Physics of fundamental Symmetries and Interactions - PSI2016

Measurement of muonic hydrogen 1S hyperfine splitting at RIKEN-RAL

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RIKEN / TITech

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Europe/Zurich timezone

As a “stranger” or “rookie” to this field,
let me introduce myself …

Size of hadrons?

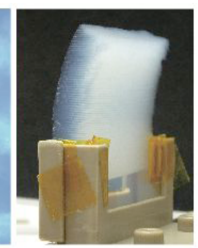
Magnetic radius of proton?

Origin of hadrons’ mass?

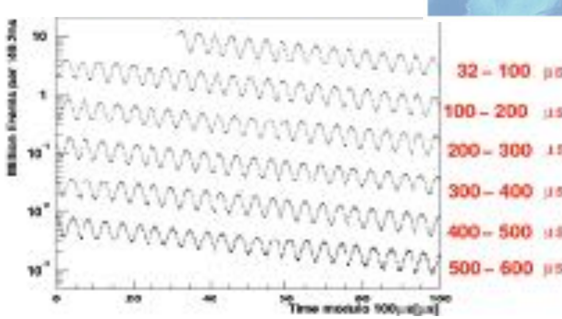
- Can kaon be a member of nuclei?
- Kaon properties change in nuclear media?

Precision Frontier

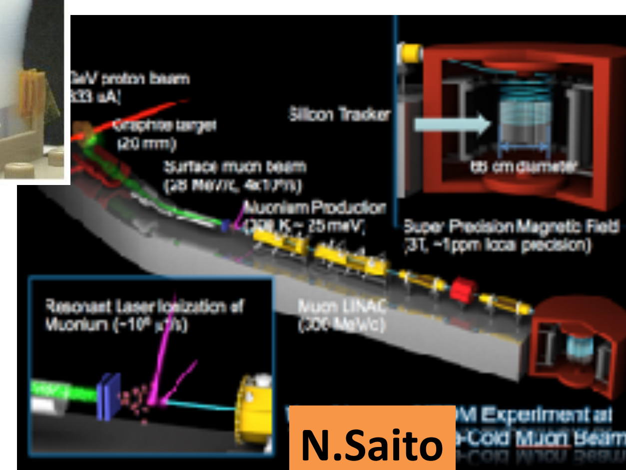
G.Marshall



BNL (g-2)μ E821

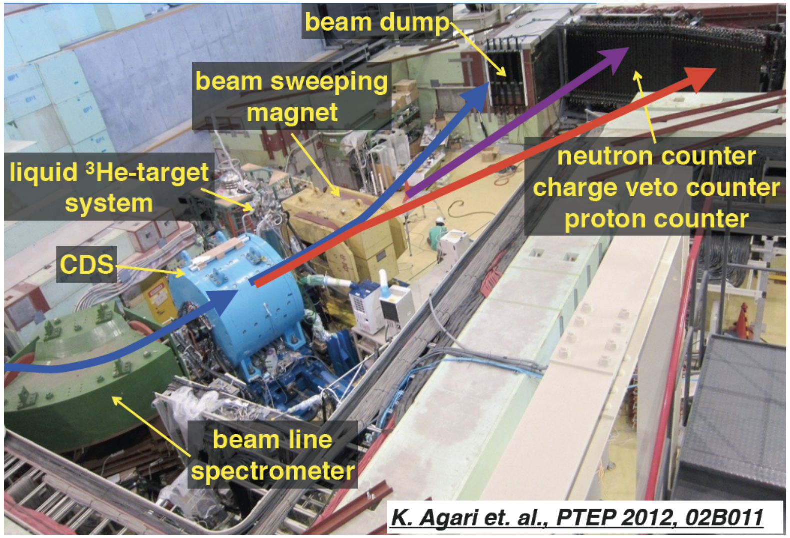


J-PARC (g-2)μ / EDMμ



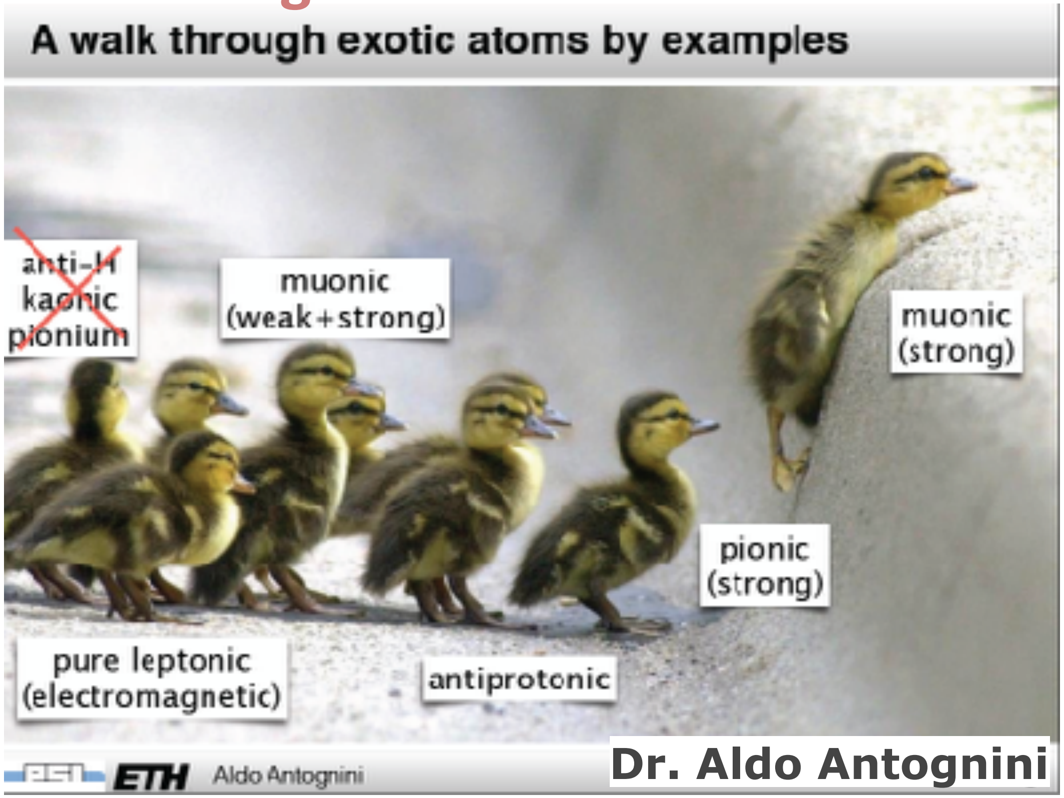
N.Saito

Kaon in nuclei



@ J-PARC

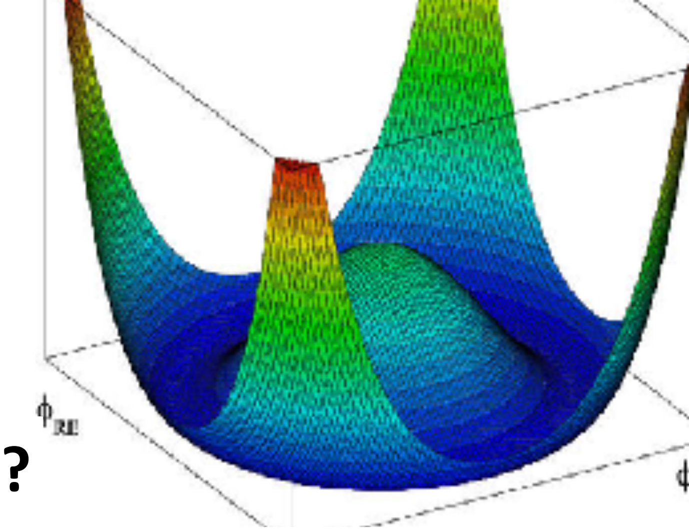
In this workshop, an (ugly) duck looking different direction…



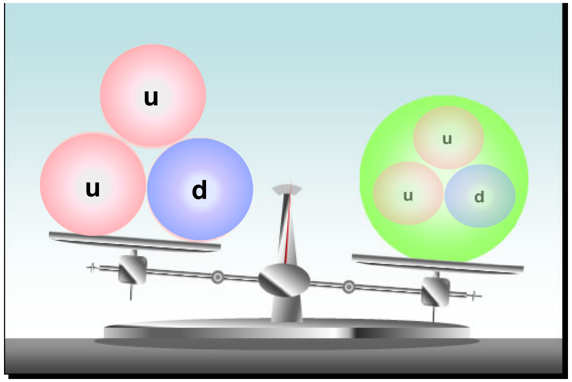
Mass origin of elementary particles

Higgs condensation!

vacuum phase transition



Origin of hadron mass?



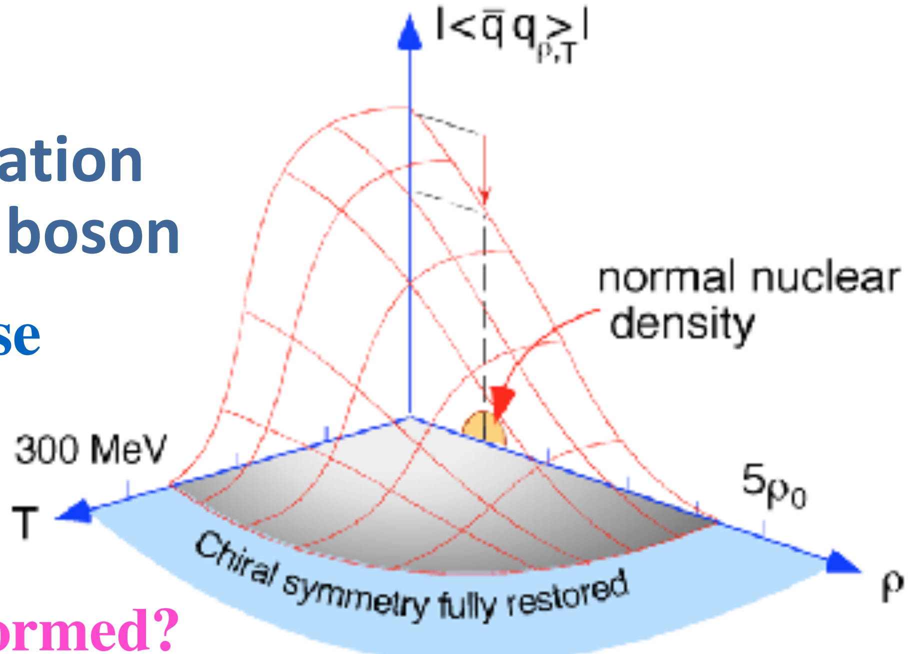
What's about proton?

Only ~1% from Higgs condensation!

Need QCD-Higgs : $\langle\bar{q}q\rangle$ condensation
scalar boson

χ-symmetry breaking of universe

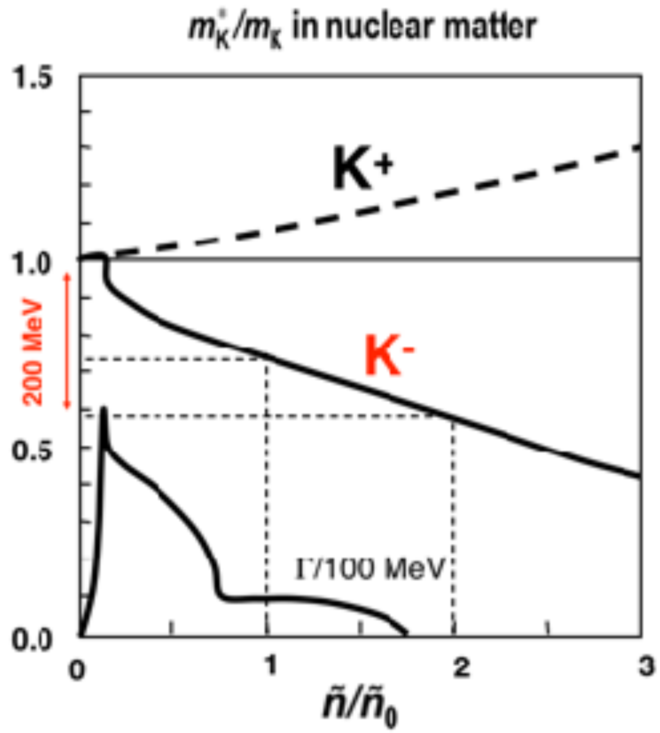
second vacuum phase transition



high density nuclear matter can be formed?

Search for Kaonic nuclear states

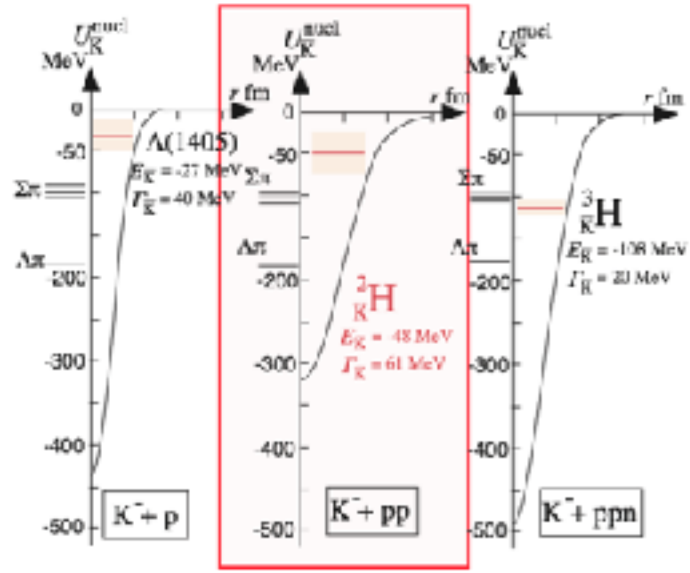
to form high density nuclear matter



T. Waas, N. Kaiser & W. Weise, Phys. Lett. B379 (1996) 34.

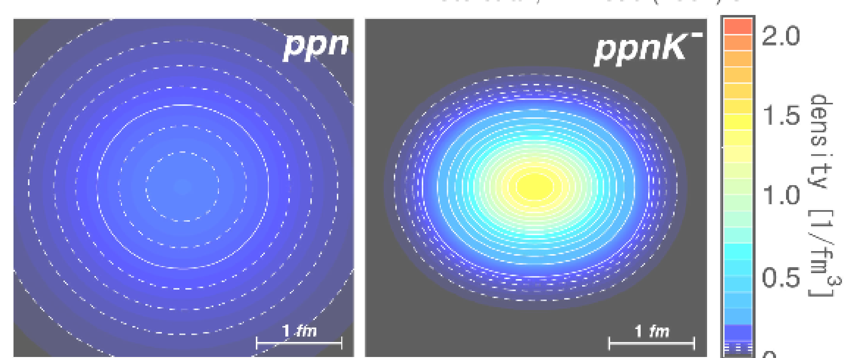
strongly attractive in I=0 channel

T. Yamazaki & Y. Akaishi, PLB 535 (2002) 70



B_K ~ 50 MeV!

Dote et al., PLB 590 (2004) 51



formation of high density matter?

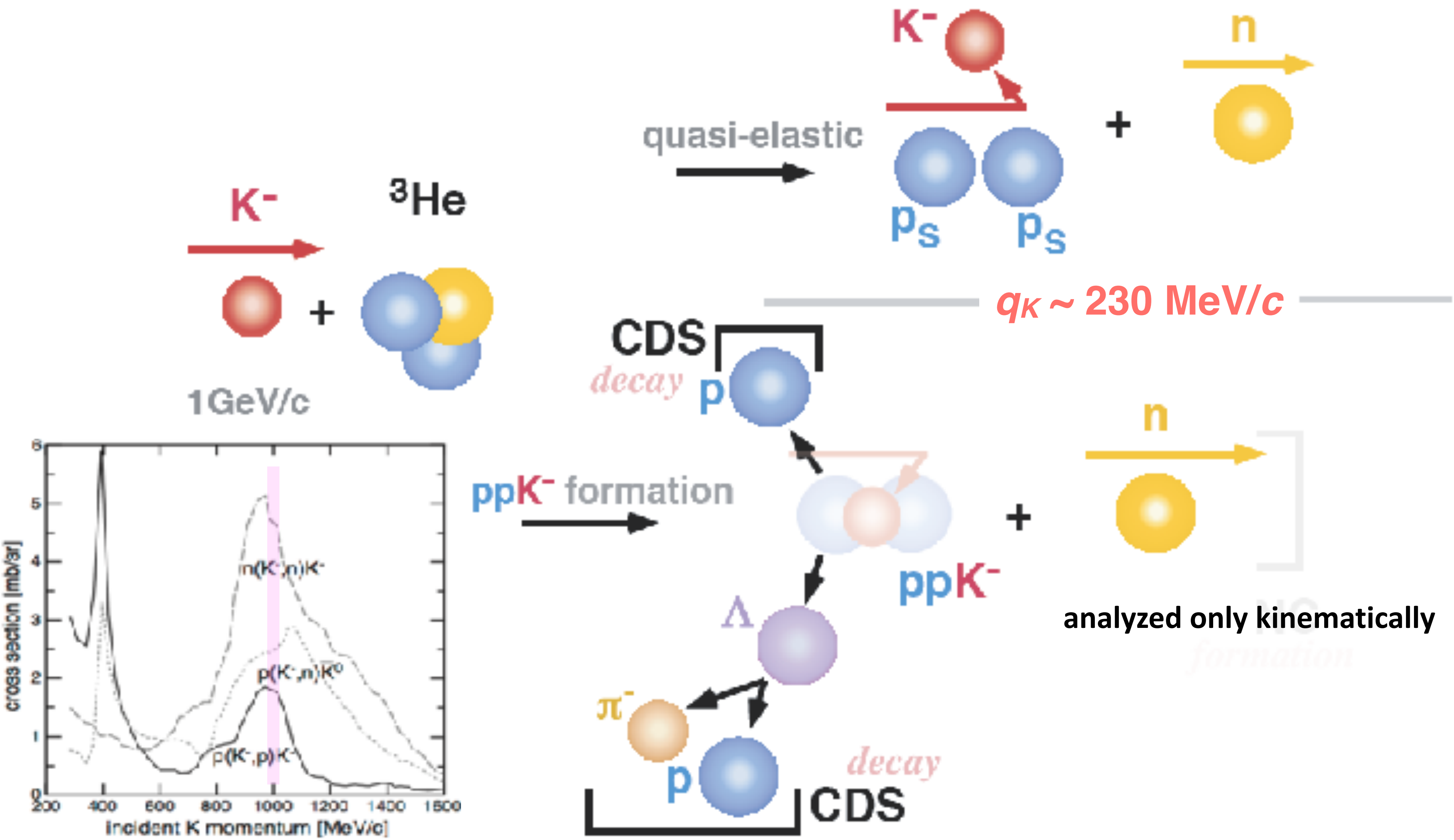
nuclear state search

- simplest system *K⁻pp*

^{3}He(K⁻, n) @ 1 GeV/*c*

"K⁻pp" search via $^3He(K^-,n)$ @p_K=1GeV/c

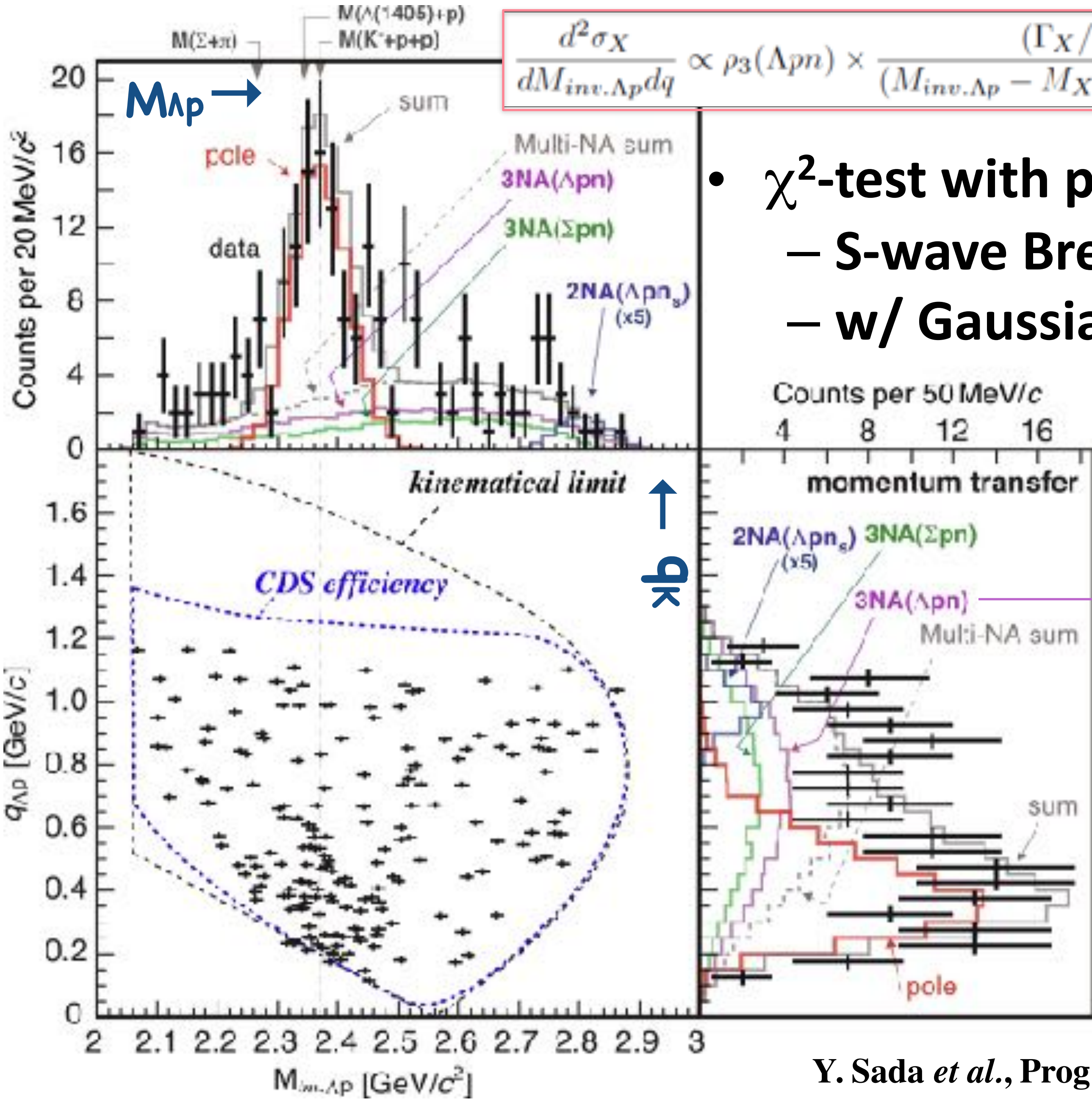
for efficient "ppK" formation $q_K \sim$ 230 MeV/c (~ p_F)



Assuming a Breit-Wigner

$K^- + {}^3He \rightarrow \Lambda + p + n_{mis.}$

$$\frac{d^2\sigma_X}{dM_{inv.\Lambda p}dq} \propto \rho_3(\Lambda pn) \times \frac{(\Gamma_X/2)^2}{(M_{inv.\Lambda p} - M_X)^2 + (\Gamma_X/2)^2} \times |\exp(-q^2/2Q_X^2)|^2,$$



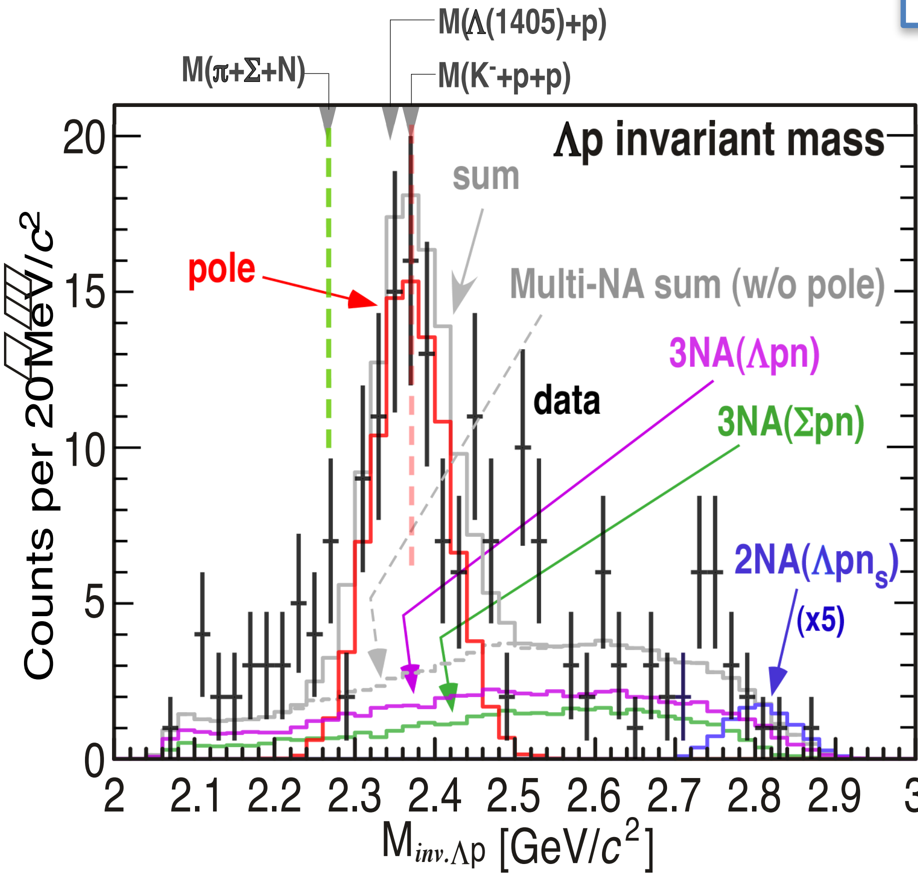
- χ^2-test with pole & 3NA(Ypn)
  - S-wave Breit-Wigner pole
  - w/ Gaussian form-factor

$$\frac{d^2\sigma_{3NA(\Lambda pn)}}{dT_n^{CM}\, d\cos\theta_n^{CM}} \propto \rho_3(\Lambda pn)$$

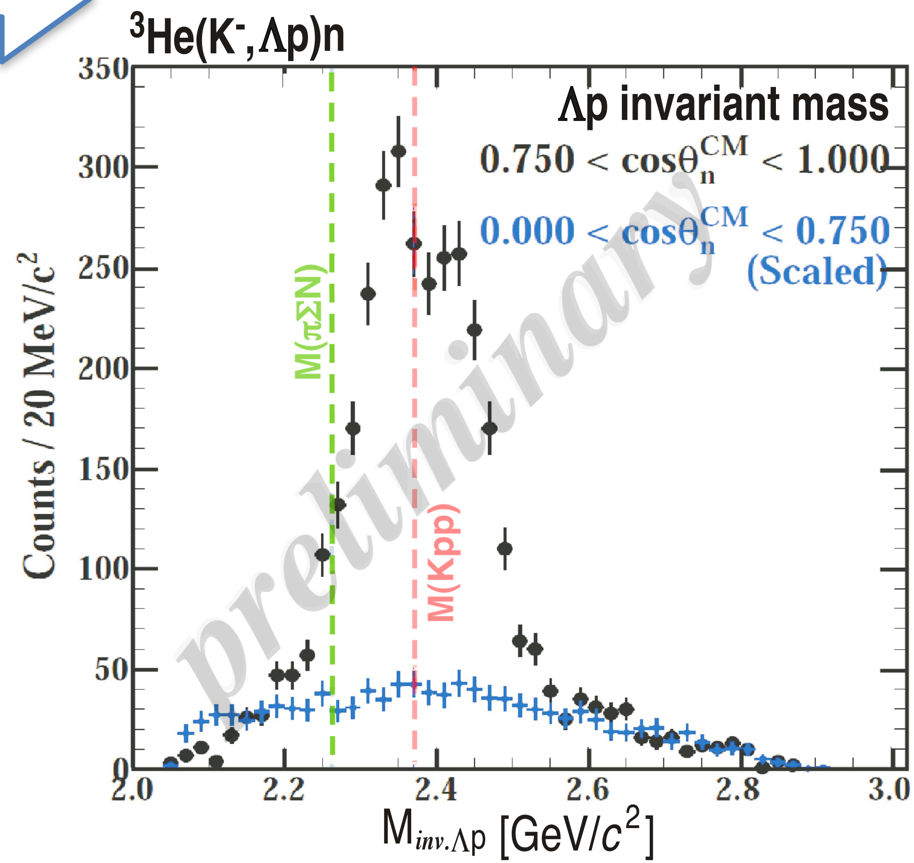
B(X) ~ 15 MeV
Γ(X) ~ 110 MeV
Q(X) ~ 400 MeV/c

Y. Sada *et al.*, Prog. Theor. Exp. Phys. 051D01 (2016)

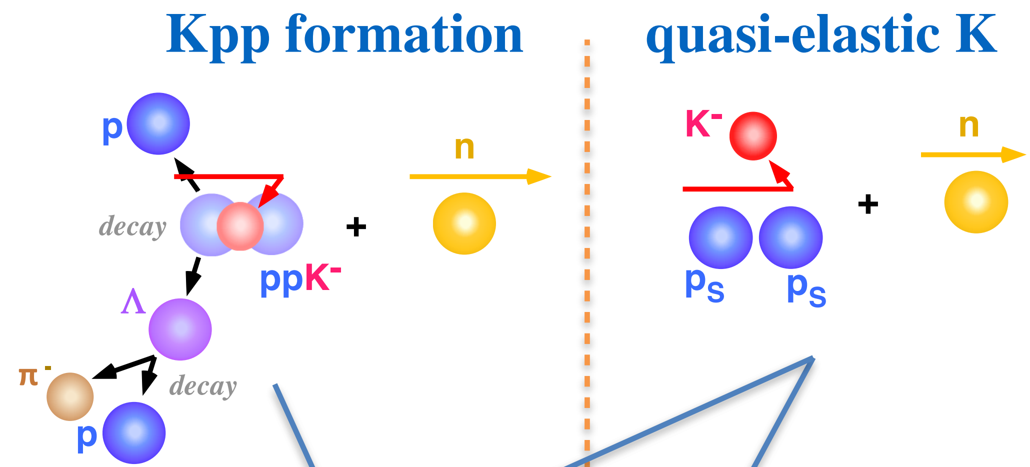
Improving statistics to study the structure near Kpp



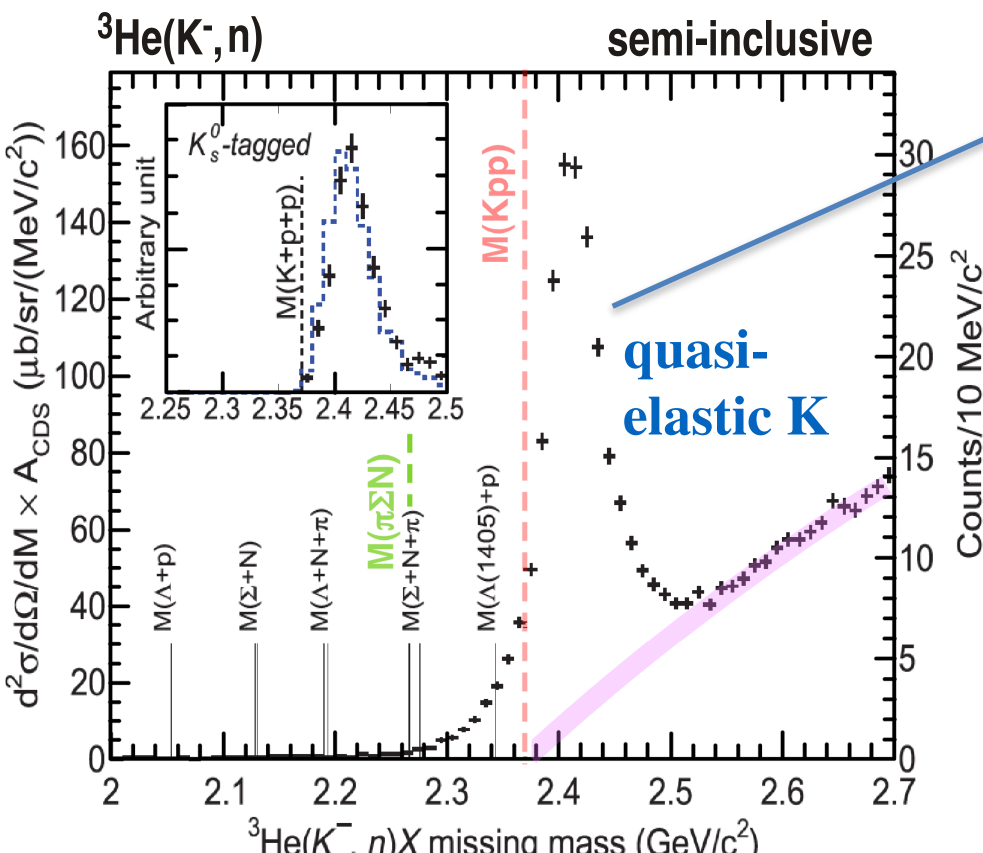
~30 times more data

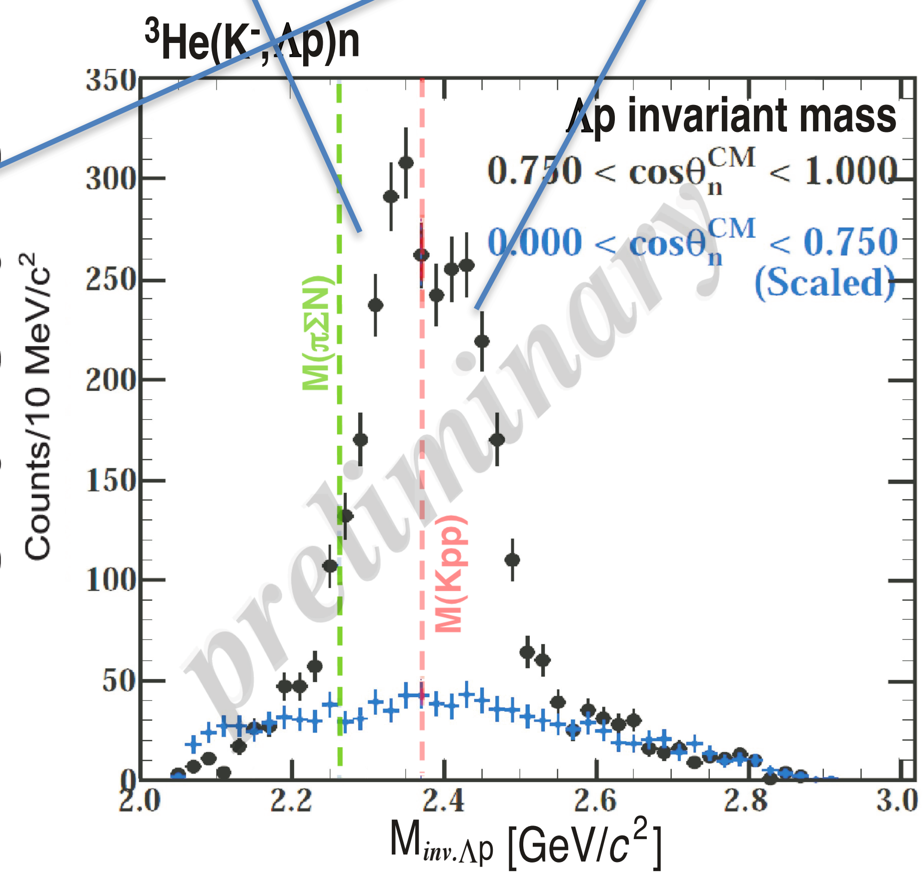


First convincing Kpp signal



T. Hashimoto *et al.*, Prog. Theor. Exp. Phys. 061D01 (2015)

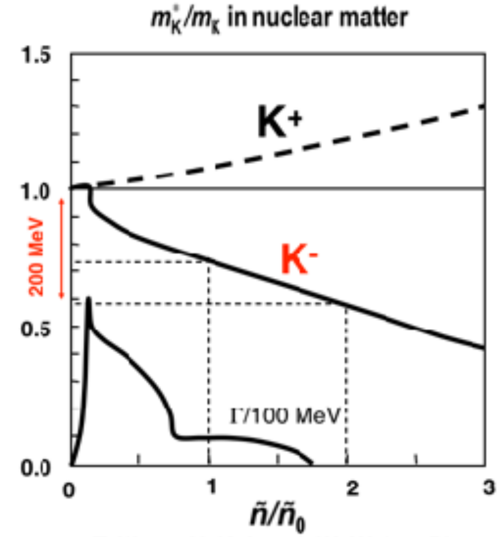




First convincing Kpp signal

If this is true, …

- Kaon can be a member of nucleus



T. Waas, N. Kaiser & W. Weise, Phys. Lett. B379 (1996) 34.

First mesonic nuclear bound state
(bound by strong-interaction)

Boson in nuclear matter (not Fermion only)

- Kaon properties change in nuclear media?

probably, $B_K \sim 50$ MeV (10% if the mass!)

