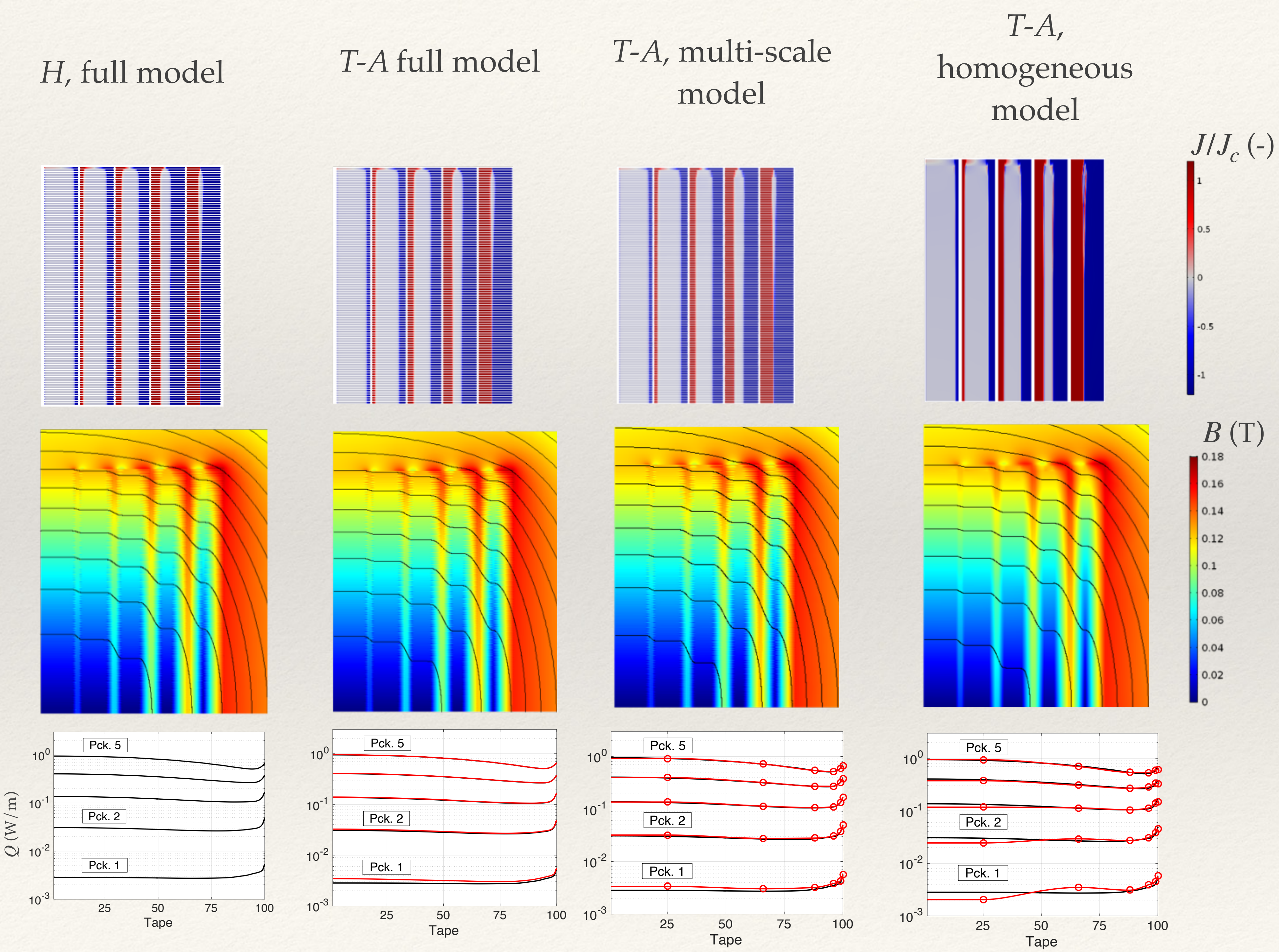


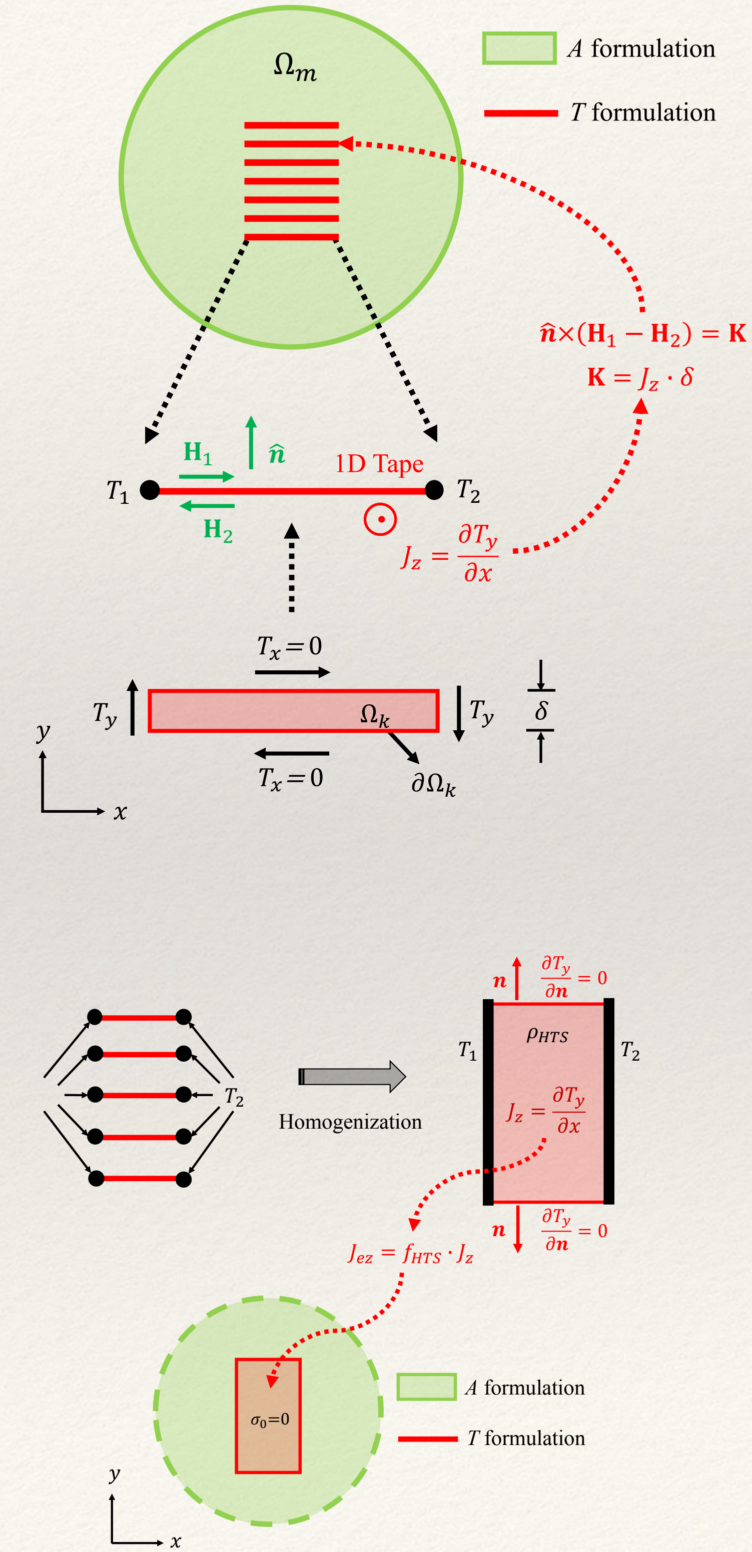




Red line: cross-checked model
Black line: *H*, full model (reference model)

Results: *T*, *A*-formulation





Red line: cross-checked model
Black line: *H*, full model (reference model)