- $Q_K \sim 400$MeV/*c* implies that the size ~ 0.5 fm

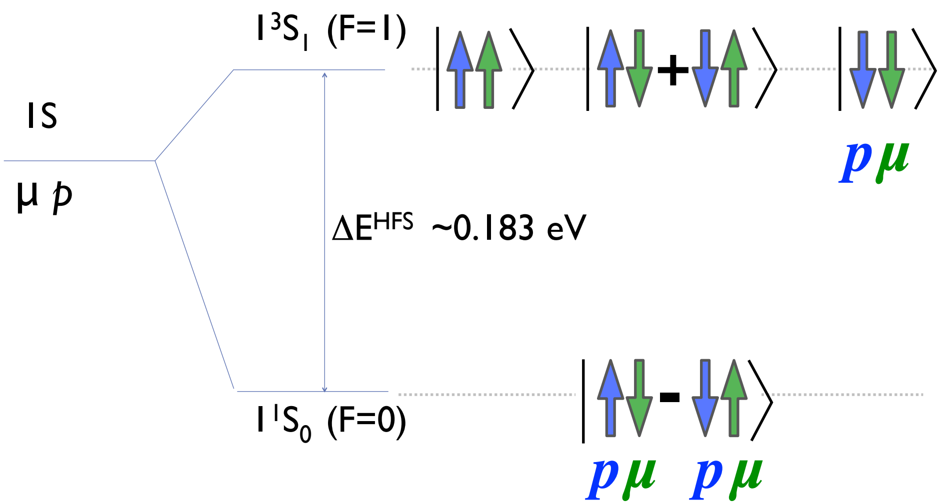
K in heavier nuclei?

New challenge!

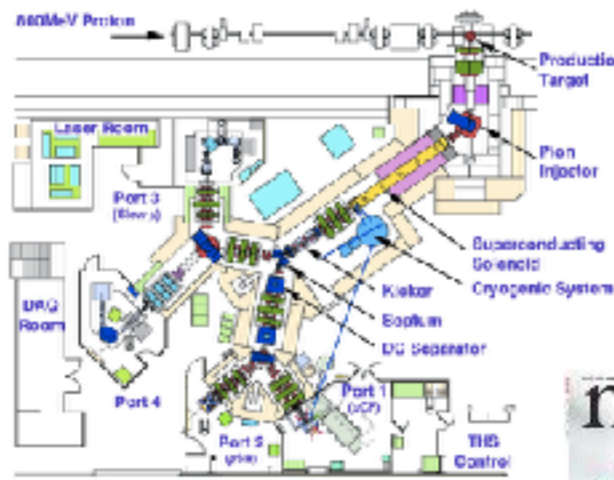
Hyperfine Splitting of Muonic Proton

@ RIKEN-RAL until 2028

1S μ −p case



F : total angular momentum



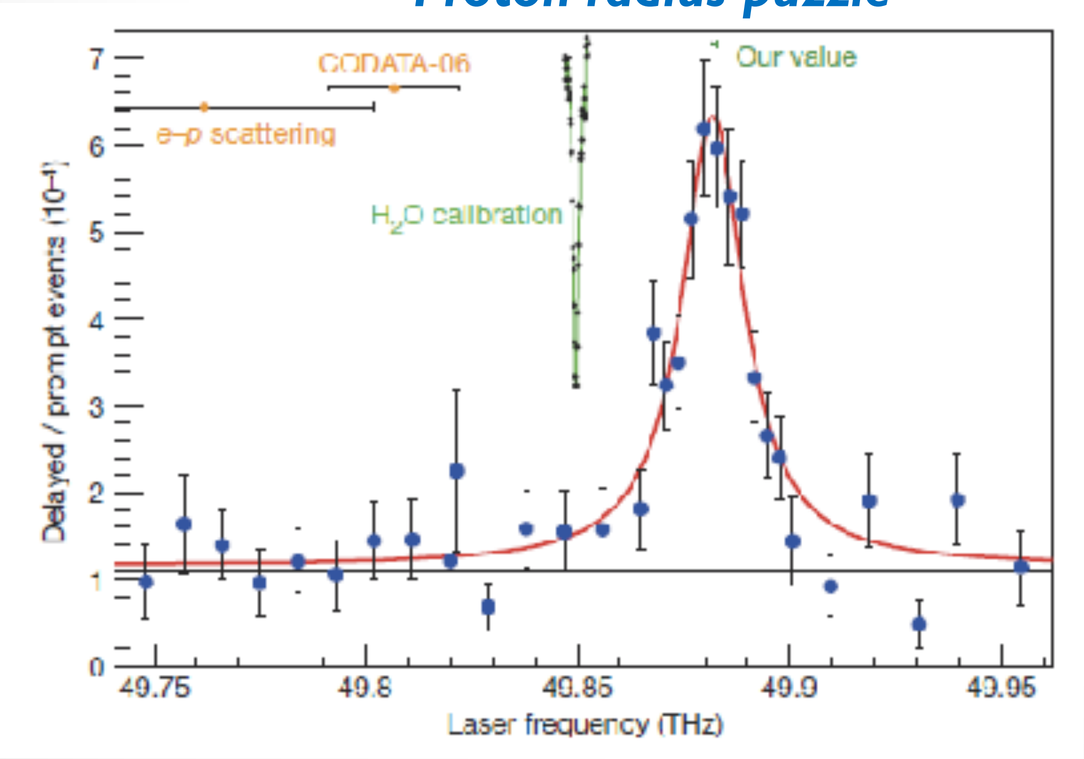
A shocking data from PSI triggered us :

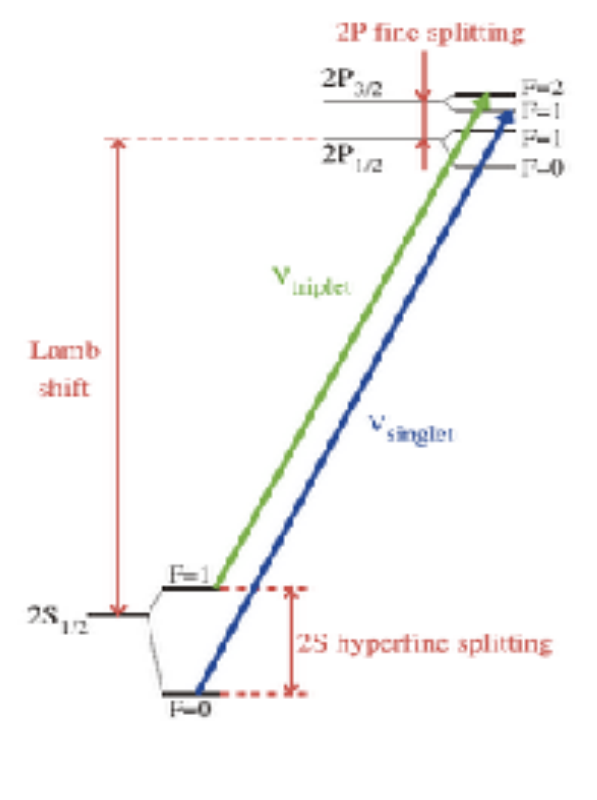
Discrepancy in proton charge radius found by;

➢*hydrogen spectroscopy / e-p scattering*
➢*muonic hydrogen Lamb shift (2s-2p level shift) @ PSI*

R. Pahl et al., Nature 466 (2010)
R. Pohl et al., Ann. Rev. Nucl. Part. Sci. 63 (2013)242001

"Proton radius puzzle"

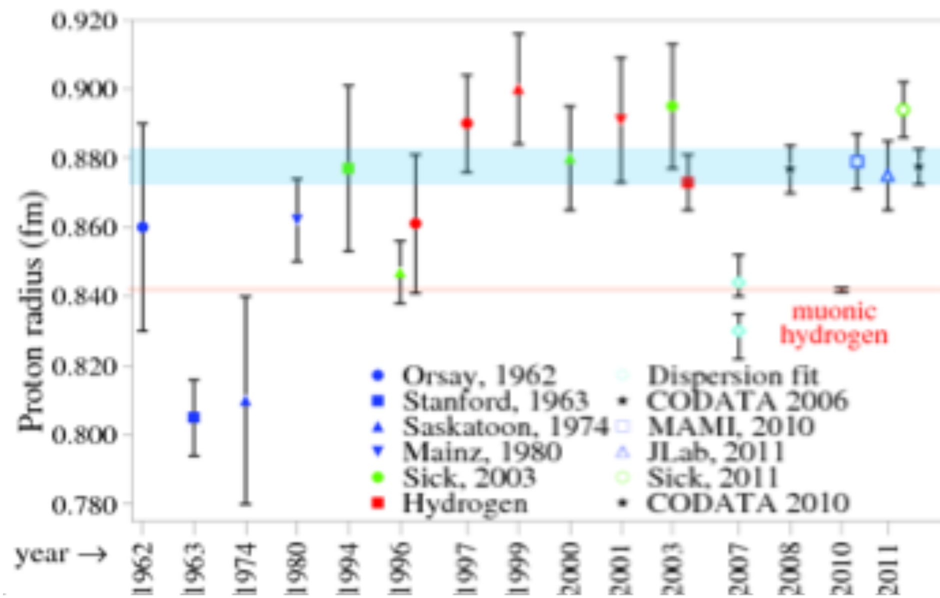




Hadron structure study by μp - atom

- BIG puzzle on Proton Charge Radius found by muon!
- muon is 10^7 more sensitive than electron

➡ why not check magnetic radius?



by hyperfine of $\mu^- p$ - atom 1S

proton @ PDG

Mass m = 938.272046 ± 0.000021 MeV

Magnetic moment μ = 2.792847356 ± 0.000000023 μ_N

Charge radius, μp Lamb shift = 0.84087 ± 0.00039 fm

Charge radius, ep CODATA value = 0.8775 ± 0.0051 fm

Magnetic radius = 0.777 ± 0.016 fm   ep data! why not μp?

μp hyperfine splitting actually gives us is:

proton Zemach radius

$$R_{\mathrm{p}} = \int \mathrm{d}^3 r\, \mathrm{d}^3 r'\, \rho_{\mathrm{E}}(\mathbf{r})\rho_{\mathrm{M}}(\mathbf{r}')|\mathbf{r}-\mathbf{r}'| = -\frac{4}{\pi}\int_0^\infty \frac{\mathrm{d}Q}{Q^2}\left[\frac{\mu_{\mathrm{N}}}{\mu_{\mathrm{p}}} G_{\mathrm{E}}(Q^2)\, G_{\mathrm{M}}(Q^2) - 1\right]$$

convolution of charge and magnetic moment distribution (ρ_E , ρ_M)

fundamental quantities of proton electronic & magnetic structure

Hyperfine splitting energy of H-like atom defined by :

1S μ –p case

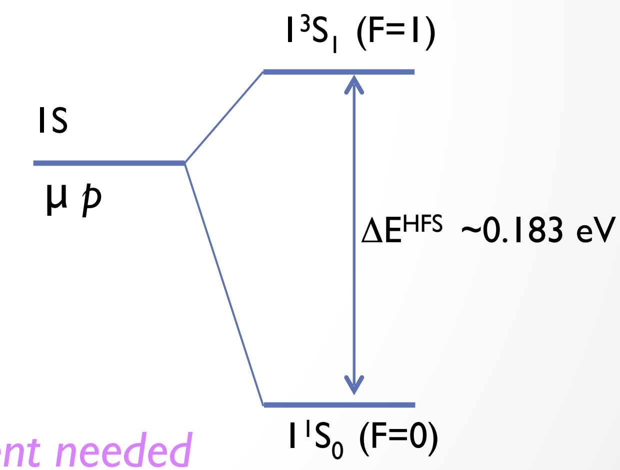
$$\Delta E^{th}_{HFS} = E_F(1 + \delta_{QED} + \delta_{str})$$

- E_F :*Fermi term* $\quad E_F = \frac{8}{3}\alpha^4 \frac{m^2_{\mu(e)}\, m^2_p}{(m_{\mu(e)} + m_p)^3}\mu_p$
- δ_{QED} :*higher order QED correction*
- δ_{str} :*proton structure correction*   *theoretical improvement needed*

$$\delta_{str} = \underline{\delta_{Zemach}} + \delta_{recoil} + \boxed{\delta_{pol}} + \delta_{hVP}$$

$$\boxed{\delta_{Zemach} = -2\alpha m_{\mu p} R_Z + O(\alpha^2)}$$

directly connected with Rz



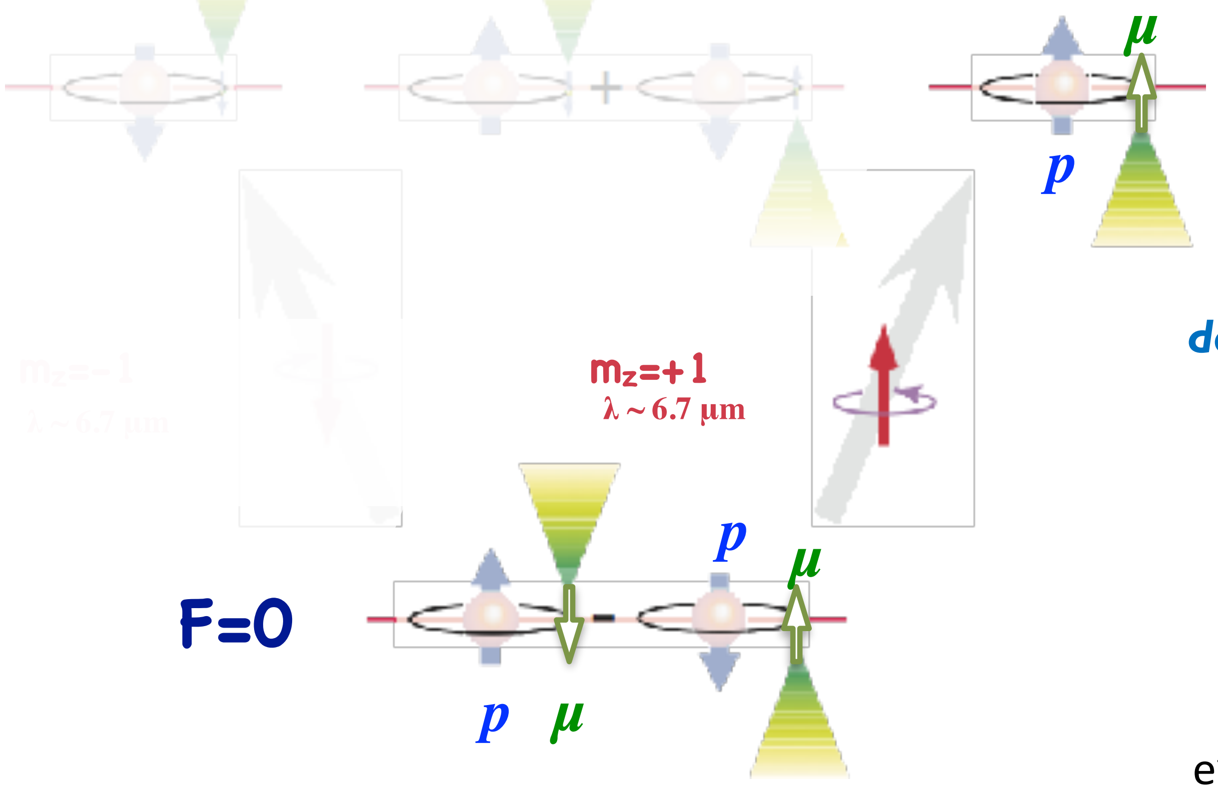
F : total angular momentum

Novel idea how to measure proton / μ⁻SR

1S Hyperfine splitting of μ^-p atom

pump by laser & probe by μ decay asymmetry!

μ polarized aligned to laser spin



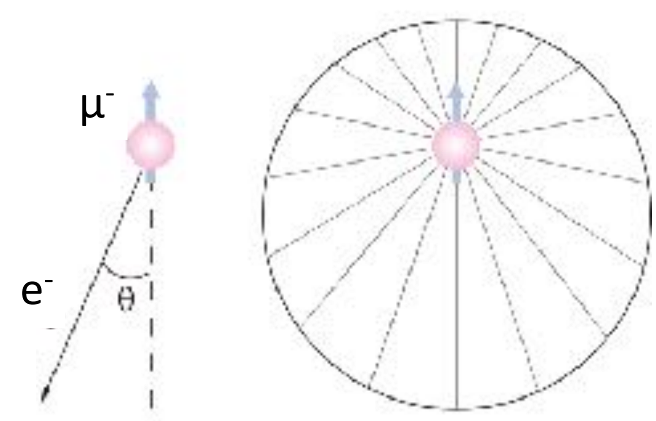
F=1, F_z=+1

F=0

μ fully depolarized in G.S.

detect decay electron!

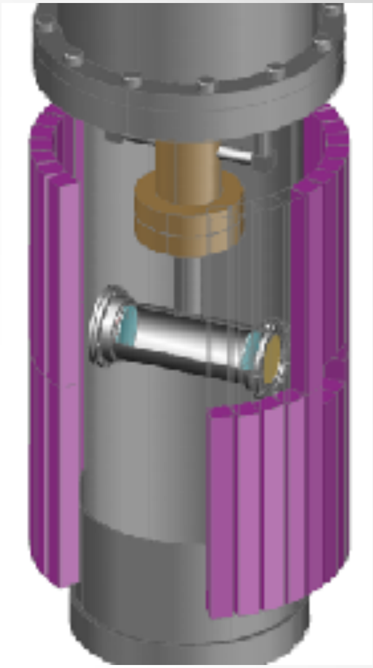
$$d\sigma_{e^-}(\theta)d\Omega \propto \left(1 - \frac{1}{3}P\cos\theta\right)d\Omega$$



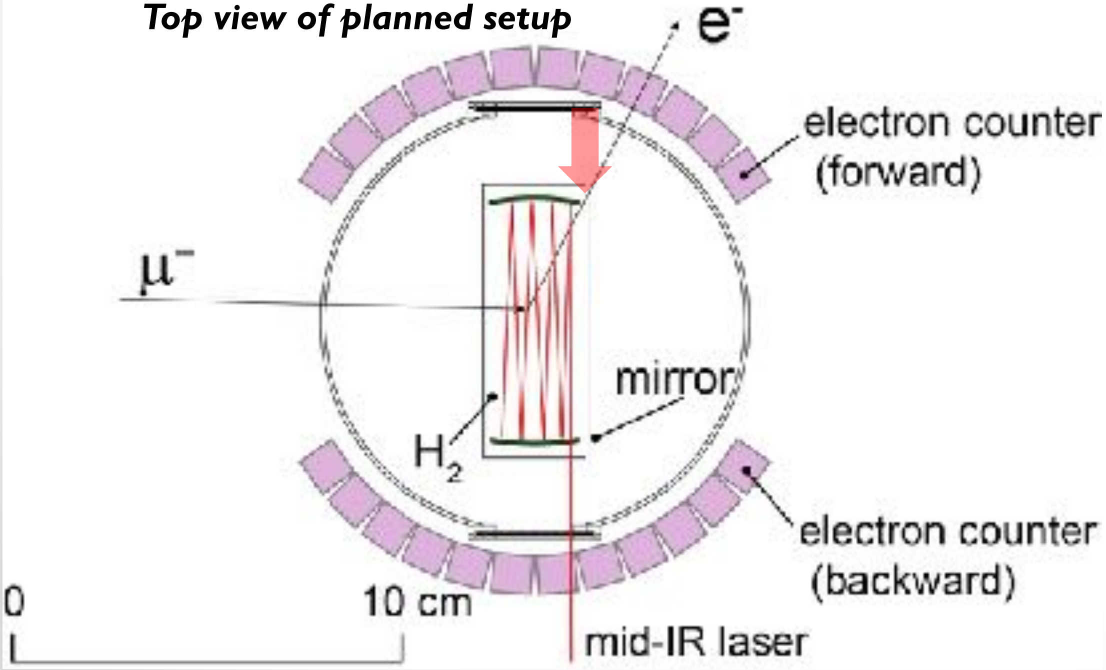
What we need :

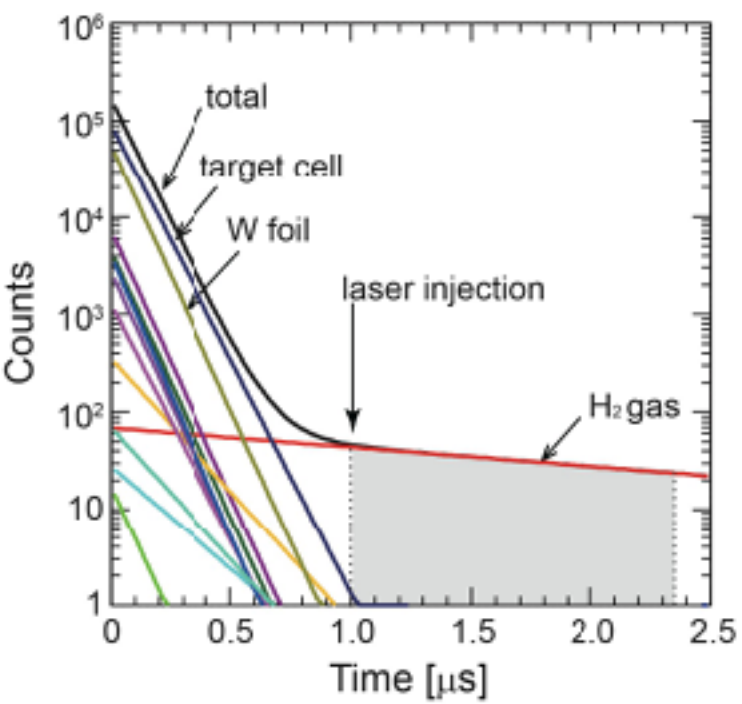
budget not secured yet …

1) *dilute H_2 target, to reduce*
   - *collisional polarization loss (quench)*
   - *collisional energy shift (density dependent)*
2) *chamber made of heavy material*
3) *large decay electron counter (forward and backward)*
4) *high-power tunable mid-infrared laser (M1)*



Top view of planned setup



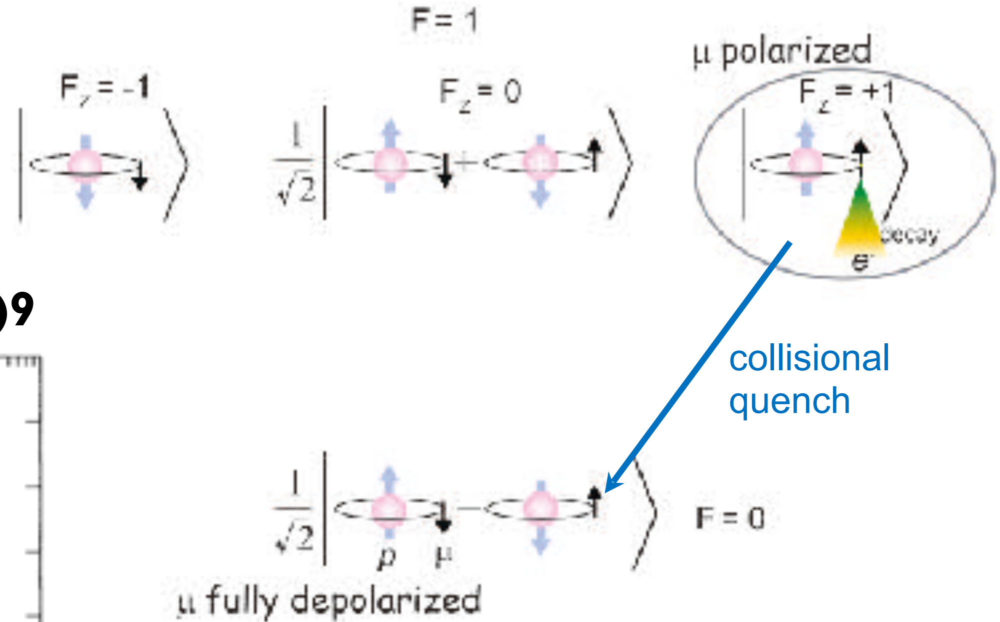


Collisional quench

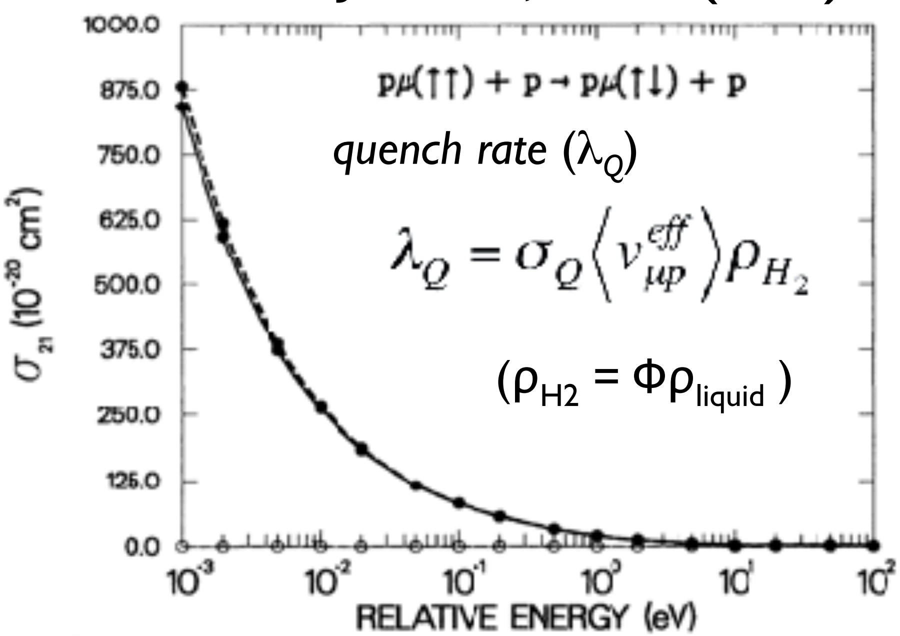
- $^3S_1 \rightarrow {}^1S_0$ collisional quench

polarization is lost…

$$\boxed{\mu p(\uparrow\uparrow) + p \rightarrow \mu p(\uparrow\downarrow) + p}$$



J. Cohen, PRA43(1991)9



proportional to H_2 density

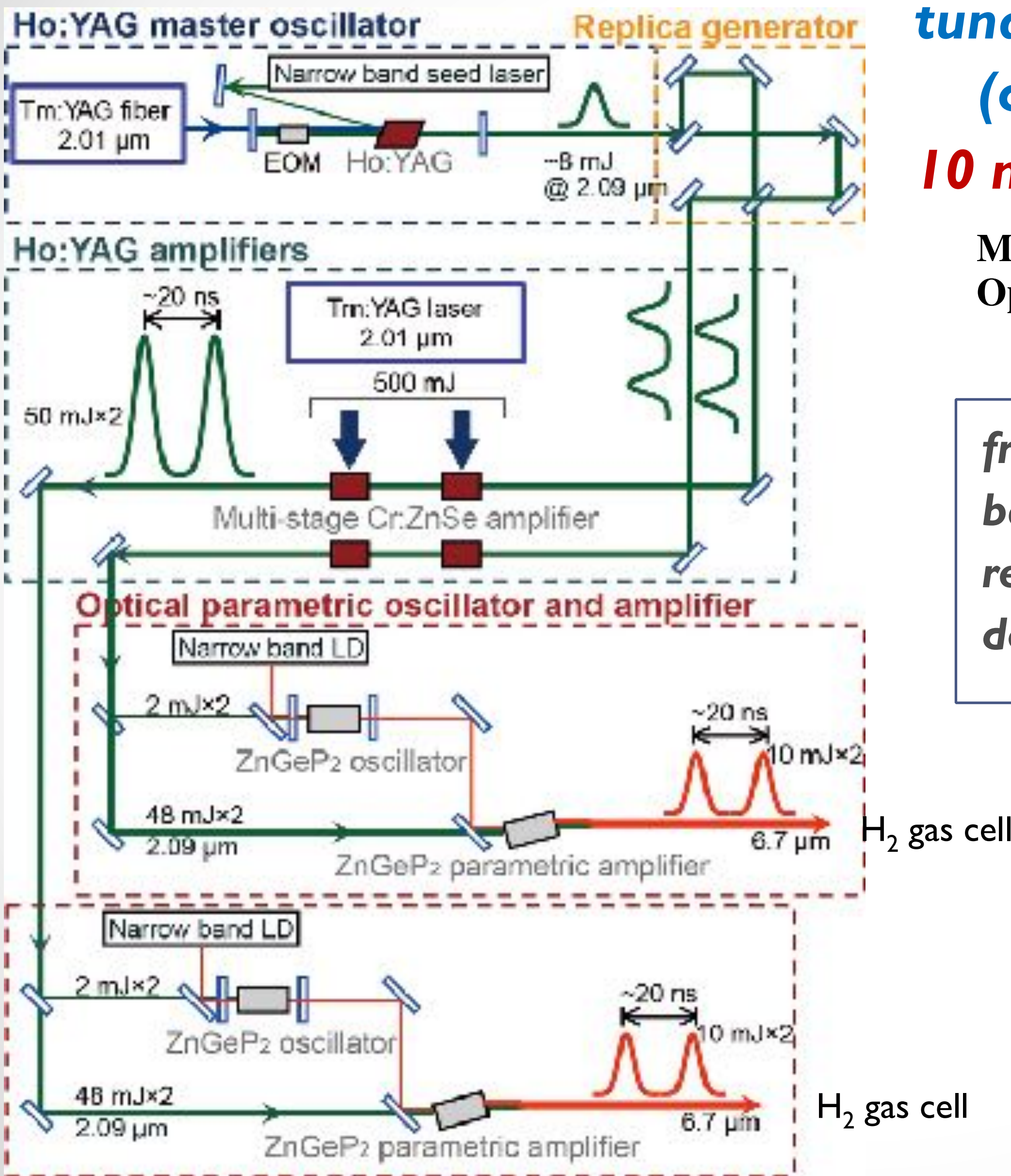
liquid target is unusable
(density is too high ← tau ~ 50 ps)

Quench rate

If Φ = 0.1% (0.01 %)LHD (liquid hydrogen density), then τ_{quench} = 50 (500) ns

Density of the hydrogen gas is very important

Laser development w/ RIKEN RAP



tunable mid-infrared laser
(developed in RAP Wada group)

10 mJ @ 6 μm system is in hand

M. Yumoto, N. Saito, U. Takagi, and S. Wada, Optics Express, 23, 25009-25016 (2015)

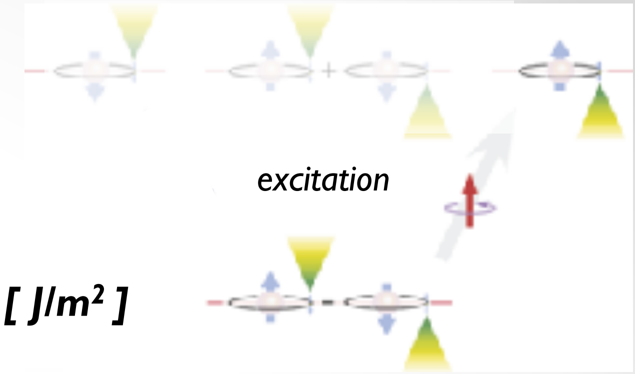
frequency ~6.8 um = ~44 THz
band width ~50 MHz
repetition ~ 50 Hz
double pulse 10 mJ x 2 set = 40 mJ

- ***Wavelength will be controlled by seeded OPO with $ZnGeP_2$ non-linear crystal.***
- ***6.8 μm seed light will be provided from Quantum cascade laser.***

40 mJ laser power is feasible

What we expect as a signal :

F=0 → F=1 transition probability



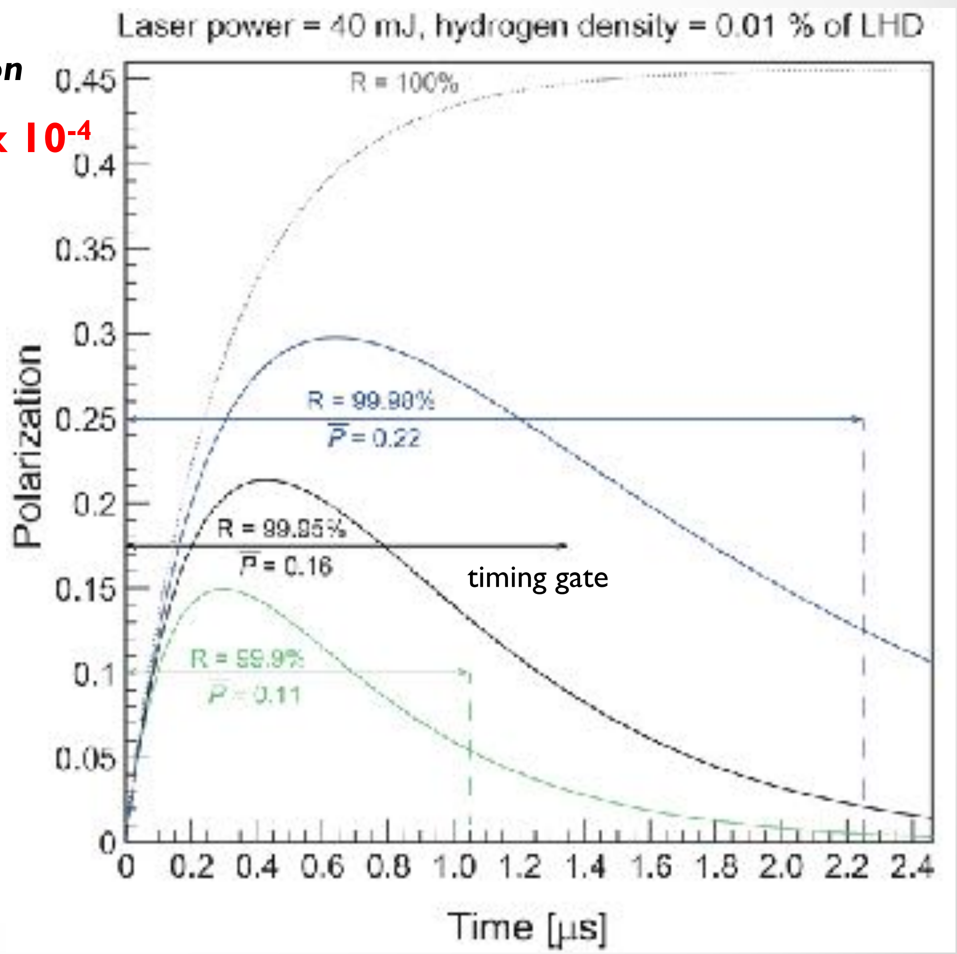
$$\overline{P} = 2 \times 10^{-5} \frac{E}{S\sqrt{T}}$$

***E/S* : laser power density [J/m²]**
***T* : temperature [K]**

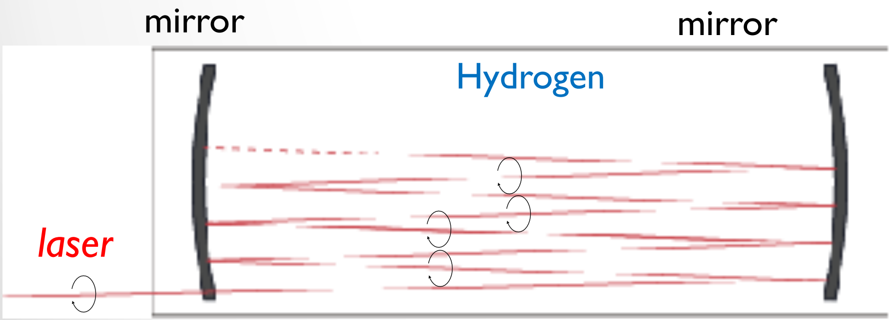
NIM B281(2012)72 & D. Bakalov, private communication

ex. *E* = 40 mJ, *S*= 4 cm², T= 20 K, then *P* = 4.5 x 10^{-4}

small probability (due to M1)
for single pass!



multi-pass cavity



reflective index R = 99.95 % → P = ~ 16%

Delayed timing laser injection

Large fraction of muons stop outside of H2 gas

Negative muon capture

lifetime of bound muon :

$\tau_{total} = 1/\Lambda_{total}$

$\Lambda_{total} = \Lambda_{capture} + Q\,\Lambda_{decay}$

Q : Huff factor

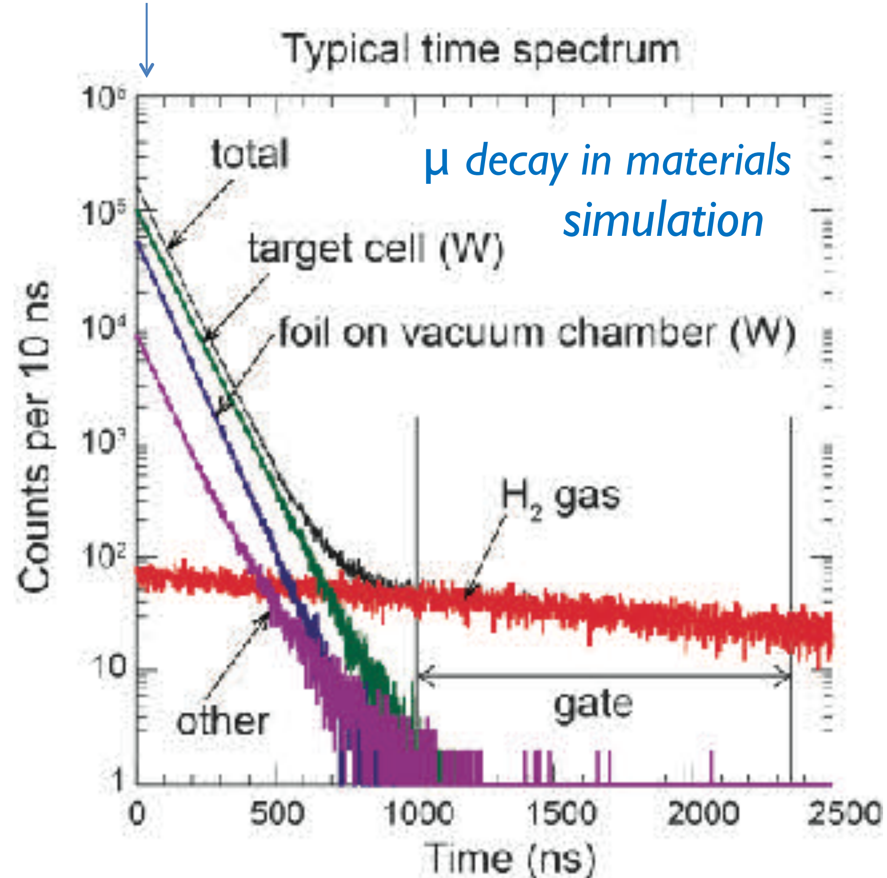
μ- capture

$$\boxed{\mu^- + \text{"}p\text{"} \rightarrow \nu_\mu + n}$$

c.f. $\Lambda_{decay} = 2.197$ us

	τ [ns](1/$\Lambda_{tota.l}$)	Q
H	2194	1.00
C	2040	1.00
Cu	160	0.967
Ag	90	0.925
W	80	0.860

μ injection



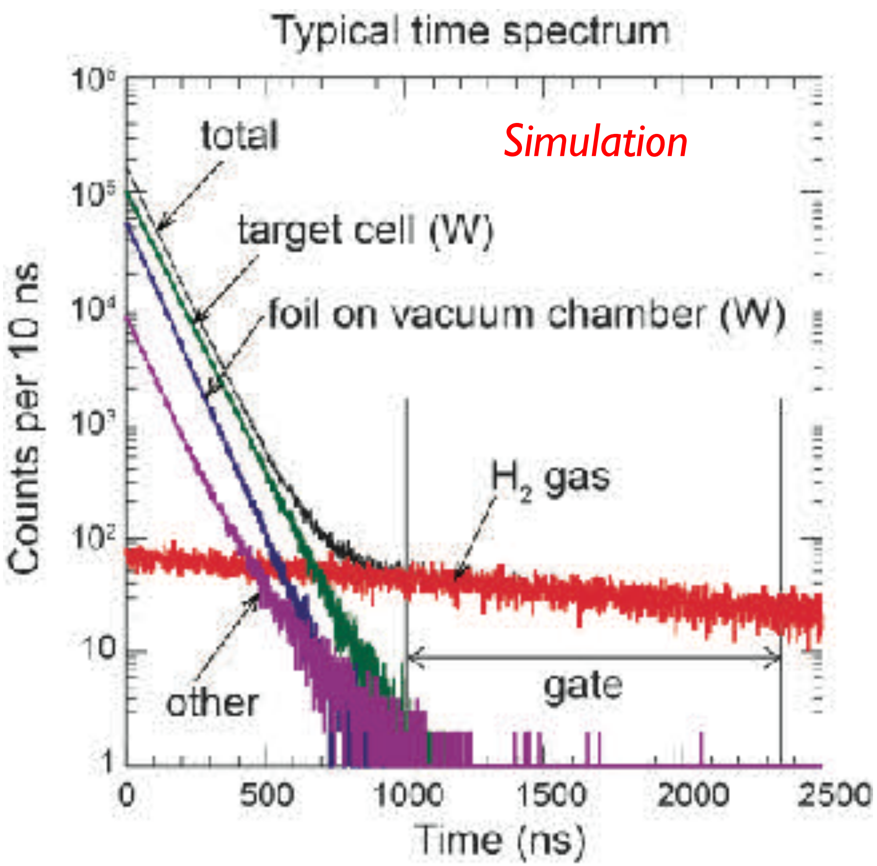
μ decay in materials simulation

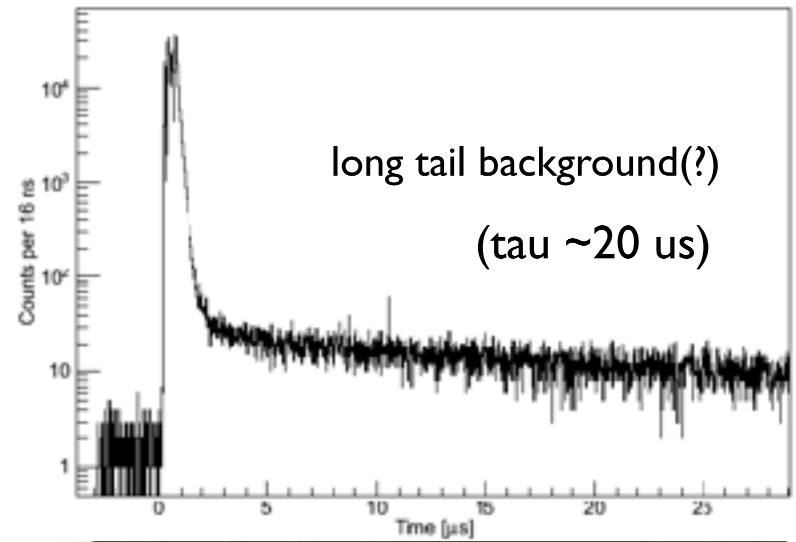
BG suppression by mu- capture & delayed laser injection

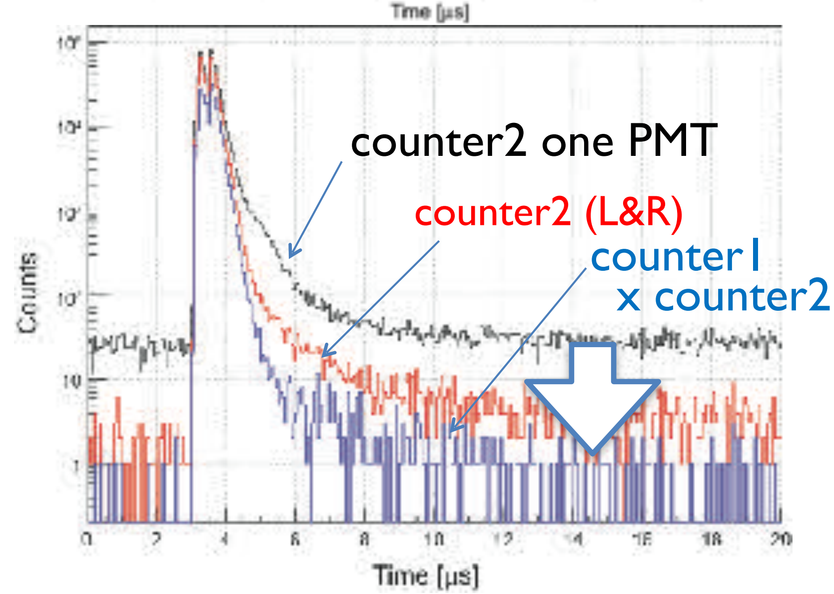
Background measurement

✓ **check BG level after μ-stop**

@ RiKEN-RAL







BG can be sufficiently suppressed by coincidence.

Summary & Outlook

- Measurement of ground state hyperfine splitting energy in muonic hydrogen with mid-infrared laser by means of spin re-polarization method.

- Accuracy of ΔE_{HFS} : ~ 2 ppm, derive proton Zemach radius < 1 % accuracy

  (need theoretical effort for further precision)

- Proposals submitted to pulse-muon facilities

  (RIKEN-RAL and J-PARC MUSE)

- Feasibility study with pulse muon beam is on-going in RAL.

Summary & Outlook

- **Experimental feasibility has been checked**

  Laser should be upgrade its intensity by factor four...

  Quenching ratio from F=1 to F=0 is based on theory

  Not experimentally verified yet J. Cohen, PRA43(1991)9

- **Came here to look for collaborator!**

  Not sufficient to support long-run

  Tell us how to convince our funding agency.

  You know, we are rookie in this field...

  comment to the last budget request:

  "It is not clear how to tackle proton charge radius"

  Anyone wish to share the budget?

  eg. laser cavity @ 20 K in hydrogen target

THANKS

Spare

Outline

Physics background

Experimental principle & feasibility

Present status

Summary and outlook

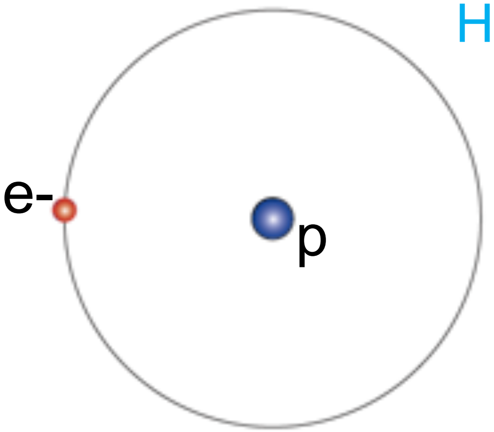
Structure of proton

Proton : a building block of the visible universe

mass = 938.272046 (21) MeV

magnetic moment = 2.792847356 (23) μN

However, its internal structure is not known well

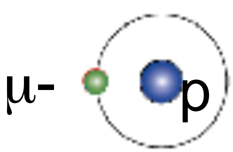


Hydrogen

Muonic hydrogen (μ-p)

exotic atom composed with μ^- & p

$m_\mu \sim 207 m_e$, $R_{\mu p} \sim a_B/207$ $\quad a_B = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2} = \frac{\hbar}{m_e c\alpha}$



Muonic Hydrogen

Atomic energy level

$$\boxed{\Delta E_{\text{finite size}}(nl) = \frac{2(Z\alpha)^4c^4}{3\hbar^2 n^3} m_r^3 R_E^2 \delta_{l0}}$$

Muon is sensitive to proton structure : $(m_\mu/m_e)^3 \sim 10^7$

Good tool to study the proton electromagnetic structure

Proton radii

Charge radius R_E :

$$R_E^2 = \int d^3r\, r^2 \rho_E(r)$$

$\rho_E(r)$: charge distribution

Magnetic radius R_M :

$$R_M^2 = \int d^3r\, r^2 \rho_M(r)$$

$\rho_M(r)$: magnetic moment distribution

Zemach radius R_Z :

$$R_Z = \int d^3r\, r \int d^3r' \rho_E(r') \rho_M(r - r')$$

Proton radii

Charge radius R_E :

$$R_E^2 = \int d^3r\, r^2 \rho_E(r)$$

$\rho_E(r)$: charge distribution

→Proton radius puzzle

Magnetic radius R_M :

$$R_M^2 = \int d^3r\, r^2 \rho_M(r)$$

$\rho_M(r)$: magnetic moment distribution

Zemach radius R_Z :

$$R_Z = \int d^3r\, r \int d^3r' \rho_E(r') \rho_M(r - r')$$

← We want to measure

R_E from electron-proton scattering

Electron-proton scattering

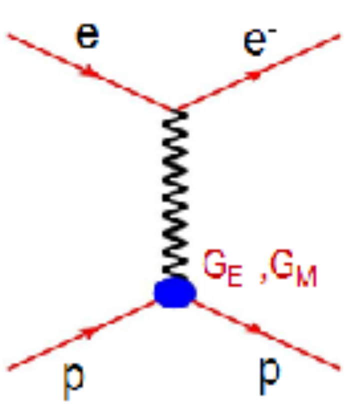
$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \frac{\epsilon G_E^2(Q^2) + \tau G_M^2(Q^2)}{\epsilon(1+\tau)}$$

$$\tau = Q^2/4m_p^2; \qquad \epsilon = 1/(1+2(1+\tau)\tan^2\frac{\theta}{2});$$

G_E , G_M : form factor

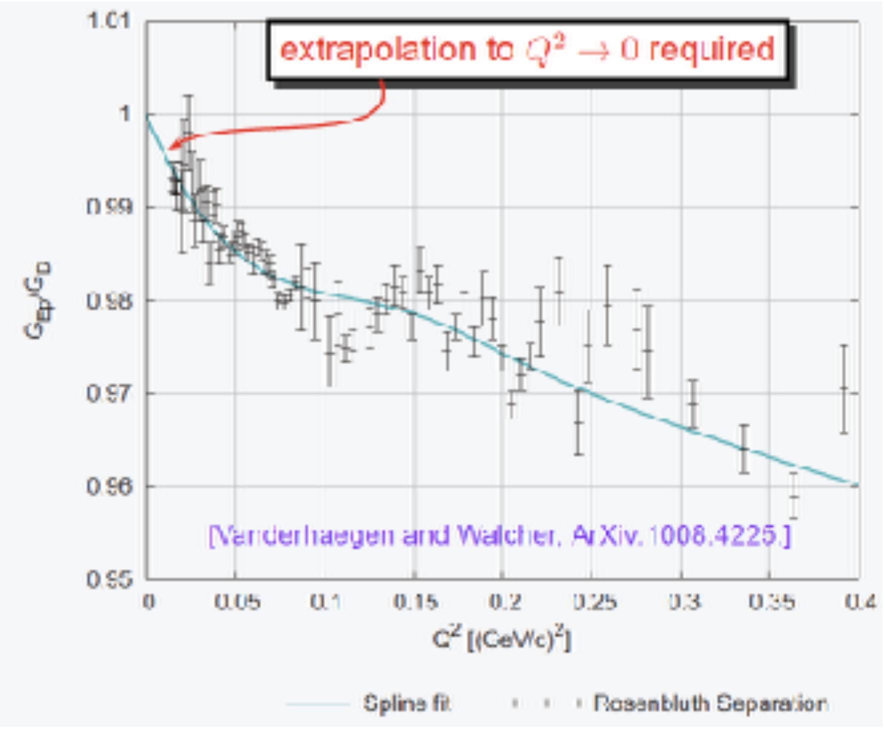
$$G_E = \int \rho_E(x)\, e^{i\,Q\cdot x}\, d^3x$$
$$= \int \left(1 + iQ\cdot x - \frac{(Q\cdot x)^2}{2} + \ \dots\right)\rho_E(x)d^3x$$
$$= 1 + \frac{1}{6}|Q^2| < R_E^2 > + \cdots$$

$$R_E^2 = 6h^2 \frac{dG_E}{dQ^2}\Big|_{Q^2=0}$$



Standard dipole form

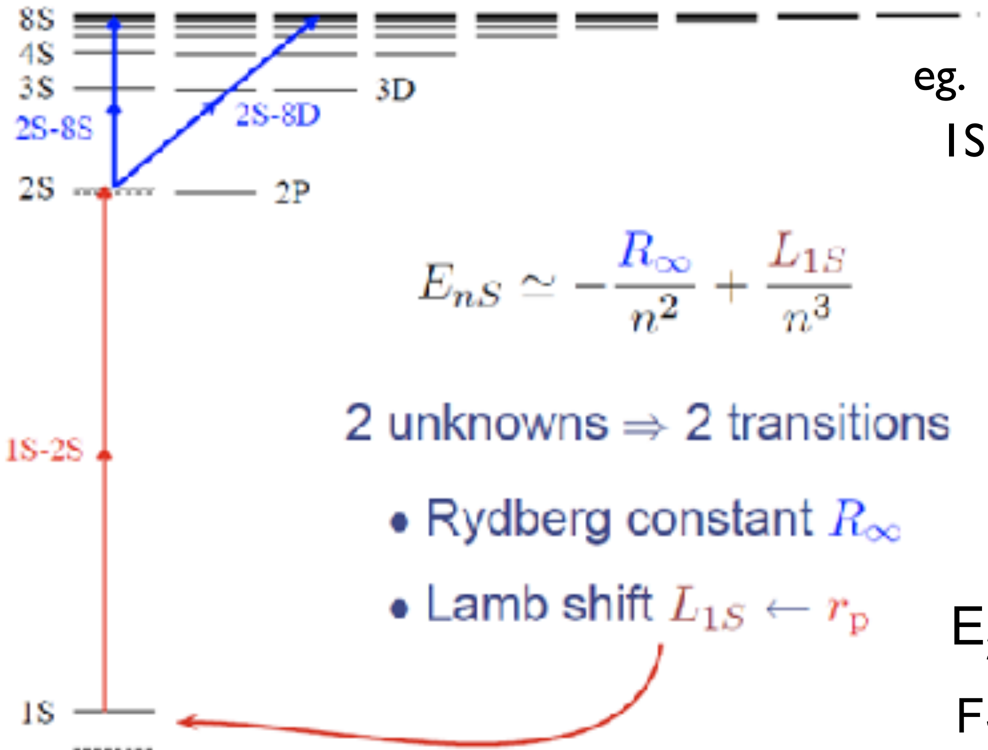
$$G_E = \frac{G_M}{\mu_p} = G_{\text{std.dip.}} = \left(1 + \frac{Q^2}{0.71\ (\text{GeV}/c)^2}\right)^{-2}$$



R_E from Hydrogen spectroscopy

atomic energy levels

$$L_{1S}(r_p) = 8171.636(4) + 1.5645 \langle r_p^2 \rangle \quad \text{MHz}$$



$$E_{nS} \simeq -\frac{R_\infty}{n^2} + \frac{L_{1S}}{n^3}$$

2 unknowns ⇒ 2 transitions

- Rydberg constant R_∞
- Lamb shift $L_{1S} \leftarrow r_p$

1S Lamb shift

~ finite size correction, VP etc

eg.

$1S_{1/2} \rightarrow 2S_{1/2}$: 2,466,061,413,187. 035(10) kHz

+ $2S \rightarrow 8D$: 770,649,561,586.6(58) kHz

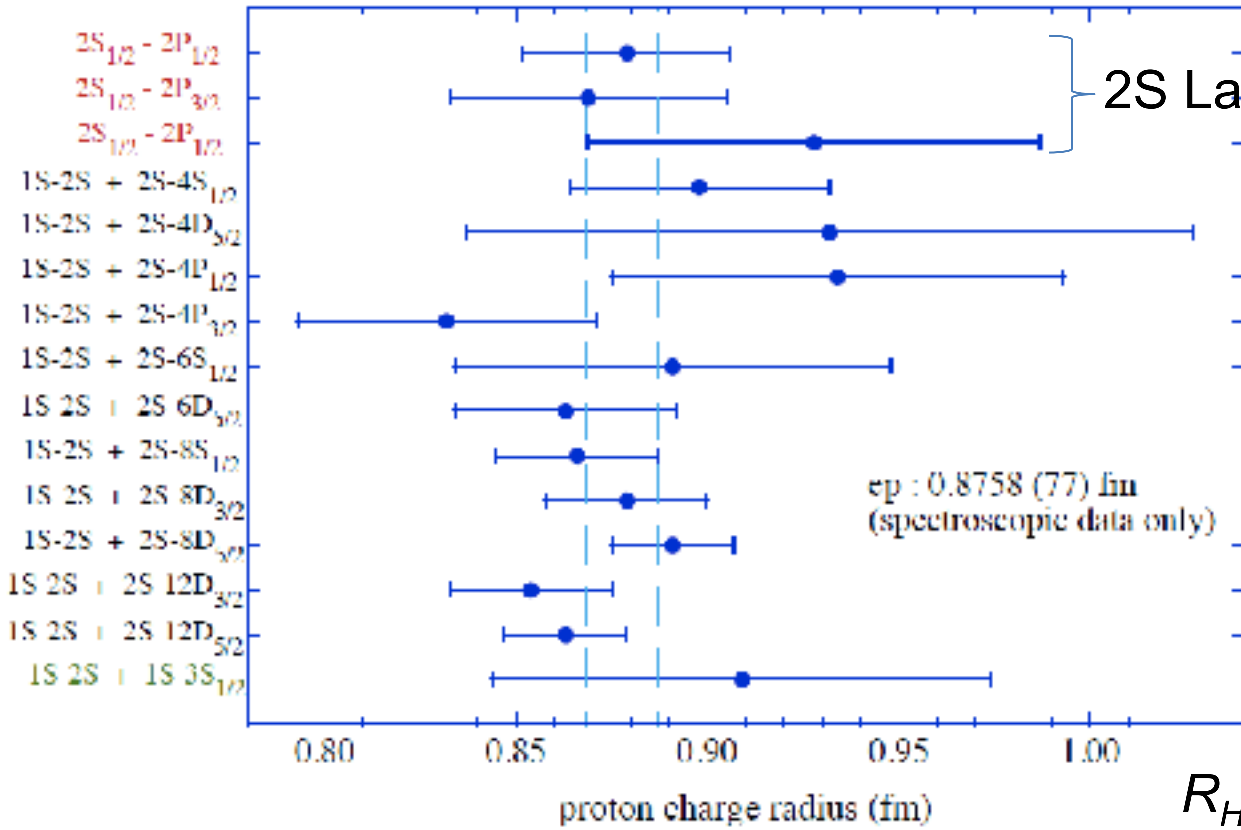
2S Lamb shift ($2S_{1/2} - 2P_{1/2}$)

$E_{2S} - E_{2P} \leftarrow$ first term ($\sim R_\infty$) cancelled

FS correction in P state is very tiny

Hydrogen spectroscopy to Rp

Summary of H spectroscopy



2S Lamb shift

R_{H_avr} = 0.8758(77) fm

CODATA 2006 (e-p scat. & H) R_E = 0.8768(69) fm

~1 %

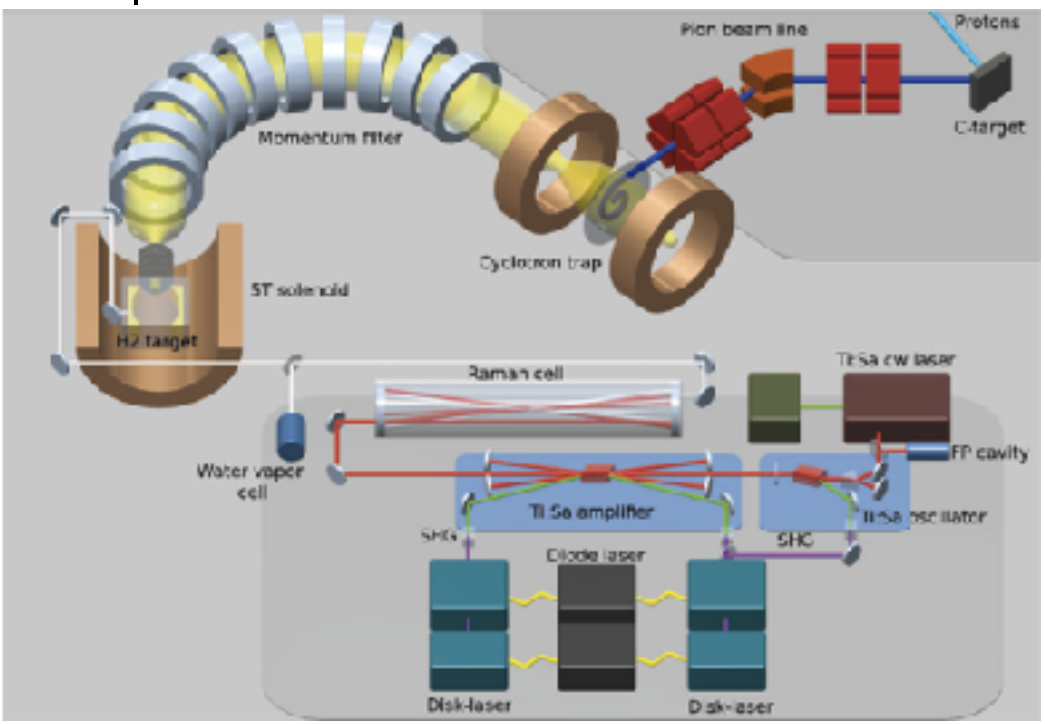
Lamb shift in muonic hydrogen

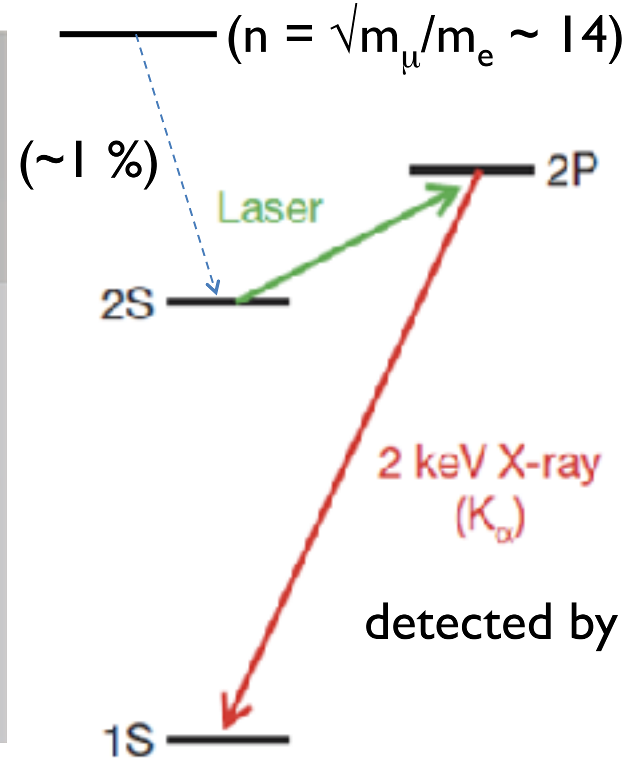
PSI CREMA (Charge Radius Experiment with Muonic Atoms) collaboration

Laser spectroscopy of $2S^{F=1}_{1/2} \rightarrow 2P^{F=2}_{3/2}$ of muonic hydrogen