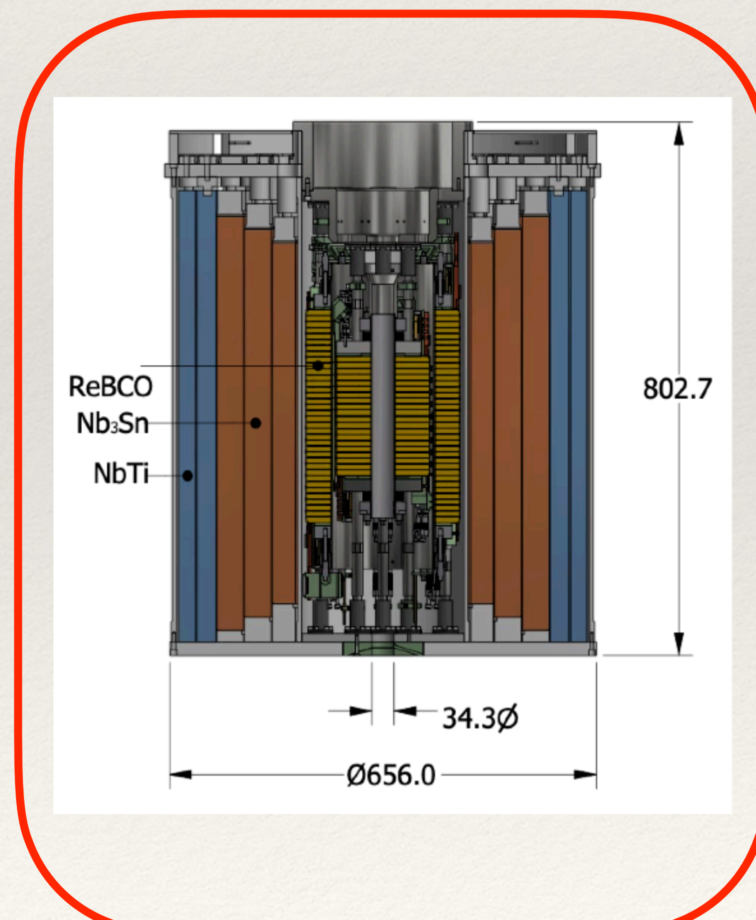
Comparison of results

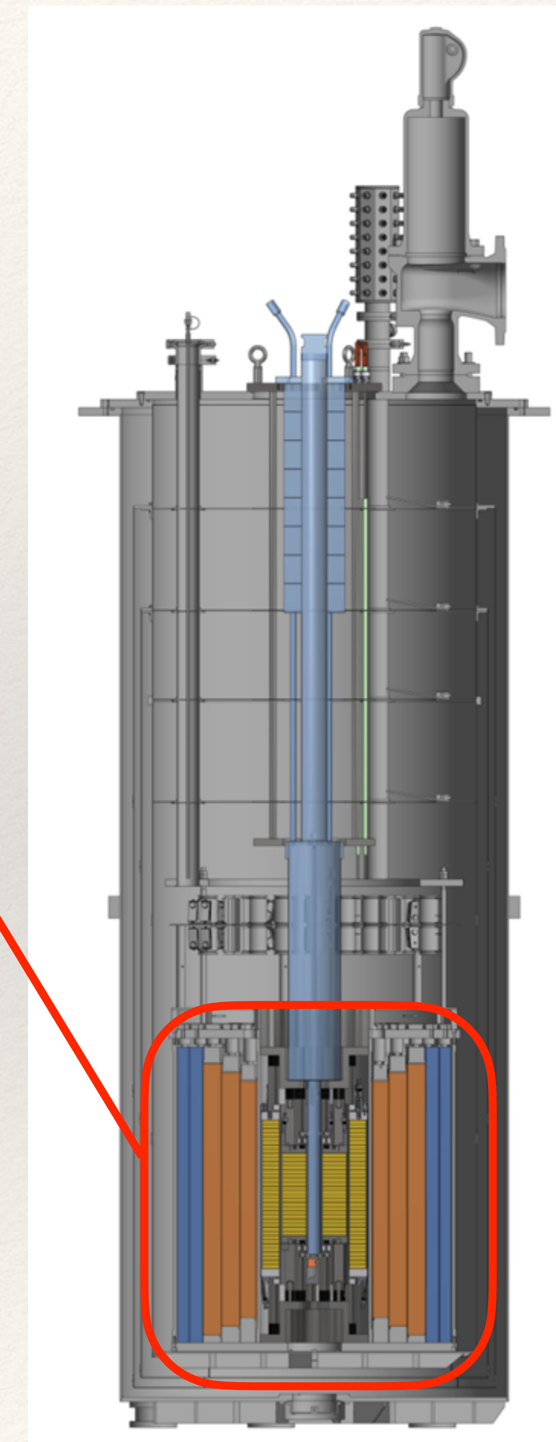
***H* full reference model: total loss = 127.24 W/m, with a computation time of 31 h 32 min (without post-processing)**