Setup in πE5

Antognini et al, Science



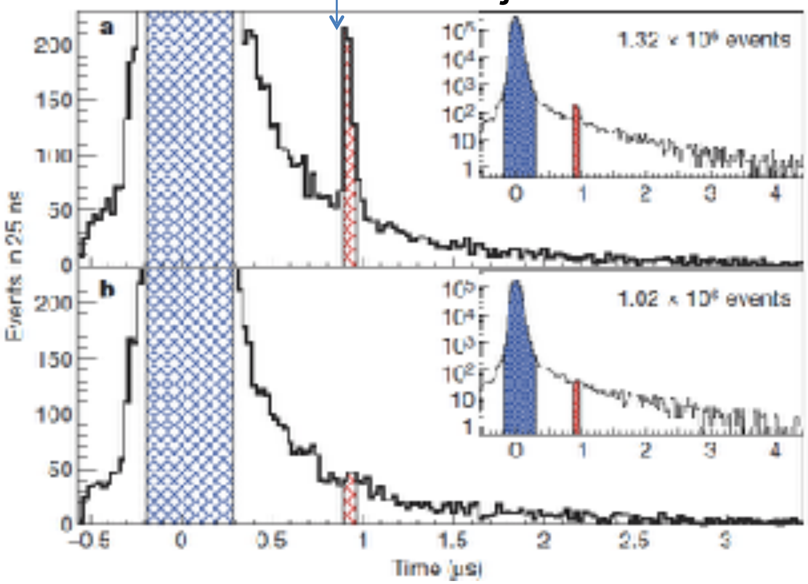


detected by LAAPD

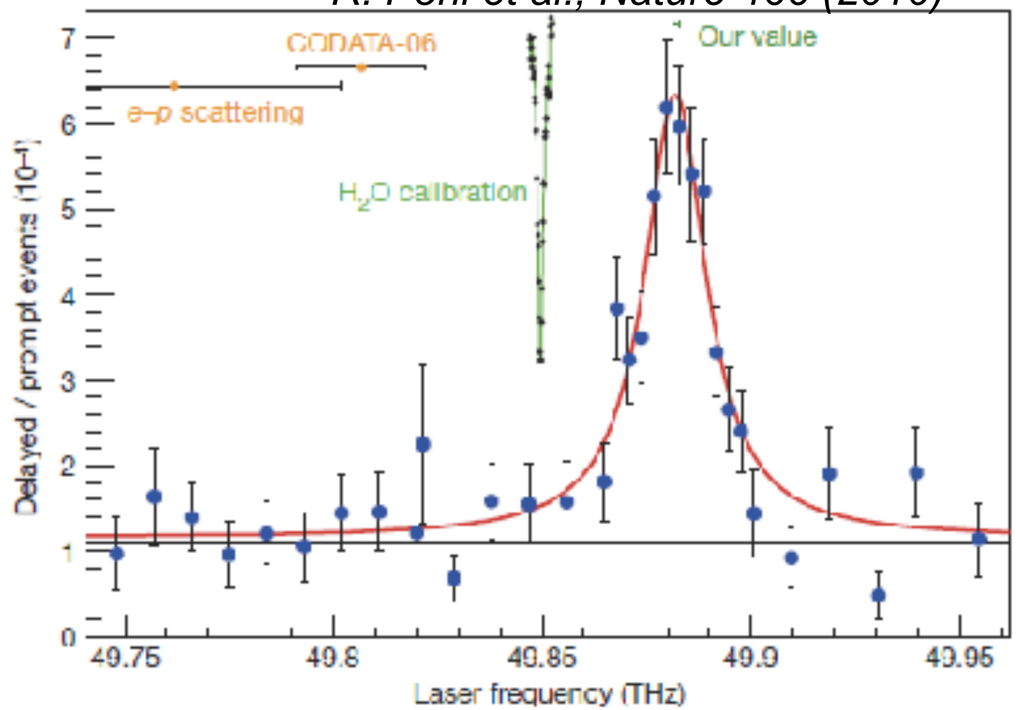
Lamb shift in muonic hydrogen

Timing spectra of LAAPD

laser injection



R. Pohl et al., Nature 466 (2010)



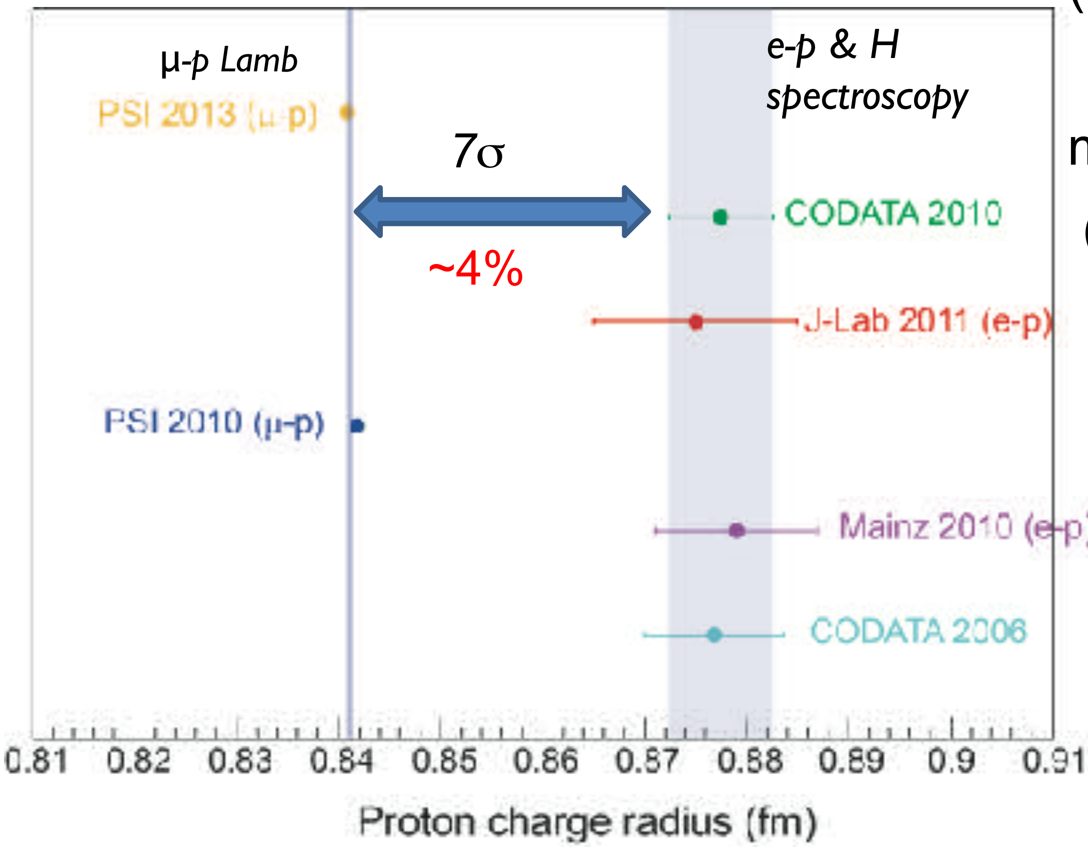
Measured value : 206.2949(32) meV

Theory: $\Delta E = : 209.9779(49) - 5.2262\ R_E^2 + 0.0347\ R_E^3$ meV

$R_E = 0.84184(67)$ fm X10 better precision

Proton radius puzzle

PSI result is x10 better precision, however central value does matter!



e-p & H spectroscopy (CODATA 2010)

0.8775(51) fm

muonic hydrogen (2010 & 2013)

0.84087(39) fm

"Proton radius puzzle" (2010~)

Possible hypothesis

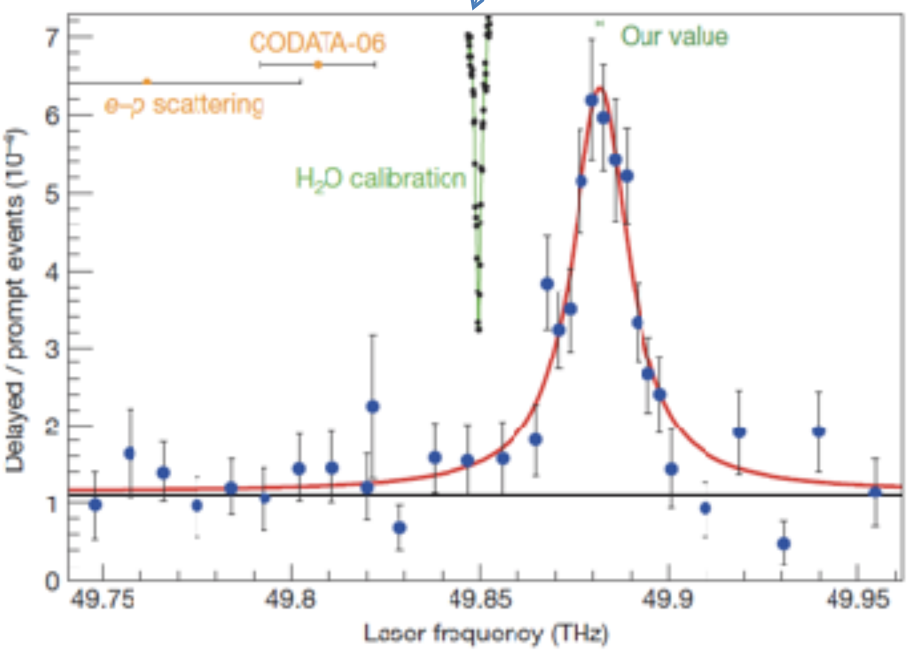
Proton radius puzzle (2010~, still unsolved)

- p *errors in the measurement(s) ?*
- p *proton structure-dependent corrections are wrong?*
- p *QED needs modification (in m-p interaction)?*
- p *Physics beyond the Standard Model?*

Are experiments correct?

PSI :Energy calibration by water vapor absorption

calibration by H2O absorption

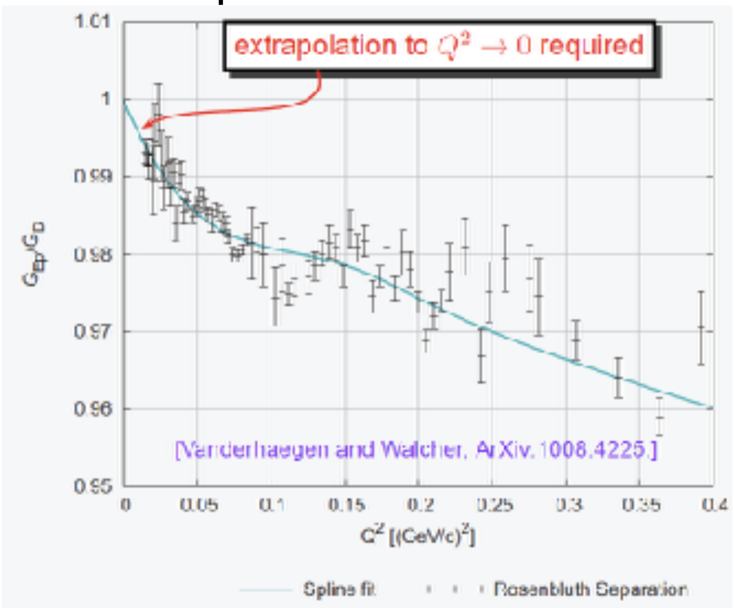


miss in the calibration is unlikely.

e-p scattering

extraction to $Q^2=0$

shape of the form factor



new data toward lower-Q^2

PRad @ Jlab with ISR

Suda san's group@Tohoku

Theory is wrong?

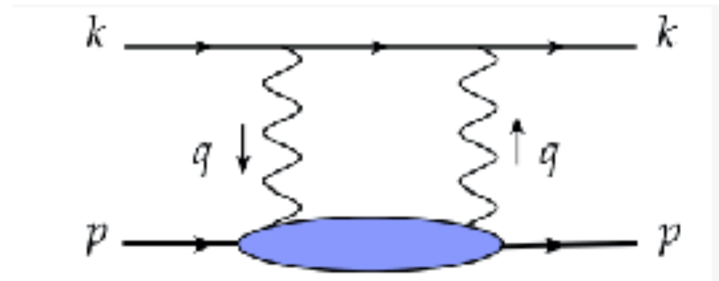
errors in the PSI measurement

Antognini,

• Statistics	
Center position uncertainty (~ 4% of Γ)	700 MHz
• Systematics	
Laser frequency (H_2O calibration)	300 MHz
AC and DC stark shift	< 1 MHz
Zeeman shift (5 Tesla)	< 30 MHz
Doppler shift	< 1 MHz
Collisional shift	2 MHz
• Total uncertainty of the line determination	760 MHz
• Theory: proton polarizability	1200 MHz
• Discrepancy with CODATA prediction	75 300 MHz

82 % of total error

proton polarizability



Carlson *et al.*, PRA 84, 020102 (2011)

uncertainty is large, but corrections value is not so large.

$$\Delta E = : 209.9779(49) - 5.2262\, R_E^2 + 0.0347\, R_E^3 \text{ meV}$$

discrepancy 0.31 meV

Theoretical uncertainty is much smaller than the observed discrepancy.

New physics?

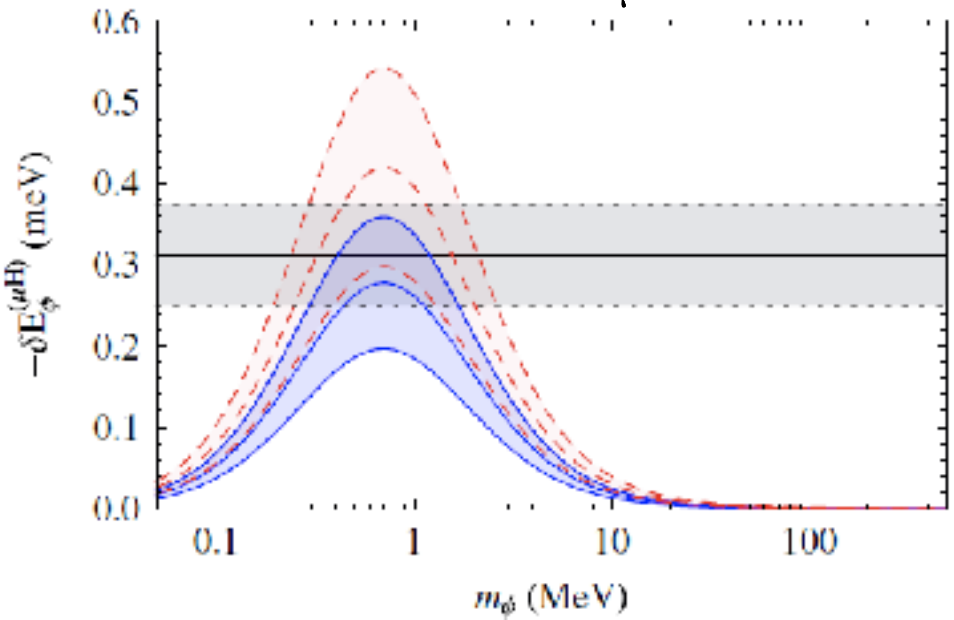
breaking of lepton universality

Tucker-Smith & Yavin (2011)

Coupling to muons and protons
(small coupling to other particles)

MeV-mass mediate particle

Lamb shift in μ-H:



may simultaneously solve the muon g-2 anomaly

predict energy shifts in mu-D, mu-He and true Muonium (μ+μ-)

FYI : MUSE (MUon proton Scattering Experiment)

talk by E.J. Downie

MUSE (Muon proton Scattering Experiment)

μ-p scattering @ PSI

	atomic spectroscopy	scattering
electron	yes	yes
muon	yes	no

μ^+p, e^+p, μ^-p, e^-p

$\theta \approx 20° - 100°$

$Q^2 \approx 0.002 - 0.07\ \mathrm{GeV}^2$

3.3 MHz total beam flux

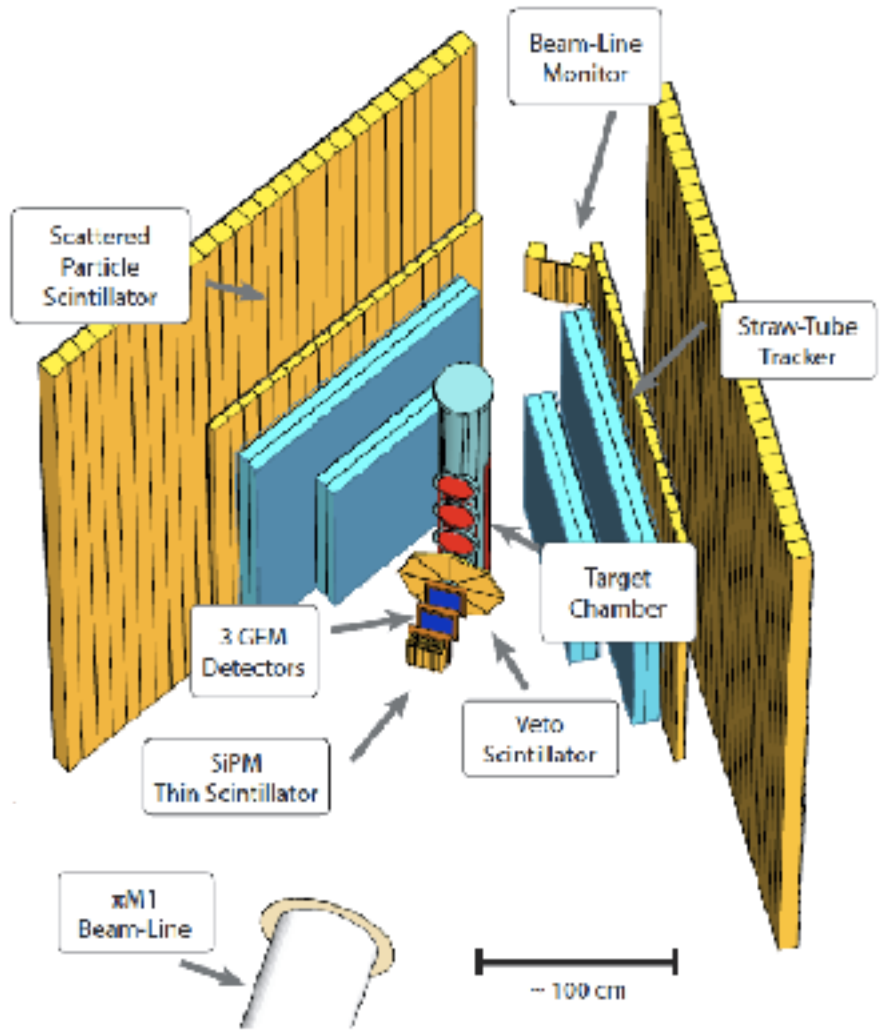
$\approx$ 2-15% μ's

$\approx$ 10-98% e's

$\approx$ 0-80% π's

precision to $R_E \sim 0.01$ fm

production run 2018-2019 (2x6 month)



Q: What about the magnetic distribution inside the proton probed with muon?

Q: What about the magnetic distribution inside the proton probed with muon?

proton Zemach radius

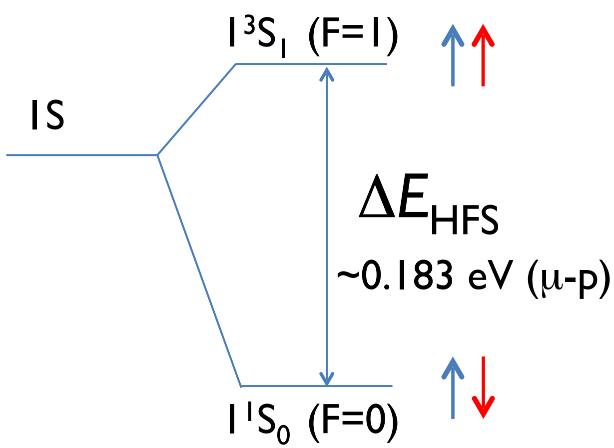
$$R_Z = \int d^3r\, r \int d^3r' \rho_E(r') \rho_M(r - r')$$

Hyperfine splitting

Zemach radius can be determined from Hyperfine splitting energy

Hyperfine splitting ($\mu^- p$ case)

Hyperfine splitting (1S)



F = I+J : atomic angular momentum

I : proton spin,

J : electron angular momentum

Theoretical expression :

common in H-like atom

$$\Delta E^{HFS} = E_F(1 + \delta^{QED} + \delta^{str})$$

- E_F :*Fermi term*

$$E_F = \frac{8}{3}\alpha^4 \frac{m_{\mu(e)}^2 m_p^2}{(m_{\mu(e)} + m_p)^3} \mu_p$$

- δ_{QED} : *QED correction (including higher order)*
- δ_{str} : *proton structure related correction*

$$\delta_{str} = \delta_{FF} + \delta_{recoil} + \delta_{pol} + \delta_{hVP}$$

$$-2\alpha m_{\mu p} R_Z + O(\alpha^2)$$

ΔE_{HFS} is directly connected with Rz

Hyperfine splitting

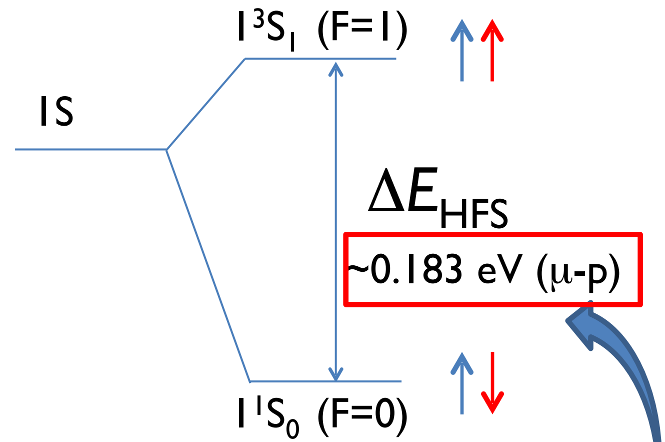
Theoretical calculations

1S HFS

Dupays et al., PRA 68

	Hydrogen		Muonic hydrogen	
	Magnitude	Uncertainty	Magnitude	Uncertainty
E^F	1418.84 MHz	0.01 ppm	182.443 meV	0.1 ppm
δ^{QED}	1.13×10^{-3}	$<0.001\times10^{-6}$	1.13×10^{-3}	10^{-6}
δ^{rigid}	39×10^{-6}	2×10^{-6}	7.5×10^{-3}	0.1×10^{-3}
δ^{recoil}	6×10^{-6}	10^{-8}	$1,7\times10^{-3}$	10^{-6}
δ^{pol}	1.4×10^{-6}	0.6×10^{-6}	0.46×10^{-3}	0.08×10^{-3}
δ^{hvp}	10^{-8}	10^{-9}	0.02×10^{-3}	0.002×10^{-3}

uncertainty ~10^{-4}



Laser spectroscopy

44.2 THz = 6.8 um

(mid-infrared laser)

Expt.

1420.4057517667(9) MHz

(well known as "21 cm line")

Not Measured before

Past measurements on Zemach radius

- *hydrogen spectroscopy*

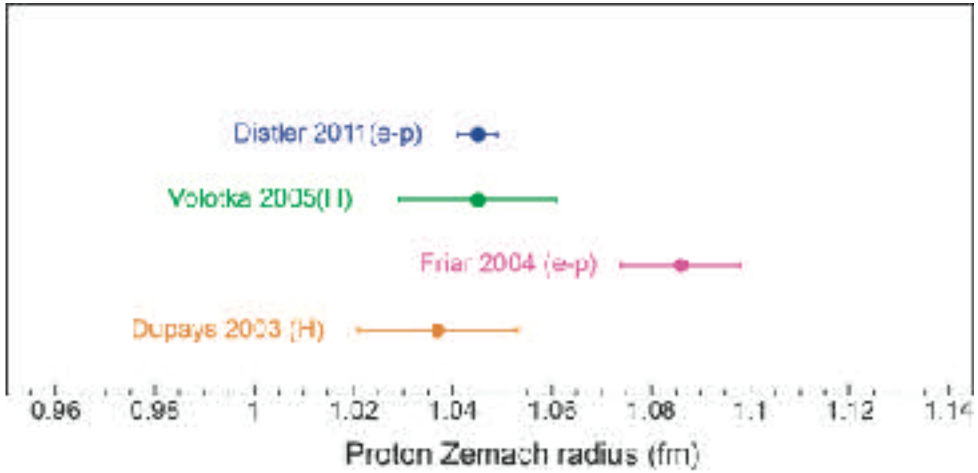
R_z =1.037(16) fm *Dupays et al., PRA(2003)*

=1.047(19) fm *Volotka et al., EPJ(2005)*

- *e-p scattering*

R_z =1.086(12) fm, *Friar & Sick, PLB(2004)*

=1.045(4) fm, *Distler et al., PLB(2011)*



Past measurements on Zemach radius

◆ *hydrogen spectroscopy*

R_z =1.037(16) fm *Dupays et al., PRA(2003)*

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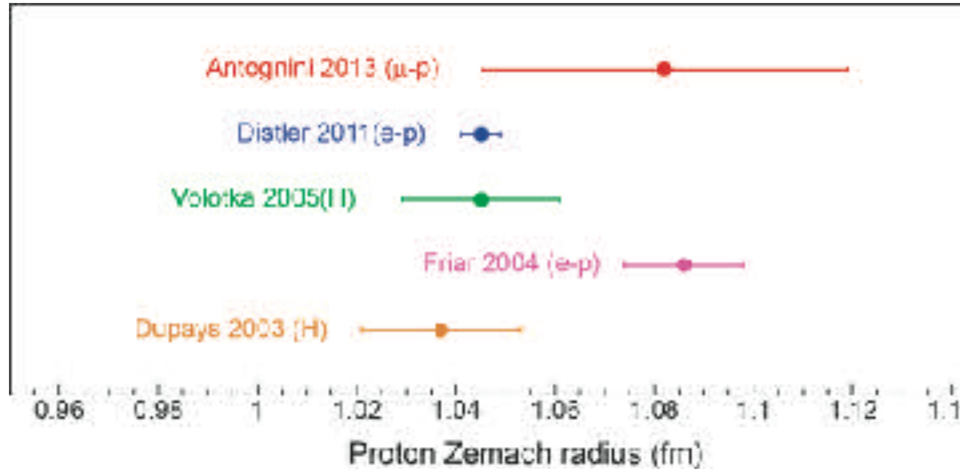


◆ *e-p scattering*

R_z =1.086(12) fm, *Friar & Sick, PLB(2004)*

=1.045(4) fm, *Distler et al., PLB(2011)*

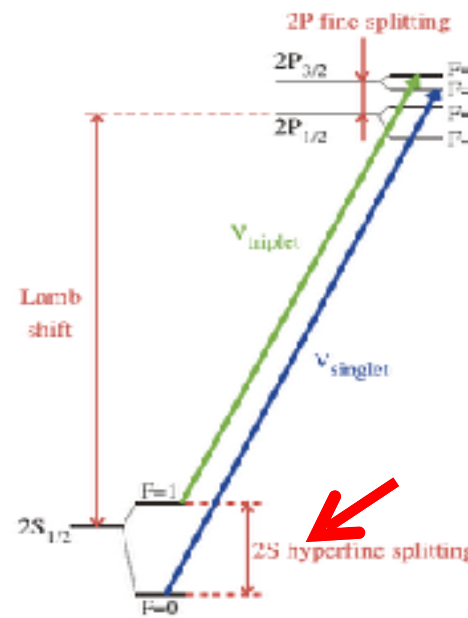
◆ *muonic hydrogen 2S HFS*

R_z = 1.082(37) fm, *Antognini et al., Science (2013)*

➢ *latest value of e-p and H spectroscopy are consistent within their errors*

➢ *μ-p value differ? But uncertainty is still large.*

Direct spectroscopy of 1S HFS



Measurement of μ-*p* ground-state ΔE_{HFS}

▶ muonic hydrogen 1S HFS energy ← *not measured before*

laser spectroscopy : 0.183 eV = ~6.78 μm, 44.2 THz (*mid-infrared*)

Our goals :

- *determine 1S* ΔE_{HFS} *with an accuracy of* ~ 2 ppm (~100 MHz)

  the 1st precise measurement of ΔE_{HFS}^{1S} *of μ-p*

  fundamental quantity of μ-p system

- *derive* *Zemach radius* *from* ΔE_{HFS}

$$\Delta E_{HFS}^{exp} = E_F(1+\delta_{QED}+\delta_{FF}+\delta_{recoil}+\delta_{pol}+\delta_{hvp})$$

$$\delta_{FF} = -2\alpha m_{\mu p} R_Z + O(\alpha^2)$$

$$R_Z = \{ (E_F (1 + \delta_{QED} + \delta_{recoil} + \delta_{pol} + \delta_{hvp})) - \Delta E_{HFS}^{exp} \}/1.281$$

theoretical corrections *measurement*

Expected precision of Zemach radius

$$R_Z = \{(E_F\,(1 + \delta_{QED} + \delta_{recoil} + \delta_{pol} + \delta_{hvp}) - \Delta E^{exp}_{HFS}\}/1.281$$

1130(1) ppm 1700(1) ppm 460(80) ppm 20(2) ppm (2) ppm

Dupays et al., PRA 2003

R_Z = 1.0XX(13) fm

δ_{pol} is dominated in precision, but improved factor ~3 from PSI results,

	Hydrogen		Muonic hydrogen	
	Magnitude	Uncertainty	Magnitude	Uncertainty
E^F	1418.84 MHz	0.01 ppm	182.443 meV	0.1 ppm
δ^{QED}	1.13×10^{-3}	$<0.001\times10^{-6}$	1.13×10^{-3}	10^{-6}
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δ^{hvp}	10^{-8}	10^{-9}	0.02×10^{-3}	0.002×10^{-3}

check with R_Z determined by "electronic" and "muonic" measurement

improvement of proton polarizability correction (δ_{pol}) drastically reduces uncertainty of Rz

e-p	*1.4(6) ppm*
μ-p	*460(80) ppm*

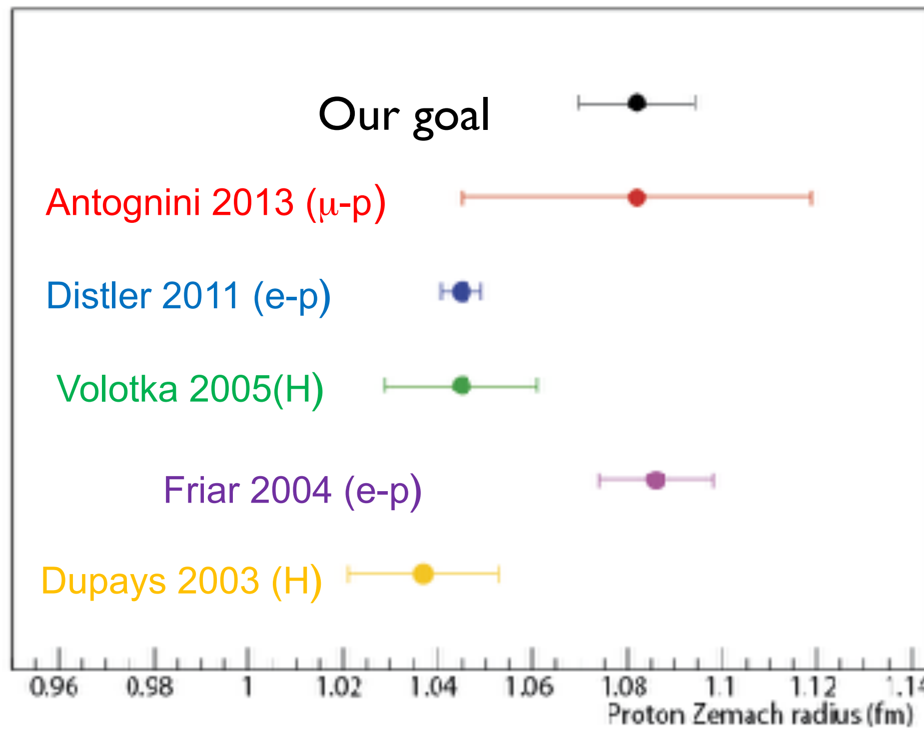
Expected precision

new chiral PT calculation on proton polarizability,
Hagelstein, arXiv 1512.03765
ECT WS 2016*

polarizability for 2S HFS

$$E^{\mathrm{pol}}_{\mathrm{HFS}}(2S) = 0.87 \pm 0.42\,\mu\mathrm{eV}.$$

x8 for $E^{pol}_{HFS}(1S)$ → 34 ueV (18 ppm)



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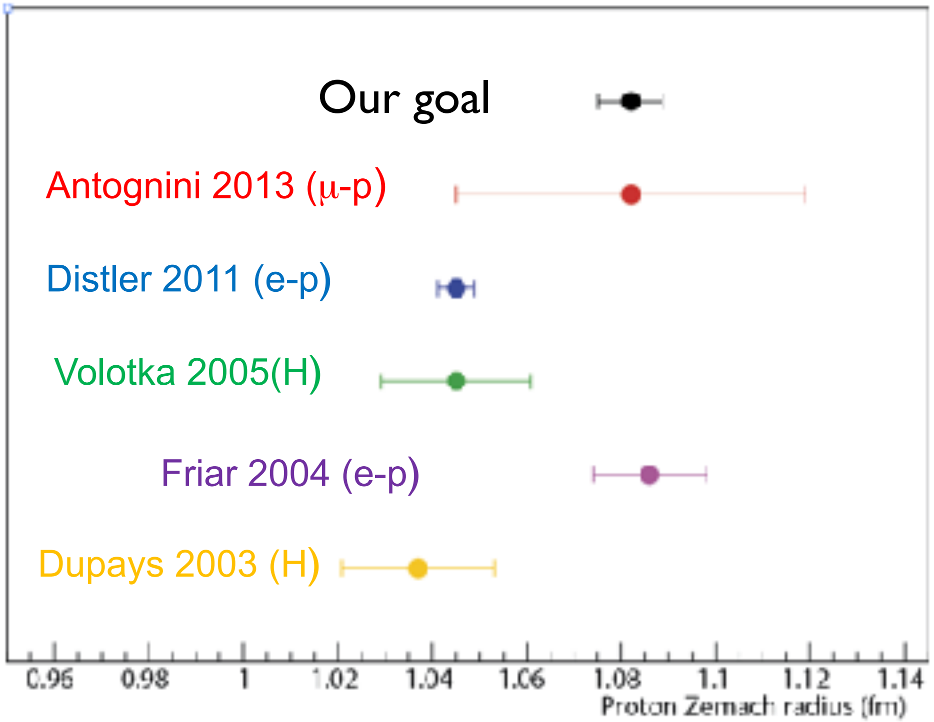


Jlab g2p collaboration

inelastic cross section with polarized e-p scattering.