Models (compared to *H* full)	Total loss [W/m]	Current density distribution	Computation time [h]
	er_Q [%]	R^2 [1]	[%]
***H* multi-scale+interpolation**	**21.7**	**0.0304**	**1.45**
H iterative multi-scale+interpolation	0.56	0.9803	10.51
H homogeneous	1.28	0.9221	1.94
T,*A* full	0.64	0.9922	10.25
T,*A* multi-scale+interpolation	0.31	0.9913	5.06
***T*,*A* homogeneous**	**0.71**	**0.9214**	**0.78**

Application: 32 T all superconducting magnet

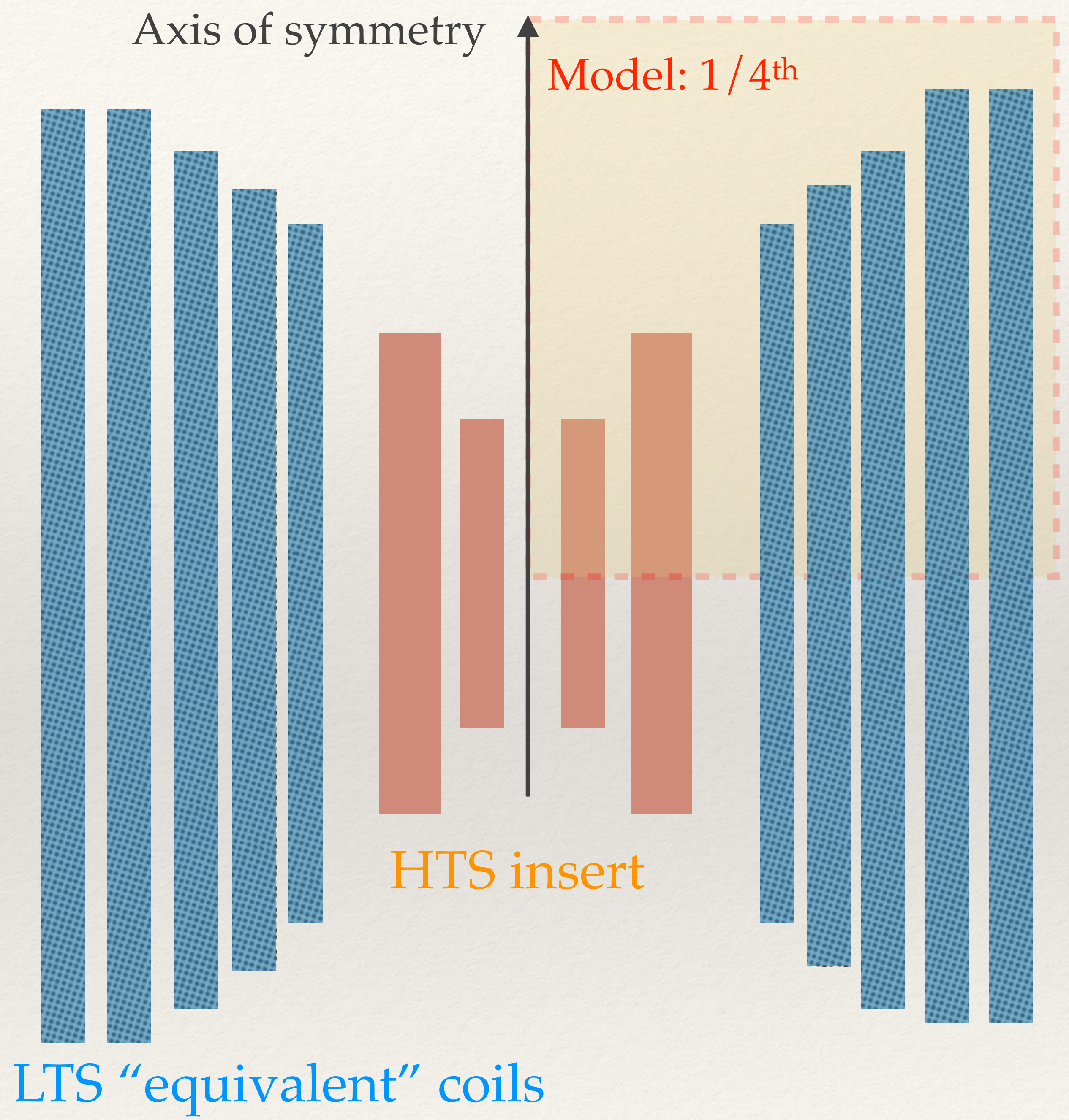
- Salient characteristics [NHMFL, 2018],[Berrospe-Juarez, IEEE-2018]:
  - Cold bore size = 34 mm
  - Target field at center 32 T
  - Full superconducting magnet (external LTS and REBCO insert)
  - Two concentric REBCO coils:
    - 112 pancakes (40 for inner coil 1 and 72 for outer coil 2)
    - Total number of turns for the full insert: ~ 20560





Model (1/2)

- **Axisymmetric dynamic *T-A* homogeneous model**
- Combined self-field (insert) and background field (LTS coils) impact on the losses during a realistic operation cycle
- Background field produced by the LTS coils represented by a set of equivalent coils. The equivalent engineering current density is such that one reproduces the expected magnetic flux density distribution over the the 2G-HTS insert.
- Detailed model of the 2G-HTS insert (turns per pancake and variation of I_c)

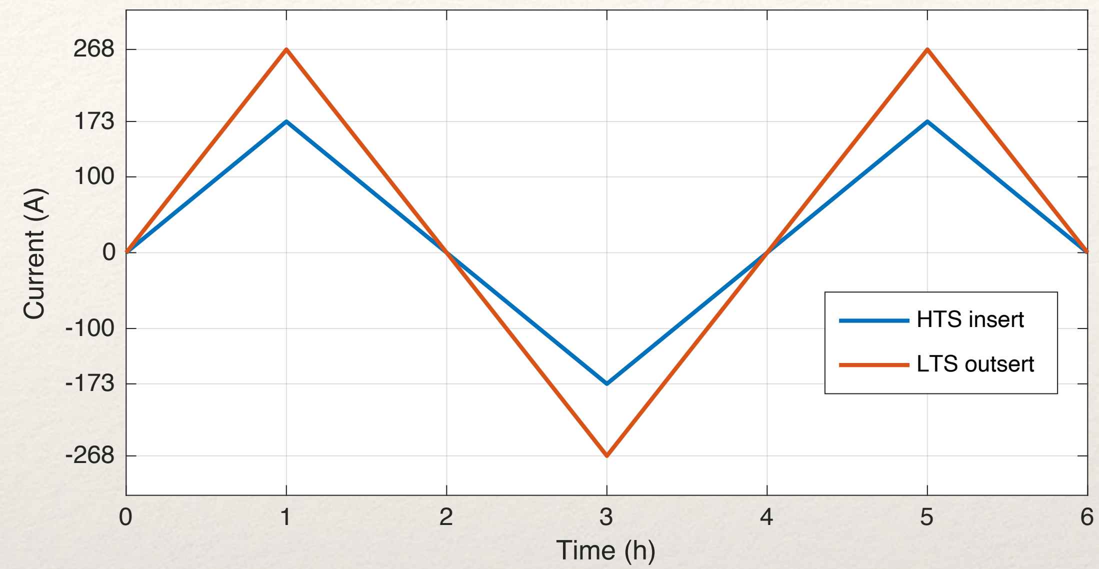


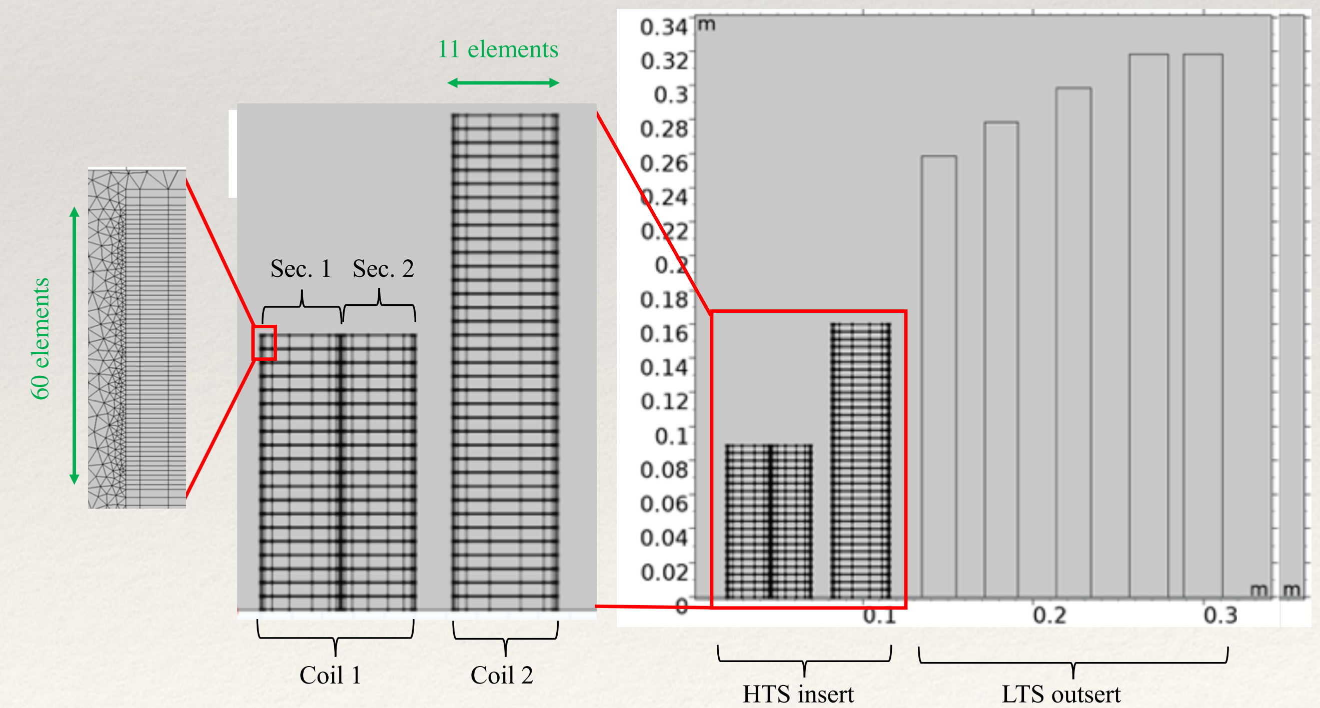
Model (2/2)

- Realistic operation cycle
- Power law with $n = 25$, and J_c depending on the magnetic flux density and its orientation
- Uneven critical current over the insert: use of a parameter β ranging from 0.65 to 1.5 to account for it [Berrospe-Juarez, IEEE-2018],

$$J_c(\mathbf{b}) = \frac{\beta J_{c0}}{\left(1 + \frac{\sqrt{k^2 b_\parallel^2 + b_\perp^2}}{B_0}\right)^\alpha}$$