ß polarized structure function

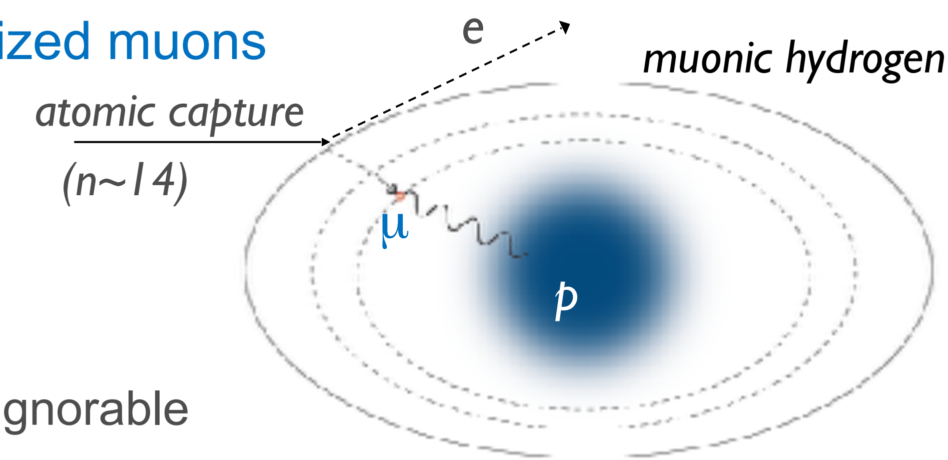
Theoretical input for proton polarizability calculation

Experimental principle

Experimental principle (1)

Laser spectroscopy : signal to detect resonance

decay asymmetry of polarized muons



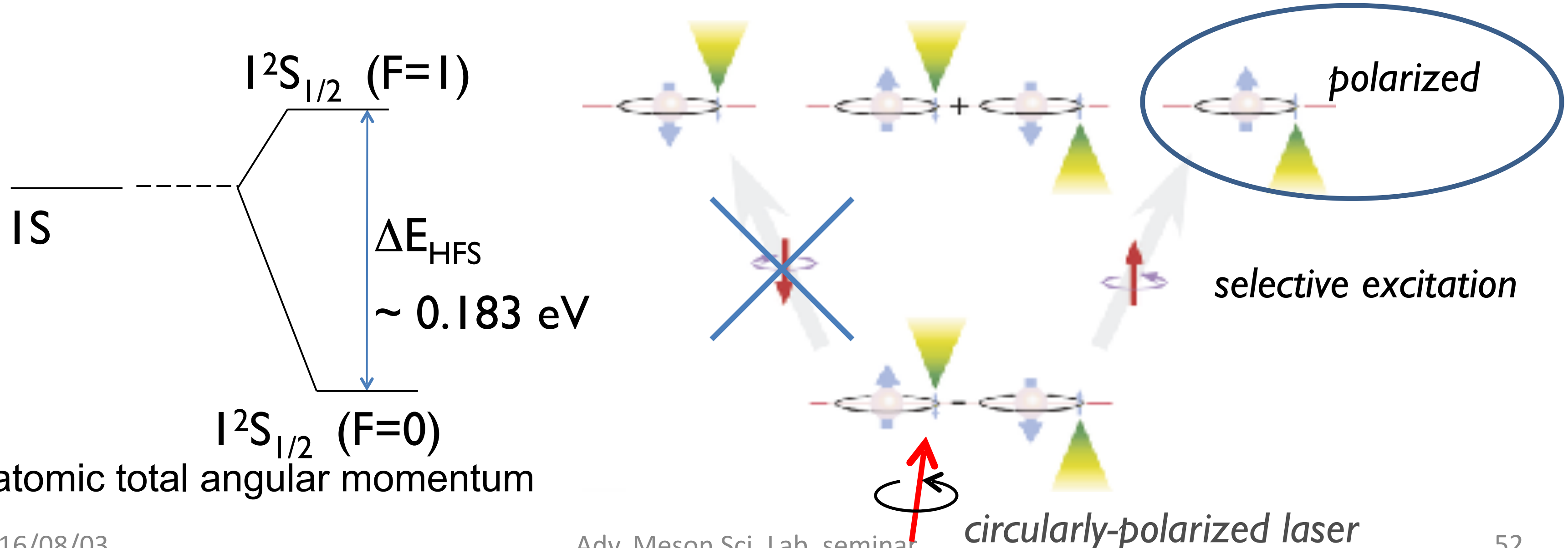
- Produce μ-p atom

  stop μ^- in hydrogen

  ⇨ de-excite to ground state

  (τ_{1S} ~ 2.2 us) nuclear capture ignorable

- Laser-induced spin repolarization



F : atomic total angular momentum

Experimental principle (2)

- Measure electrons from muon decay

$\mu^- \rightarrow e^- + \overline{\nu_e} + \nu_\mu$

muon decay asymmetry with polarization (P)

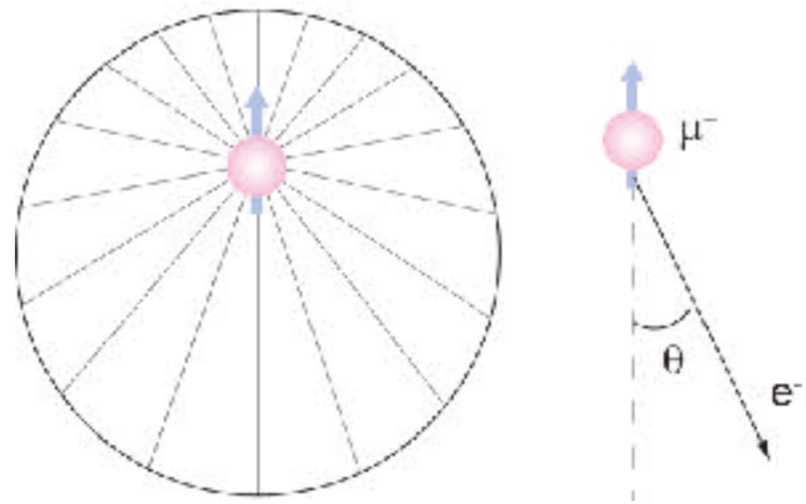
$$d\sigma_{e^-}(\theta)d\Omega \propto \left(1 - \frac{1}{3} P \cos\theta\right) d\Omega$$

Laser frequency matches the HFS energy

spin polarization in $^3S_1$ state

detect electron decay asymmetry

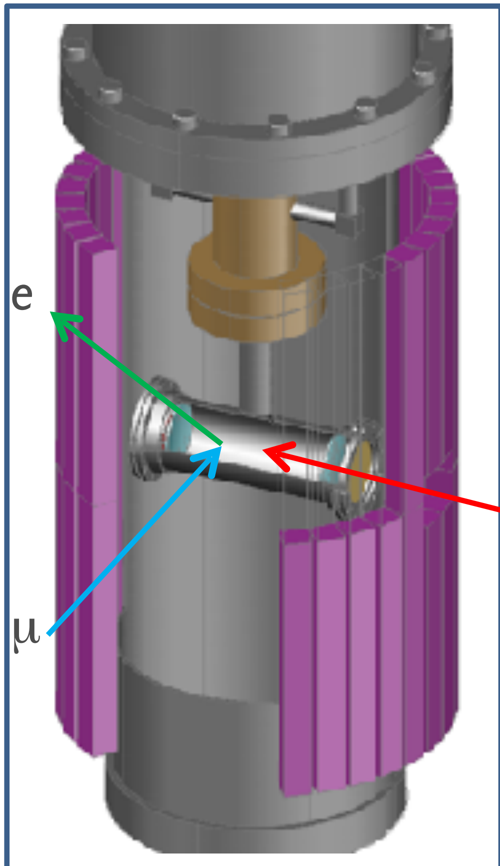
polarized ($P \neq 0$)



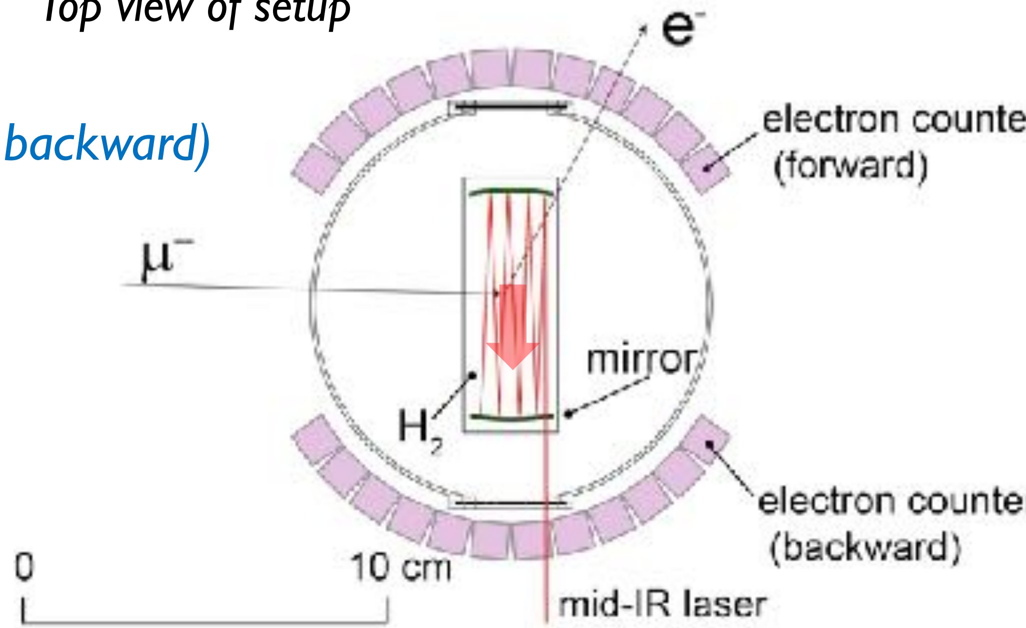
more decay electrons in opposite direction of muon spin

Conceptual design of experimental setup

1) *gas H_2 target*
2) *tunable mid-infrared laser*
3) *decay electron counter(forward and backward)*

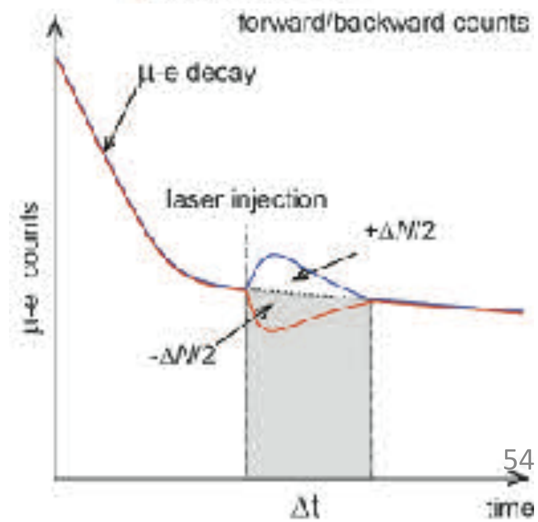


Top view of setup



detect forward/backward electrons

Asymmetry = N_F - N_B

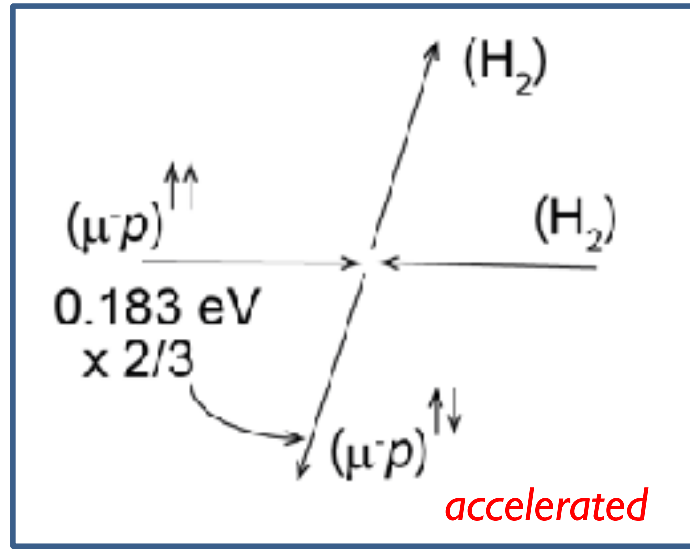


Competitor 1 : Italian (A. Vacci's et. al.,) group

A. Vacchi, ECT WS 2016*

Laser spectroscopy, but different method of detection of resonance

"muon transfer" method



1. $\mu^- p(\uparrow\downarrow)$ absorbs a photon of resonance wavelength

   $\lambda_0 = hc/\Delta E^{1S}_{HFS}$ ~ 6.8 μ

   Converts the spin state of the $(^-\mu p)$ atoms from $^1S_0$ to $^3S_1$

   $\mu^- p(\uparrow\downarrow) \rightarrow \mu^- p(\uparrow\uparrow)$

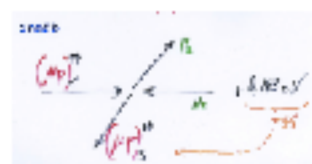
2. $\mu^- p(\uparrow\uparrow)$ $\quad$ $^3S_1$ atoms are collisionally de-excited to

   $\mu^- p(\uparrow\downarrow)$ $\quad$ $^1S_0$ and accelerated by

   ~0.12 eV ~ 2/3 ΔE^{HFS}_{1S}

Energy-dependent muon transfer rates change the time distribution of the events λ_0 is recognized by maximal response

D. Bakalov, et al., Phys. Lett. A172 (1993).
D. Bakalov, et al., NIM B281 (2012).



H_2 target + ~ % O_2 mixture

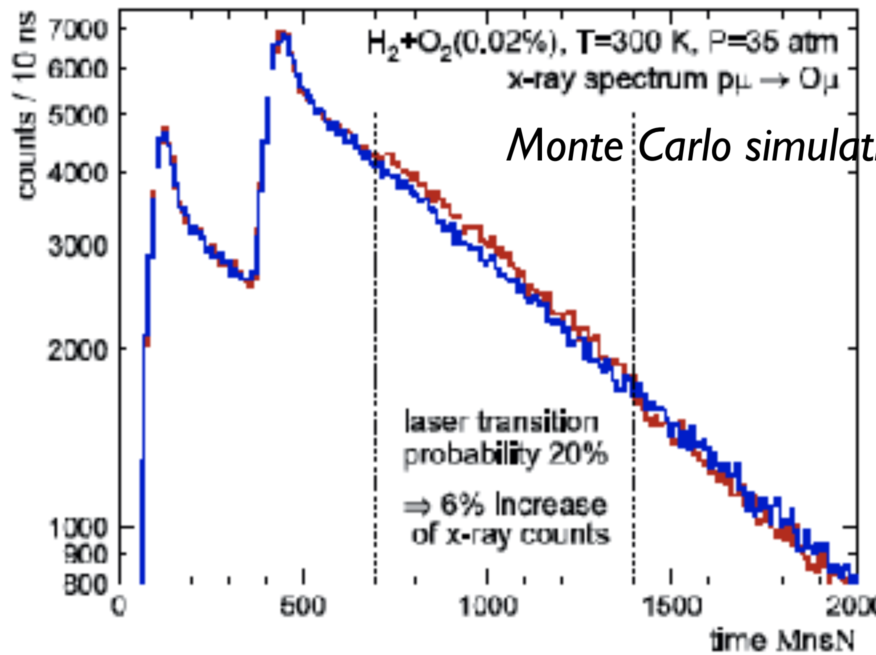
increase muon transfer rate: $\quad \mu p + Z \Rightarrow \mu Z + p$ $\quad$ *detect muonic X-rays*

Competitor 1 : Italian (A. Vacci's et. al.,) group

A. Vacchi, ECT WS 2016*

Recent simulations of μ-p HFS expt.

Phys. Lett. A 379 151 2015



$\rho = (N_A - N_B) / \sqrt{2(N_A + N_B)}$

10^6 muonic atoms "shot", $\rho = 6\times10^4/\sqrt{4.10^6} = 30$.

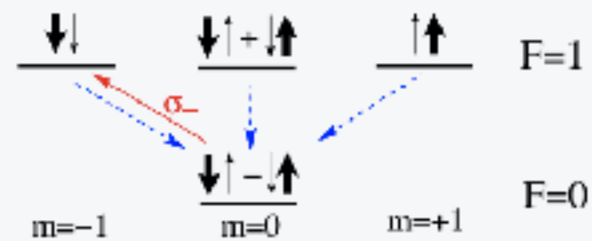
try to measure in RIKEN RAL

Competitor 2 : PSI group

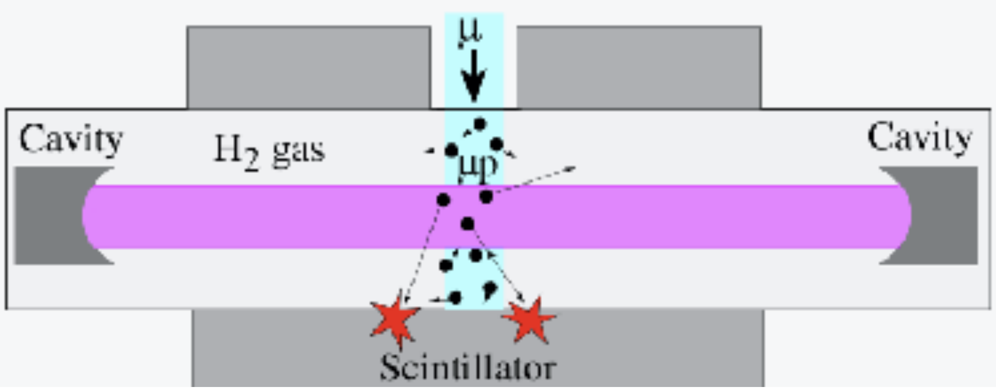
Same with Lamb shift group (CREMA to HyperMu collaboration) *R. Pohl, ECT* WS 2016*

Principle of the μp HFS experiment

- μ^- of 10 MeV/c are detected $\longrightarrow$ trigger the laser
  - μ^- stops in H_2 gas (500 mbar, 50 K) $\longrightarrow$ μp(F=0) formation
- Laser pulse: μp(F=0)$\longrightarrow$$\mu p$(F=1)
  - Collision: μp(F=1)+ H_2 $\longrightarrow$ H_2 +μp(F=0) + E_{kin}
    - Diffusion: the faster μp reach the target walls



- At the wall: μ^- transfer to high-Z atom $\longrightarrow$ $(\mu Z)^*$ formation
  - $(\mu Z)^*$ de-excitation → MeV X-rays, e^- and μ^- capture
    - Resonance: Number of X-rays/e^-/capture signals after laser excitation versus laser frequency



Signal events:

Laser excited μp reach wall in $t \in [t_{\text{laser}}, t_{\text{laser}} + \Delta t]$

Background events:

Thermalized μp reach wall in $t \in [t_{\text{laser}}, t_{\text{laser}} + \Delta t]$

$\Rightarrow$ Cool target to 50 K

Feasibility of the measurement

Transition probability

$^1S_0 \rightarrow {}^3S_1$ transition probability

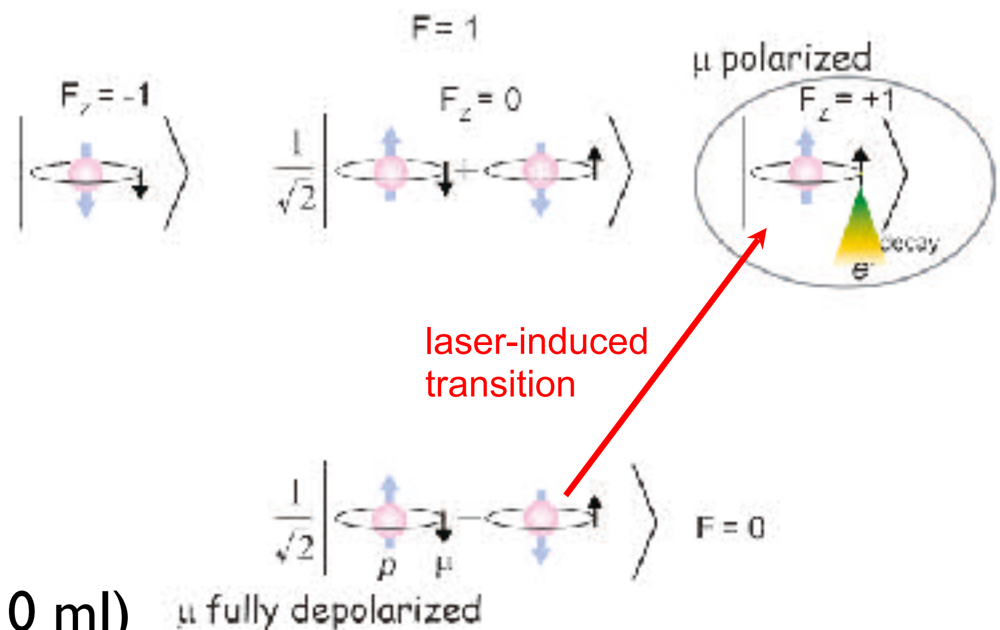
$$\boxed{P = 2\times 10^{-5}\,\frac{E}{S\sqrt{T}}}$$

E/S : laser fluence [J/m^2],
T : temperature [K]

NIM B281(2012)72

$\propto$ *laser power*

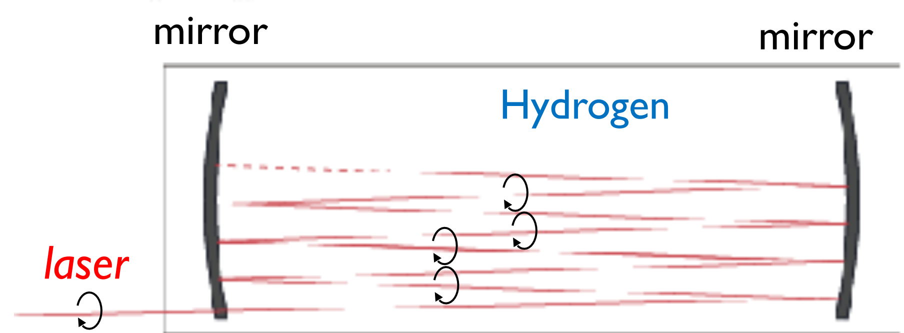
➡ *need high-power mid IR laser (> 10 mJ)*



multi-pass cavity

enhance effective laser power by multiple reflection by mirrors

refractive index R= 99.95 %



Mid-infrared laser system

Tunable mid-infrared laser (under development by Wada san's group in RIKEN)

wavelength ~6.8 um (= ~44.2 THz)

bandwidth ~50 MHz

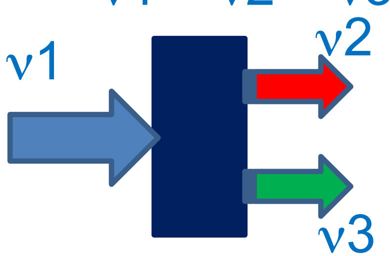
repetition ~ 50 Hz

mid-IR by seeded optical parametric oscillation (OPO) with $ZnGeP_2$ non-linear crystal (2 um→ 6.8 um)

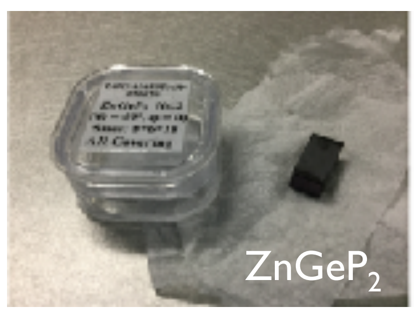
double pulse 10 mJ x 2 sets = 40 mJ

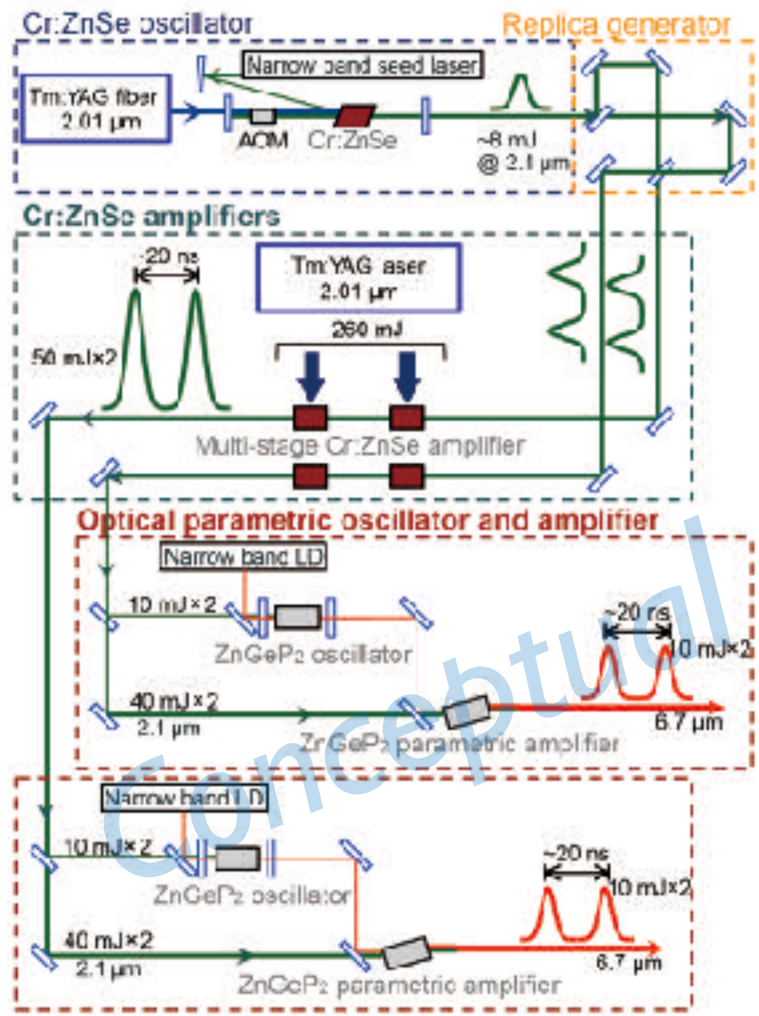
OPO

$\nu1 = \nu2 + \nu3$



non-linear crystal





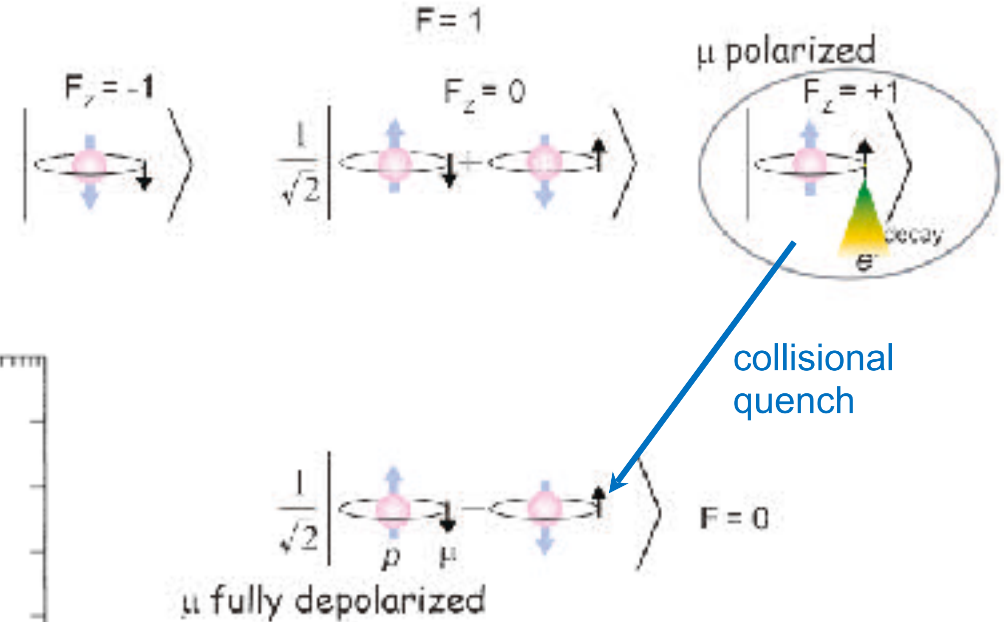
40 mJ laser power is possible

Collisional quench

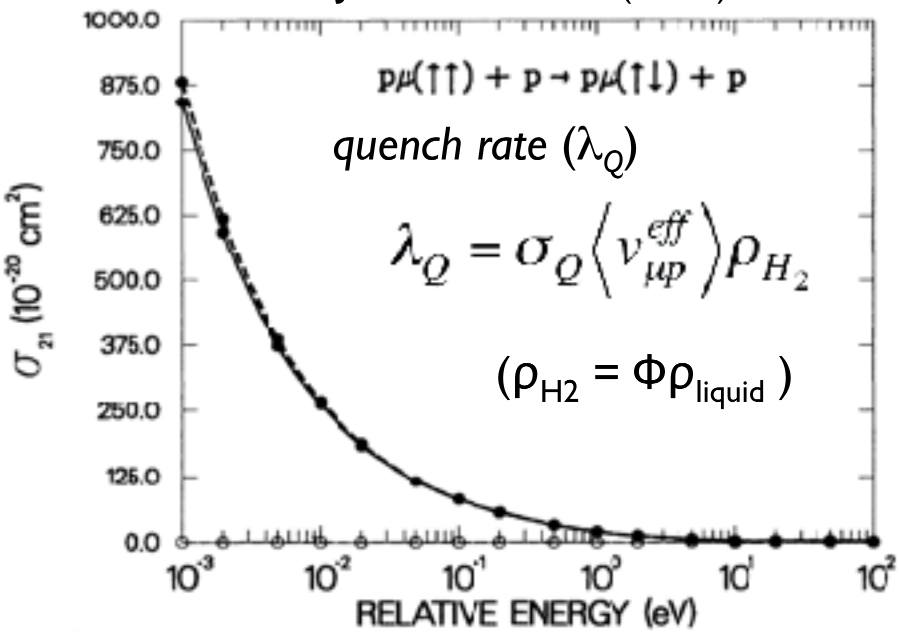
$^3S_1 \rightarrow {}^1S_0$ collisional quench

polarization is lost…

$$\boxed{\mu p(\uparrow\uparrow) + p \rightarrow \mu p(\uparrow\downarrow) + p}$$



J. Cohen, PRA43(1991)9



quench rate (λ_Q)

$$\lambda_Q = \sigma_Q \left\langle v_{\mu p}^{eff} \right\rangle \rho_{H_2}$$

$(\rho_{H2} = \Phi\rho_{liquid})$

proportional to H_2 density

liquid target is unusable

(density is too high ← tau ~ 50 ps)

Quench rate

If Φ = 0.1% (0.01 %)LHD (liquid hydrogen density), then τ_{quench} = 50 (500) ns

Density of the hydrogen gas is very important

Population of spin polarization

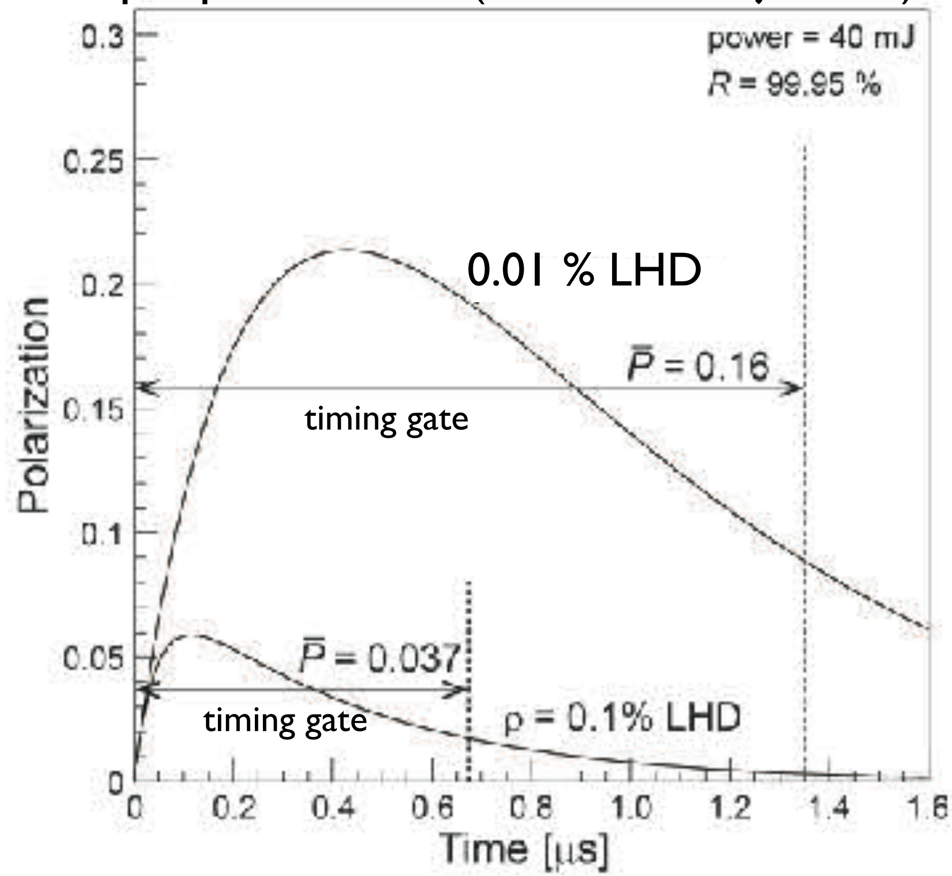
Condition

- ✓ *Laser power 40 mJ*
- ✓ *gas H2 target*
  (t_{quench} ~ 50 or 500ns)
- ✓ *multi-pass cavity*
  (reflective index ~ 99.95 %)