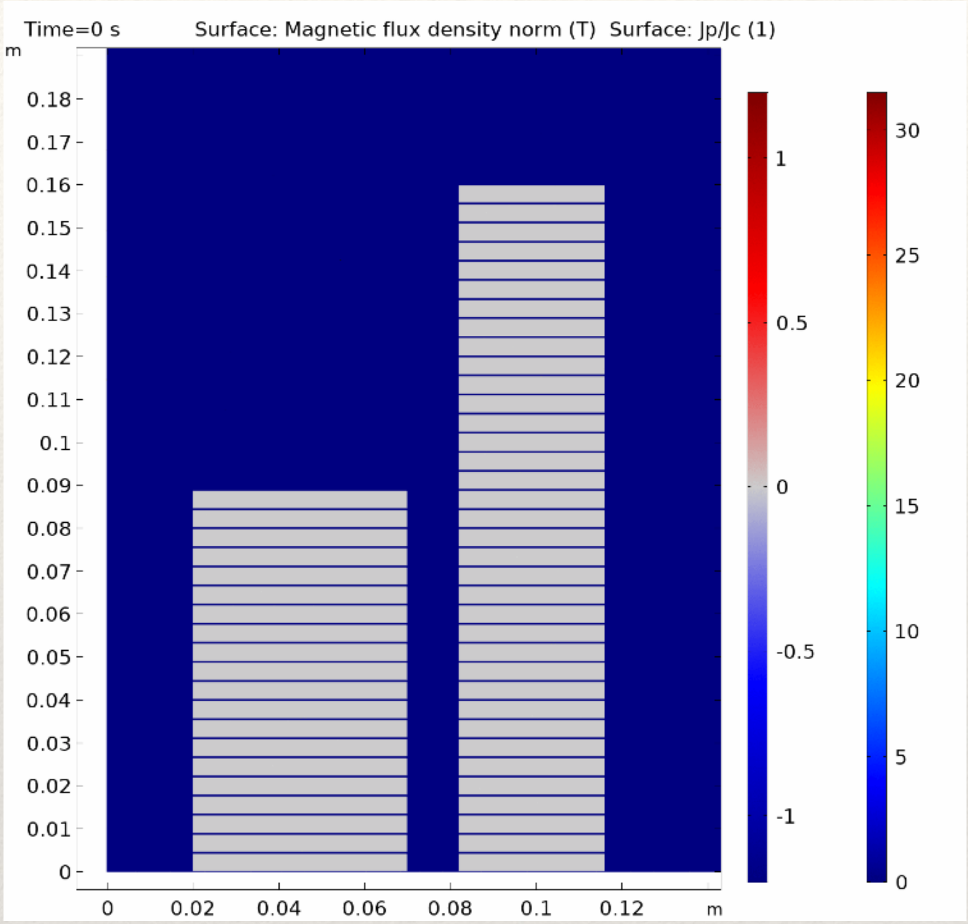
with, $J_{c0} = 2.896 \times 10^{12}$ A/m², $B_0 = 0.4674$ T, $k = 9.13 \times 10^{-3}$, $\alpha = 0.7518$.