Set proper timing gate after laser injection :
averaged polarization

$$\bar{P} = \sim 3.7\% \quad (0.1\ \%\ \mathrm{LHD})$$
$$\sim 16\ \% \quad (0.01\ \%\ \mathrm{LHD})$$

spin polarization (after laser injection)



Delayed timing laser injection

Large fraction of muons stop outside of H2 gas

Negative muon capture

lifetime of bound muon :

$\tau_{total} = 1/\Lambda_{total}$

$\Lambda_{total} = \Lambda_{capture} + Q\,\Lambda_{decay}$

Q : Huff factor

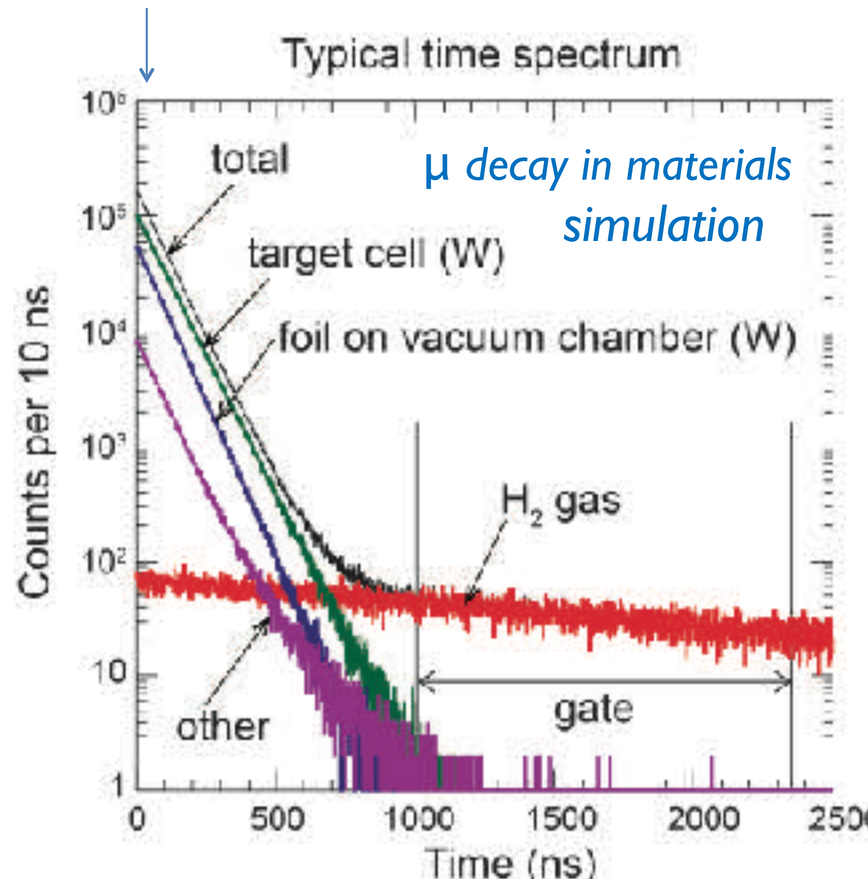
μ- capture

$$\boxed{\mu^- + \text{"}p\text{"} \rightarrow \nu_\mu + n}$$

c.f. $\Lambda_{decay} = 2.197$ us

	τ [ns]($1/\Lambda_{tota.l}$)	Q
H	2194	1.00
C	2040	1.00
Cu	160	0.967
Ag	90	0.925
W	80	0.860

μ injection



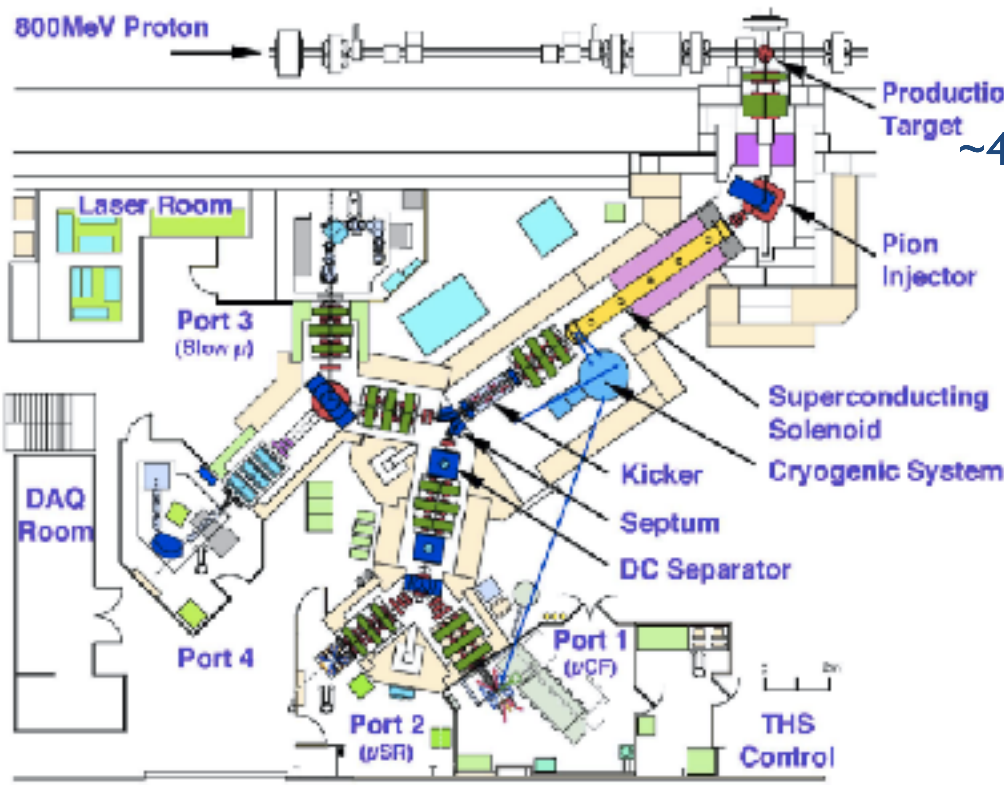
BG suppression by mu- capture & delayed laser injection

proposal to RIKEN-RAL muon facility

proton synchrotron : ISIS

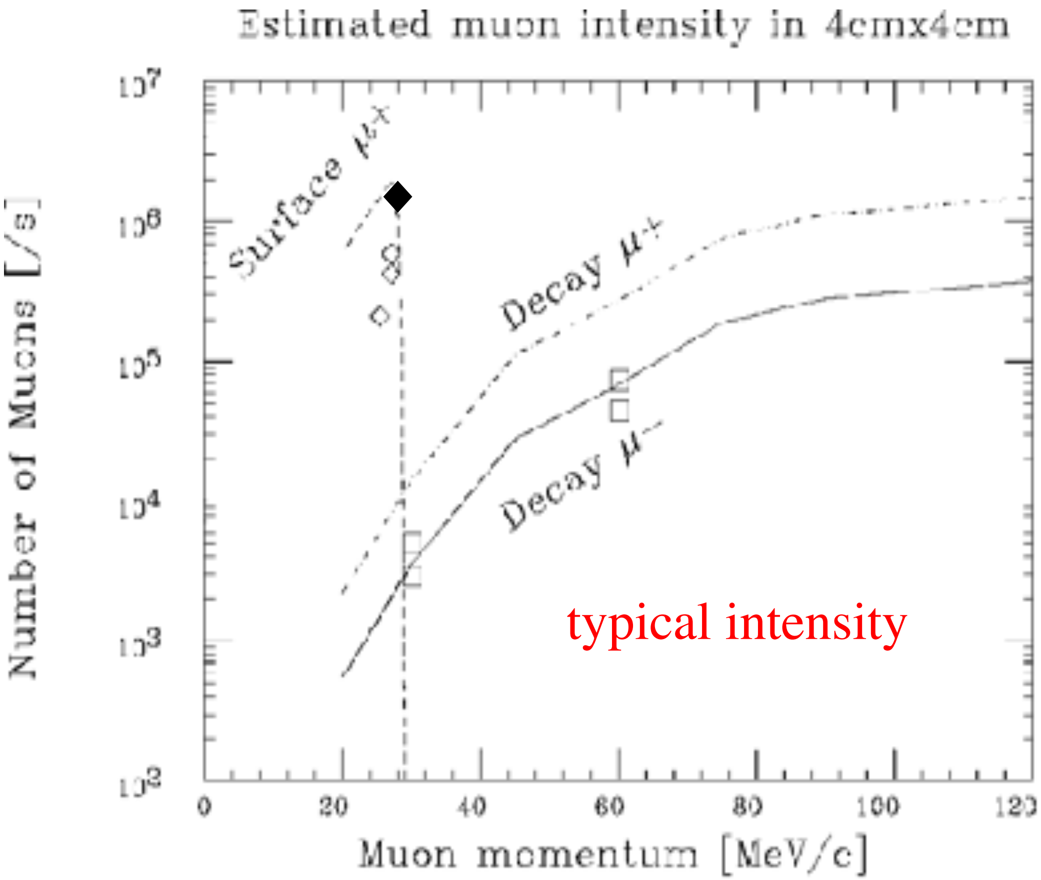
800 MeV proton with 50 Hz repetition (2 bunches)

2 uA (1.6 uA for Target station 1)



Didcot, Oxfordshire

pulse muon yield



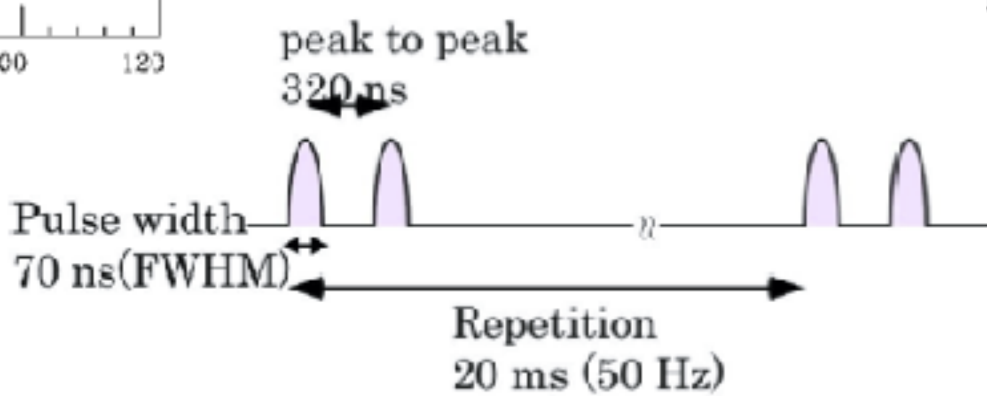
Negative muon yield

P_μ = 40 MeV/c

dp/p = ±4 %

2.4×10^4 [s^{-1}]

T. Matsuzaki et al., NIMA465



Resonance hunting

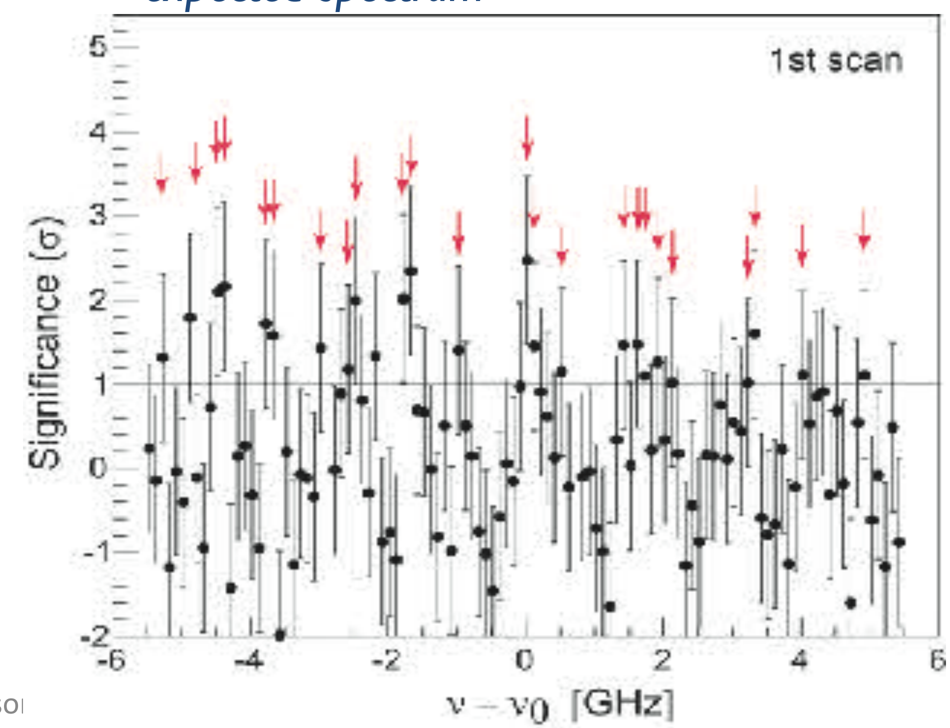
- *scanning region and steps*

  scan interval : 100 MHz *(Doppler ~ Bandwidth ~ 50 MHz)*

  scan region: ±5.7 GHz *(theoretical uncertainty)*

$$\boxed{Significance(\sigma) = \frac{signal}{fluctuation} = \frac{(N_F - N_B)}{\sqrt{(N_F + N_B)}}}$$

expected spectrum



N_F, N_B forward/backward electron counts

if off resonance, $N_F - N_B = 0$

$N_F - N_B = 0$ should be identified over the statistical fluctuation (~3 sigma eqiv .time)

pick up > 1 sigma

25 days

Resonance hunting

- *scanning region and steps*

  scan interval : 100 MHz *(Doppler ~ Bandwidth ~ 50 MHz)*

  scan region: ±5.7 GHz *(theoretical uncertainty)*

$$Significance(\sigma) = \frac{signal}{fluctuation} = \frac{(N_F - N_B)}{\sqrt{(N_F + N_B)}}$$

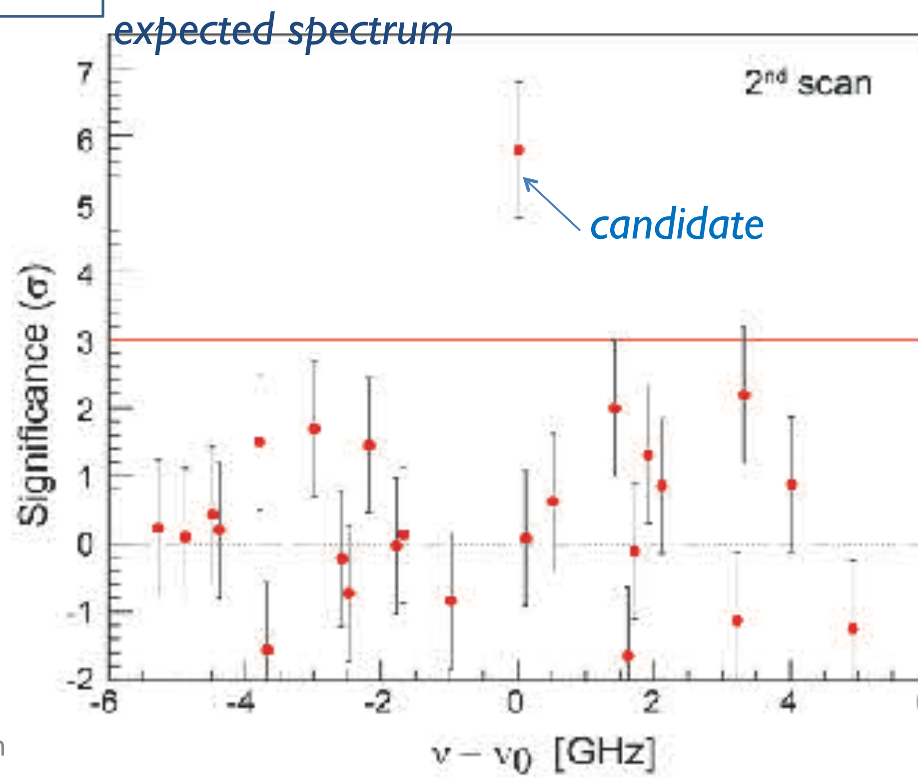
N_F, N_B *forward/backward electron counts*

if off resonance, $N_F - N_B = 0$

$N_F - N_B = 0$ *should be identified over the statistical fluctuation (~5 sigma eqiv. time)*

pickup > 3 sigma

11 days



Resonance hunting

- *scanning region and steps*

  scan interval : 100 MHz *(Doppler ~ Bandwidth ~ 50 MHz)*

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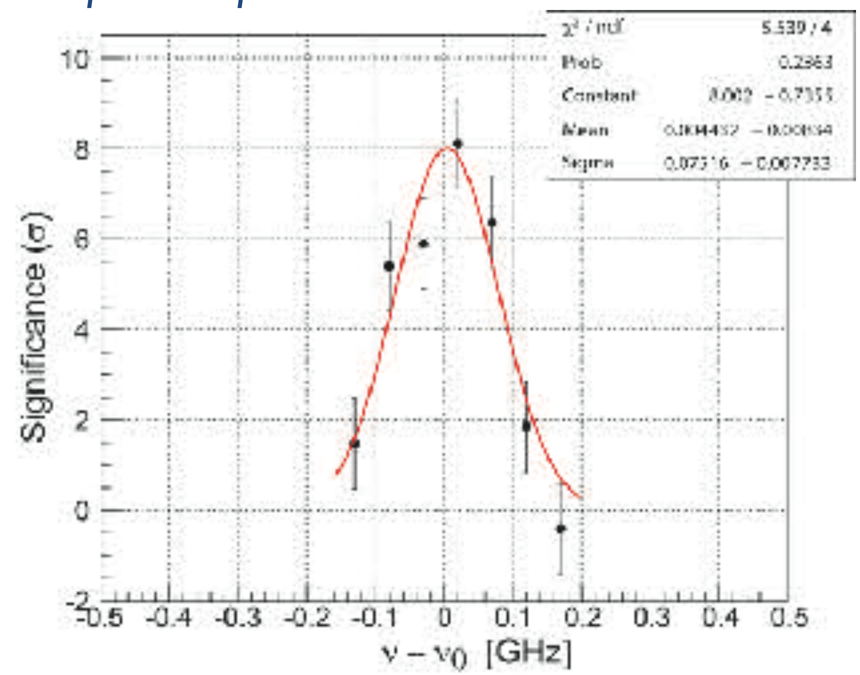
expected spectrum



N_F, N_B *forward/backward electron counts*

if off resonance, $N_F - N_B = 0$

$N_F - N_B = 0$ *should be identified over the statistical fluctuation (~7 sigma eqiv time)*

beam time estimation

resonance finding : *36 days*

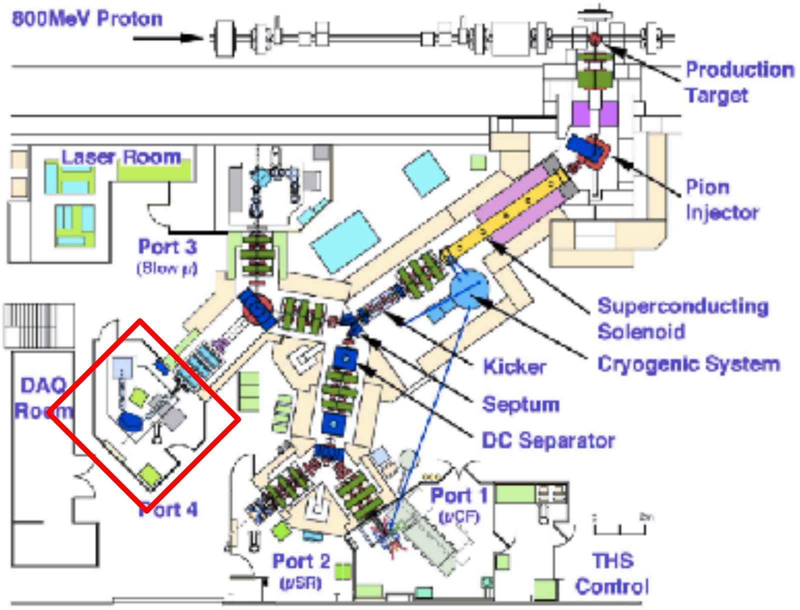
frequency determination. : *8 days*

statistical error : ~10 MHz

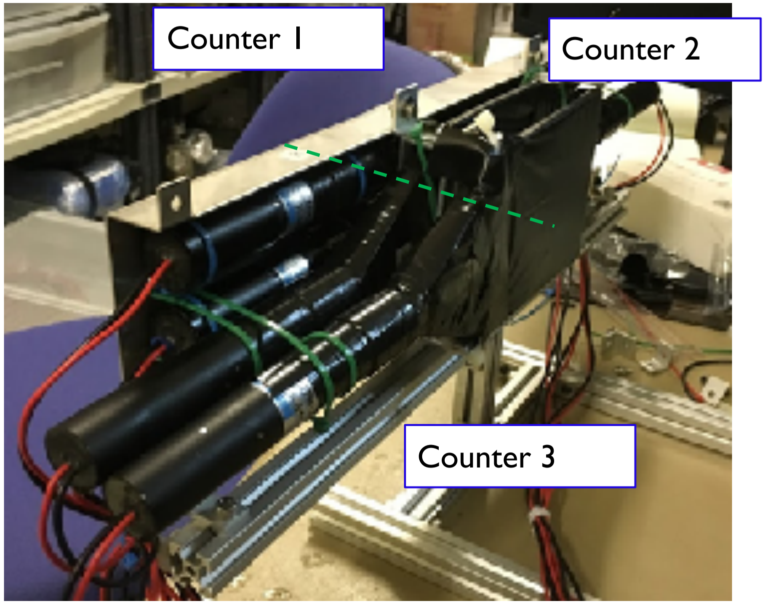
(~ 0.2 ppm)

beam test @ RIKEN-RAL

RIKEN-RAL Port-4

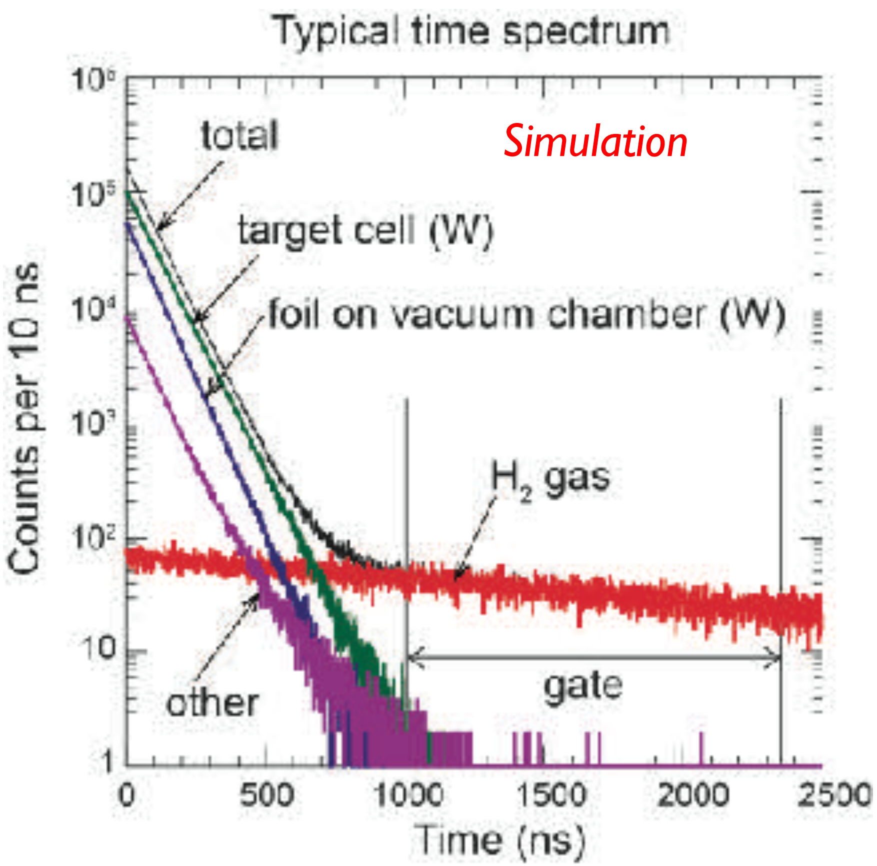


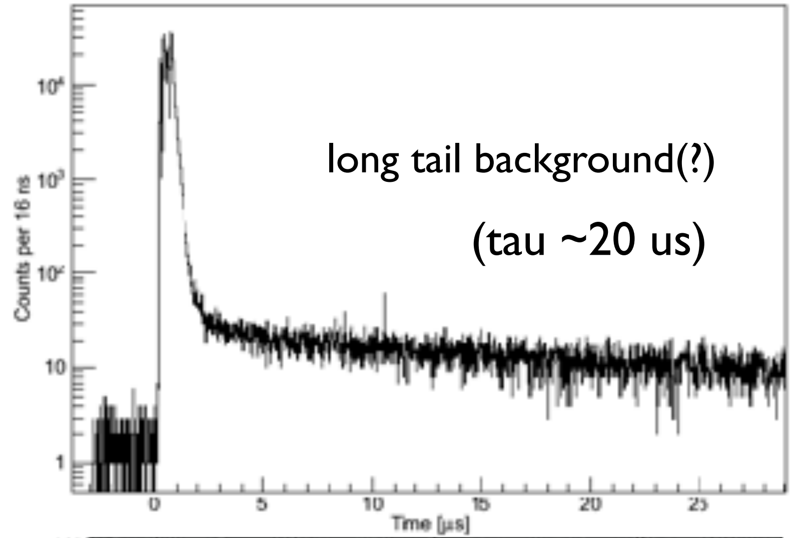
Coincidence counter

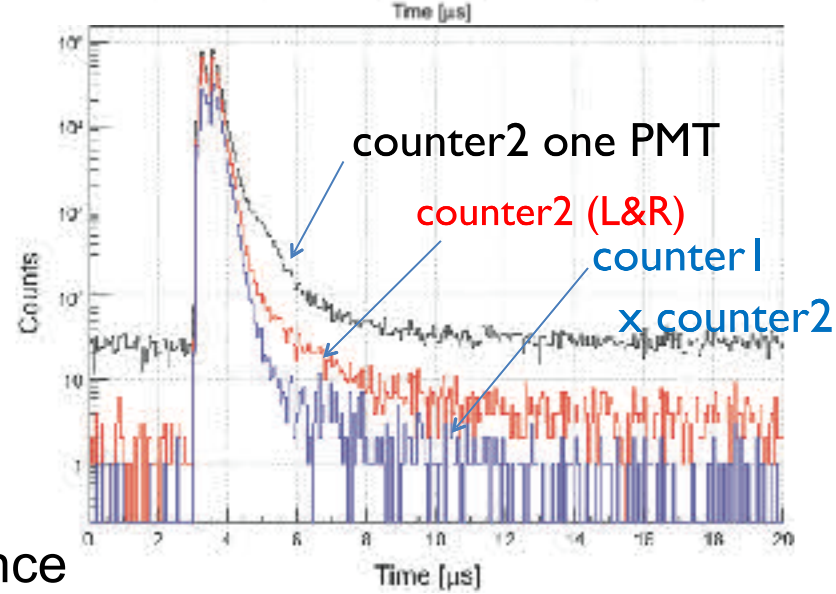


Background measurement

- ✓ check BG level after μ-stop







BG can be suppressed by coincidence

Summary & Outlook

- Measurement of ground state hyperfine splitting energy in muonic hydrogen with mid-infrared laser by means of spin-repolarization method.

- Accuracy of ΔE_{HFS} : ~ 2 ppm, derive proton Zemach radius < 1 % accuracy

  (need theoretical effort for further precision)

- Proposals submitted to pulse-muon facilities

  (RIKEN-RAL and J-PARC MUSE)

- Feasibility study with pulse muon beam is on-going in RAL.

Proton Zemach radius from Hydrogen HFS

theoretical calculation of correction value

	Value	Error	Ref.
ΔE_{exp}	1 420 405 751 767	0.000 000 001	[10]
E_F	1 418 840 08	0.000 02	[6]
$\Delta E_{exp}/E_F$	1.001 103 49	0.000 000 01	
δ^{Dirac}	0.000 079 88		[17]
δ^{QED}	0.001 056 21	0.000 000 001	[18–23]
δ^{FS}	−0.000 040 11	0.000 000 61	
δ^{recoil}	0.000 005 97	0.000 000 06	[25,31], this work
δ^{Pol}	0.000 001 4	0.000 000 6	[24]
$\delta^{\mu VP}$	0.000 000 07	0.000 000 02	[25]
δ^{hVP}	0.000 000 01		[26,27]
δ^{weak}	0.000 000 06		[28,29]

Volotka et al., EPJ 2005

- *Spectroscopy of hydrogen HFS*

$$\Delta E^{HFS}_{exp} = 1420405751.7667(9)\ \text{Hz}$$

theory of hydrogen HFS

$$\Delta E^{HFS} = E_F(1 + \delta^{QED} + \delta^{str})$$

$$\delta^{str} = \delta^{pol} + \delta^{\mu VP} + \delta^{hVP} + \delta^{weak} + \delta^{size} + \delta^{recoil}$$

$$\delta^{size} = 1.0154(2)\delta^{Zemach} + 1.4\times10^{-8}$$
$$= 1.0154(2) \times 2m_{ep}\alpha R_z + 1.4\times10^{-8}$$

input

$$R_Z = \frac{\frac{E_{exp}}{E_F} - 1 - \delta^{Dirac} - \delta^{QED} - \delta^{pol} - \delta^{\mu VP} - \delta^{hVP} - \delta^{weak} - \delta^{recoil} - 1.4\times10^{-8}}{1.0154\times 2m_{ep}\alpha}$$

R_z = 1.037(16) fm, *Dupays et al., PRA(2003)*
1.045(16) fm, *Volotka et al., EPJ(2005)*

Uncertainty (16) mainly comes from proton polarizability effect :

δ^{pol} = 1.4(6) ppm

EPJC 24(2002)24

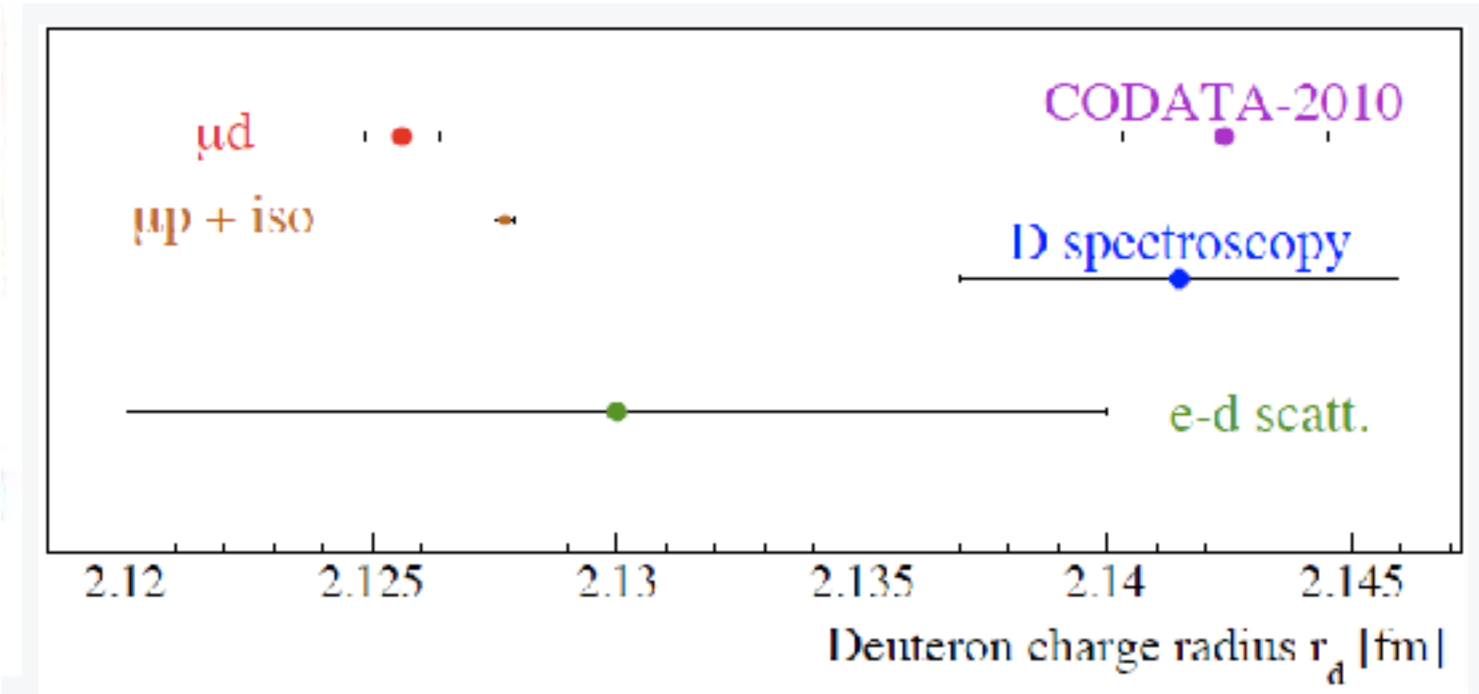
μ-D in PSI

A. Antognini PSPS 2016

arXiv 1607.03165

r_d(CODATA2010) = 2.1424(21) fm, H2 isotope

r_d(CODATA2010) = 2.121(25) fm, D2 spec. only



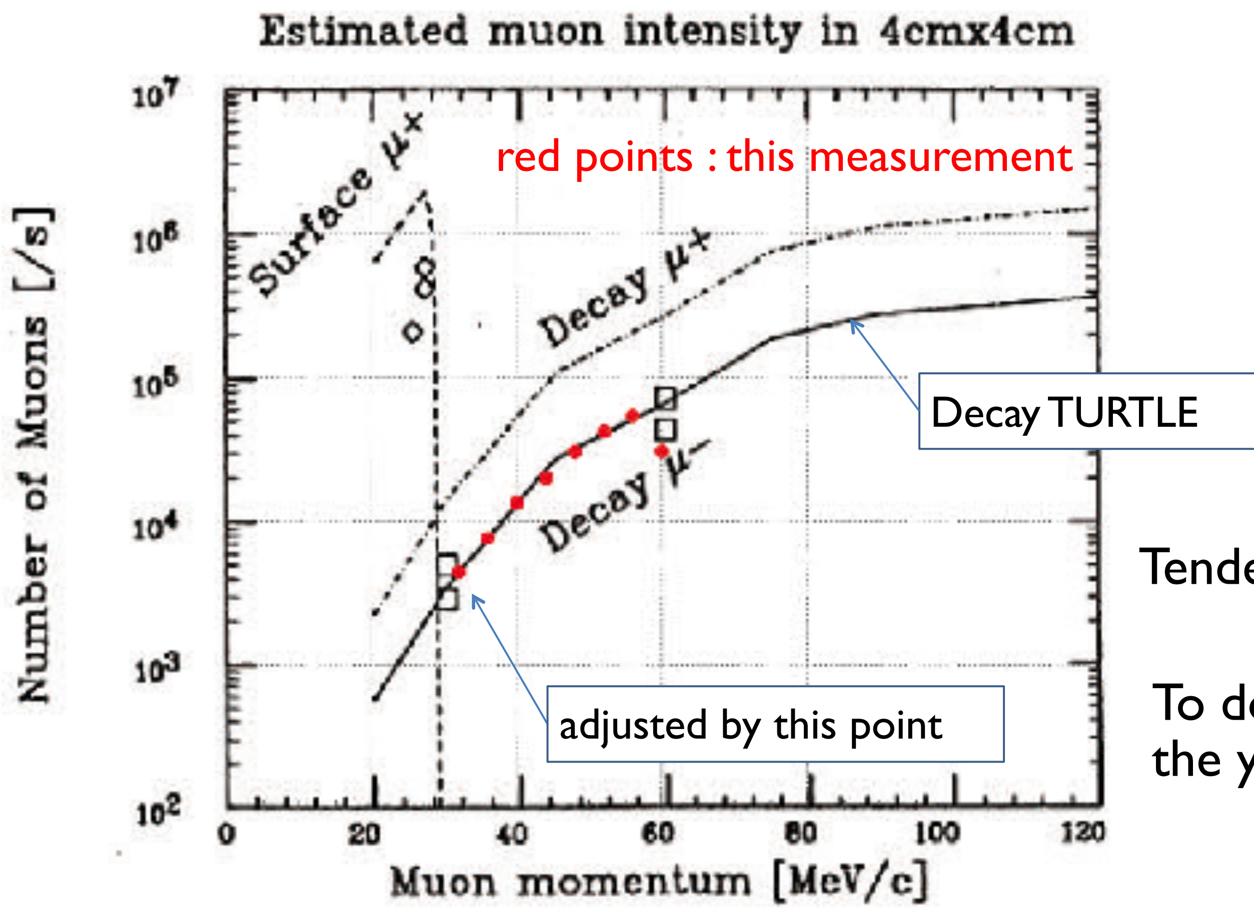
isotope shift of 1S->2S in H and D (PRL 104)

$$r_{\mathrm{d}}^2 - r_{\mathrm{p}}^2 = 3.82007(65)\,\mathrm{fm}^2.$$

Deuteron radius puzzle?

Comparison with past measurement

T. Matsuzaki et al., NIM 465(2001) , 200 uA, prod.T thickness 10 mm

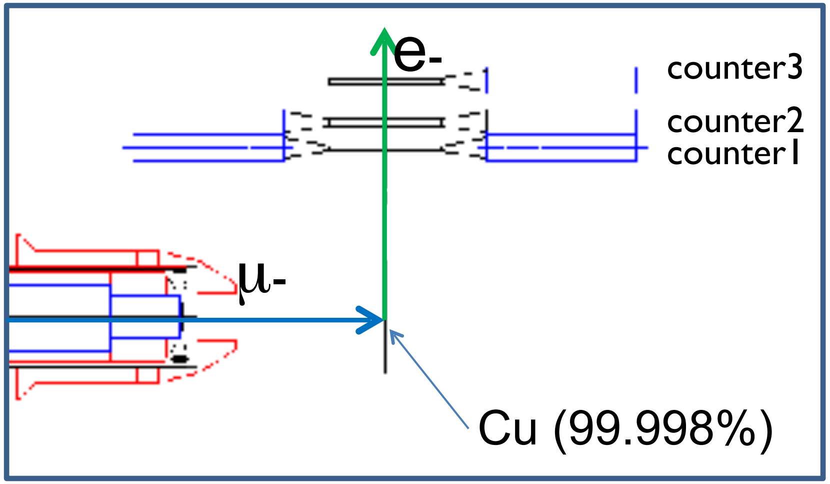


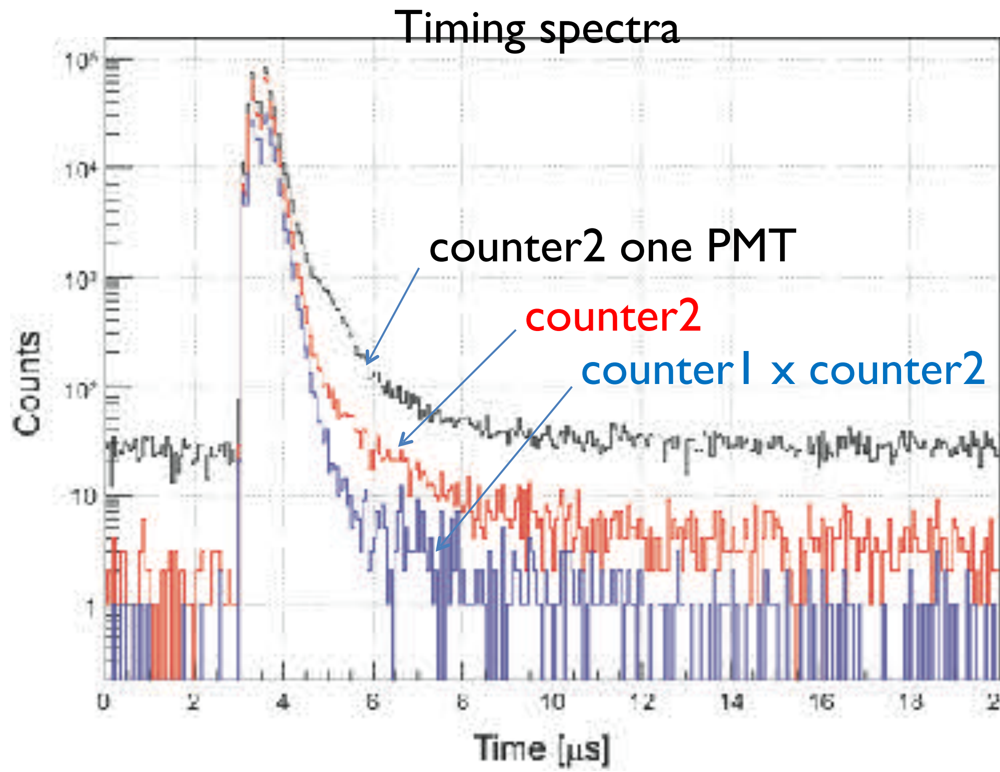
Tendency is consistent

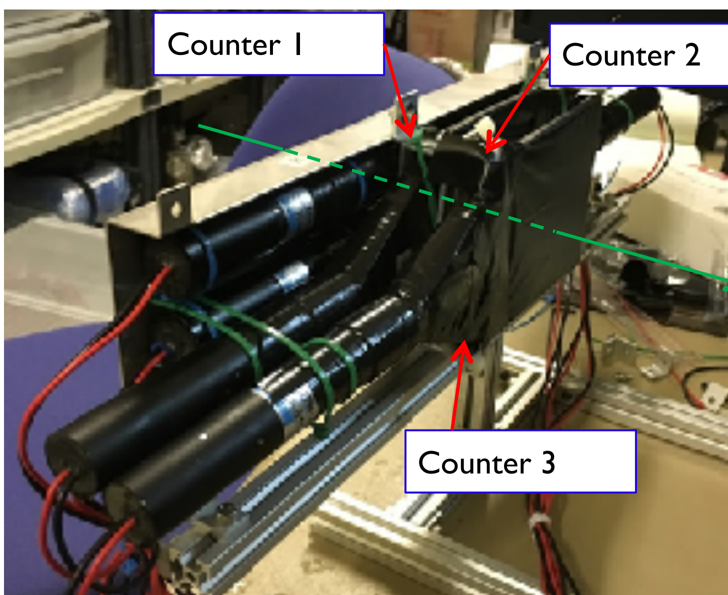
To do: absolute value of the yield

Negative decay muon beam study at PORT-4

Beam time in May (5/11-13) : CHRONUS → coincidence counters







BG can be suppressed with coincidence : ~ 10^{-4}

- ✓ design the prototype of counters
- ✓ μ-stop beam test with dilute gas

➡ beam time in this autumn

Detector : CHRONUS @ PORT-4

CHRONUS @ PORT-4 (already equipped for μSR)

D. Tomono et. al, NIMA600 (2009)44

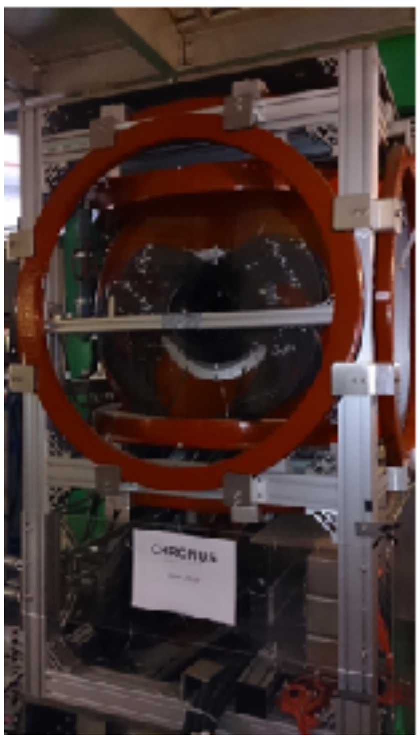
- 303 counter x 2 arms

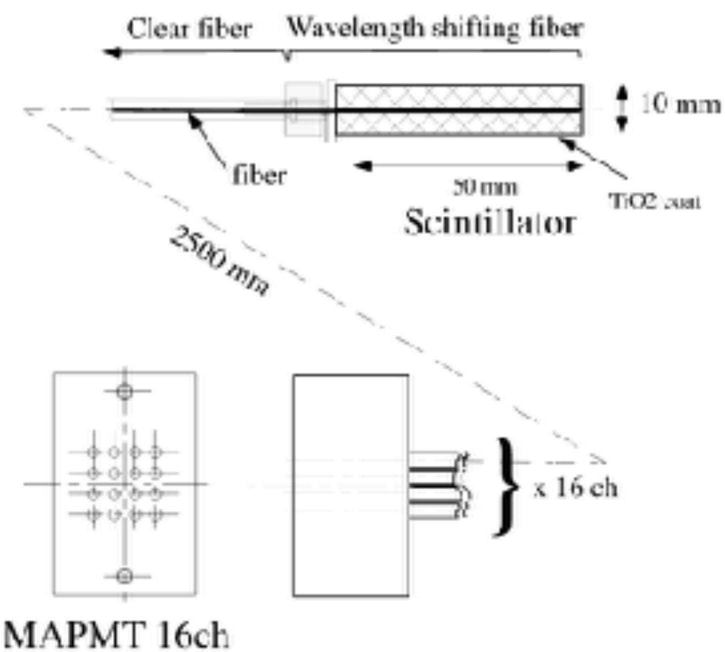
➢ plastic scintillator : 10x10x 50 (or 40) mm^3 coated with TiO_2

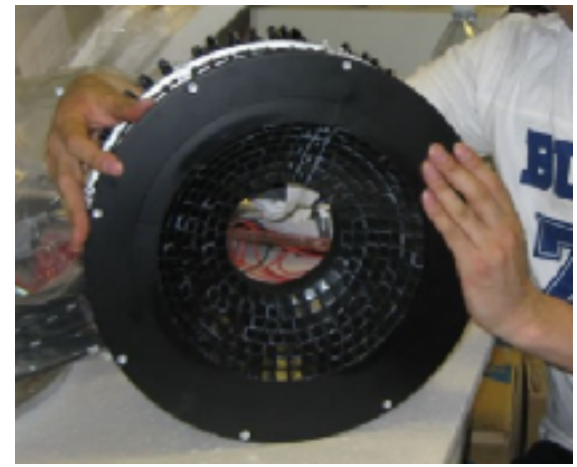
WSF : 1.5 m (Kuraray Y-11(400)MS)

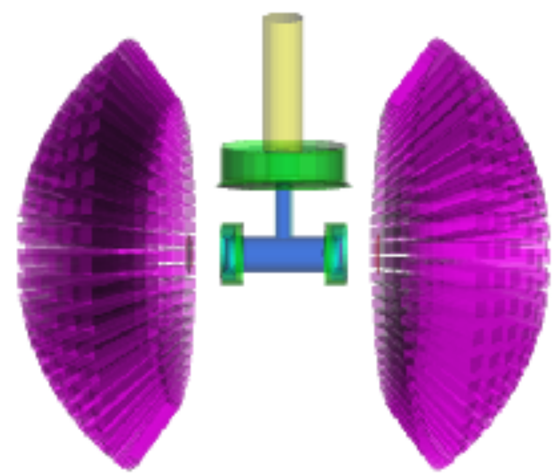
➢ light guide : fiber 2.5 m (Kuraray)

➢ PMT: 16-ch. multi-anode PMT (H6568-10-200)







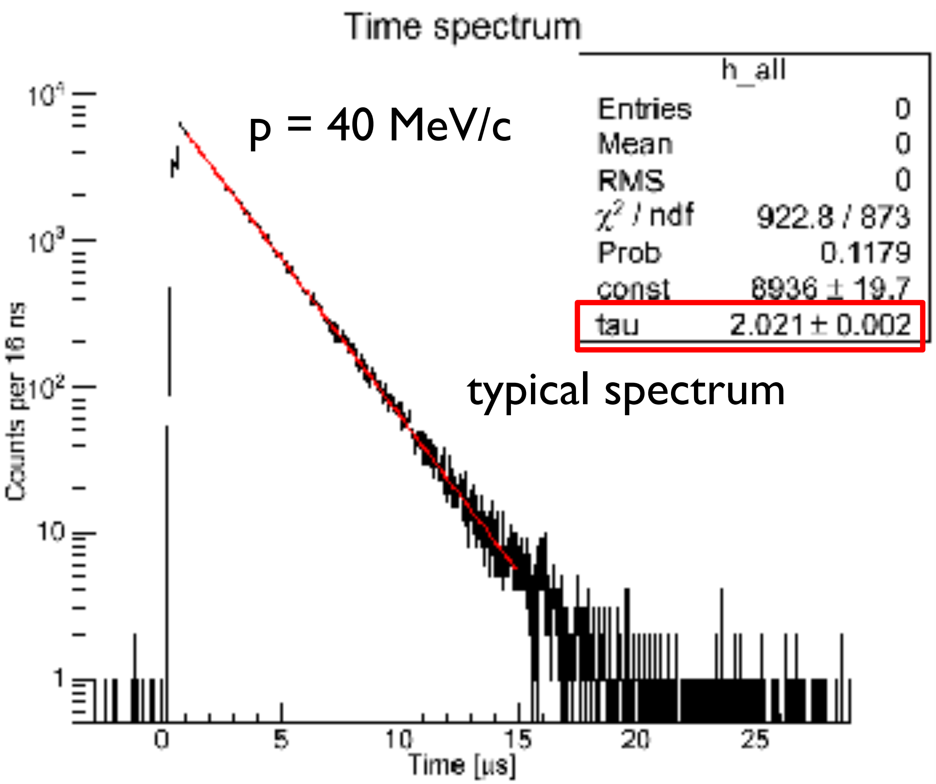


Results

muon momentum : 32/36/40/44/48/52/56/60 MeV/c

effective decay time in C : 2026.3 ±1.5 ns

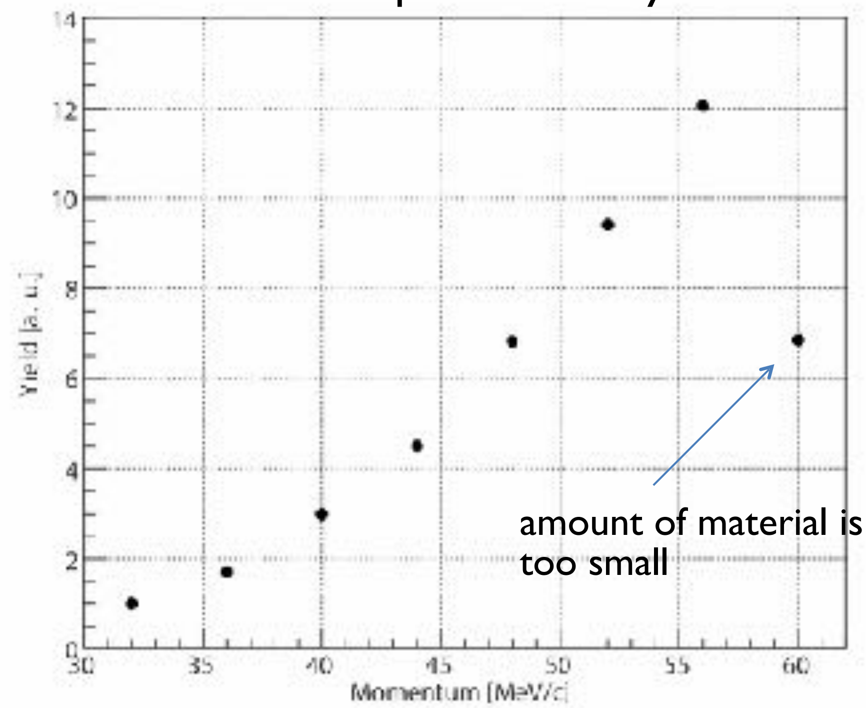
T. Suzuki et al., PRC35(1987)



p = 40 MeV/c

typical spectrum

Momentum dependence of yield



amount of material is too small

Proton Zemach radius from *e-p* scattering

- *electron – proton scattering*

$$\left(\frac{d\sigma}{d\Omega}\right) = \underbrace{\left(\frac{d\sigma}{d\Omega}\right)_{Mott}}_{\text{point like}} \underbrace{\frac{\epsilon G_E^2 + \tau G_M^2}{\epsilon(1+\tau)}}_{\text{proton size}}$$

G_E, G_M : form factor

$$Q^2 = 4EE'\sin^2\frac{\theta}{2}, \tau = \frac{Q^2}{4m_p^2c^2}, \ \epsilon = \left[1 + 2(1+\tau)\tan^2\left(\frac{\theta}{2}\right)\right]^{-1}$$

$$R_Z = \frac{2\alpha m_{\mu p}}{\pi^2}\int \frac{d^3p}{Q^4}\left(1/\mu_p G_E(Q^2)G_M(Q^2) - 1\right)$$

- Friar & Sick, PLB(2004)
  - *G_E and G_M by fitting with (old) data*

$$\boxed{R_z = 1.086(12) \text{ fm}}$$

error : normalization of e-p cross section & statistics of data

- Distler, PLB(2011)
  - *new high precision data from Mainz (Bernauer et al., PRL 2010)*
  - *direct fit of G_E and G_M (not a classical Rosenbluth separation)*

$$\boxed{R_z = 1.045(4) \ fm}$$

Proton Zemach radius from Hydrogen HFS

- *Spectroscopy of hydrogen HFS*

ΔE^{HFS}_{exp} = 1420405751.7667(9) Hz

theory of hydrogen HFS

$$\Delta E^{HFS} = E_F(1 + \delta^{QED} + \delta^{str})$$

$$\delta^{str} = \delta^{pol} + \delta^{\mu VP} + \delta^{hVP} + \delta^{weak} + \delta^{size} + \delta^{recoil}$$

$$\delta^{size} = 1.0154(2)\delta^{Zemach} + 1.4\times 10^{-8}$$
$$= 1.0154(2) \times 2m_{ep}\alpha R_z + 1.4 \times 10^{-8}$$

theoretical calculation of correction value

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δ^{weak}	0.000 000 06		[28,29]

Volotka et al., EPJ 2005

input

$$R_Z = \frac{\frac{E_{exp}}{E_F} - 1 - \delta^{Dirac} - \delta^{QED} - \delta^{pol} - \delta^{\mu VP} - \delta^{hVP} - \delta^{weak} - \delta^{recoil} - 1.4\times 10^{-8}}{1.0154\times 2m_{ep}\alpha}$$

R_z = 1.037(16) fm, *Dupays et al., PRA(2003)*
1.045(16) fm, *Volotka et al., EPJ(2005)*

Uncertainty (16) mainly comes from proton polarizability effect :

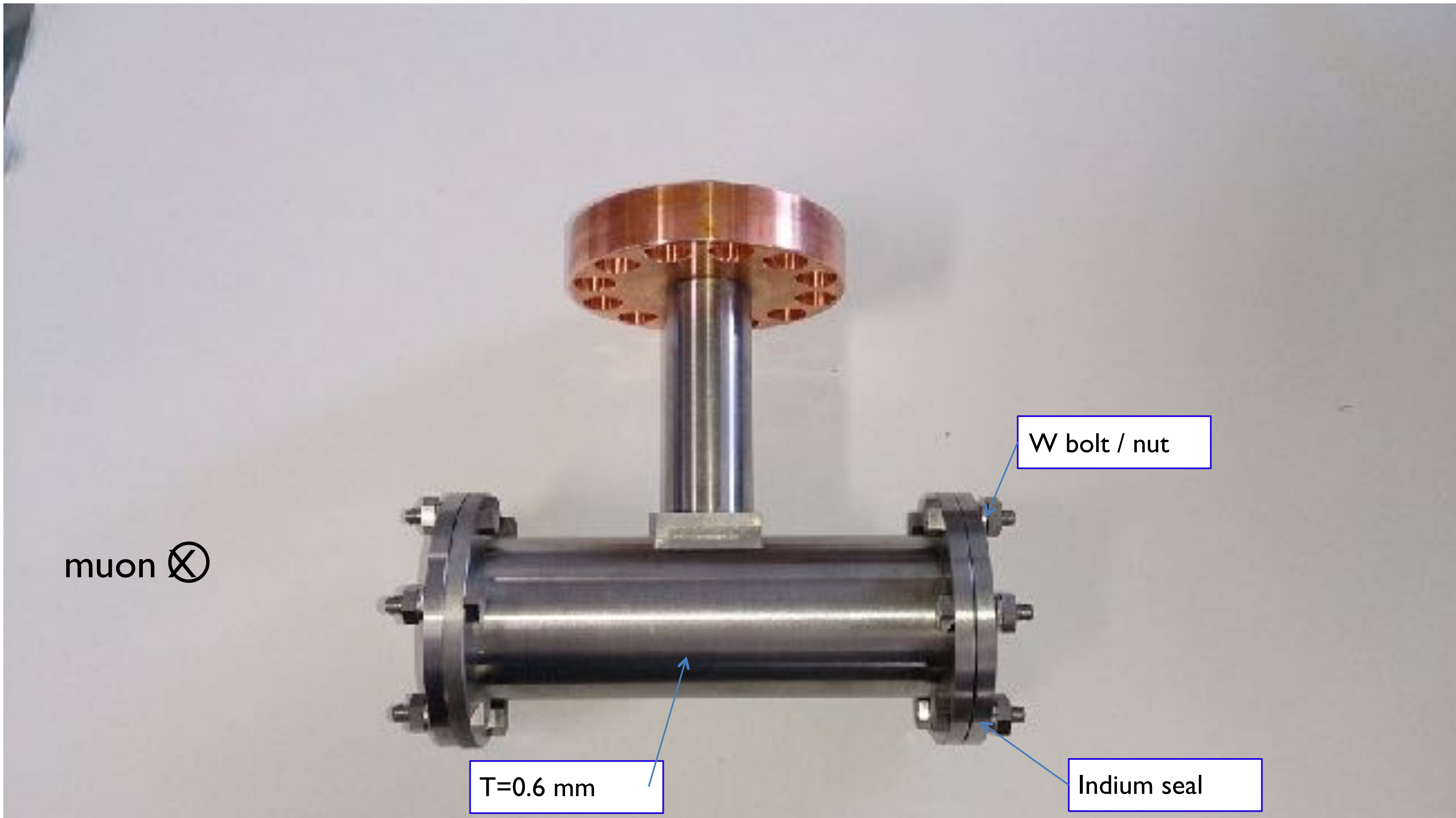
δ^{pol} = 1.4(6) ppm

EPJC 24(2002)24

Tungsten H_2 gas cell

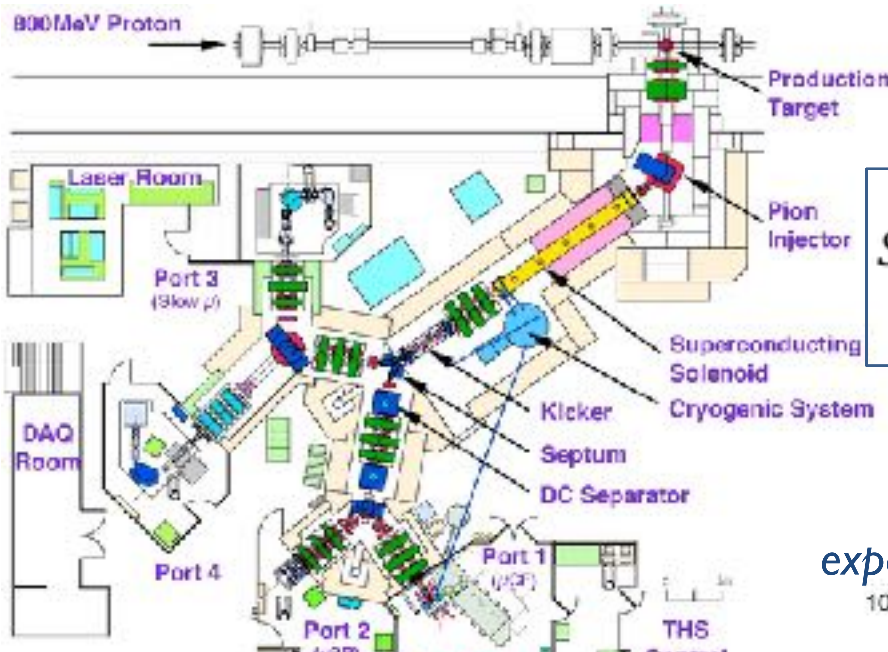
glued with STYCAST (epoxy)

Indium : tau = ~85 ns



Time for resonance hunting

In the case of RIKEN-RAL muon facility



Parameter for estimation

- negative muon
  - 2.4×10^{4} [s^{-1}]
    - (50 Hz repetition)
  - P_{μ} = 40 MeV/c
  - dp/p = ±4 %

- *scanning region and steps*

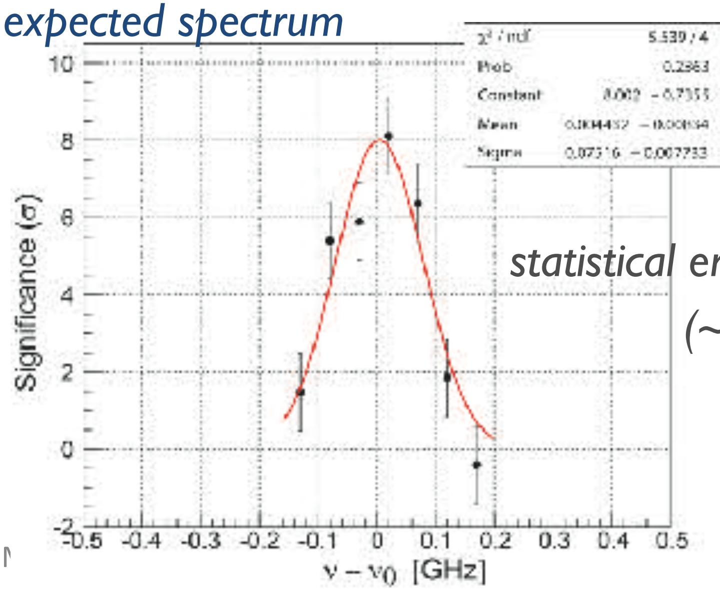
  scan interval : 100 MHz

  scan region: ±5.7 GHz (~ $\delta^{Zemach} + \delta^{pol}$)

$$Significance(\sigma) = \frac{signal}{fluctuation} = \frac{(N_F - N_B)}{\sqrt{(N_F + N_B)}}$$

beam time for resonance finding : 3 months

expected spectrum



statistical error : ~10 MHz
(~ 0.2 ppm)

Expected precision of Zemach radius

$$R_Z = \{(E_F\,(1 + \delta_{QED} + \delta_{recoil} + \delta_{pol} + \delta_{hvp}) - \Delta E^{exp}_{HFS}\}/1.281$$

1130(1) ppm　1700(1) ppm　460(80) ppm　20(2) ppm　(2) ppm

Dupays et al., PRA 2003

$\boxed{R_Z = 1.0XX(13)\ \mathrm{fm}}$

δ_{pol} is dominated in precision, but improved factor ~3 from PSI results,

	Hydrogen		Muonic hydrogen	
	Magnitude	Uncertainty	Magnitude	Uncertainty
E^F	1418.84 MHz	0.01 ppm	182.443 meV	0.1 ppm
δ^{QED}	1.13×10^{-3}	$<0.001\times10^{-6}$	1.13×10^{-3}	10^{-6}
δ^{rigid}	39×10^{-6}	2×10^{-6}	7.5×10^{-3}	0.1×10^{-3}
δ^{recoil}	6×10^{-6}	10^{-8}	1.7×10^{-3}	10^{-6}
δ^{pol}	1.4×10^{-6}	0.6×10^{-6}	0.46×10^{-3}	0.08×10^{-3}
δ^{hvp}	10^{-8}	10^{-9}	0.02×10^{-3}	0.002×10^{-3}

check with R_Z determined by "electronic" and "muonic" measurement

e-p	*1.4(6) ppm*
μ-p	*460(80) ppm*

improvement of proton polarizability correction (δ_{pol}) drastically reduces uncertainty of Rz

Doppler width

✓ Estimation of Doppler width broadening

Maxwell distribution (1 direction)

$$f_1(v) = \sqrt{\frac{mc^2}{2\pi kT}} exp\left(-\frac{mv^2}{2kT}\right)$$

$$\sigma_f = \sqrt{\frac{kT}{mc^2}} = 1.28 \times 10^{-6}$$

kT = 8.617 x 10^{-5} eV/K x 20K = 1.72 x 10^{-3} eV
mc^2 = (M_p + M_μ) MeV = 1.044x 10^9 eV

Hyperfine splitting energy:
E = 0.183 eV = 44.2THz

Doppler width
σ(f) = 44.2 THz x 1.28 x 10^{-6} = 56.5 MHz

Our case :	
line width of 6.7 um laser	50 MHz (σ)
Doppler broadening(20 K)	56.5MHz (σ)
total	75.7 MHz (σ)

Proton Zemach radius (μ-p 2S HFS)

- *PSI : 2-transition of 2S-2P of μ-p*

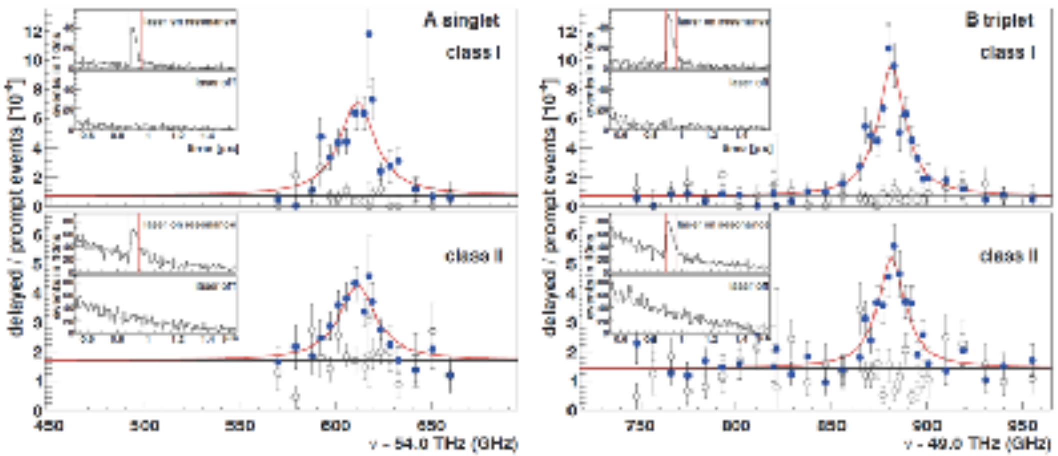
A. Antognini, Science 339(2013)417

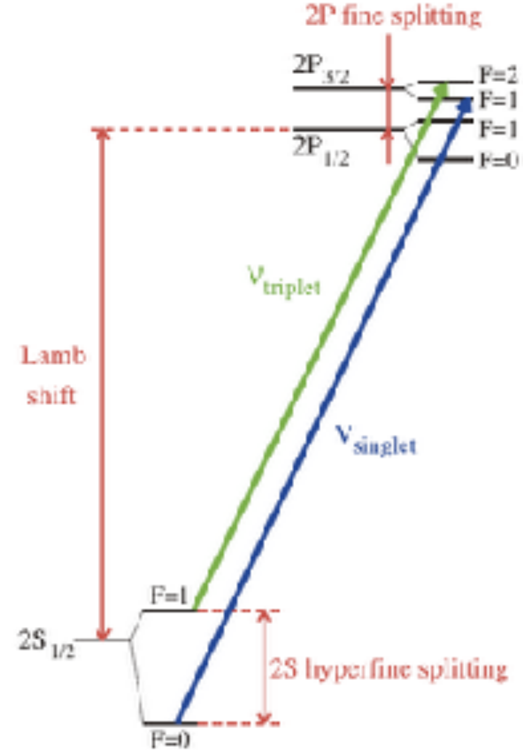
$f_1 : 2S_{1/2}(F=1) \rightarrow 2P_{3/2}\ (F=2)$

$f_2 : 2S_{1/2}(F=0) \rightarrow 2P_{3/2}\ (F=1)$

⇨ $f_2 - f_1 : \Delta E^{HFS}(2S)$

singlet *triplet*





$$R_z = 1.082(31)^{exp}(20)^{th}\ \text{fm}$$
$$= 1.082(37)\ \text{fm}$$

uncertainty :

✓ *experimental(statistical) error due to large 2P width*

Proton Zemach radius from *e-p* scattering

- *electron – proton scattering*

$$\left(\frac{d\sigma}{d\Omega}\right) = \underbrace{\left(\frac{d\sigma}{d\Omega}\right)_{Mott}}_{\text{point like}} \underbrace{\frac{\epsilon G_E^2 + \tau G_M^2}{\epsilon(1+\tau)}}_{\text{proton size}}$$

G_E, G_M : form factor

$$Q^2 = 4EE'\sin^2\frac{\theta}{2}, \tau = \frac{Q^2}{4m_p^2c^2}, \ \epsilon = \left[1 + 2(1+\tau)\tan^2\left(\frac{\theta}{2}\right)\right]^{-1}$$

$$R_Z = \frac{2\alpha m_{\mu p}}{\pi^2}\int \frac{d^3p}{Q^4}\left(1/\mu_p G_E(Q^2)G_M(Q^2) - 1\right)$$

- Friar & Sick, PLB(2004)
  - *G_E and G_M by fitting with (old) data*

$\boxed{R_z = 1.086(12) \text{ fm}}$

error : normalization of e-p cross section & statistics of data

- Distler, PLB(2011)
  - *new high precision data from Mainz (Bernauer et al., PRL 2010)*
  - *direct fit of G_E and G_M (not a classical Rosenbluth separation)*

$\boxed{R_z = 1.045(4) \ fm}$

Zemach radius from Hydrogen & muonium HFS

E_{HFS}(e- p) = 1420.405751 7667(9)MHz

E_{HFS}(e- μ+) = 4463.302765(53)MHz

μ_μ / μ_p = 3.183345118(89)

$$E_{\rm HFS}(e^-p) = (1 + \Delta_{\rm QED} + \Delta_R^p + \Delta_S)E_F^p,$$

$$E_{\rm HFS}(e^-\mu^+) = (1 + \Delta_{\rm QED} + \Delta_R^\mu)E_F^\mu.$$

Δ_R : recoil

difference due to the proton structure

(after correcting magnetic moment and reduced mass effect)

Brodsky, PRL94 (2005)022001 (&erratum 169902)

$$\Delta_{\rm HFS} = \frac{E_{\rm HFS}(e^-p)}{E_{\rm HFS}(e^-\mu^+)}\frac{\mu_\mu}{\mu_p}\frac{(1+m_e/m_p)^3}{(1+m_e/m_\mu)^3} - 1$$
$$= \frac{E_{\rm HFS}(e^-p)/E_F^p}{E_{\rm HFS}(e^-\mu^+)/E_F^\mu} - 1.$$

Δ_{HFS} = 145.54(4) ppm

$$\frac{E_{\rm HFS}(e^-p)/E_F^p}{E_{\rm HFS}(e^-\mu^+)/E_F^\mu} = \frac{(1 + \Delta_{\rm QED} + \Delta_R^p + \Delta_S)}{(1 + \Delta_{\rm QED} + \Delta_R^\mu)}.$$

$$\delta_Z^{\rm rad} = (\alpha/3\pi)[2\ln(\Lambda^2/m_e^2) - 4111/420].$$

$$\Delta_S = \Delta_{\rm HFS} + \Delta_R^\mu - \Delta_R^p + \Delta_{\rm HFS}(\Delta_{\rm QED} + \Delta_R^\mu).$$

0.71 GeV^2, this yields $\delta_Z^{\rm rad} = 0.0153$.

Δ_S = Δ_Z +Δ_{pol} = -37.66(16) ppm Δ_{pol} = -1.4(6) ppm

Δ_Z = -39.1(6) ppm

depends shape of form factor?