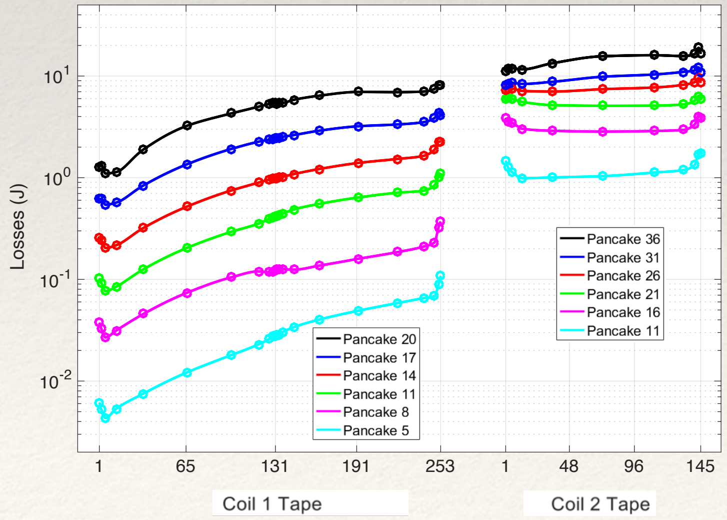


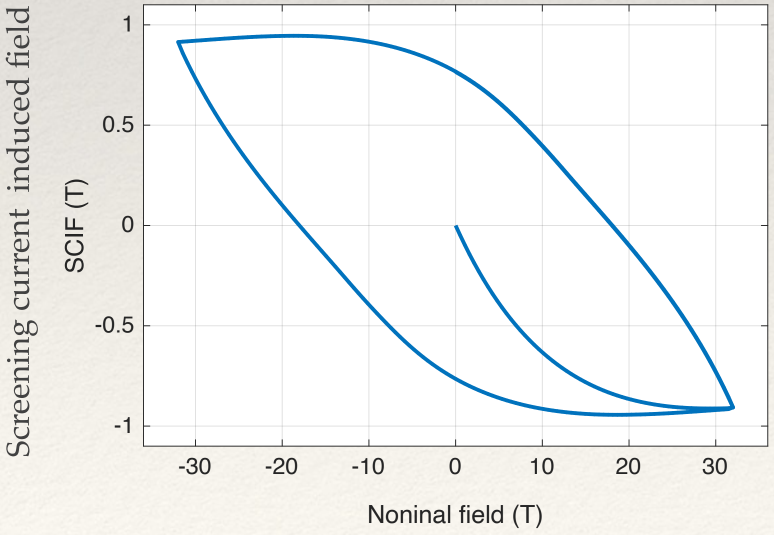
Results

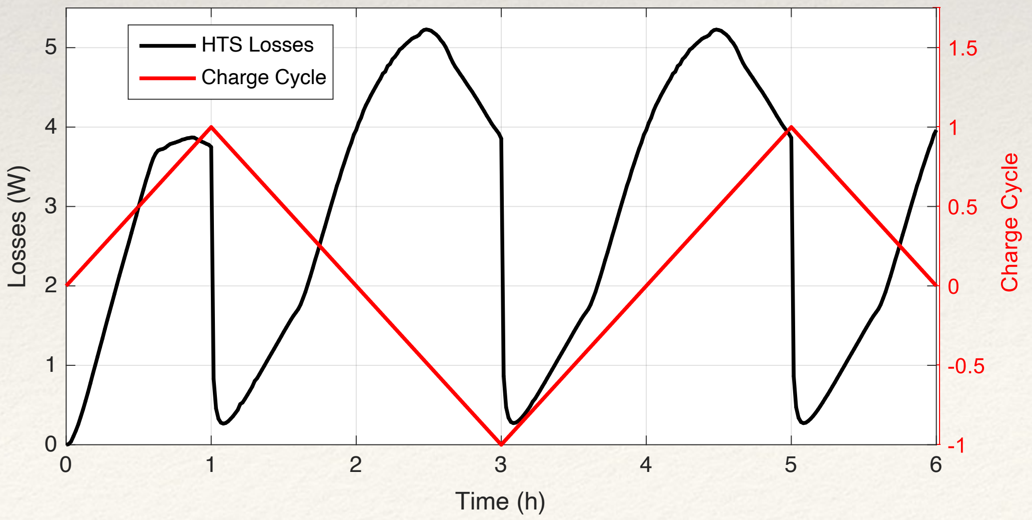
- Total loss: 61.08 kJ
- Computation time:
  - *T,A* homogeneous model: 5 h 29 min
  - *H*-formulation iterative multi-scale: 19 days



$$\text{SCIF} = B_{sim} - B_{unif.J}$$







Conclusion

1. Development of modeling tools for large 2G-HTS systems (planar and axisymmetric) based on the Finite Element solver COMSOL Multiphysics with Matlab liveLink
2. Improvements on the *H*-formulation multi-scale model by including an iterative scheme => better resolution on the current density distribution and magnetic field distribution, but "slow"
3. With the possibility to mix formulations, the *T-A* formulation is a step further to allow simulations in real time (computation time < actual operation cycle of magnets)
4. For practical cases, the best trade-off between accuracy and fast computation in 2D is the *T-A* formulation combined with the homogenization technique
5. 3D models are being developed based on the *T-A* formulation. Not yet widely employed due to its large computational load, only models involving short tape length exist.

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