Δ_R^p = 6.01(15) ppm

$$\boxed{R_p = 1.019(16)\ \rm fm}$$

De Rujula's idea

- *Advocated as possible solution of Proton Radius Puzzle*

  "QED is not endangered by the proton's size", PLB693, 555 (2010)

form factor with "dipole" and "single pole"

large third Zemach moment

$$\rho_{(2)}(r) = \int d^3 r_2\, \rho_{\rm charge}(|\vec{r} - \vec{r}_2|)\, \rho_{\rm charge}(r_2)$$

$$\langle r^3 \rangle_{(2)} = \int d^3 r\, r^3 \rho_{(2)}(r) = 36.2\,\mathrm{fm}^3$$

$$\left(209.9779(49) - 5.2262\,\frac{\langle r^2 \rangle}{\mathrm{fm}^2} + 0.00913\,\frac{\langle r^3 \rangle_{(2)}}{\mathrm{fm}^3}\right)\,\mathrm{meV}$$

and get $r_p = 0.878\,\mathrm{fm}$

PSI: $\langle r^3 \rangle = f \langle r^2 \rangle^{3/2}$
w/ f=3.79

Such a large third Zemach moment is impossible.

$\langle r_p^3 \rangle_{(2)}$ (De Rujula) $=$ 36.6 ± 6.9 fm^3

$\langle r_p^3 \rangle_{(2)}$ (Sick) $=$ 2.71 ± 0.13 fm^3

$\langle r_p^3 \rangle_{(2)}$ (Mainz 2010) $=$ 2.85 ± 0.08 fm^3

e-p scattering data are inconsistent with De Rujula's hypothesis

Comparison between RIKEN-RAL & J-PARC

- *RIKEN RAL*
- *J-PARC MUSE*

	RIKEN-RAL	J-PARC
Beam power [kW]	160	300
Repetition [Hz]	50	25
Proton energy [GeV]	0.8	3
Prod. target thickness [mm]	?	?x2
Momentum bite [%]	4	10?
Double pulse interval [ns]	320	600

Proton Zemach radius (μ-p 2S HFS)

- *PSI : 2-transition of 2S-2P of μ-p*

A. Antognini, Science 339(2013)417

$\nu_1 : 2S_{1/2}(F=1) \rightarrow 2P_{3/2}\ (F=2)$

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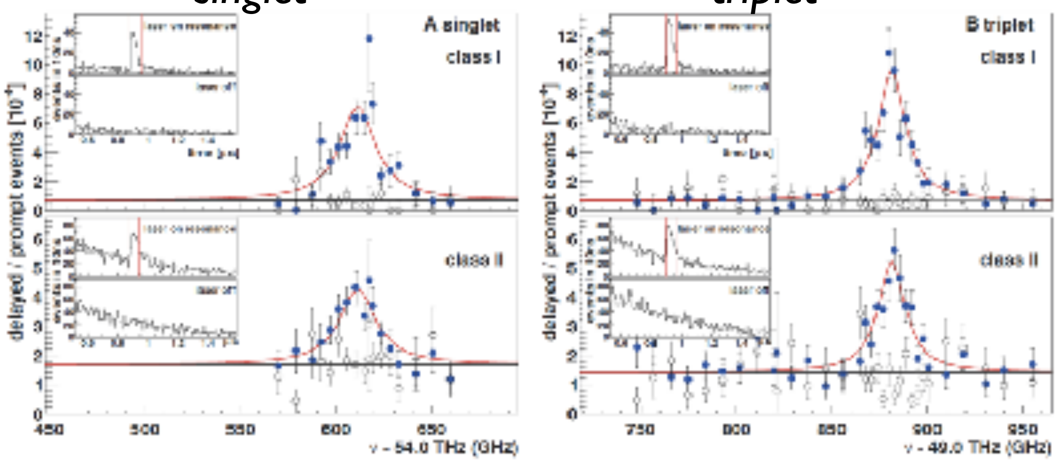

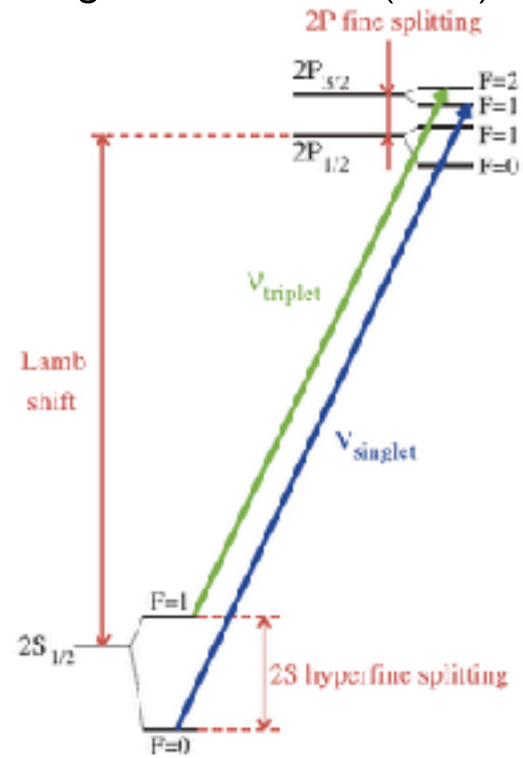

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$$= 1.082(37)\ \text{fm}$$

uncertainty :

✓ *experimental(statistical) error due to large 2P width*

Proton Zemach radius from Hydrogen HFS

- *Spectroscopy of hydrogen HFS*

$$\Delta E^{HFS}_{exp} = 1420405751.7667(9)\ \text{Hz}$$

theory of hydrogen HFS

$$\Delta E^{HFS} = E_F(1 + \delta^{QED} + \delta^{str})$$

$$\delta^{str} = \delta^{pol} + \delta^{\mu VP} + \delta^{hVP} + \delta^{weak} + \delta^{size} + \delta^{recoil}$$

theoretical calculation of correction value

	Value	Error	Ref.
ΔE_{exp}	1 420 405 751 767	0.000 000 001	[10]
E_F	1 418 840 08	0.000 02	[6]
$\Delta E_{exp}/E_F$	1.001 103 49	0.000 000 01	
δ^{Dirac}	0.000 079 88		[17]
δ^{QED}	0.001 056 21	0.000 000 001	[18–23]
δ^{FS}	−0.000 040 11	0.000 000 61	
δ^{recoil}	0.000 005 97	0.000 000 06	[25,31], this work
δ^{pol}	0.000 001 4	0.000 000 6	[24]
$\delta^{\mu VP}$	0.000 000 07	0.000 000 02	[25]
δ^{hVP}	0.000 000 01		[26,27]
δ^{weak}	0.000 000 06		[28,29]

Volotka et al., EPJ 2005

input

$$\delta^{size} = 1.0154(2)\delta^{Zemach} + 1.4\times10^{-8}$$
$$= 1.0154(2)\times 2m_{ep}\alpha R_z + 1.4\times10^{-8}$$

$$R_Z = \frac{\frac{E_{exp}}{E_F} - 1 - \delta^{Dirac} - \delta^{QED} - \delta^{pol} - \delta^{\mu VP} - \delta^{hVP} - \delta^{weak} - \delta^{recoil} - 1.4\times10^{-8}}{1.0154\times 2m_{ep}\alpha}$$

R_z = 1.037(16) fm, *Dupays et al., PRA(2003)*

1.045(16) fm, *Volotka et al., EPJ(2005)*

Uncertainty (16) mainly comes from proton polarizability effect :

δ^{pol} = 1.4(6) ppm

EPJC 24(2002)24

Hydrogen spectrosocpy

#	$(n,\ell,j)-(n',\ell',j')$	ν_{meas} (kHz)	rel. unc.	Source	Ref.
H1	$2S_{1/2} \rightarrow 2P_{1/2}$	-1 057 862(20)	1.9×10^{-5}	Sussex 1979	[25] *
H2		-1 057 845.0(9.0)	8.5×10^{-6}	Harvard 1986	[26] *
H3	$2S_{1/2} \rightarrow 2P_{3/2}$	9 911 200(12)	1.2×10^{-6}	Harvard 1994	[27] *
H4	$2S_{1/2} \rightarrow 8S_{1/2}$	770 649 350 012.0(8.6)	1.1×10^{-11}	LKB 1997	[28] *
H5	$2S_{1/2} \rightarrow 8D_{3/2}$	770 649 504 450.0(8.3)	1.1×10^{-11}	LKB 1997	[28] *
H6	$2S_{1/2} \rightarrow 8D_{5/2}$	770 649 561 584.2(6.4)	8.3×10^{-12}	LKB 1997	[28] *
H7	$2S_{1/2} \rightarrow 12D_{3/2}$	799 191 710 472.7(9.4)	1.1×10^{-11}	LKB 1999	[29] *
H8	$2S_{1/2} \rightarrow 12D_{5/2}$	799 191 727 403.7(7.0)	8.7×10^{-12}	LKB 1999	[29] *
H9	$1S_{1/2} \rightarrow 2S_{1/2}$	2 466 061 413 187.103(46)	1.9×10^{-14}	MPQ 2000	[30]
H10		2 466 061 413 187.080(34)	1.4×10^{-14}	MPQ 2004	[31] *
H11		2 466 061 413 187.035(10)	4.2×10^{-15}	MPQ 2011	[32]
H12		2 466 061 413 187.018(11)	4.5×10^{-15}	MPQ 2013	[33]
H13	$1S_{1/2} \rightarrow 3S_{1/2}$	2 922 743 278 678(13)	4.4×10^{-12}	LKB 2010	[34] *
H14		2 922 743 278 659(17)	5.8×10^{-12}	MPQ 2016	[35]

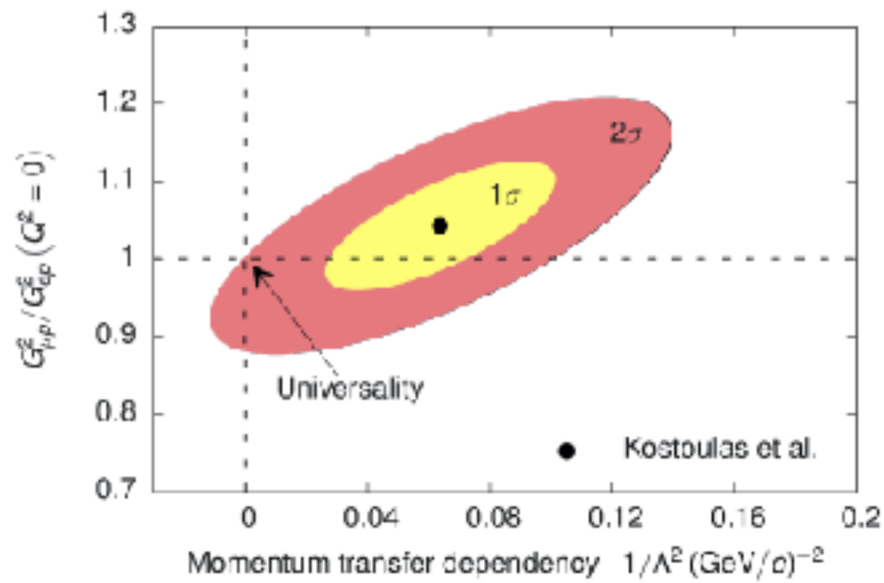
Physics beyond the standard model

Miha Mihovilovic talk in MESON 2014

- Puzzle could be explained by **breaking the e-μ universality**.

- New interaction could also explain the $(g\text{-}2)_\mu$ puzzle.

- The universality tested, but constrains loose enough for such explanations.



- Various new interactions proposed. Constraints on new forces limit the possibilities.

- Most interesting candidate: A new U(1) gauge boson mediating the interaction between dark matter and the Standard model particles.

De Rujula's idea on the puzzle

- *Advocated as possible solution of Proton Radius Puzzle*

  "QED is not endangered by the proton's size", PLB693, 555 (2010)

form factor with "dipole" and "single pole"

large third Zemach moment

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and get $r_p = 0.878 \, \mathrm{fm}$

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e-p scattering data are inconsistent with De Rujula's hypothesis

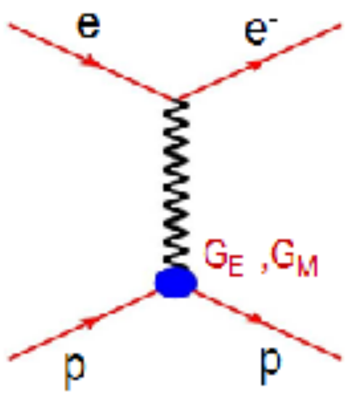
Proton charge radius from *e-p* scattering

In the limit of first Borm approximation the elastic e-p scattering (one photon exchange)

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \frac{\epsilon G_E^2 + \tau G_M^2}{\epsilon(1+\tau)}$$

G_E, G_M : form factor

$$Q^2 = 4EE'\sin^2\frac{\theta}{2}, \tau = \frac{Q^2}{4m_p^2c^2}, \ \epsilon = \left[1 + 2(1+\tau)\tan^2\left(\frac{\theta}{2}\right)\right]^{-1}$$



structure less proton

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \frac{\alpha^2\left[1 - \beta^2\sin^2\frac{\theta}{2}\right]}{4k^2\sin^4\frac{\theta}{2}}$$

G^E and G^M were extracted using the Rosenbluth separations (ot at extreme low Q^2 the G_M can be ignored.)

Definition of the Proton Radius :

Tayler expansion at low Q^2

$$G_E^p(Q^2) = 1 - \frac{Q^2}{6} < r^2 > + \frac{Q^4}{120} < r^4 > + \ \ldots$$

$$< r^2 > = -6 \left.\frac{dG_E^p(Q^2)}{dQ^2}\right|_{Q^2=0}$$

Polarization transfer method

$$\vec{e} + p \rightarrow e + \vec{p}$$

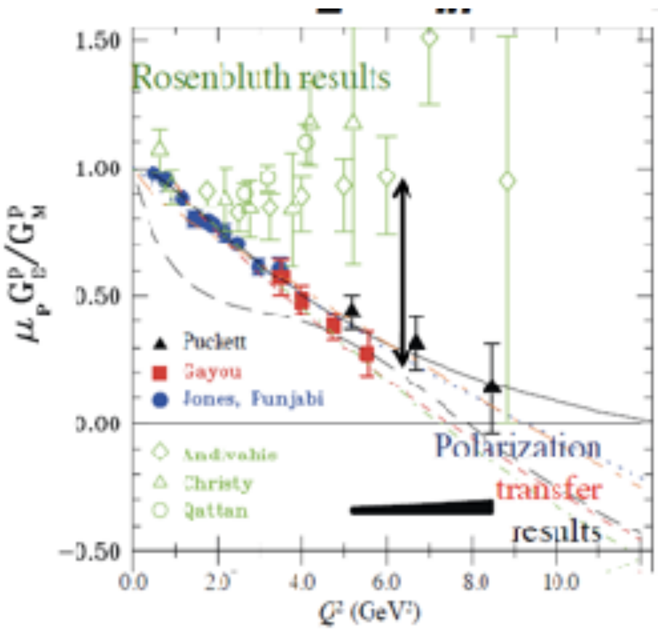
Measure transverse (P_T) and longitudinal (P_L) polarization outgoing proton.

$$\frac{G_E}{G_M} = -\frac{P_T}{P_L}\frac{(E+E')}{2m_p}\tan\frac{\theta}{2}$$

Many recent experiments conducted in Halls A & C at Jefferson Lab (to name a few):

M.K. Jones *et al.*, ***Phys. Rev. Lett. 84, 1398 (2000)***
O. Gayou *et al.*, ***Phys. Rev. C64, 038202 (2001)***
O. Gayou *et al.*, ***Phys. Rev. Lett. 88, 092301 (2002)***
V. Punjabi *et al.*, ***Phys. Rev. C71, 055202 (2005)***
M.K. Jones *et al.*, ***Phys. Rev. C74, 035201 (2006)***
G. MacLachlan *et. al.*, ***Nucl. Phys. A764, 261 (2006)***
A. J. R. Puckett *et. al.*, ***Phys. Rev. Lett. 104, 242301 (2010)***

Clearly a high pric
for the nuclear
physics communit



A. J. R. Puckett *et. al.*, ***Phys. Rev. Lett. 104, 242301 (2010)***

Proton Zemach radius from 2S HFS

Antognini Ann. Phys.

$$\Delta E^{th}_{HFS} = E_F(1+\delta_{QED}+\delta_{str})$$
$$= E_F(1+\delta_{QED}+\delta_Z+\delta_{recoil}+\delta_{pol}+\delta_{HVP})$$

$$\Delta E^{th}_{HFS} = 22.9778(2) - 0.0022(5)\, r_E^2 - 0.1621(10)\, r_Z + \Delta E^{pol}_{HFS}\ \mathrm{meV}$$

substitute $r_E = 0.84087(39)\ \mathrm{fm}$

$$0.0022(5) \times 0.84087(39)^2 = 0.00156(35)$$

$$\Delta E^{th}_{HFS} = 22.9763(15) - 0.1621(10)\, r_Z + \Delta E^{pol}_{HFS}\ \mathrm{meV}$$

(15) comes from uncertainties of different theoretical calculation

Fermi	22.807995
μ anomalous M.M.	0.02659
all-order eVP	0.07437
2-loop to E_F	0.00056
1-loop eVP in 1g	0.04818
2-loop eVP in 1g	0.00037
further 2-loop eVP	0.00037
muVP 2-loop	0.00091
vertex	-0.00311
higher order	-0.00017
hadron VP	0.00060(10)
weak	0.00027
higher order size to EF	0.0009
recoil	0.02123
~~eVP + p structure~~	~~-0.0001~~
eVP cor to size	-0.00114(20)
total W/O δpol	22.97783(22)

$$\Delta E^{th}_{HFS} = 22.9858(26) - 0.1621(10)\, r_Z - 0.0022\, r_E^2$$

substitute E^{exp} into E^{th}

substitute $\Delta E^{exp}_{HFS} = 22.8089(51)\ \mathrm{meV}$ Antognini, Science(2013)

$\Delta E^{pol}_{HFS} = 0.0080(26)\ \mathrm{meV}$ PRA83(2011) 042509

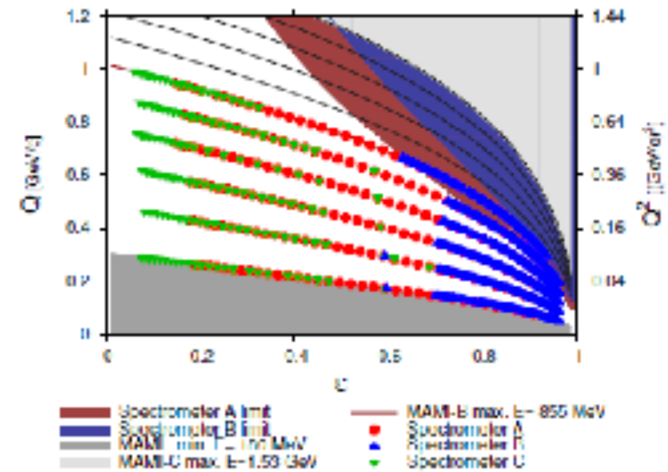
$$r_Z = \frac{22.9763(15) + 0.0080(26) - 22.8089(51)}{0.1621(10)} = 1.082(31)^{\mathrm{exp}}(20)^{\mathrm{th}} = 1.082(37)$$

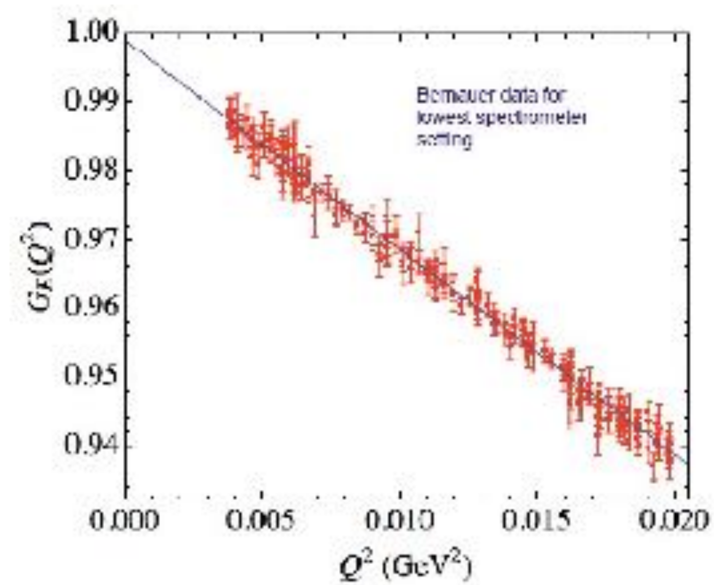
MAINZ A1

$$\tau = \frac{Q^2}{4m_p^2}, \quad \varepsilon = \left(1 + 2(1+\tau)\tan^2\frac{\theta_e}{2}\right)^{-1}$$

Three spectrometer facility of the A1 collaboration:

J. Bernauer, PRL 105,242001, 2010





- ✓ Q^2 = [0.004 – 1.0] (GeV/c)^2 range
- ✓ Large amount of overlapping data sets (~1400)
- ✓ Statistical error ≤ 0.2%
- ✓ Luminosity monitoring with spectrometer
- ✓ Additional beam current measurements

$r_p = 0.879(5)_{stat}(4)_{sys}(2)_{mod}(4)_{group}$

- ✓ Confirms the previous results from *ep*→*ep* scattering;
- ✓ Consistent with CODATA06 value: (r_p=0.8768(69) fm)

non-linear crystal

Zinc Germanium Phosphide (ZGP)

Overview | Orientations & Sizes | Optical Characteristics

Click image to view gallery

ZGP Single Crystals

Zinc Germanium Phosphide (ZGP) exhibits a large nonlinear coefficient—160 times larger than potassium dihydrogen phosphate (KDP) —making it one of the most efficient nonlinear crystals available. Other properties include:

- **Optical transparency** from 1 to 12 μm, high thermal conductivity
- **High laser damage threshold**
- **Tolerance of high average power**
- **Stable mechanical properties**
- **Phasematching ability** over a broad spectral region.
- **Wide operating temperature range** from -40° to 180° C temperature range.

Cavity mirror

mid-infrared mirror
CRD Optics, Inc.

Model Number	Center Wavelength	Reflectivity	Bandwidth (nm)	Diameter (in)	ROC (m)	Price per Pair	Details
901-0010-3300	3300	99.99%	3000-3400	1	1	$2500	View
901-0010-4000	4000	99.98%	3580-4350	1	1	$3000	
901-0008-4000	4000	99.98%	3580-4350	0.8	6	$3000	
901-0008-4600	4600	99.98%	4280-4740	0.8	1	$3000	
901-0010-4860	4860	99.98%	4685-5290	1	1	$3000	
901-0020-5200	5200	99.98%	5018-5382	2	1	$6000	
901-0010-5200	5200	99.98%	5018-5382	1	1	$3000	
901-0010-6200	6200	99.98%	5800-6550	1	1	$3000	
901-0008-6800	6800	99.98%	6500-7200	0.8	1	$3500	
901-1010-6800	6800	99.96%	6500-7200	1.0	1	$3500	View
901-0010-7350	7350	99.98%	7020-8150	1	1	$3500	
901-0010-7800	7800	99.98%	7500-8100	1	1	$3500	View
901-0010-8300	8300	99.99%	7950-8650	1	1	$3500	
901-0010-8600	8600	99.98%	8300-8900	1	1	$3500	
901-0010-9600	9600	99.99%	9200-10000	1	1	$3500	View
901-0010-DB10	11000	99.99%	9800-14000	1	1	$3500	View

Ionization effect by laser

6.7 um = 0.183 eV

Ionization potential of Hydrogen : 13.6 eV

Ionization potential of MuonicHydrogen : ~ 2.6 keV

Pulse energy : 40 mJ

Pulse duration : ~ 10 ns

Beam size at focus : ø100 μm (middle of multipass cell)

Focused Intensity :

5.2×10^{8} W/cm^2 << 10^{14} W/cm^2 (multiphoton ionization threshold)

Focused intensity is significantly smaller than multiphoton ionization threshold

uncertainty in Zemach radius from μp atom

Uncertainty of ΔE^{2S}_{HFS} and R_z (PSI case)

ΔE^{2S}_{HFS} (theory) = 22.9843(30) – 0.1621(10) R_z

$$R_p = -\left(\Delta E^{HFS}_{exp}/E^F - 1 - \delta^{QED} - \delta^{recoil} - \delta^{pol} - \delta^{hvp}\right)/(2 m_{ep} \alpha)$$

R_z = (22.9843(30) - ΔE^{2S}_{HFS} *(exp)*) / 0.1621(10)

R_z = (22.9763(15)+0.0080(26) - ΔE^{2S}_{HFS} *(exp)*) / 0.1621(10)

ΔE^{2S}_{HFS} by PSI : ΔE^{2S}_{HFS} =22.8089(51) (Antognini 2012)

R_z = 0.1754(59) / 0.1621(10) = 1.082 (37) *(37) = (31)^{exp}(20)th

3.4x10-2 6.2x10-3 3.4x10^-2

* uncertainty mainly comes from *proton polarizability*

ΔE_{pol} = 0.0080(26) meV

ΔE^{1S}_{HFS} *case:*

ΔE^{1S}_{HFS} *(theory)* = 182.725(62)

R_z = (184.087 (15) - ΔE^{1S}_{HFS} *(exp)*)/ 1.281(YY)

(YY) <10?

	Hydrogen		Muonic Hydrogen	
	magnitude	uncertainty	magnitude	uncertainty
E^F	1420 MHz	0.01 ppm	182.443 meV	0.1 ppm
δ^{QED}	$1.16\ 10^{-3}$	$< 0.001\ 10^{-6}$	$1.16\ 10^{-3}$	10^{-6}
δ^{rigid}	$39\ 10^{-6}$	$2\ 10^{-6}$	$7.5\ 10^{-3}$	$0.1\ 10^{-3}$
δ^{recoil}	$6\ 10^{-6}$	10^{-8}	$1,7\ 10^{-3}$	10^{-6}
δ^{pol}	$1.4\ 10^{-6}$	$0.6\ 10^{-6}$	$0.46\ 10^{-3}$	$0.08\ 10^{-3}$
δ^{hvp}	10^{-8}	10^{-9}	$0.02\ 10^{-3}$	$0.002\ 10^{-3}$

* uncertainty of proton polarizability (1S)

ΔE_{pol}(1S) = 0.084(15) meV (460(80) ppm)

muonic hydrogen molecular ion

muonic molecular ion formation:

$$(\mu^- p) + \mathrm{H}_2 \rightarrow \left[(p\mu p)_J pee\right]^*$$

rate : $\lambda_{p\mu p} = \phi \times 2.2 \times 10^6$ [s^{-1}]

ϕ : density [/ρ_{LHD}]

0.001LHD case : rate = 2.2×10^3 [s^{-1}] << muon life

magnetic field effect on transition energy

Magnetic field effect on transition energy
For muonium,
$W = -(1/4)\Delta W - \mu_B^{\mu}\, g_{\mu}\, M_F\, H \pm (1/2)\Delta W\, (1 + 2M_F\, x + x^2)^{1/2}$
ΔW = hyperfine splitting
$x = (g_J\, \mu_B^{e} + g_{\mu}\, \mu_B^{\mu})H/\Delta W = H/1585$
We could use similar equation for pμ.
However, for simpler estimates...

Energy by Magnetic moment under field
$\mu_N = e\hbar/2m_p = 5.05 \times 10^{-27}$ J/T = 3.16×10^{-8} eV/T = 0.00316 μeV/kG

$\mu_p = 2.792\ \mu_N$ = 0.00882 μeV/kG
$\mu_n = -1.913\ \mu_N$ = -0.00604 μeV/kG
$\mu_{\mu} = -4.49 \times 10^{-26}$ J/T = 0.0281 μeV/kG
$\mu_e = -928 \times 10^{-26}$ J/T = 5.80 μeV/kG

Hyperfine splitting
Muonium ΔE^{Mu} = 4.463 GHz = 18.4 μeV
Muonic proton $\Delta E^{\mu p}$ (1S) = 45 THz = 185 meV (h = 4.136 μeV/GHz)
This is ~10,000 of Mu, because of atomic size (x $(m_{\mu}^{*}/m_e)^3 = 6.5 \times 10^6$) and the magnetic moment ($2.792(m_e/m_p)=0.0015$)

Thus μ_{μ} H/ $\Delta E^{\mu p}$ (1S) = 0.11 μeV / 185 meV < 10^{-4} even at 4 kG.
This is quite different from Mu, where the ratio >1.
Magnetic field effect on μp is negligible.

Test of bound-state QED

Test of bound-state QDDD

Sternheim interval

$8\ dE_{2S} - dE_{1S}$ = -0.120 meV with accuracy of 10^-6

PSI 2S HFS : 22.8080(51) meV
1S HFS : 183.XXXX(37) meV

PSI 2S HFS : 22.8080(51) meV
1S HFS : 183.XXXX(37) meV
-0.120 (41) meV